Department of Energy
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Final Test of 30-06-2015 – Problems

Exercise 1. (9pt)

Let us consider a system composed by two components: a burner (a) and a boiler (b). In the burner, a flow rate of gaseous propane undergoes a combustion process with excess air. The fuel and the air enter separately in the burner, at the conditions of 75°C and 2 bar, while flue gases exit the burner at 1920 K and 2 bar. After the combustion process, flue gases enter the boiler, where they undergo a cooling process till 420 K, heating a flow of 92 t/h of steam.

	h_f kJ/kmol	Δh (1920-298)K kJ/kmol	Δs (1920-298)K kJ(kmolK	Δh (420-298)K kJ/kmol	Δs (420-298)K kJ(kmolK	ex_{ch} kJ/kmol
C3H8	-103850					2163190
CO2	-393520	86631	93,0660	4842	13,5730	20108
H2O	-241820	68623	73,7770	4139	11,6300	8667
O2	0	56186	62,0820	3632	10,2080	3947
N2	0	53267	59,0000	3556	9,9970	691

	unit	w_{in}	w_{out}
T	°C	50	175
p	bar	5	5
h	kJ/kg	210	2801
s	kJ/kgK	0,7036	6,9427

Isobaric heat capacities of the fuel and the air are respectively equal to 1.673 kJ/kg-K and 29.17 kJ/kmol-K. With reference to the data given in the tables, and considering environmental reference temperature and pressure of 25°C and 1 bar, it is required to:

a. (2pt) Draw the scheme of the process. Apply the energy balance to the burner applying proper hypotheses, and derive the excess air λ. Is the excess air a function of the reactants temperature and pressure?

b. (3pt) Apply the exergy balance to the burner applying proper hypotheses, and calculate the exergy destruction (per unit of molar flow of C3H8) and the rational exergy efficiency of the burning process, considering the physical and the chemical exergy for both the reactants and the products of combustion;

c. (1pt) Calculate again the rational exergy efficiency of the combustion process neglecting the physical exergy of reactants and the chemical exergy of products. In this case, are these components essential in the evaluation of the rational efficiency?

d. (1pt) Apply the energy balance to the boiler, and determine the molar flow rate of fuel required to produce the mass flow rate of steam;

e. (2pt) Apply the exergy balance to the whole system, calculating the total exergy destructions and the functional exergy efficiency (the exergy of flue gases and the change in exergy of the steam are considered as useful products).

Exercise 1. Solution

a. **<u>Draw the scheme of the process. Apply the energy balance to the burner applying proper hypotheses, and derive the excess air λ. Is the excess air a function of the reactants temperature and pressure?</u>**

The combustion reaction can be written as follows:

$$C_3H_8 + \lambda \cdot 5\left(O_2 + 3.76N_2\right) \rightarrow 3CO_2 + 4H_2O + \lambda \cdot 5 \cdot 3.76N_2 + \left(\lambda - 1\right) \cdot 5 \cdot O_2$$

Assumptions for the application of the energy balance:

- The combustion chamber is adiabatic and rigid;
- Steady state operation is assumed throughout;
- The balance is written per unit of kmol of C3H8;
- Both the air and the fuel are modelled as perfect gases;

$$\frac{dE}{dt} = \sum \dot{W}^{\leftarrow} + \sum \dot{Q}^{\leftarrow} + \dot{\bar{H}}_R - \dot{\bar{H}}_P \qquad \rightarrow \qquad \dot{\bar{H}}_R = \dot{\bar{H}}_P$$

Enthalpies of reactants and products can be evaluated as follows:

$$\bar{h}_R\left(T_R, p_R\right) = 1\left(\bar{h}_f + \Delta\bar{h}\right)_{C3H8} + \lambda \cdot 5 \cdot \left(\cancel{\bar{h}_f} + \Delta\bar{h}\right)_{air} \qquad \rightarrow \qquad \begin{cases} \Delta\bar{h}_{air} = c_{p,air}\left(T_{air} - T_0\right) = 1371\,kJ/kmol \\ \Delta\bar{h}_{C3H8} = c_{p,C3H8}\left(T_{C3H8} - T_0\right) = 3689\,kJ/kmol \end{cases}$$

$$\bar{h}_P\left(T_P, p_P\right) = 3\left(\bar{h}_f + \Delta\bar{h}\right)_{CO2} + 4\left(\bar{h}_f + \Delta\bar{h}\right)_{H2O} + \lambda \cdot 5 \cdot 3.76\left(\cancel{\bar{h}_f} + \Delta\bar{h}\right)_{N2} + \left(\lambda - 1\right) \cdot 5 \cdot \left(\cancel{\bar{h}_f} + \Delta\bar{h}\right)_{O2}$$

The energy balance becomes:

$$1\left(\bar{h}_f + \Delta\bar{h}\right)_{C3H8} + \lambda \cdot 5 \cdot 4.76 \cdot \Delta\bar{h}_{air} = 3\left(\bar{h}_f + \Delta\bar{h}\right)_{CO2} + 4\left(\bar{h}_f + \Delta\bar{h}\right)_{H2O} + \lambda \cdot 5 \cdot 3.76 \cdot \Delta\bar{h}_{N2} + \left(\lambda - 1\right) \cdot 5 \cdot \Delta\bar{h}_{O2}$$

The excess air λ is the only unknown of the energy balance:

$$\lambda = \frac{\bar{h}_{f,C3H8} + \Delta\bar{h}_{C3H8} - 3\left(\bar{h}_f + \Delta\bar{h}\right)_{CO2} - 4\left(\bar{h}_f + \Delta\bar{h}\right)_{H2O} + 5\Delta\bar{h}_{O2}}{-5 \cdot 4.76 \cdot \Delta\bar{h}_{air} + 5 \cdot \Delta\bar{h}_{O2} + 5 \cdot 3.76 \cdot \Delta\bar{h}_{N2}} = 1.436$$

An increase in temperature of reactants results in an increase in the enthalpies of them: if the adiabatic flame temperature is kept constant, the excess air will decrease and the composition of the products will change. A change in pressure do not affect the combustion process and the composition of the products.

b. **Apply the exergy balance to the burner applying proper hypotheses, and calculate the exergy destruction (per unit of molar flow of C3H8) and the rational exergy efficiency of the burning process, considering the physical and the chemical exergy for both the reactants and the products of combustion;**

Assumptions for the application of the exergy balance:

- The combustion chamber is adiabatic and rigid;
- Steady state operation is assumed throughout;
- The balance is written per unit of kmol of C3H8;

$$\frac{dEx_A}{dt} = \sum_j \dot{E}x_{W,j}^{A\leftarrow} + \sum_k \dot{E}x_{Q,k}^{A\leftarrow} + \sum_i \left(\dot{N}^{A\leftarrow}\overline{ex}\right)_i - \dot{E}x_{des} \rightarrow \qquad \overline{ex}_R\left(T_R, p_R\right) - \overline{ex}_P\left(T_P, p_P\right) - \overline{ex}_{D,comb} = 0$$

$$\overline{ex}_R\left(T_R, p_R\right) = \overline{ex}_{ph} + \overline{ex}_{ch} = 2227637\,kJ/kmol_{C3H8}$$

$$\overline{ex}_{ph} = \sum_i \nu_i\left[\bar{c}_{p,i}\left(T_1 - T_0\right) - T_0\left(\bar{c}_{p,i}\ln\frac{T_1}{T_0} - R\ln\frac{p_1}{p_0}\right)\right] = \left(1 \cdot 1995 + 5 \cdot \lambda \cdot 4.76 \cdot 1828\right)kJ/kmol_{C3H8} = 64447\,kJ/kmol_{C3H8}$$

$$\overline{ex}_{ch} = \underbrace{\nu_{C3H8}\overline{ex}_{ch,C3H8}}_{tables} + \cancel{\nu_{air}\overline{ex}_{ch,air}} = 2163190\,kJ/kmol_{C3H8}$$

$$\overline{ex}_P\left(T_P,p_P\right)=\overline{ex}_{ph}+\overline{ex}_{ch}=1647864\,kJ/kmol_{C3H8}$$

$$\overline{ex}_{ph}=\sum_i\nu_i\left\{\Delta\bar{h}\left(1920-420\right)-T_0\Delta\bar{s}\left(1920-420\right)+T_0R\ln\frac{p_1}{p_0}\right\}=1519949\,kJ/kmol_{C3H8}$$

$$\overline{ex}_{ch}=\nu_{tot,P}\cdot\left(\sum_i x_i ex_{ch,i}+T_0R\sum_i x_i\ln x_i\right)=127916\,kJ/kmol_{C3H8}$$

$$\overline{ex}_{D,\text{comb}}=707688\,kJ/kmol_{C3H8}\rightarrow\quad\eta_{ex,r}=\frac{\overline{ex}_P}{\overline{ex}_R}=0.740$$

Differently from the previous question, both temperature and pressure of reactants affects values of exergy destruction and exergy efficiency, since physical exergy of ideal gases is a function of both temperature and pressure. Specifically, exergy destructions increase if temperature and pressure increase.

c. **Calculate again the rational exergy efficiency of the combustion process neglecting the physical exergy of reactants and the chemical exergy of products. In this case, are these components essential in the evaluation of the rational efficiency?**

$$\overline{ex}_R\left(T_R,p_R\right)=\cancel{\overline{ex}_{ph}}+\overline{ex}_{ch}=2163190\,kJ/kmol_{C3H8}$$

$$\overline{ex}_{ch}=\underbrace{\nu_{C3H8}\overline{ex}_{ch,\text{C3H8}}}_{tables}+\cancel{\nu_{air}\overline{ex}_{ch,air}}=2163190\,kJ/kmol_{C3H8}$$

$$\overline{ex}_P\left(T_P,p_P\right)=\overline{ex}_{ph}+\cancel{\overline{ex}_{ch}}=1519949\,kJ/kmol_{C3H8}$$

$$\overline{ex}_{ph}=\sum_i\nu_i\left\{\Delta\bar{h}\left(1920-420\right)-T_0\Delta\bar{s}\left(1920-420\right)+T_0R\ln\frac{p_1}{p_0}\right\}=1519949\,kJ/kmol_{C3H8}$$

$$\overline{ex}_{D,\text{comb}}=643241\,kJ/kmol_{C3H8}\rightarrow\quad\eta_{ex,r}=\frac{\overline{ex}_P}{\overline{ex}_R}=0.703$$

Neglecting such contributions results in an underestimation of the rational efficiency: the contribution due to the chemical exergy of products is higher with respect to the contribution of the physical exergy of reactants.

d. **Apply the energy balance to the boiler, and determine the molar flow rate of fuel required to produce the mass flow rate of steam;**

Assumptions for the application of the energy balance:

- The boiler is adiabatic and rigid;
- Steady state operation is assumed throughout;
- The balance is written per unit of kmol of C3H8;
- Enthalpies of the fluids are taken from the tables;

$$\frac{dE}{dt}=\sum\dot{W}^{\leftarrow}+\sum\dot{Q}^{\leftarrow}+\Delta\dot{H}_{fg}+\Delta\dot{H}_w\quad\rightarrow\qquad\Delta\dot{H}_{fg}=\Delta\dot{H}_w$$

$$\dot{N}_{C3H8}\Delta h_{fg}-\dot{m}_w\Delta h_w=0$$

$$\Delta\bar{h}_{fg}=\sum_i\nu_i\left[\Delta\bar{h}_i\left(1920-298\right)-\Delta\bar{h}_i\left(420-298\right)\right]=2030092\,kJ/kmol_{C3H8}$$

$$\Delta h_w=h_{w,\text{out}}-h_{w,\text{in}}=2592\,kJ/kg_w$$

$$\dot{N}_{C3H8} = \frac{\dot{m}_w \Delta h_w}{\Delta \bar{h}_{fg}} = \frac{25.26\, kg/s \cdot 2592\, kJ/kg}{2030092\, kJ/kmol_{C3H8}} = 0.033\, kg/kmol_{C3H8}$$

e. **Apply the exergy balance to the whole system, calculating the total exergy destructions and the functional exergy efficiency (the exergy of flue gases and the change in exergy of the steam are considered as useful products).**

Assumptions for the application of the exergy balance:

- The boiler is adiabatic and rigid;
- Steady state operation is assumed throughout;

$$\dot{N}_{fg}\left[\overline{ex}_{fg}(1920K, 2bar) - \overline{ex}_{fg}(420K, 2bar)\right] - \dot{m}_w \Delta ex_w (175-50K, 5bar) - \dot{E}x_{D,boiler} = 0$$

$$\overline{ex}_{fg}(1920K, 2bar) = 1647864\, kJ/kmol_{C3H8} \quad \rightarrow \quad Ex_{fg,in} = 72675kW$$

$$\overline{ex}_{fg}(420K, 2bar) = \overline{ex}_{ph} + \overline{ex}_{ch} = 212271\, kJ/kmol_{C3H8} \quad \rightarrow \quad Ex_{fg,out} = 6925kW$$

$$\overline{ex}_{ph} = \sum_i \nu_i \left\{ \Delta \bar{h}(420-420) - T_0 \Delta \bar{s}(420-420) + T_0 R \ln \frac{p_1}{p_0} \right\} = 84355\, kJ/kmol_{C3H8}$$

$$\overline{ex}_{ch} = \nu_{tot,P} \cdot \left(\sum_i x_i ex_{ch,i} + T_0 R \sum_i x_i \ln x_i \right) = 127916\, kJ/kmol_{C3H8}$$

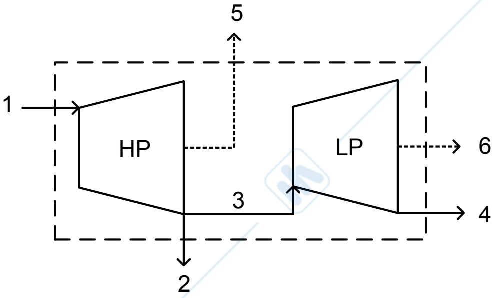
$$ex_w(175-50K, 5bar) = h_{w,\mathrm{out}} - h_{w,\mathrm{in}} - T_0\left(s_{w,\mathrm{out}} - s_{w,\mathrm{in}}\right) = 731.4\, kJ/kg_w \quad \rightarrow \quad Ex_w = 18691kW$$

$$\overline{ex}_{D,boiler} = 47058kW \quad \rightarrow \quad \eta_{ex,r} = \frac{\overline{ex}_P}{\overline{ex}_R} = 0.35$$

Exercise 2. (8pt)

Let's consider a steam turbine of a large Rankine power plant. Steam mass flow rate 1 expand in the high pressure (HP) turbine. Then, one portion of the steam 2 is extracted in order to be used inside the power plant, and the mass flow rate 3 expands in the low pressure (LP) turbine. The steam flow rate 4 exits the LP turbine and goes into the condenser.
Steam mass flow rate 1 has a total enthalpy of 150 MW (measured with respect to the reference enthalpy 0), and the net electric power produced by the turbine is equal to 25 MW_{el}.
Total investment cost of HP and LP turbines are respectively 250 €/h and 375 €/h. The cost of inlet steam 1 is equal to 1.5 c€ per kWh of energy.
Considering an environmental temperature T_0 of 20°C, and with reference to the data given in the table, it is required to:

a. (1pt) Drawn the expansion processes in the T-s diagram (Temperature – specific entropy). Evaluate the mass flow rates of steam 1, 2, 3 and 4, and the specific and total exergy of each energy and bulk flow interaction of the system;
b. (2pt) Write exergy balances for the whole system and for the two turbines according to the inlet/outlet paradigm. Evaluate the rational exergy efficiencies for the whole system and for the two turbines;
c. (3pt) Write the thermoeconomic system of equations for the system at its lower level of aggregation. Introducing proper auxiliary relations, evaluating specific and total costs of outlet flows;
d. (2pt) Assuming that the system works at constant electric power production, derive and evaluate the structure of the cost of useful outlet flows for the two expanders, and the cost of exergy destructions and losses. Comment the obtained results.



