Global Macroeconomics

Revision Lecture - Spring 2022

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Revision lecture outline

- Recap of the topics seen during the course
- Description of the *tools* you need to know for the exam
- Good and bad practices at the exam
- Answer to some of your questions
- Solve together two exam-type questions

Topics covered in this course

(1) Growth theories: Solow and Romer models

(2) Neoclassical Consumption model and consumption theories

(3) Centralized economy (Ramsey model) with and without taxation

(4) Decentralized economy (RBC model) with and without taxation

(5) Overlapping generation model and pension systems

A broad view of the topics covered

- The aim of this course was to present you some of the main modern macroeconomic theories.
- We studied different theoretical models because they are used to address different policy questions.
- **Key**: being able formalize the features of an economic problem, to select the most appropriate theoretical framework to tackle the issue.
- Each topic is connected with the others, there is no *watertight compartment division* between the models.
- Some models are not only related, but they present some overlap with others in terms of questions asked, approach used, assumptions made or results found.

A broad view of the topics covered (ii)

- The course is divided into two broad parts and five sub-sections.
- We can related the two sections about growth with each section about the medium run.
- This is because we can include economic growth into OLG or RBC models. We can also micro-found some parts of the growth theories.
- The neoclassical consumption model and the consumption theories studied are actually fully integrated into the RBC framework.
- We studied them separately to build up step by step all the elements necessary to approach RBC models.

The "tools" you need to know for the exam

- How to assess the strengths and weaknesses of a model:
  - identify the key assumptions and their implications
  - are these assumptions reasonable (from an economic point of view)?
  - strength: the model answers the initial question
  - weakness: the model fails to answer the initial question
  - understand the specific aim of the model
  - the role of other details of a model: is more complexity beneficial?
- Compare models:
  - do they try to answer the same question?
  - which are the main elements in common and of difference?
  - what are the outcomes of the models?

How to *solve* a model

- Identify agents' objectives and constraints:
  - households' objectives are defined by the utility function
  - firms's objective is profit maximization (in the models we have seen)
  - government's objective is the maximization of economy's welfare
- Describe in words the problem and the solution *path* you will adopt
- Set up and solve a constrained maximization problem
  - define the appropriate Lagrangian function
  - maximize for the objective(s) of the agent, using the right timing
  - derive the first order conditions
  - use the FOCs to get the optimality condition(s)

Exam questions

General structure of the exam questions:

- Answer each question
- Each question has an essay-oriented and a quantitative component
- Try to alway write an answer: if you leave the paper blank, there is nothing I can evaluate

Good and bad practices at the exam

- Write clearly and do not 'overwrite'.
- Even if you are not sure about the answer...write something.
- If you find something that you think is a typo (do not panic!), just state what you think is wrong and make the assumptions you need to go on in the solution.
- Write your answers in a decent calligraphy to maximize your chances of getting a proper evaluation!

How to set up a Lagrangian function

- The utility/objective function is the first part of the Lagrangian.
- If you do not use substitution techniques, you need a Lagrangian multiplier for each constraint.

How to derive the Euler Equation:

- The Euler Equation (EE) is a necessary, but not sufficient optimality condition.
- It provides us a relation between consumption and investment today and consumption tomorrow

  ⇒ if the EE holds, the household is indifferent between consumption today and tomorrow, give the personal discount factor and the exogenous interest rate.

More about the Euler Equation

- **Key point**: the EE depends on households' preferences but also on the investment conditions of the economy!
- If the interest rate is *high*, a small saving (a smaller reduction of c_t) leads to a higher level of c_{t+1} tomorrow compared to the case with *low* interest rate (given a certain discount factor).
- This means that; depending on β and r_t, an household will change his indifference condition.
- Also, the type of utility function matters: e.g. the log-utility has the property that *income* and *substitution* effects cancel out each other.
- We derive the EE combining the FOC with respect to consumption with the FOC with respect to the investment asset available.

Implications of closed economy and perfect competition

Closed economy:

- all output in the economy needs to serve a purpose
- e.g. output can be consumed or saved: $y_t = c_t + i_t$

Perfect competition:

- each input is payed its marginal product. So:
- cost of labor (w) is the marginal product of labor
- cost of capital (r) is the (net) marginal product of capital

Some of your questions:

- Derive the Euler Equation of Lecture 6, slide 33.
- Derive capital in steady state of Lecture 7, slide 11.

Derivation of the Euler Equation in a decentalizzed RBC

We want to derive the Euler Equation of a decentralized RBC model.

