Advance Microeconomics 2023/2024 - EFE

Homework I

Maximization - Minimization

Exercise 1 (Utility Maximization I)
Alessandro lives in New York and has m of his income to spend to build a small wine cellar. He enjoys two vintages in particular: an expensive 2005 Italian Nero d'Avola (x_I) at price p_I a bottle and a less expensive Argentinian 1997 Malbec (x_A) at price p_A per bottle. Alessandro's preferences are represented by the following utility function.

$$u(x_I, x_A) = x_I^{2/3} x_A^{1/3}$$

1. Set up the utility maximization problem that Alessandro is facing. Represent it on a graph.
2. Solve the utility maximization problem, explicitly using the Lagrangian method (hence find the two ordinary/Marshallian demands for the Italian and the Argentinian wines).
3. Find the indirect utility function and show that is decreasing in $p_I()$.
4. Take $m = €300$, $p_I = €25$ and $p_A = €4$. Find the values of ordinary demands and indirect utility function.
5. For an unclear reason, the value of the Euro has increased and the price of Alessandro's beloved Nero d'Avola becomes €40 per bottle. If the price of the Malbec stays constant at €4 per bottle, how much of each wine should Alessandro buy under these altered conditions?

Exercise 2 (Expenditure Minimization I)
Meanwhile, in a different version of this universe ...

Alessandro lives in New York and wishes to build a small wine cellar. He enjoys two vintages in particular: an expensive 2005 Italian Nero d'Avola (x_I) at price p_I a bottle and a less expensive Argentinian 1997 Malbec (x_A) at price p_A per bottle. Alessandro's preferences are represented by the following utility function

$$u(x_I, x_A) = x_I^{2/3} x_A^{1/3}.$$

Alessandro has a considerable level of income to spend, but at the same time he is careful about his expenses. To combine his desire of having a nice wine cellar that does not ruin his finances, he decided that the minimum requirements it should have are given by the utility level $\overline{u}$.

1. Set up the expenditure minimization problem and and find the two compensated/Hicksian demands for the Italian and the Argentinian wines.
2. Find the expenditure function.
3. Prove that $E(p, V(p, m)) = m$.