	T °C	p bar	h kJ/kg	s kJ/kg-K
0	20	1,01	84	0,30
1	460	35	3361	7,04
2	311	10	3075	7,16
3	311	10	3075	7,16
4	153	2	2774	7,29

Exercise 2. Solution

a. **Drawn the expansion processes in the T-s diagram (Temperature – specific entropy). Evaluate the mass flow rates of steam 1, 2, 3 and 4, and the specific and total exergy of each energy and bulk flow interaction of the system;**

The mass flow rates of steam can be evaluated through the application of the energy balance on the two turbines. Main assumptions:

- Turbines work at steady state;
- Turbines are adiabatic and negligible changes in velocity and height of the fluids occur;

$$\dot{W}_T^{\leftarrow} + \dot{m}_i\left(h_{i,IN} - h_{i,OUT}\right) = 0 \quad \rightarrow \quad \begin{cases} \dot{W}_{HP}^{\rightarrow} = \dot{m}_1\left(h_1 - h_2\right) \\ \dot{W}_{LP}^{\rightarrow} = \dot{m}_3\left(h_3 - h_4\right) \end{cases}$$

Therefore:

$$\begin{cases} \dot{W}_{HP}^{\rightarrow} + \dot{W}_{LP}^{\rightarrow} = 25MW \\ \dot{m}_1\left(h_1 - h_0\right) = 150MW \end{cases} \rightarrow \begin{cases} \dot{m}_1\left(h_1 - h_2\right) + \dot{m}_3\left(h_3 - h_4\right) = 25MW \\ \dot{m}_1\left(h_1 - h_0\right) = 150MW \end{cases} \rightarrow \begin{cases} \dot{m}_1 = 45.8\,kg/s \\ \dot{m}_3 = \dot{m}_4 = 39.6\,kg/s \end{cases}$$

Considering the mass balance at the HP turbine, the mass flow rate of steam 2 can be derived:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \quad \rightarrow \quad \dot{m}_2 = 6.2\,kg/s$$

Power produced by each one of the turbines is equal to:

$$\dot{W}_{HP}^{\rightarrow} = \dot{m}_1 \left(h_1 - h_2 \right) = 13.1 MW$$
$$\dot{W}_{LP}^{\rightarrow} = \dot{m}_3 \left(h_3 - h_4 \right) = 11.9 MW$$

	ex kJ/kg	Ex MW
0	0	0
1	1300	59,5
2	977	6,0
3	977	38,7
4	639	25,3

Specific and total exergy of the streams are derived as follows:

$$ex_i = h_i - h_0 - T_0 \left(s_i - s_0 \right) \qquad \rightarrow \qquad \dot{E}x_i = \dot{m}_i \cdot ex_i$$

b. Write exergy balances for the whole system and for the two turbines according to the inlet/outlet paradigm. Evaluate the rational exergy efficiencies for the whole system and for the two turbines;

The exergy balance can be written as follows:

$$\frac{dEx}{dt} = \sum \dot{E}x_W^{\leftarrow} + \sum \dot{E}x_Q^{\leftarrow} + \sum \dot{m}_i^{\leftarrow} ex_i - \dot{E}x_D = 0 \qquad \rightarrow \qquad \begin{cases} \sum\limits_{IN} \dot{E}x_{IN} - \sum\limits_{OUT} \dot{E}x_{OUT} - \dot{E}x_D = 0 \\ \eta_{ex,R} = \sum\limits_{OUT} \dot{E}x_{OUT} \Big/ \sum\limits_{IN} \dot{E}x_{IN} \end{cases}$$

Therefore:

	HP	LP	HP+LP
	1	3	1
in	59,5	38,7	59,5
	2+W_HP	4+W_LP	W_tot+2+4
out	57,8	37,2	56,3
dest	1,7	1,5	3,2
eta_R	0,97	0,96	0,95

c. Write the thermoeconomic system of equations for the system at its lower level of aggregation. Introducing proper auxiliary relations, evaluating specific and total costs of outlet flows;

The cost of the outlet flows of the HP turbine can be derived once n auxiliary relations have been defined:

$$n_{aux,HP} = m_{flows} - l_{components} \rightarrow n_{aux,HP} = 4 - 1 = 3$$

The unique product of the HP turbine is the mechanical power W_{HP}, whether the steam flows 2 and 3 are classified as by-products, since they will be respectively used in the power plant and in the LP turbine.
The cost of steam per unit of energy must be converted in the cost per unit of exergy considering the equality of total cost:

$$\dot{C}_{1,en} = \dot{C}_{1,ex} \qquad \rightarrow \qquad c_{1,en} \left(\dot{H}_1 - \dot{H}_0 \right) = c_{1,ex} \dot{E}x_1 \qquad \rightarrow \qquad c_{1,ex} = c_{1,en} \frac{\left(\dot{H}_1 - \dot{H}_0 \right)}{\dot{E}x_1} = 0.0378 \frac{€}{kWh}$$

$$\begin{cases} \dot{E}x_1 - \dot{E}x_2 - \dot{E}x_3 - \dot{W}_{HP} - \dot{E}x_{D,HP} = 0 \\ \dot{C}_1 + \dot{Z}_{HP} = \dot{C}_{W,HP} + \dot{C}_2 + \dot{C}_3 \\ \dot{C} = c \cdot \dot{E}x \end{cases} \qquad \rightarrow \qquad \begin{aligned} & c_1 \dot{E}x_1 + \dot{Z}_{HP} = c_{W,HP} \dot{W}_{HP} + c_2 \dot{E}x_2 + c_3 \dot{E}x_3 \\ & c_2 = c_1 \\ & c_3 = c_1 \\ & c_1 = 0.0378 \, € / kWh \end{aligned}$$

The cost of mechanical power is the only unknown:

$$c_{W,HP}=\frac{c_1\left(\dot{E}x_1-\dot{E}x_2-\dot{E}x_3\right)+\dot{Z}_{HP}}{\dot{W}_{HP}}=0.0618\frac{€}{kWh}\qquad\rightarrow\qquad c_{W,HP}=61.78\frac{€}{MWh}$$

The cost of the outlet flows of the LP turbine can be derived once n auxiliary relations have been defined:

$$n_{aux,LP}=m_{flows}-l_{components}\ \rightarrow\ n_{aux,LP}=3-1=2$$

Only mechanical power is considered as useful product of the LP turbine, while steam 4 is considered as a co-product.

$$\begin{cases}\dot{E}x_3-\dot{E}x_4-\dot{W}_{LP}-\dot{E}x_{D,LP}=0\\ \dot{C}_3+\dot{Z}_{LP}=\dot{C}_{W,LP}+\dot{C}_4\\ \dot{C}=c\cdot\dot{E}x\end{cases}\qquad\rightarrow\qquad \begin{array}{l}c_3\dot{E}x_3+\dot{Z}_{LP}=c_{W,LP}\dot{W}_{LP}+c_4\dot{E}x_4\\ c_3=c_1=c_4=0.0378\,€/kWh\end{array}$$

$$c_{W,LP}=\frac{c_1\dot{E}x_3+\dot{Z}_{LP}-c_1\dot{E}x_4}{\dot{W}_{LP}}=0.0740\frac{€}{kWh}\qquad\rightarrow\qquad c_{W,LP}=74\frac{€}{MWh}$$

	Ex MW	c €/MWh	C €/h
0	0,0	-	-
1	59,5	37,8	2250
2	6,0	37,8	227
3	38,7	37,8	1464
4	25,3	74,0	1875
W_HP	13,1	61,8	809
W_LP	11,9	74,0	882

d. <u>Assuming that the system works at constant electric power production, derive and evaluate the structure of the cost of useful outlet flows for the two expanders, and the cost of exergy destructions and losses. Comment the obtained results.</u>

This system has no losses. All the outlet flows are products or by-products. Substituting the exergy balance into the cost balance, considering a demand driven model, the cost structure is derived.

For the HP turbine:

$$\begin{cases}c_1\left(\dot{E}x_1-\dot{E}x_2-\dot{E}x_3\right)+\dot{Z}_{HP}=c_{W,HP}\dot{W}_{HP}\\ \dot{E}x_1-\dot{E}x_2-\dot{E}x_3=\dot{W}_{HP}+\dot{E}x_{D,HP}\end{cases}\qquad\rightarrow\qquad c_1\left(\dot{W}_{HP}+\dot{E}x_{D,HP}\right)+\dot{Z}_{HP}=c_{W,HP}\dot{W}_{HP}$$

$$c_{W,HP}=c_1+c_1\frac{\dot{E}x_{D,HP}}{\dot{W}_{HP}}+\frac{\dot{Z}_{HP}}{\dot{W}_{HP}}=\left(37.80+4.88+19.10\right)\frac{€}{MWh}=61.78\frac{€}{MWh}$$

$$\dot{C}_{D,HP}=c_1\dot{E}x_{D,HP}=63.9\,€/h$$

For the LP turbine:

$$\begin{cases}c_3\dot{E}x_3+\dot{Z}_{LP}=c_{W,LP}\dot{W}_{LP}+c_4\dot{E}x_4\\ \dot{E}x_3=\dot{E}x_4+\dot{W}_{LP}+\dot{E}x_{D,LP}=0\end{cases}\qquad\rightarrow\qquad c_3\left(\dot{E}x_4+\dot{W}_{LP}+\dot{E}x_{D,LP}-\dot{E}x_4\right)+\dot{Z}_{LP}=c_{W,LP}\dot{W}_{LP}$$

$$c_{W,LP}=c_3+c_3\frac{\dot{E}x_{D,LP}}{\dot{W}_{LP}}+\frac{\dot{Z}_{LP}}{\dot{W}_{LP}}=\left(37.80+1.52+17.85\right)\frac{€}{MWh}=57.17\frac{€}{MWh}$$

$$\dot{C}_{D,LP}=c_3\dot{E}x_{D,LP}=56.4\,€/h$$

Exercise 3. (5pt)

Let us consider the Brazilian economy in 2013.