- The Lagrangian function:

$$\max_{\{c_t, k_{t+1}\}} \mathcal{L} = \sum_{t=0}^{\infty} \{\beta^t U(c_t) + \lambda_t [F(k_t) - c_t - k_{t+1} + (1-\delta)k_t]\}$$

- We need to maximize w.r.t. two choice variables: c_t and k_{t+1}, hence we will have two first order conditions:

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t U'(c_t) - \lambda_t = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = \lambda_{t+1}[F'(k_{t+1}) + (1-\delta)] - \lambda_t = 0 \quad (2)$$

- Eq. (2) is obtained recognizing that the problem has an infinite horizon and in the IBC two elements are function of $k_{t=1}$ if we bring the IBC one period forward!

Derivation of the Euler Equation in a decentalizzed RBC

- Our aim is to combine the FOCs to obtain an equation that tells us at which conditions we are indifferent between consumption today and in the future.
- The first step: substitute λ_t from (1) into (2):

$$\lambda_{t+1}[F'(k_{t+1}) + (1-\delta)] - \beta^t U'(c_t) = 0$$

- Second step: get rid of λ_{t+1}. We do it by recognizing that FOC can be re-written one period forward, thanks to the recursivity nature of the problem:

$$\beta^{t+1} U'(c_{t+1}) - \lambda_{t+1} = 0$$

- Now we can get rid off also of λ_{t+1}:

$$\beta^{t+1} U'(c_{t+1})[F'(k_{t+1}) + (1-\delta)] - \beta^t U'(c_t) = 0$$

- β^t can be simplified and we can re-arrange the equation:

$$\frac{\beta^t U'(c_{t+1})}{U'(c_t)}[F'(k_{t+1}) + (1-\delta)] = 1 \qquad (3)$$

- Eq. (3) is the Euler Equation.

Derivation of capital & consumption in SS in a RBC

- We assume to have a CRRA utility function and a Cobb-Douglas production function.
- The Lagrangian of the problem reads:

$$\mathcal{L} = \sum_{t=0}^{\infty} \left\{ \beta^t \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda_t [A k_t^{\alpha} - c_t - k_{t+1} + (1-\delta) k_t] \right\}$$

- The FOCs in this case are: (same steps as before)

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t c_t^{-\sigma} - \lambda_t = 0 \quad (4)$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = \lambda_{t+1} [\alpha A k_{t+1}^{\alpha-1} + (1-\delta)] - \lambda_t = 0 \quad (5)$$

Derivation of capital & consumption in SS in a RBC

- Substituting λ_t from eq.(4) into (5) and bringing one period forward the FOC w.r.t. c_t, we get:

$$\beta^{t+1} c_{t+1}^{-\sigma}[\alpha A k_{t+1}^{\alpha-1} + (1-\delta)] = \beta^t c_t^{-\sigma}$$

- Rearranging we get the Euler Equation with CRRA utility:

$$\beta \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \left[\alpha A k_{t+1}^{(\alpha-1)} + (1-\delta)\right] = 1$$

- In steady-state $k_t = k_{t+1} = k^*$, $c_t = c_{t+1} = c^*$, so we get:

$$\beta \left(\frac{c*}{c*}\right)^{-\sigma} \left[\alpha A k^{*(\alpha-1)} + (1-\delta)\right] = 1$$

Derivation of capital & consumption in SS in a RBC

- Simplifying we get capital in SS:

$$\alpha A k^{*(\alpha-1)} + (1-\delta) = \frac{1}{\beta}$$

$$\alpha A k^{*(\alpha-1)} = 1 + \theta - (1-\delta)$$

$$\alpha A k^{*(\alpha-1)} = \theta + \delta$$

$$k^{*(\alpha-1)} = \frac{\theta + \delta}{A\alpha}$$

$$k^{*} = \left(\frac{A\alpha}{\theta + \delta}\right)^{\frac{1}{1-\alpha}}$$

Derivation of capital & consumption in SS in a RBC

- To obtain the level of consumption in steady state we can plug k^* into the IBC in its steady state version:

$$Ak^{*\alpha} - c^* - k^* + (1-\delta)k^* = 0$$

$$Ak^{*\alpha} - c^* - \delta k^* = 0$$

- Hence:

$$\begin{aligned} c^* =& Ak^{*\alpha} - \delta k^* \\ =& A\left(\frac{A\alpha}{\theta+\delta}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{A\alpha}{\theta+\delta}\right)^{\frac{1}{1-\alpha}} \\ =& \underbrace{AA^{\frac{\alpha}{1-\alpha}}}_{A^{\frac{1}{1-\alpha}}}\left(\frac{\alpha}{\delta+\theta}\right)^{\frac{\alpha}{1-\alpha}} - \delta A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\delta+\theta}\right)^{\frac{1}{1-\alpha}} \\ =& A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\delta+\theta}\right)^{\frac{1}{1-\alpha}}\left[\left(\frac{\alpha}{\delta+\theta}\right)^{\alpha} - \delta\right] \end{aligned}$$

Do you have other questions?

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Good luck!