Brazil	**2013**
Total Primary Energy Supply	Mtoe
coal	16,45
oil - oil products	121,87
natural gas	32,07
nuclear	3,82
hydro	33,62
biomass	81,23
other RES	1,12
electricity Import - Export	
Total	293,68
Gross electricity production	TWh
Coal	21,78
Oil	26,56
Natural gas	69,00
Nuclear	
Hydro	
Biomass	
other RES	6,92
Final electricity consumption	487,38
Electricity losses/own uses	122,51

Relying on the IEA data provided by the table, it is required to:

a. (1pt) Evaluate the electric energy Import/Export, the electric energy production from nuclear, and the electric energy production from hydro. Express all the results in TWh;

b. (2pt) Starting from the value of final electricity consumption, evaluate the gross electric energy production. Derive the electricity production from hydro and nuclear starting from their primary energy requirements. Finally, derive the electricity production from Biomass. Express all the results in TWh;

c. (2pt) Evaluate the electric penetration of renewable sources only (hydro, biomass, other RES);

Exercise 3. Solution

a. **Evaluate the changes in: electric energy Import/Export, electric energy production from nuclear, and electric energy production from hydro, between years 1990 and 2013. Express all the results in TWh;**

The changes in electric energy Import/Export can be derived by subtracting the total primary energy supply of all the voices in the table form the TPES:

$$EE_{I/E,2013} = TPES - \sum_i TPES_i = \left[293.68 - (16.45 + 121.87 + 32.07 + 3.82 + 33.62 + 81.23 + 1.12)\right] Mtoe = 3.50 Mtoe$$

$$\frac{3.50 Mtoe}{0.086 Mtoe/TWh} = 40.7 TWh$$

The electric energy production by nuclear and hydro can be calculated considering the IEA conventions for primary energy calculation:

$$EE_{NU} = TPES_{NU} \cdot 0.33 = 1.26 Mtoe (14.65 TWh)$$

$$EE_H = TPES_H = 33.62 Mtoe (390.93 TWh)$$

b. **<u>Starting from the value of final electricity consumption, evaluate the gross electric energy production. Derive the electricity production from hydro and nuclear starting from their primary energy requirements. Finally, derive the electricity production from Biomass. Express all the results in TWh;</u>**

The electric energy production results as the sum of the final electricity consumption, plus the electricity losses/own uses, minus the positive net value of electricity import/export (the last term should be properly converted from Mtoe to TWh dividing by 0.086 Mtoe/TWh):

$$EE_{gross} = EE_{net} + EE_{loss} - EE_{I/E} = 569.19 TWh$$

Hydro and Nuclear electric energy production results from the table (these terms should be properly converted from Mtoe to TWh dividing by 0.086 Mtoe/TWh), while the electric energy production by biomass can be derived indirectly, subtracting all the known values of electricity production from the gross electric energy production:

$$EE_H = 390.93 TWh$$
$$EE_{NU} = 14.66 TWh$$
$$EE_B = EE_{gross} - EE_H - EE_{NU} - \sum_i EE_i = 39.34 TWh$$

c. **<u>Evaluate the electric penetration of renewable sources only (hydro, biomass, other RES);</u>**

The electric penetration of renewables is derived by dividing the primary energy requirements of renewables and the TPES:

$$EP_{RES} = \frac{E_H + E_B + E_{others}}{TPES} = 0.39$$

Politecnico di Milano
Dipartimento di Energia

POLITECNICO DI MILANO

Milano –15 July 2015

Final Test of 30-06-2015 – Problems

Exercise 1. (8pt)

Let's consider an adiabatic combustion chamber where a complete and isobaric combustion of acetylene with excess air takes place (C_2H_2: with LHV=1256460 kJ/kmol and ex_{ch}=1269310).
Both the air and the acetylene are available at 353 K and environmental pressure. With the given excess air, the adiabatic flame temperature results as 2250 K.
After combustion, flue gases enter a counter-flow heat exchanger and are cooled at constant pressure till 673 K, heating a flow of water that enters the heat exchanger as liquid at 323 K and 500 kPa (h=209.3 kJ/kg, s=0.7027 kJ/kg-K) and exits as superheated steam at 873 K and 500 kPa (h=3701 kJ/kg, s=8.3513 kJ/kg-K).

	Δh_R (353 K) kJ/kmol	Δh_P (2250 K) kJ/kmol	Δh_P (673 K) kJ/kmol	$ex_{ph,R}$ (353 K) kJ/kmol	$ex_{ph,P}$ (2250 K) kJ/kmol	$ex_{ph,P}$ (673 K) kJ/kmol
C2H2	3101			260		
N2	1600	65205	11121	132	45905	3944
CO2		106640	16450		76029	5995
H2O		86162	13164		61636	4697
O2	1622	68771	11647	134	48453	4164

With reference to the data given in the table (R: reactants, P: products) and assuming environmental temperature and pressure respectively as T_0=298 K and p_0=101.3 kPa, it is required to:

a. (2pt) Apply the energy balance to the combustion process and, recalling the definition of Lower Heating Value, derive the excess air λ. Is the excess air a function of the reactants' temperature and pressure?

b. (3pt) Apply the exergy balance to the combustion process, evaluating the exergy destructions (per kmol of C2H2) and the rational exergy efficiency, assuming air and fuel enter as separate substance in the combustion chamber. Are these values functions of the reactants' temperature and pressure? Neglect the chemical exergy of products of combustion.

c. (2pt) Apply the exergy balance to the heat exchanger, evaluating the exergy destructions (per kmol of C_2H_2) and the functional exergy efficiency. Neglect the chemical exergy of products of combustion.;

d. (1pt) Evaluate the mass flow rate of acetylene required to produce 180 kg/h of superheated steam.

Exercise 1. Solution

a. **<u>Apply the energy balance to the combustion process and, recalling the definition of Lower Heating Value, derive the excess air λ. Is the excess air a function of the reactants' temperature and pressure?</u>**

The combustion reaction can be written as follows:

$$C_2H_2+\lambda\cdot 2.5\left(O_2+3.76N_2\right)\rightarrow 2CO_2+H_2O+\lambda\cdot 2.5\cdot 3.76N_2+\left(\lambda-1\right)\cdot 2.5\cdot O_2$$

Assumptions for the application of the energy balance:

- The combustion chamber is adiabatic and rigid;
- Steady state operation is assumed throughout;
- The balance is written per unit of kmol of acetylene;

$$\frac{dE}{dt}=\sum \dot{W}^{\leftarrow}+\sum \dot{Q}^{\leftarrow}+\dot{\bar{H}}_R-\dot{\bar{H}}_P \qquad \rightarrow \qquad \dot{\bar{H}}_R=\dot{\bar{H}}_P$$

Enthalpies of reactants and products can be evaluated as follows:

$$\bar{h}_R\left(T_R,p_0\right)=1\left(\bar{h}_f+\Delta\bar{h}\right)_{C2H2}+\lambda\cdot 2.5\cdot\left[1\left(\cancel{\bar{h}_f}+\Delta\bar{h}\right)_{O2}+3.76\left(\cancel{\bar{h}_f}+\Delta\bar{h}\right)_{N2}\right]$$
$$\bar{h}_P\left(T_P,p_0\right)=2\left(\bar{h}_f+\Delta\bar{h}\right)_{CO2}+1\left(\bar{h}_f+\Delta\bar{h}\right)_{H2O}+\lambda\cdot 2.5\cdot 3.76\left(\cancel{\bar{h}_f}+\Delta\bar{h}\right)_{N2}+\left(\lambda-1\right)\cdot 2.5\cdot\left(\cancel{\bar{h}_f}+\Delta\bar{h}\right)_{O2}$$

Recalling the definition of Lower Heating Value for a stoichiometric combustion reaction in air:

$$C_2H_2+2.5\left(O_2+3.76N_2\right)\rightarrow 2CO_2+H_2O+2.5\cdot 3.76N_2$$

$$LHV=\sum_R \nu_i\bar{h}_{f,i}-\sum_P \nu_j\bar{h}_{f,j}\rightarrow$$
$$LHV_{C2H2}=\bar{h}_{f,\mathrm{C2H2}}+2.5\left(\nu_{O2}\bar{h}_{f,O2}+\nu_{N2}\bar{h}_{f,N2}\right)-\left(\nu_{CO2}\bar{h}_{f,CO2}+\nu_{H2O}\bar{h}_{f,H2O}+2.5\cdot\nu_{N2}\bar{h}_{f,N2}\right)$$
$$LHV_{C2H2}=\bar{h}_{f,\mathrm{C2H2}}-\nu_{CO2}\bar{h}_{f,CO2}-\nu_{H2O}\bar{h}_{f,H2O}$$
$$LHV_{C2H2}=\bar{h}_{f,\mathrm{C2H2}}-2\cdot\bar{h}_{f,CO2}-1\cdot\bar{h}_{f,H2O}$$

Substituting the last expression in the first energy balance, the following expression is obtained:

$$LHV_{C2H2}+\Delta\bar{h}_{C2H2}\left(T_R\right)+\lambda\cdot 2.5\cdot\left[\Delta\bar{h}_{O2}\left(T_R\right)+3.76\cdot\Delta\bar{h}_{N2}\left(T_R\right)\right]=$$
$$=2\cdot\Delta\bar{h}_{CO2}\left(T_P\right)+\Delta\bar{h}_{H2O}\left(T_P\right)+\lambda\cdot 2.5\cdot 3.76\cdot\Delta\bar{h}_{N2}\left(T_P\right)+\left(\lambda-1\right)\cdot 2.5\cdot\Delta\bar{h}_{O2}\left(T_P\right)$$

In the last equation, all the enthalpy differences can be derived from the table, and the only unknown is the excess air:

$$\lambda=\frac{2\cdot\Delta\bar{h}_{CO2}\left(T_P\right)+\Delta\bar{h}_{H2O}\left(T_P\right)-2.5\cdot\Delta\bar{h}_{O2}\left(T_P\right)-LHV_{C2H2}-\Delta\bar{h}_{C2H2}}{2.5\cdot\Delta\bar{h}_{O2}\left(T_R\right)+2.5\cdot 3.76\cdot\Delta\bar{h}_{N2}\left(T_R\right)-2.5\cdot 3.76\cdot\Delta\bar{h}_{N2}\left(T_P\right)-2.5\cdot\Delta\bar{h}_{N2}\left(T_P\right)}$$

$$\lambda=\frac{2\cdot 106640+86162-2.5\cdot 68771-1256460-3101}{2.5\cdot 1622+2.5\cdot 3.76\cdot 1600-2.5\cdot 3.76\cdot 1600-2.5\cdot 65205}=1.49$$

An increase in temperature of reactants results in an increase in the enthalpies of them: if the adiabatic flame temperature is kept constant, the excess air will decrease and the composition of the products will change.
A change in pressure do not affect the combustion process and the composition of the products.

b. **Apply the exergy balance to the combustion process, evaluating the exergy destructions (per kmol of C2H2) and the rational exergy efficiency, assuming air and fuel enter as separate substance in the combustion chamber. Are these values functions of the reactants' temperature and pressure? Neglect the chemical exergy of products of combustion.**

Assumptions for the application of the exergy balance:
- The combustion chamber is adiabatic and rigid;
- Steady state operation is assumed throughout;
- The balance is written per unit of kmol of acetylene;

$$\frac{dEx_A}{dt}=\sum_j \dot{E}x^{A\leftarrow}_{W,j}+\sum_k \dot{E}x^{A\leftarrow}_{Q,k}+\sum_i\left(\dot{N}^{A\leftarrow}\overline{ex}\right)_i-\dot{E}x_{des} \qquad \rightarrow \qquad \overline{ex}_R\left(T_R,p_0\right)-\overline{ex}_P\left(T_P,p_0\right)-\overline{ex}_{D,comb}=0$$

$$\overline{ex}_R\left(T_R,p_0\right)=\left[\overline{ex}_{ch}+\overline{ex}_{ph}\left(T_R,p_0\right)\right]_{C2H2}+\lambda\cdot 2.5\cdot\left[\overline{ex}_{ph,O2}\left(T_R,p_0\right)+3.76\cdot\overline{ex}_{ph,\mathrm{N}2}\left(T_R,p_0\right)\right]=1271900\,kJ/kmol_{C2H2}$$

$$\overline{ex}_P\left(T_P,p_0\right)=\sum\nolimits_P\left[\nu_i\overline{ex}_{ph,i}\left(T_P,p_0\right)\right]=909546\,kJ/kmol_{C2H2}$$

$$\overline{ex}_{D,\mathrm{comb}}=362354\,kJ/kmol_{C2H2}\rightarrow\quad \eta_{ex,r}=\frac{\overline{ex}_P}{\overline{ex}_R}=0.7151$$

Differently from the previous question, both temperature and pressure of reactants affects values of exergy destruction and exergy efficiency, since physical exergy of ideal gases is a function of both temperature and pressure.
Specifically, exergy destructions increase if temperature and pressure increase.

c. <u>Apply the exergy balance to the heat exchanger, evaluating the exergy destructions (per kmol of C2H2) and the functional exergy efficiency. Neglect the chemical exergy of products of combustion;</u>

Before applying exergy balance, it is required to derive the ratio between the mass flow rate of water and the molar flow rate of flue gases. The heat released per molar unit of fuel is equal to:

$$\Delta\overline{h}_{fg}\left(2250K\rightarrow 673K,p_0\right)=\sum_P\nu_i\cdot\left[\Delta\overline{h}\left(2250K\right)-\Delta\overline{h}\left(673K\right)\right]_i=1073257\,kJ/kmol_{C2H2}$$

The heat absorbed by a mass unit of water in the heat exchanger is equal to:

$$\Delta h_w\left(873K\rightarrow 323K,p_w\right)=h_{w,OUT}-h_{w,\mathrm{IN}}=3492\,kJ/kg_w$$

Therefore:

$$r=\frac{\Delta\overline{h}_{fg}\left(2250K\rightarrow 673K,p_0\right)}{\Delta h_w\left(873K\rightarrow 323K,p_w\right)}=307\,kg_w/kmol_{C2H2}$$

Assumptions for the application of the exergy balance:
- The heat exchanger is adiabatic and rigid;
- Steady state operation is assumed throughout;
- The balance is written per unit of kmol of acetylene;

$$\underbrace{\overline{ex}_{fg,IN}\left(2250K,p_0\right)-\overline{ex}_{fg,\mathrm{OUT}}\left(673\mathrm{K},p_0\right)}_{\Delta ex_{fg}\left(2250K\rightarrow 673\mathrm{K},p_0\right)}+\underbrace{\overline{ex}_{w,IN}\left(323\mathrm{K},p_w\right)-\overline{ex}_{w,\mathrm{OUT}}\left(873\mathrm{K},p_w\right)}_{-\Delta ex_w\left(873K\rightarrow 323\mathrm{K},p_w\right)}-\overline{ex}_{D,\mathrm{HE}}=0$$

$$\Delta\overline{ex}_{fg}\left(2250K\rightarrow 673\mathrm{K},p_0\right)=\sum_P\nu_i\cdot\left[\overline{ex}\left(2250K,p_0\right)-\overline{ex}\left(673K,p_0\right)\right]_i=833072\,kJ/kmol_{C2H2}$$

$$\Delta ex_w\left(873K\rightarrow 323\mathrm{K},p_w\right)=\left[h_{w,\mathrm{OUT}}-h_{w,\mathrm{IN}}-T_0\left(s_{w,\mathrm{OUT}}-s_{w,\mathrm{IN}}\right)\right]kJ/kg_w=1211.3\,kJ/kg_w$$

$$\Delta\overline{ex}_w=\Delta ex_w\cdot r=372313\,kJ/kmol_{C2H2}$$

$$\overline{ex}_{D,\mathrm{HE}}=\Delta\overline{ex}_{fg}\left(2250K\rightarrow 673\mathrm{K},p_0\right)-\Delta\overline{ex}_w\left(873K\rightarrow 323\mathrm{K},p_w\right)=460760\,kJ/kmol_{C2H2}$$

$$\eta_{ex,\mathrm{f}}=\frac{\overline{ex}_P}{\overline{ex}_F}=\frac{\Delta\overline{ex}_w\left(873K\rightarrow 323\mathrm{K},p_w\right)}{\Delta\overline{ex}_{fg}\left(2250K\rightarrow 673\mathrm{K},p_0\right)}=0.4469$$

d. <u>Evaluate the mass flow rate of acetylene required to produce 180 kg/h of superheated steam.</u>

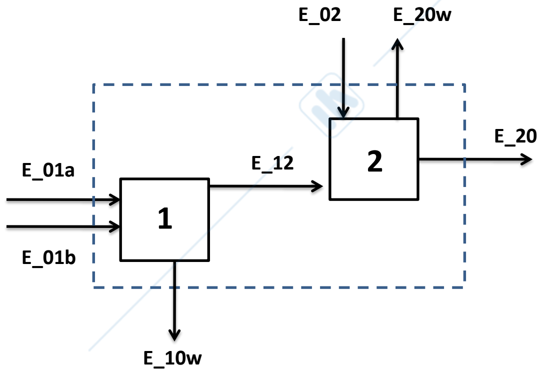
$$\dot{m}_{C2H2}=\dot{m}_w\cdot\frac{1}{3600}\cdot\frac{1}{r}\cdot MW_{C2H2}=180\frac{kg_w}{h}\cdot\frac{1}{3600}\frac{h}{s}\cdot\frac{1}{307}\frac{kmol_{C2H2}}{kg_w}\cdot 26\frac{kg_{C2H2}}{kmol_{C2H2}}=0.0042\frac{kg_{C2H2}}{s}$$

Exercise 2. (8pt)

The system presented in the figure represent a plant for the production of bio-methanol.

Flow	Symbol	Ex [MW]
Electricity from renewables	E_01a	35
Electricity from grid	E_01b	65
Hydrogen	E_12	60
Syngas from other processes	E_02	40
Flow of bio-methanol	E_20	40
Flow of oxygen	E_10w	10
Flue gases	E_20w	10

Process	no	Z [€/h]
Process 1	1	1000
Process 2	2	400



The plant is formed by two sub-processes:

1. Electrolyzer: it receives one flow of water from the environment (zero exergy) and two flows of electric energy, the first produced by a renewable system and the second bought by the national grid. The process produces hydrogen, considered as the useful product, and oxygen, which is vented in the atmosphere without any additional cost;
2. Chemical reactor: in this process, the hydrogen reacts with a syngas in order to produce a constant amount of bio-methanol (final product) and flue gases, which are discharged in the environment with an additional cost of 10 €/MWh_{ex};

Notes:

- E_xy: represent the flow of exergy directed from process x to process y. Index "0" represent all the processes outside the considered system (including the environoment);
- Specific cost per unit of exergy of the flow E_02 is equal to 30 €/MWh_{ex};
- Specific cost per unit of exergy of the flow E_01a is equal to 0 €/MWh_{ex}, while is 40 €/MWh_{ex} for the flow E_01b.

With reference to the data given in the table, it is required to:

a. (1pt) Write the exergy balance for the two processes according to the Inlet/Outlet paradigm, evaluating exergy destructions and exergy rational efficiencies in the two processes and in the whole system;
b. (2pt) Introducing proper auxiliary relations, write thermoeconomic system for the two processes according to the Inlet/Outlet paradigm. Evaluate total and specific economic cost of the products;
c. (2pt) Introducing proper hypotheses, evaluate the cost of exergy destruction and losses, and derive the structure of the specific economic cost of products;
d. (3pt) Apply the first iteration of the Design Evaluation procedure, evaluating the related thermoeconomic indexes for the two processes. Discuss the obtained results and propose a possible way to improve the economic efficiency of the system.

Exercise 2. Solution

a. **Write the exergy balance for the two processes according to the Inlet/Outlet paradigm, evaluating exergy destructions and exergy rational efficiencies in the two processes and in the whole system;**

The exergy balance can be written as follows:

$$\dot{E}_I = \dot{E}_O + \dot{E}_D \qquad \rightarrow \qquad \eta_{ex,r} = \frac{\dot{E}_O}{\dot{E}_I}$$

For the considered system:

Component	Inlet MW	Outlet MW	Destr. MW	$\eta_{ex,r}$ -
1 - Electrolyzer	100 E01a+E01b	70 E12+E10w	30	0.70
2 - Chemical reactor	100 E12+E02	50 E20+E20w	50	0.50
Whole system	140 E01a+E01b+E02	60 E20+E10w+E20w	80	0.43

b. **Introducing proper auxiliary relations, write thermoeconomic system for the two processes according to the Inlet/Outlet paradigm. Evaluate total and specific economic cost of the products;**

The cost of the products of the Electrolyzer can be derived once n auxiliary relations have been defined:

$$n_{aux,1}=m_{flows}-l_{components} \quad \rightarrow \quad n_{aux,1}=4-1=3$$

For the Electrolyzer, the thermoeconomic system can be written as follows:

$$\begin{cases}\dot{E}x_{01a}+\dot{E}x_{01b}=\dot{E}x_{12}+\dot{E}x_{10w}+\dot{E}x_{D1}\\ \dot{C}_{01a}+\dot{C}_{01b}+\dot{Z}_1=\dot{C}_{12}+\dot{C}_{10w}\\ \dot{C}=c\cdot\dot{E}x\end{cases} \quad \rightarrow \quad \begin{array}{l} c_{01a}\dot{E}x_{01a}+c_{01b}\dot{E}x_{01b}+\dot{Z}_1=c_{12}\dot{E}x_{12}+c_{10w}\dot{E}x_{10w}\\ c_{01a}=0\,€/MWh\\ c_{01b}=40\,€/MWh\\ c_{10w}=0\,€/MWh\end{array}$$

$$c_{01b}\dot{E}x_{01b}+\dot{Z}_1=c_{12}\dot{E}x_{12} \quad \rightarrow \quad \begin{array}{l} c_{12}=\dfrac{c_{01b}\dot{E}x_{01b}+\dot{Z}_1}{\dot{E}x_{12}}=60\,€/MWh\\ \dot{C}_{12}=c_{12}\dot{E}x_{12}=3600\,€/h\end{array}$$

The cost of the products of the Chemical reactor can be derived once n auxiliary relations have been defined:

$$n_{aux,2}=m_{flows}-l_{components} \quad \rightarrow \quad n_{aux,2}=4-1=3$$

For the Electrolyzer, the thermoeconomic system can be written as follows:

$$\begin{cases}\dot{E}x_{12}+\dot{E}x_{02}=\dot{E}x_{20}+\dot{E}x_{20w}+\dot{E}x_{D2}\\ \dot{C}_{12}+\dot{C}_{02}+\dot{Z}_2=\dot{C}_{20}+\dot{C}_{20w}\\ \dot{C}=c\cdot\dot{E}x\end{cases} \quad \rightarrow \quad \begin{array}{l} c_{12}\dot{E}x_{12}+c_{02}\dot{E}x_{02}+\dot{Z}_2=c_{20}\dot{E}x_{20}+c_{20w}\dot{E}x_{20w}\\ c_{02}=30\,€/MWh\\ c_{12}=60\,€/MWh\\ c_{20w}=-10\,€/MWh\end{array}$$

$$c_{12}\dot{E}x_{12}+c_{02}\dot{E}x_{02}+\dot{Z}_2=c_{20}\dot{E}x_{20}+c_{20w}\dot{E}x_{20w} \quad \rightarrow \quad \begin{array}{l} c_{20}=\dfrac{c_{12}\dot{E}x_{12}+c_{02}\dot{E}x_{02}+\dot{Z}_2-c_{20w}\dot{E}x_{20w}}{\dot{E}x_{20}}=132.5\,€/MWh\\ \dot{C}_{20}=c_{20}\dot{E}x_{20}=5300\,€/h\end{array}$$

c. **Introducing proper hypotheses, evaluate the cost of exergy destruction and losses, and derive the structure of the specific economic cost of products;**

The text suggest that the system produces a constant amount of final product E_20. Therefore, it can be assumed that any change in efficiency of the system will result in a change of the inlet flows for both the processes. In order to derive the cost structure and the cost of exergy destructions, the average costs of inlet flows are evaluated. For the Electrolyzer:

$$\begin{cases} \dot{Ex}_{01a} + \dot{Ex}_{01b} = \dot{Ex}_{12} + \dot{Ex}_{10w} + \dot{Ex}_{D1} \\ c_{I,1}\left(\dot{Ex}_{01a} + \dot{Ex}_{01b}\right) + \dot{Z}_1 = c_{12}\dot{Ex}_{12} \end{cases}$$

The average cost of inlet flows is:

$$c_{I,1} = \frac{c_{01a}\dot{Ex}_{01a} + c_{01b}\dot{Ex}_{01b}}{\dot{Ex}_{01a} + \dot{Ex}_{01b}} = 26\,€/MWh$$

$$\begin{aligned} & c_{I,1}\left(\dot{Ex}_{12} + \dot{Ex}_{10w} + \dot{Ex}_{D1}\right) + \dot{Z}_1 = c_{12}\dot{Ex}_{12} \\ \rightarrow \qquad & c_{12} = c_{I,1} + c_{I,1}\left(\frac{\dot{Ex}_{10w} + \dot{Ex}_{D1}}{\dot{Ex}_{12}}\right) + \frac{\dot{Z}_1}{\dot{Ex}_{12}} = (26.0 + 17.3 + 16.7)\frac{€}{MWh} = 60\frac{€}{MWh} \\ & \dot{C}_{DL,1} = c_{I,1}\left(\dot{Ex}_{10w} + \dot{Ex}_{D1}\right) = 1040\,€/h \end{aligned}$$

For the Chemical reactor:

$$\begin{cases} \dot{Ex}_{12} + \dot{Ex}_{02} = \dot{Ex}_{20} + \dot{Ex}_{20w} + \dot{Ex}_{D2} \\ c_{I,2}\left(\dot{Ex}_{12} + \dot{Ex}_{02}\right) + \dot{Z}_2 = c_{20}\dot{Ex}_{20} + c_{20w}\dot{Ex}_{20w} \quad [c_{20w} < 0] \end{cases}$$

The average cost of inlet flows is:

$$c_{I,2} = \frac{c_{12}\dot{Ex}_{12} + c_{02}\dot{Ex}_{02}}{\dot{Ex}_{12} + \dot{Ex}_{02}} = 48\,€/MWh$$

$$\begin{aligned} & c_{I,2}\left(\dot{Ex}_{20} + \dot{Ex}_{20w} + \dot{Ex}_{D2}\right) + \dot{Z}_2 - c_{20w}\dot{Ex}_{20w} = c_{20}\dot{Ex}_{20} \\ \rightarrow \qquad & c_{20} = c_{I,2} + c_{I,2}\left(\frac{\dot{Ex}_{20w} + \dot{Ex}_{D2}}{\dot{Ex}_{20}}\right) + \left(\frac{\dot{Z}_2}{\dot{Ex}_{20}} - c_{20w}\frac{\dot{Ex}_{20w}}{\dot{Ex}_{20}}\right) = (48.0 + 72.0 + 12.5)\frac{€}{MWh} = 132.5\frac{€}{MWh} \\ & \dot{C}_{DL,2} = c_{I,2}\left(\dot{Ex}_{20w} + \dot{Ex}_{D2}\right) = 2880\,€/h \end{aligned}$$

Notice that the cost of waste treatment is accounted as an additional investment cost.

d. **Apply the first iteration of the Design Evaluation procedure, evaluating the related thermoeconomic indexes for the two processes. Discuss the obtained results and propose a possible way to improve the economic efficiency of the system.**

Application of DE consists in the evaluation of the following indexes for each one of the plant components:

$\dot{Z}_i + \dot{C}_{(D,L)k}$	Sum of cost of exergy destruction and losses, and investment cost;
$r_k = (c_O - c_I)/c_I$	Relative difference of average cost of outlet and inlet flows;
$f_k = \dfrac{\dot{Z}_k}{\dot{Z}_k + \dot{C}_{(D,L)k}}$	Exergoeconomic Factor;

Relative cost difference is evaluated as follows. Notice that in order to evaluate the relative cost difference, the average specific costs of inlet and outlet flows must be evaluated:

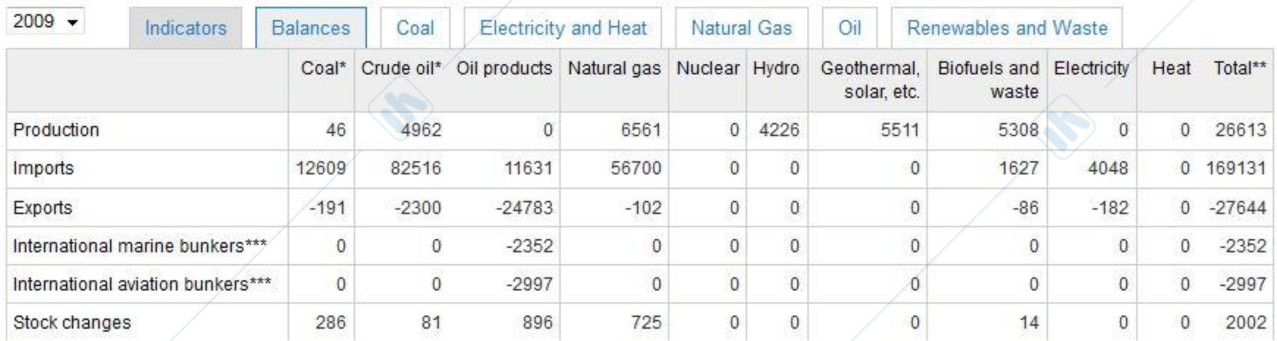
$$1: \begin{cases} c_I = \left(\dfrac{\dot{C}_{01a} + \dot{C}_{01b}}{\dot{E}x_{01a} + \dot{E}x_{01b}}\right) = 26.0 \dfrac{€}{MWh} \\ c_O = \left(\dfrac{\dot{C}_{12} + \dot{C}_{10w}}{\dot{E}x_{12} + \dot{E}x_{10w}}\right) = 51.4 \dfrac{€}{MWh} \end{cases} \quad \rightarrow \quad r_1 = 0.98$$

$$2: \begin{cases} c_I = \dfrac{\dot{C}_{12} + \dot{C}_{02}}{\dot{E}x_{12} + \dot{E}x_{02}} = 48\,€/MWh \\ c_O = c_{20} = 132.5 \dfrac{€}{MWh} \end{cases} \quad \rightarrow \quad r_2 = 1.76$$

Component	$\dot{C}_{(D,L)k}$ [€/h]	$\dot{Z}_k + \dot{C}_{(D,L)k}$ [€/h]	r_k [-]	f_k [-]
1 - Electrolyzer	1040	2040	0.98	0.49
2 - Chemical reactor	2880	3280	1.76	0.12

Exercise 3. (5pt)

Let's consider the energy statistics for the Italian country in 2009 (IEA data, values in ktoe).



	Coal*	Crude oil*	Oil products	Natural gas	Nuclear	Hydro	Geothermal, solar, etc.	Biofuels and waste	Electricity	Heat	Total**
Production	46	4962	0	6561	0	4226	5511	5308	0	0	26613
Imports	12609	82516	11631	56700	0	0	0	1627	4048	0	169131
Exports	-191	-2300	-24783	-102	0	0	0	-86	-182	0	-27644
International marine bunkers***	0	0	-2352	0	0	0	0	0	0	0	-2352
International aviation bunkers***	0	0	-2997	0	0	0	0	0	0	0	-2997
Stock changes	286	81	896	725	0	0	0	14	0	0	2002

Imagine to substitute a portion of the electric energy production from Natural Gas Combined Cycle power plants (NGCC, with η_I=0.55 and LHV_{ng}=50 MJ/kg, assumed as pure methane) with the energy produced by 1400 MW_{el} of Nuclear power plants (NU, with an average load factor f_{NU}=0.9).
With reference to the data given in the table, it is required to calculate:

a. (1pt) The Total Primary Energy Supply of Italy in 2009, according to the data provided by the table;
b. (2pt) The amount of natural gas saved and CO2 emissions avoided thanks to the production of nuclear energy in one year (in Mton);
c. (2pt) The new value of the TPES, considering the natural gas save from imports and the production of electric energy by nuclear plants;

Exercise 3. Solution

a. **The Total Primary Energy Supply of Italy in 2009, according to the data provided by the table;**

The TPES for Italy in 2009 is evaluated through the algebraic sum of the last column of the given table.

$$TPES = \begin{matrix} endogenous \\ production \end{matrix} + imports - exports - bunkers \pm \begin{matrix} stock \\ changes \end{matrix}$$

$$TPES = \left[26613 + 169131 - 27644 - \left(2352 + 2997\right) + 2002\right] ktoe = 164753 ktoe$$

b. **The amount of natural gas saved and CO2 emissions avoided thanks to the production of nuclear energy in one year (in Mton);**

The electric energy produced by the nuclear plants in one year is equal to:

$$E_{NU} = P_{NU} \cdot f_{NU} \cdot H_y = 1400 MW_{el} \cdot 0.9 \frac{h}{h_y} \cdot 8760 h_y = 11037600 MWh_{el} \left(11.038 TWh_{el}\right)$$

If the same energy is no more produced by NGCC power plant, the natural gas saved in one year can be evaluate as:

$$E_{NU} = E_{NGCC} = m_{ng} \cdot LHV_{ng} \cdot \eta_{NGCC} \quad \rightarrow \quad m_{ng} = \frac{E_{NGCC}}{LHV_{ng} \cdot \eta_{NGCC}}$$

$$m_{ng} = \frac{E_{NGCC}}{LHV_{ng} \cdot \eta_{NGCC}} = 11037600 \frac{MJh}{s} \cdot \frac{1}{50} \frac{kg}{MJ} \cdot \frac{1}{0.55} \cdot 3600 \frac{s}{h} \cdot 10^{-9} \frac{Mton}{kg} = 1.445 Mton$$

Similarly, the avoided CO2 emissions are:

$$m_{CO2} = m_{ng} \cdot \frac{44}{16} \frac{kg_{CO2}}{kg_{CH4}} = 3.974 Mtoe$$

c. The new value of the TPES, considering the natural gas save from imports and the production of electric energy by nuclear plants;

Considering the electric energy production by nuclear plants, the table given will change. Specifically:

- the amount of natural gas (produced or imported) will decrease by the quantity of 1444.9 Mtoe, converted in ktoe on a net calorific value;
- the nuclear production will become a positive value, equal to the electric energy produced by nuclear plant properly converted into primary energy (ktoe) through the IEA conventions (η_{NU}=0.33);

$$\text{NG imports} = \left(1.445 Mton_{NG} \cdot 10^9 \frac{kg_{NG}}{Mton_{NG}} \cdot 50 \frac{MJ}{kg_{NG}} \cdot \frac{1}{41860} \frac{toe}{MJ} \cdot 10^{-3} \frac{ktoe}{toe}\right) = 1726 ktoe$$

$$\text{Tot. Imports} = (169131 - 1726) ktoe = 167405 ktoe$$

$$\text{NU production} = 11.038 TWh \cdot \frac{0.086}{0.33} \frac{Mtoe}{TWh} \cdot 10^3 \frac{ktoe}{Mtoe} = 2876 ktoe$$

$$\text{Tot. production} = (26613 + 2876) ktoe = 29489 ktoe$$

$$TPES' = \left[29489 + 167405 - 27644 - (2352 + 2997) + 2002\right] ktoe = 165900 ktoe$$

Department of Energy
Politecnico di Milano

Milan, 29th June 2016

Final Test of 21-07-2015 – Problems

Exercise 1. (9pt)

A vapor-compression refrigeration system is designed to cool down a mass flow rate of a fluid for an industrial process. The cycle works with ammonia and it is characterized by the following transformations:

- 1-2: vapor enters the compressor (CMP, with isentropic efficiency of 0.6) as saturated vapor;
- 2-3: vapor condensates in the condenser (COND) and it exits as saturated liquid. The heat is rejected to the environment at 25 °C and 1 bar (reference conditions);
- 3-4: liquid faces an isenthalpic lamination process through a lamination valve (LAM);
- 4-1: the multi-phase flow evaporates into the evaporator (EVA), exiting as saturated vapor and cooling down a liquid mass flow rate from 15°C to 7°C;

Ammonia	T °C	p MPa	quality -	h kJ/kg	s kJ/kg-K	ex kJ/kg	h_ls kJ/kg	h_vs kJ/kg	s_ls kJ/kg-K	s_vs kJ/kg-K
0	25	0.1013	-	1689.8	7.0732	0.0				
1	-10.0	0.2908		1593.9	6.2284					
2s	88.6	1.1670		1792.4						
2	141.7	1.1670			6.5699					
3	30.0	1.1670		484.9	1.9596					
4	-10.0	0.2908					297.2	1593.9	1.3010	6.2284

With reference to the properties collected in the table, it is required to:

a. (3pt) Draw the inverse cycle in the P-h diagram. Draw the scheme of the system, with all of its components, highlighting directions of work, heat and bulk flow streams. Evaluate all the missing values in the table (quality, enthalpy, entropy and exergy in the grey cells);

b. (2pt) Define boundaries of the whole system and apply the energy and the exergy balances, deriving the *analytical* expressions for the COP and the functional exergy efficiency $\eta_{ex,f}$ of the system. What are the differences among the two performance indicators?

c. (2pt) Apply the exergy balance to all the components of the system, deriving the analytical expressions for the exergy destructions and the functional exergy efficiencies;

d. (2pt) Derive numerical results for questions b and c.

Notice that:

- Irreversibilities occurring during the heat transfer processes must be included in the evaluation of the exergy efficiencies;
- State 0 refers to the reference state of ammonia;
- Transformation 1-2s refers to the isentropic compression process between evaporation and condensation pressures;
- Values of entropy includes the effects of both temperature and pressure;
- Subscripts _ls and _vs refers respectively to the saturated liquid and the saturated vapor;

Exercise 1. Solution

a. **<u>Draw the inverse cycle in the P-h diagram. Draw the scheme of the system, with all of its components, highlighting directions of work, heat and bulk flow streams. Evaluate all the missing values in the table (quality, enthalpy, entropy and exergy in the grey cells);</u>**

$s_{2s} = s_1 = 6.2284\,kJ/kgK$ the isentropic compression does not change the entropy of the fluid;

$q_1 = 1$ saturated vapor;
$q_{2s} = q_2 = nd$ the quality of superheated vapor is not defined;
$q_3 = 1$ saturated liquid;

$$\eta_{iso,CMP} = \frac{h_{2s} - h_1}{h_2 - h_1} \quad \rightarrow \quad h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{iso,CMP}} = 1924.8\,kJ/kg$$

$h_4 = h_3$ isenthalpic lamination process (assuming valve as adiabatic);

$$q_4 = \frac{h_4 - h_{4,ls}}{h_{4,vs} - h_{4,ls}} \quad \rightarrow \quad q_4 = 0.145$$

$$q_4 = \frac{s_4 - s_{4,ls}}{s_{4,vs} - s_{4,ls}} \quad \rightarrow \quad s_4 = s_{4,ls} + q_4\left(s_{4,vs} - s_{4,ls}\right) = 2.0142\,kJ/kgK$$

$$ex_i = h_i - h_0 - T_0\left(s_i - s_0\right)$$

$$ex_1 = 156.0\,kJ/kg$$
$$ex_2 = 385.1\,kJ/kg$$
$$ex_3 = 319.8\,kJ/kg$$
$$ex_4 = 303.5\,kJ/kg$$

b. **<u>Define boundaries of the whole system and apply the energy and the exergy balances, deriving the analytical expressions for the COP and the functional exergy efficiency ηex,f of the system. What are the differences among the two performance indicators?</u>**

$$COP = \frac{\dot{Q}^{\leftarrow}_{EVA}}{\dot{W}^{\leftarrow}_{CMP}}$$

Application of the energy balance to EVA:

- Steady state;
- Adiabatic and rigid component;
- No relevant changes in elevation and speed of fluids;

$$\sum_i \left(\dot{m}^{\leftarrow} h\right)_i = 0 \quad \rightarrow \quad \dot{m}_f\left(h_{f,IN} - h_{f,OUT}\right) + \dot{m}_r\left(h_{r,4} - h_{r,1}\right) = 0 \quad \rightarrow \quad \dot{Q}^{\leftarrow}_{EVA} = \dot{m}_r\left(h_{r,1} - h_{r,4}\right)$$

Application of the energy balance to CMP:

- Steady state;
- Adiabatic component;
- No relevant changes in elevation and speed of fluids;

$$\sum_i \left(\dot{m}^{\leftarrow} h\right)_i + \sum_i \dot{W}^{\leftarrow}_i = 0 \quad \rightarrow \quad \dot{W}^{\leftarrow} + \dot{m}_r\left(h_{r,1} - h_{r,2}\right) = 0 \quad \rightarrow \quad \dot{W}^{\leftarrow} = \dot{m}_r\left(h_{r,2} - h_{r,1}\right)$$

Therefore:

$$COP=\frac{\dot{Q}^{\leftarrow}_{EVA}}{\dot{W}^{\leftarrow}_{CMP}}=\frac{\cancel{\dot{m}_r}\left(h_{r,1}-h_{r,4}\right)}{\cancel{\dot{m}_r}\left(h_{r,2}-h_{r,1}\right)}=\frac{h_{r,1}-h_{r,4}}{h_{r,2}-h_{r,1}}$$

Functional exergy efficiency:

$$\eta_{ex,f}=\frac{\dot{Ex}^{\rightarrow}_{Q,EVA}}{\dot{W}^{\leftarrow}_{CMP}}$$

Application of the exergy balance to EVA:

- Steady state;
- Adiabatic and rigid component;
- No relevant changes in elevation and speed of fluids;

$$\sum_i\left(\dot{m}^{\leftarrow}ex\right)_i-\dot{Ex}_{D,EVA}=0 \quad\rightarrow\quad \dot{m}_f\left(ex_{f,IN}-ex_{f,OUT}\right)+\dot{m}_r\left(ex_{r,4}-ex_{r,1}\right)-\dot{Ex}_{D,EVA}=0$$

Properties of the cold fluid are unknown. However, the change in exergy of the cold fluid is equal to the exergy of the heat provided to the fluid between the same inlet and outlet temperatures:

$$\dot{Q}^{\leftarrow}_{EVA}\left(1-\frac{T_0}{T_{ml,f}}\right)+\dot{m}_r\left(ex_{r,4}-ex_{r,1}\right)-\dot{Ex}_{D,EVA}=0 \quad\rightarrow\quad \dot{Ex}^{\leftarrow}_{Q,EVA}=\dot{Q}^{\leftarrow}_{EVA}\left(1-\frac{T_0}{T_{ml,f}}\right)=\dot{m}_r\left(h_{r,1}-h_{r,4}\right)\left(1-\frac{T_0}{T_{ml,f}}\right)$$

Application of the energy balance to CMP:

- Steady state;
- Adiabatic component;
- No relevant changes in elevation and speed of fluids;

$$\sum_i\left(\dot{m}^{\leftarrow}ex\right)_i+\sum_i\dot{W}^{\leftarrow}_i=0 \;\rightarrow\; \dot{W}^{\leftarrow}_{CMP}+\dot{m}_r\left(ex_{r,1}-ex_{r,2}\right)=0 \quad\rightarrow\quad \dot{W}^{\leftarrow}_{CMP}=\dot{m}_r\left(ex_{r,2}-ex_{r,1}\right)$$

Therefore:

$$T_{ml,f}=\frac{T_{f,IN}-T_{f,OUT}}{\ln\frac{T_{f,IN}}{T_{f,OUT}}}=284.1K$$

$$\eta_{ex,f}=\frac{\dot{Ex}^{\rightarrow}_{Q,EVA}}{\dot{W}^{\leftarrow}_{CMP}}=\frac{-\cancel{\dot{m}_r}\left(h_{r,1}-h_{r,4}\right)\left(1-\frac{T_0}{T_{ml,f}}\right)}{\cancel{\dot{m}_r}\left(ex_{r,2}-ex_{r,1}\right)}=\frac{\left(h_{r,4}-h_{r,1}\right)\left(1-\frac{T_0}{T_{ml,f}}\right)}{ex_{r,2}-ex_{r,1}}$$

c. **Apply the exergy balance to all the components of the system, deriving the analytical expressions for the exergy destructions and the functional exergy efficiencies;**

Application to the compressor (CMP):

- Steady state;
- Adiabatic component;
- No relevant changes in elevation and speed of fluids;

$$\dot{Ex}^{\leftarrow}_{W,CMP}+\dot{m}_r\left(ex_1-ex_2\right)-\dot{Ex}_{D,CMP}=0$$

$$\dot{Ex}^{\leftarrow}_{W,CMP}=\dot{W}^{\leftarrow}_{CMP}=\dot{m}_r\left(h_2-h_1\right)$$

$$ex_{D,CMP}=\dot{Ex}_{D,CMP}/\dot{m}_r$$

$$ex_{D,CMP} = ex_1 - ex_2 + (h_2 - h_1)$$

$$\eta_{ex,f,CMP} = \frac{\dot{m}_r (ex_1 - ex_2)}{\dot{m}_r (h_2 - h_1)} = \frac{ex_1 - ex_2}{h_2 - h_1}$$

Application to the condenser (COND):

- Steady state;
- Adiabatic and rigid component;
- No relevant changes in elevation and speed of fluids;
- Heat discharged into the environment at T0;

$$\dot{E}x^{\leftarrow}_{Q,COND} + \dot{m}_r (ex_2 - ex_3) - \dot{E}x_{D,COND} = 0$$

$$\dot{E}x^{\leftarrow}_{Q,COND} = \dot{Q}^{\leftarrow}_{COND} (1 - T_0/T_{COND}) \qquad \left[T_{COND} = T_0 \quad \rightarrow \quad \dot{E}x^{\leftarrow}_{Q,COND} = 0\right]$$

$$ex_{D,COND} = \dot{E}x_{D,COND} / \dot{m}_r$$

$$ex_{D,COND} = ex_2 - ex_3$$

$$\eta_{ex,r,COND} = \frac{\dot{m}_r ex_3}{\dot{m}_r ex_2} = \frac{ex_3}{ex_2}$$

Application to the lamination valve (LAM):

- Steady state;
- Adiabatic and rigid component;
- No relevant changes in elevation and speed of fluids;

$$\dot{m}_r (ex_3 - ex_4) - \dot{E}x_{D,LAM} = 0$$

$$ex_{D,LAM} = \dot{E}x_{D,LAM} / \dot{m}_r$$

$$ex_{D,LAM} = ex_3 - ex_4$$

$$\eta_{ex,r,LAM} = \frac{\dot{m}_r ex_4}{\dot{m}_r ex_3} = \frac{ex_4}{ex_3}$$

Application to the evaporator (EVA):

- Steady state;
- No relevant changes in elevation and speed of fluids;
- Since properties of the cold fluid are unknown, the exergy of heat transfer is considered;

$$\dot{E}x^{\leftarrow}_{Q,EVA} + \dot{m}_r (ex_4 - ex_1) - \dot{E}x_{D,EVA} = 0$$

$$\dot{E}x^{\leftarrow}_{Q,EVA} = \dot{Q}^{\leftarrow}_{EVA} (1 - T_0/T_{EVA})$$

$$\dot{Q}^{\leftarrow}_{EVA} = \dot{m}_r (h_1 - h_4)$$

$$ex_{D,EVA} = \dot{E}x_{D,EVA} / \dot{m}_r$$

$$ex_{D,\mathrm{EVA}} = (h_1 - h_4)(1 - T_0/T_{EVA}) + (ex_4 - ex_1)$$

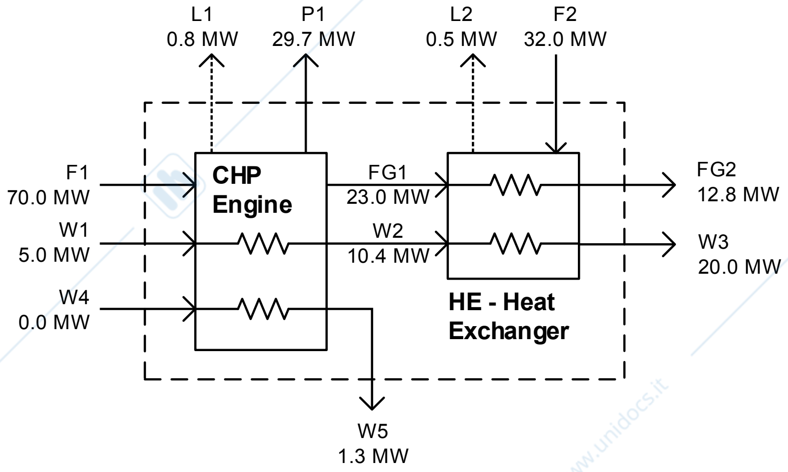
$$\eta_{ex,\mathrm{f,EVA}} = \frac{\dot{m}_r (h_1 - h_4)(1 - T_0/T_{EVA})}{\dot{m}_r (ex_4 - ex_1)} = \frac{(h_1 - h_4)(1 - T_0/T_{EVA})}{ex_4 - ex_1}$$

d. **Derive numerical results for questions b and c.**

	COP -	ex_d kJ/kg	eta_ex,f -
CMP		101.8	0.69
COND		81.6	0.83
LAM		16.3	0.95
EVA		92.7	0.37
TOT	3.351	292.5	0.165

Exercise 2. (8pt)

The system depicted in the figure is composed by an combined heat and power engine (CHP) and an heat exchanger endowed with an additional post-combustion chamber. Values in the figure quantify the exergy of the material and energy interactions represented by the arrows.
The CHP absorbs a flow of low quality fuel (c_{F1} = 20 €/MWh), a flow of pressurized water (c_{W1} = 2 €/MWh), and a flow of water at environmental conditions (c_{W4} = 0 €/MWh), producing useful flows of electric power P1 and hot water W5 and losing high temperature heat exergy L1 through the mantle.
The HE absorbs high quality fuel (c_{F2} = 40 €/MWh) and the by-products of the CHP (flue gases FG1 and pressurized water W2), producing a flow of steam W3 and losing high temperature heat exergy L2 through the mantle. The flue gases FG2 need to be treated causing additional economic expenses of 5 €/MWh.
Total investment cost of CHP is 1850 €/h, while for the HE is 350 €/h.



It is required to:

a. (1pt) Define the thermoeconomic systems of equations for the CHP and the HE in analytical way;
b. (2pt) Derive the number of the auxiliary relations required to solve the thermoeconomic systems. Appropriately set the required auxiliary relations, justifying each assumption with a brief comment;
c. (2pt) Solve the defined thermoeconomic systems, calculating specific and total economic costs of useful products of both CHP and HE. Then calculate the average specific costs of their inlet and outlet flows;
d. (3pt) Assuming that the system works at constant production (demand driven model), calculate the costs of exergy destructions for CHP and HE. Apply the first iteration of the Design Evaluation process: comment the obtained results and propose one possible way to increase the economic efficiency of the system.

Exercise 2. Solution

a. **Define the thermoeconomic systems of equations for the CHP and the HE in analytical way;**

For the CHP:

$$\begin{cases} \dot{E}x_{F1}+\dot{E}x_{W1}+\dot{E}x_{F4}=\dot{E}x_{L1}+\dot{E}x_{P1}+\dot{E}x_{W5}+\dot{E}x_{FG1}+\dot{E}x_{W2}+\dot{E}x_{D,CHP} \\ \dot{C}_{F1}+\dot{C}_{W1}+\dot{C}_{F4}+\dot{Z}_{CHP}=\dot{C}_{L1}+\dot{C}_{P1}+\dot{C}_{W5}+\dot{C}_{FG1}+\dot{C}_{W2} \\ \dot{C}=c\cdot\dot{E}x \end{cases}$$

$$\rightarrow \qquad c_{F1}\dot{E}x_{F1}+c_{W1}\dot{E}x_{W1}+c_{F4}\dot{E}x_{F4}+\dot{Z}_{CHP}=c_{L1}\dot{E}x_{L1}+c_{P1}\dot{E}x_{P1}+c_{W5}\dot{E}x_{W5}+c_{FG1}\dot{E}x_{FG1}+c_{W2}\dot{E}x_{W2}$$

For the HE:

$$\begin{cases} \dot{E}x_{FG1}+\dot{E}x_{W2}+\dot{E}x_{F2}=\dot{E}x_{L2}+\dot{E}x_{FG2}+\dot{E}x_{W3}+\dot{E}x_{D,HE} \\ \dot{C}_{FG1}+\dot{C}_{W2}+\dot{C}_{F2}+\dot{Z}_{HE}=\dot{C}_{L2}+\dot{C}_{FG2}+\dot{C}_{W3} \\ \dot{C}=c\cdot\dot{E}x \end{cases}$$

$$\rightarrow \qquad c_{FG1}\dot{E}x_{FG1}+c_{W2}\dot{E}x_{W2}+c_{F2}\dot{E}x_{F2}+\dot{Z}_{HE}=c_{L2}\dot{E}x_{L2}+c_{FG2}\dot{E}x_{FG2}+c_{W3}\dot{E}x_{W3}$$

b. **Derive the number of the auxiliary relations required to solve the thermoeconomic systems. Appropriately set the required auxiliary relations, justifying each assumption with a brief comment;**

The cost of the useful flows can be derived once n auxiliary relations have been defined:

$$n_{aux,HP}=m_{flows}-l_{components} \qquad \rightarrow \qquad n_{aux,HP}=12-2=10$$

1.	$c_{F1}=20\,€/MWh$	Given data;
2.	$c_{F2}=40\,€/MWh$	Given data;
3.	$c_{W1}=2\,€/MWh$	Given data;
4.	$c_{W4}=0\,€/MWh$	Given data;
5.	$c_{L1}=0\,€/MWh$	Loss released with no additional costs;
6.	$c_{L2}=0\,€/MWh$	Loss released with no additional costs;
7.	$c_{W2}=c_{W1}=2\,€/MWh$	Pressurized water is a by-product of the CHP engine;
8.	$c_{FG1}=c_{F1}=20\,€/MWh$	Flue gases are by-product of the CHP engine;
9.	$c_{P1}=c_{W5}$	Useful products of the CHP engine;
10.	$c_{FG2}=-5\,€/MWh$	Flue gases are released by HE with additional treatment cost;

c. **Solve the defined thermoeconomic systems, calculating specific and total economic costs of useful products of both CHP and HE. Then calculate the average specific costs of their inlet and outlet flows;**

Considering the products of CHP:

$$c_{products,CHP}=\frac{c_{F1}\dot{E}x_{F1}+c_{W1}\dot{E}x_{W1}+c_{F4}\dot{E}x_{F4}+\dot{Z}_{CHP}-\left(c_{L1}\dot{E}x_{L1}+c_{W2}\dot{E}x_{W2}+c_{FG1}\dot{E}x_{FG1}\right)}{\dot{E}x_{P1}+\dot{E}x_{W5}}=89.7\,\frac{€}{MWh}$$

$$c_{P1}=c_{W5}=89.7\,\frac{€}{MWh} \qquad \rightarrow \qquad \begin{cases} \dot{C}_{P1}=2662.7\,€/h \\ \dot{C}_{W5}=116.5\,€/h \end{cases}$$

$$c_{in,CHP}=\frac{c_{F1}\dot{E}x_{F1}+c_{W1}\dot{E}x_{W1}+c_{F4}\dot{E}x_{F4}}{\dot{E}x_{F1}+\dot{E}x_{W1}+\dot{E}x_{F4}}=18.8\,\frac{€}{MWh}$$

$$c_{out,CHP}=\frac{c_{L1}\dot{E}x_{L1}+c_{P1}\dot{E}x_{P1}+c_{W5}\dot{E}x_{W5}+c_{FG1}\dot{E}x_{FG1}+c_{W2}\dot{E}x_{W2}}{\dot{E}x_{L1}+\dot{E}x_{P1}+\dot{E}x_{W5}+\dot{E}x_{FG1}+\dot{E}x_{W2}}=50\,\frac{€}{MWh}$$

Considering the products of HE:

$$c_{products,HE}=c_{W3}=\frac{c_{FG1}\dot{E}x_{FG1}+c_{W2}\dot{E}x_{W2}+c_{F2}\dot{E}x_{F2}+\dot{Z}_{HE}-\left({}_{L2}\dot{E}x_{L2}+c_{FG2}\dot{E}x_{FG2}\right)}{\dot{E}x_{W3}}=108.7\frac{€}{MWh}$$

$$c_{W3}=108.7\frac{€}{MWh}\quad\rightarrow\quad\dot{C}_{W3}=2174.8\,€/h$$

$$c_{in,\mathrm{HE}}=\frac{c_{FG1}\dot{E}x_{FG1}+c_{W2}\dot{E}x_{W2}+c_{F2}\dot{E}x_{F2}}{\dot{E}x_{FG1}+\dot{E}x_{W2}+\dot{E}x_{F2}}=26.9\frac{€}{MWh}$$

$$c_{out,HE}=\frac{c_{L2}\dot{E}x_{L2}+c_{FG2}\dot{E}x_{FG2}+c_{W3}\dot{E}x_{W3}}{\dot{E}x_{L2}+\dot{E}x_{FG2}+\dot{E}x_{W3}}=67.2\frac{€}{MWh}$$

d. **Assuming that the system works at constant production (demand driven model), calculate the costs of exergy destructions for CHP and HE. Apply the first iteration of the Design Evaluation process: comment the obtained results and propose one possible way to increase the economic efficiency of the system.**

Considering the CHP:

$$\dot{E}x_{F1}+\dot{E}x_{W1}+\dot{E}x_{F4}=\dot{E}x_{L1}+\dot{E}x_{P1}+\dot{E}x_{W5}+\dot{E}x_{FG1}+\dot{E}x_{W2}+\dot{E}x_{D,CHP}$$

$$\dot{E}x_{D,CHP}=43.2MW$$

$$\dot{C}_{D,CHP}=c_{in,CHP}\dot{E}x_{D,CHP}=812.2\,€/h$$

Considering the HE:

$$\dot{E}x_{FG1}+\dot{E}x_{W2}+\dot{E}x_{F2}=\dot{E}x_{L2}+\dot{E}x_{FG2}+\dot{E}x_{W3}+\dot{E}x_{D,HE}$$

$$\dot{E}x_{D,HE}=32.1MW$$

$$\dot{C}_{D,\mathrm{HE}}=c_{in,\mathrm{HE}}\dot{E}x_{D,\mathrm{HE}}=864.2\,€/h$$

Application of DE consists in the evaluation of the following indexes for each one of the plant components:

$\dot{Z}_i+\dot{C}_{D,i}$	Sum of cost of exergy destruction and losses, and investment cost;
$r_i=(c_{out}-c_{in})/c_{in}$	Relative difference of average cost of outlet and inlet flows;
$f_i=\frac{\dot{Z}_i}{\dot{Z}_i+\dot{C}_{D,\mathrm{i}}}$	Exergoeconomic Factor;

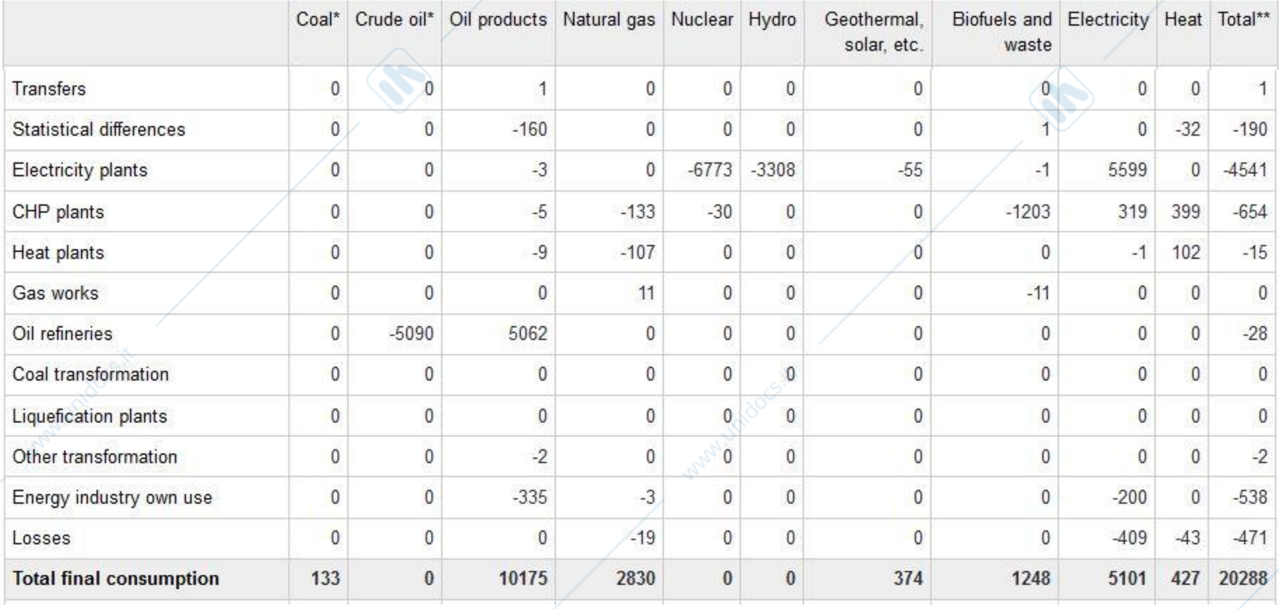
Component	$\dot{C}_{D,\mathrm{i}}$ [€/h]	$\dot{Z}_i+\dot{C}_{D,\mathrm{i}}$ [€/h]	r_i [-]	f_i [-]
CHP	812.2	2662.2	1.66	0.69
HE	864.2	1214.2	1.50	0.29

Comments:

- The CHP show higher impact of investment and irreversibilities on economic cost of products. Then attention should be devoted to CHP first;
- Relative cost difference is very high for CHP and HE, revealing high margins of improvements for both components;
- Exergoeconomic factor for CHP reveal that investment costs are very high compared to the cost of irreversibilities: a decrease in investment costs may be beneficial, even if this imply a reduction of thermodynamic performance. The opposite trend holds for the HE.

Exercise 3. (5pt)

Let us consider the energy balance of Switzerland in 2013 (the table reports only the energy transformation from TPES to TFC).



	Coal*	Crude oil*	Oil products	Natural gas	Nuclear	Hydro	Geothermal, solar, etc.	Biofuels and waste	Electricity	Heat	Total**
Transfers	0	0	1	0	0	0	0	0	0	0	1
Statistical differences	0	0	-160	0	0	0	0	1	0	-32	-190
Electricity plants	0	0	-3	0	-6773	-3308	-55	-1	5599	0	-4541
CHP plants	0	0	-5	-133	-30	0	0	-1203	319	399	-654
Heat plants	0	0	-9	-107	0	0	0	0	-1	102	-15
Gas works	0	0	0	11	0	0	0	-11	0	0	0
Oil refineries	0	-5090	5062	0	0	0	0	0	0	0	-28
Coal transformation	0	0	0	0	0	0	0	0	0	0	0
Liquefication plants	0	0	0	0	0	0	0	0	0	0	0
Other transformation	0	0	-2	0	0	0	0	0	0	0	-2
Energy industry own use	0	0	-335	-3	0	0	0	0	-200	0	-538
Losses	0	0	0	-19	0	0	0	0	-409	-43	-471
Total final consumption	**133**	**0**	**10175**	**2830**	**0**	**0**	**374**	**1248**	**5101**	**427**	**20288**

With reference to the data given in the table, it is required to calculate:

a. (2pt) The Total Primary Energy Supply and the Electric penetration of Switzerland in 2013. Then calculate the average efficiency of electricity production as the ratio between the net electricity production and the primary energy devoted to electricity production;

In the next year, one new Natural Gas Combined Cycle plant (NGCC, with η_I=0.55 and LHV_{ng}=50 MJ/kg of pure methane, load factor 0.45) of 1.2 GW net electric power will be installed to substitute the portion of the electricity produced by nuclear plants (NU) under disposal. It is then required to calculate:

b. (2pt) The change in natural gas consumption and CO2 emissions due to the new production of energy from NGCC in one year (in Mton);
c. (1pt) The change in national TPES.

Exercise 3. Solution

a. **The Total Primary Energy Supply and the Electric penetration of Switzerland in 2013. Then calculate the average efficiency of electricity production as the ratio between the net electricity production and the primary energy devoted to electricity production;**

The TPES for Switzerland in 2013 is evaluated through the sum of the last column of the given table (pay attention to the signs!).

$$TPES = \left[20288 + \left(471+538+2+28+15+654+4541+190\text{-}1\right)\right] ktoe = 26726\, ktoe$$

The electric penetration is defined as the patio between the primary energy devoted to electric energy production [ktoe] and the TPES [ktoe]:

$$EP = \frac{\sum_i E_{P,i}}{TPES} = \frac{\left(3+6773+3308+55+1+5+133+30+1203\right) ktoe}{26726\, ktoe} = 0.431$$

The average efficiency of electricity production is equal to :

$$\eta_{el}=\frac{\sum_i E_{el,i}-E_{el,\text{own use}}-E_{el,losses}}{\sum_i E_{P,el,i}}=\frac{(5599+319\text{-}200\text{-}409)\,ktoe}{(3+6773+3308+55+1+5+133+30+1203)\,ktoe}=0.514$$

b. The amount of natural gas consumed and CO2 emitted due to the new production of energy from NGCC in one year (in Mton);

Electricity production from NGCC:

$$E_{\text{ng},NGCC}=P_{\text{el},NGCC}\cdot h_y\cdot f=4730400\,MWh/y$$

Natural gas requirements and CO2 emissions:

$$E_{\text{P,ng},NGCC}=\frac{E_{\text{ng},NGCC}}{\eta_{NGCC}}=\frac{4730400\,MWh/y}{0.55}=8600727\frac{MWh}{y}(739.5ktoe)$$

$$m_{CH4,NGCC}=\frac{E_{\text{P,ng},NGCC}}{LHV_{\text{ng}}}\cdot 3600\cdot 10^{-6}=619.3\frac{kton_{CH4}}{y}\quad\rightarrow\quad m_{CO2,NGCC}=m_{CH4,NGCC}\frac{44}{16}=1702.9\frac{kton_{CO2}}{y}$$

c. The change in national TPES.

$$E_{\text{P},NU}=\frac{E_{\text{ng},NGCC}}{0.33}=14334545\frac{MWh}{y}(1233ktoe)$$

$$\Delta TPES=E_{\text{P,ng},NGCC}-E_{\text{P},NU}=(739.5-1233)\,ktoe=-493ktoe$$

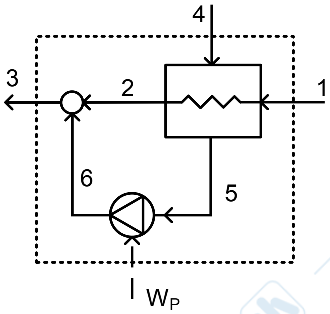
POLITECNICO DI MILANO

Milano –25 July 2015

Final Test of 20-07-2015 – Problems

Exercise 1. (8pt)

A feedwater heater used in a steam power plant is shown in figure. The flow of saturated steam 4 enters the heat exchanger, heating the subcooled liquid water 1. The steam exits the heat exchanger as saturated liquid 5 and enters the pump, where it is brought to the same pressure of water 2 absorbing mechanical power W_P. Pressure losses in the cold and hot side of the heat exchanger are respectively 10% of p_1 and 5% of p_4.



number	p	T	h	s
	bar	°C	kJ/kg	kJ/kg-K
1	27	38	161,6	
2				
3		53	224,0	
4	1.4	109.3	2690.0	7.2461

With reference to the data provided in the tables, and considering an environmental reference temperature T_0 of 298 K it is required to:

a. (1pt) Approximately drawn the transformations 4-5-6 and 1-2-3 in the T-s diagram (Temperature-specific entropy). Evaluate the pressure for all the states of the process and the pressure ratio of the pump, considering that the mixing process is isobaric;

b. (2pt) Introducing proper hypotheses, evaluate the ratio between the mass flow rates 4 and 1, then evaluate the mechanical energy absorbed by the pump per unit of feedwater mass flow rate 1;

c. (3pt) Evaluate the exergy destructions in the whole system (dotted line) per unit of feedwater mass flow rate 1. Then evaluate the entropy generations in the whole system per unit of saturated steam 4;

d. (2pt) Consider a process where the feedwater mass flow rate 1 is heated from conditions 1 to 3 thanks to a heat flow Q_H produced by a thermal reservoir at constant temperature of 500K. Evaluate the exergy destructions (per unit of feedwater mass flow rate 1) caused by such process and comment the obtained results.

Additional notes:

- All the data required to solve the problem are given in the text;
- Saturated and subcooled water can be modeled as perfect liquid with c=4.2 kJ/kg-K and v=0.001 m^3/kg;
- Enthalpy and entropy of the saturated liquid at $p=p_4$ are respectively: 458.4 kJ/kg and 1.4110 kJ/kg-K;

Exercise 1. Solution

a. **Approximately drawn the transformations 4-5-6 and 1-2-3 in the T-s diagram (Temperature-specific entropy). Evaluate the pressure for all the states of the process and the pressure ratio of the pump, considering that the mixing process is isobaric;**

The pressure of 2, 6 and 3 must be the same, therefore:

$P_2 = P_1\left(1-\Delta P_{cold}/P_1\right) = 24.3bar \qquad \rightarrow P_2 = P_3 = P_6$

$P_5 = P_4\left(1-\Delta P_{hot}/P_4\right) = 1.3bar$

$\beta_P = P_6/P_5 = 18.3$ pressure ratio of the pump.

b. Introducing proper hypotheses, evaluate the ratio between the mass flow rates 4 and 1, then evaluate the mechanical energy absorbed by the pump per unit of feedwater mass flow rate 1;

The pump is assumed as:

- Operating in steady state;
- Perfectly insulated;
- With rigid boundaries;
- With no large differences in velocity and height of the fluid that cross it;
- Water is assumed as perfect liquid;
- Isentropic and isothermal compression process is assumed;

The energy balance applied to the pump only then results as:

$$\begin{cases} \dot{W}_P^{\leftarrow} + \dot{m}_5 h_5 - \dot{m}_6 h_6 = 0 \\ \dot{m}_5 = \dot{m}_6 = \dot{m}_4 \end{cases} \rightarrow \qquad \dot{W}_P^{\leftarrow} + \dot{m}_4\left(h_5 - h_6\right) = 0$$

Dividing by mass flow rate 4 and introducing the perfect liquid model (pressures must be converted in kPa):

$$\underbrace{\frac{\dot{W}_P^{\leftarrow}}{\dot{m}_4}}_{w_{P4}^{\leftarrow}} + c\left(\cancel{T_5 - T_6}\right) + v\left(p_5 - p_6\right) = 0 \quad \rightarrow \qquad w_{P,4}^{\leftarrow} = v\left(p_6 - p_5\right) = 2.30\, kJ/kg_4$$

The ratio between mass flow rates 4 and 1 can be derived through the application of the mass and the energy balance to the whole system:

$$\begin{cases} w_{P,4}^{\leftarrow}\dot{m}_4 + \dot{m}_1 h_1 + \dot{m}_4 h_4 - \dot{m}_3 h_3 = 0 \\ \dot{m}_1 + \dot{m}_4 - \dot{m}_3 = 0 \end{cases} \rightarrow \qquad w_{P,4}^{\leftarrow}\dot{m}_4 + \dot{m}_1 h_1 + \dot{m}_4 h_4 - \left(\dot{m}_1 + \dot{m}_4\right) h_3 = 0$$

Rearranging the equation and dividing by mass flow rate 1:

$$\dot{m}_4\left(w_{P,4}^{\leftarrow} + h_4 - h_3\right) + \dot{m}_1\left(h_1 - h_3\right) = 0 \quad \rightarrow \quad \frac{\dot{m}_4}{\dot{m}_1} = \frac{h_3 - h_1}{w_{P,4}^{\leftarrow} + h_4 - h_3} = 0.0253$$

$$w_{P,1}^{\leftarrow} = w_{P,4}^{\leftarrow} \cdot \frac{\dot{m}_4}{\dot{m}_1} = 0.0581\, kJ/kg_1$$

c. Evaluate the exergy destructions in the whole system (dotted line) per unit of feedwater mass flow rate 1. Then evaluate the entropy generations in the whole system per unit of saturated steam 4;

The application of the exergy balance to the whole system results as:

$$\cancel{\frac{dEx}{dt}} = \sum \dot{Ex}_W^{\leftarrow} + \cancel{\sum \dot{Ex}_Q^{\leftarrow}} + \sum \dot{m}_i^{\leftarrow} ex_i - \dot{Ex}_D = 0$$

Introducing the mass balance and rearranging the exergy balance:

$$\begin{cases} w^{\leftarrow}_{P,1}\dot{m}_1 + \dot{m}_1 ex_1 + \dot{m}_4 ex_4 - \dot{m}_3 ex_3 - \dot{Ex}_D = 0 \\ \dot{m}_1 + \dot{m}_4 - \dot{m}_3 = 0 \end{cases} \rightarrow \begin{array}{l} w^{\leftarrow}_{P,1}\dot{m}_1 + \dot{m}_1 ex_1 + \dot{m}_4 ex_4 - (\dot{m}_1 + \dot{m}_4) ex_3 - \dot{Ex}_D = 0 \\ w^{\leftarrow}_{P,1} + (ex_1 - ex_3) + \dfrac{\dot{m}_4}{\dot{m}_1}(ex_4 - ex_3) - \dfrac{\dot{Ex}_D}{\dot{m}_1} = 0 \end{array}$$

Exergy differences 1-3 and 4-3 can be evaluated as follows:

$$ex_1 - ex_3 = \Delta h - T_0 \Delta s = c(T_1 - T_3) + v(p_1 - p_3) - T_0 \cdot c \cdot \ln\left(\frac{T_1}{T_3}\right) = -3.8\,\frac{kJ}{kg}$$

subcooled liquid is a perfect liquid

$$ex_4 - ex_3 = \underbrace{(ex_4 - ex_{4SL})}_{from\,tables} + \underbrace{(ex_{4SL} - ex_3)}_{perfect\,liquid} \rightarrow \begin{array}{l} ex_4 - ex_{4SL} = h_4 - h_{4SL} - T_0(s_4 - s_{4SL}) = 492.7\,kJ/kg \\ ex_{4SL} - ex_3 = c(T_4 - T_3) + v(p_4 - p_3) - T_0 \cdot c \cdot \ln\left(\dfrac{T_4}{T_3}\right) = 34.9\,kJ/kg \\ ex_4 - ex_3 = 527.6\,kJ/kg \end{array}$$

$$\frac{\dot{Ex}_D}{\dot{m}_1} = w^{\leftarrow}_{P,1} + (ex_1 - ex_3) + \frac{\dot{m}_4}{\dot{m}_1}(ex_4 - ex_3) = 9.59\,kJ/kg_1$$

Entropy generations per unit of mass flow rate 4 can be derived through the Gouy-Stodola relation:

$$s_{irr,4} = \frac{\dot{Ex}_D}{\dot{m}_1} \cdot \frac{\dot{m}_1}{\dot{m}_4} \cdot \frac{1}{T_0} = 1.2733\,kJ/kg_4 K$$

d. **Consider a process where the feedwater mass flow rate 1 is heated from conditions 1 to 3 thanks to a heat flow Q_H produced by a thermal reservoir at constant temperature of 500K. Evaluate the exergy destructions (per unit of feedwater mass flow rate 1) caused by such process and comment the obtained results.**

The heat absorbed by the mass flow rate 1 between the conditions 1 and 3 can be evaluated through the application of the energy balance, assuming:

- Steady state operation;
- Rigid boundaries;
- No changes in velocity and height of the flows;

$$\dot{Q}^{\leftarrow}_H + \dot{m}_1 (h_1 - h_3) = 0 \qquad \rightarrow \qquad q^{\leftarrow}_H = h_3 - h_1 = 62.4\,kJ/kg$$

The application of the exergy balance to the whole system results as:

$$\cancel{\frac{dEx}{dt}} = \cancel{\sum \dot{Ex}^{\leftarrow}_W} + \sum \dot{Ex}^{\leftarrow}_{QH} + \sum \dot{m}^{\leftarrow}_i ex_i - \dot{Ex}_D = 0 \quad \rightarrow \quad \dot{Q}^{\leftarrow}_H \left(1 - \frac{T_0}{T_H}\right) + \dot{m}_1 (ex_1 - ex_3) - \dot{Ex}_D = 0$$

Dividing by mass flow rate 1 and rearranging the exergy balance:

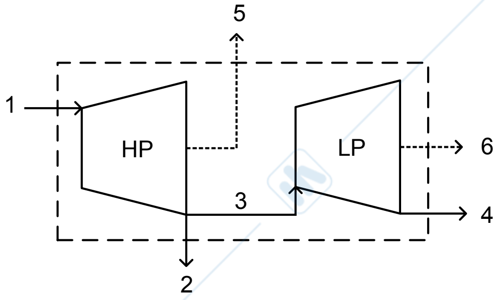
$$q^{\leftarrow}_H \left(1 - \frac{T_0}{T_H}\right) + ex_1 - ex_3 - \frac{\dot{Ex}_D}{\dot{m}_1} = 0 \quad \rightarrow \quad \frac{\dot{Ex}_D}{\dot{m}_1} = 21.4\,kJ/kg$$

This process results as less efficient from the thermodynamic point of view.

Exercise 2. (8pt)

Let's consider a steam turbine of a large Rankine power plant operating as CHP. Steam mass flow rate 1 expand in the high pressure (HP) turbine. Then, one portion of the steam 2 is then extracted in order to be used inside the power plant, and the mass flow rate 3 expands in the low pressure (LP) turbine. The steam flow rate 4 exits the LP turbine and it is used for cogeneration purposes.
Steam mass flow rate 1 has a total enthalpy of 100 MW (measured with respect to the reference enthalpy 0), and the net electric power produced by the turbine is equal to 15 MW$_{el}$.
Total investment cost of HP and LP turbines are respectively 210 €/h and 350 €/h. The cost of inlet steam 1 is equal to 1.5 c€ per kWh of energy.
Considering an environmental temperature T$_0$ of 20°C, and with reference to the data given in the table, it is required to:

a. (1pt) Drawn the expansion processes in the T-s diagram (Temperature – specific entropy). Evaluate the mass flow rates of steam 1, 2, 3 and 4, and the specific and total exergy of each energy and bulk flow interaction of the system;
b. (2pt) Write exergy balances for the whole system and for the two turbines according to the inlet/outlet paradigm. Evaluate the rational exergy efficiencies for the whole system and for the two turbines;
c. (3pt) Write the thermoeconomic system of equations for the system at its lower level of aggregation. Introducing proper auxiliary relations, evaluating specific and total costs of outlet flows;
d. (2pt) Assuming that the system works at constant electric power production, derive and evaluate the structure of the cost of useful outlet flows for the two expanders, and the cost of exergy destructions and losses. Comment the obtained results.



	T	p	h	s
	°C	bar	kJ/kg	kJ/kg-K
0	20	1,01	84	0,30
1	460	35	3361	7,04
2	311	10	3075	7,16
3	311	10	3075	7,16
4	153	2	2774	7,29

Exercise 2. Solution

a. **Drawn the expansion processes in the T-s diagram (Temperature – specific entropy). Evaluate the mass flow rates of steam 1, 2, 3 and 4, and the specific and total exergy of each energy and bulk flow interaction of the system;**

The mass flow rates of steam can be evaluated through the application of the energy balance on the two turbines. Main assumptions:

- Turbines work at steady state;
- Turbines are adiabatic and negligible changes in velocity and height of the fluids occur;

$$\dot{W}_T^{\leftarrow} + \dot{m}_i\left(h_{i,IN} - h_{i,OUT}\right) = 0 \quad \rightarrow \quad \begin{cases} \dot{W}_{HP}^{\rightarrow} = \dot{m}_1\left(h_1 - h_2\right) \\ \dot{W}_{LP}^{\rightarrow} = \dot{m}_3\left(h_3 - h_4\right) \end{cases}$$

Therefore:

$$\begin{cases} \dot{W}_{HP}^{\rightarrow} + \dot{W}_{LP}^{\rightarrow} = 15MW \\ \dot{m}_1\left(h_1 - h_0\right) = 100MW \end{cases} \rightarrow \begin{cases} \dot{m}_1\left(h_1 - h_2\right) + \dot{m}_3\left(h_3 - h_4\right) = 15MW \\ \dot{m}_1\left(h_1 - h_0\right) = 100MW \end{cases} \rightarrow \begin{cases} \dot{m}_1 = 30.5\,kg/s \\ \dot{m}_3 = \dot{m}_4 = 20.9\,kg/s \end{cases}$$

Considering the mass balance at the HP turbine, the mass flow rate of steam 2 can be derived:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \quad \rightarrow \quad \dot{m}_2 = 9.6\,kg/s$$

Power produced by each one of the turbines is equal to:

$$\dot{W}^{\rightarrow}_{HP} = \dot{m}_1 \left(h_1 - h_2\right) = 8.7 MW$$

$$\dot{W}^{\rightarrow}_{LP} = \dot{m}_3 \left(h_3 - h_4\right) = 6.3 MW$$

	ex kJ/kg	Ex MW
0	0	0
1	1300	39,7
2	977	9,4
3	977	20,4
4	639	13,3

Specific and total exergy of the streams are derived as follows:

$$ex_i = h_i - h_0 - T_0\left(s_i - s_0\right) \qquad \rightarrow \qquad \dot{E}x_i = \dot{m}_i \cdot ex_i$$

b. **Write exergy balances for the whole system and for the two turbines according to the inlet/outlet paradigm. Evaluate the rational exergy efficiencies for the whole system and for the two turbines;**

The exergy balance can be written as follows:

$$\frac{dEx}{dt} = \sum \dot{E}x^{\leftarrow}_W + \sum \dot{E}x^{\leftarrow}_Q + \sum \dot{m}^{\leftarrow}_i ex_i - \dot{E}x_D = 0 \qquad \rightarrow \qquad \begin{cases} \sum\limits_{IN} \dot{E}x_{IN} - \sum\limits_{OUT} \dot{E}x_{OUT} - \dot{E}x_D = 0 \\ \eta_{ex,R} = \sum\limits_{OUT} \dot{E}x_{OUT} \Big/ \sum\limits_{IN} \dot{E}x_{IN} \end{cases}$$

Therefore:

	HP	LP	HP+LP
in	1	3	1
	39,7	20,4	39,7
out	2+W_HP	4+W_LP	W_tot+2+4
	38,6	19,6	37,8
dest	1,1	0,8	1,9
eta_R	0,97	0,96	0,95

c. **Write the thermoeconomic system of equations for the system at its lower level of aggregation. Introducing proper auxiliary relations, evaluating specific and total costs of outlet flows;**

The cost of the outlet flows of the HP turbine can be derived once n auxiliary relations have been defined:

$$n_{aux,HP} = m_{flows} - l_{components} \;\rightarrow\; n_{aux,HP} = 4 - 1 = 3$$

The unique product of the HP turbine is the mechanical power W_{HP}, whether the steam flows 2 and 3 are classified as by-products, since they will be respectively used in the power plant and in the LP turbine.
The cost of steam per unit of energy must be converted in the cost per unit of exergy considering the equality of total cost:

$$\dot{C}_{1,en} = \dot{C}_{1,ex} \quad \rightarrow \quad c_{1,en}\left(\dot{H}_1 - \dot{H}_0\right) = c_{1,ex}\dot{E}x_1 \quad \rightarrow \quad c_{1,ex} = c_{1,en}\frac{\left(\dot{H}_1 - \dot{H}_0\right)}{\dot{E}x_1} = 0.0378\frac{€}{kWh}$$

$$\begin{cases} \dot{E}x_1 - \dot{E}x_2 - \dot{E}x_3 - \dot{W}_{HP} - \dot{E}x_{D,HP} = 0 \\ \dot{C}_1 + \dot{Z}_{HP} = \dot{C}_{W,HP} + \dot{C}_2 + \dot{C}_3 \\ \dot{C} = c \cdot \dot{E}x \end{cases} \quad \rightarrow \quad \begin{array}{l} c_1\dot{E}x_1 + \dot{Z}_{HP} = c_{W,HP}\dot{W}_{HP} + c_2\dot{E}x_2 + c_3\dot{E}x_3 \\ c_2 = c_1 \\ c_3 = c_1 \\ c_1 = 0.0378\,€/kWh \end{array}$$

The cost of mechanical power is the only unknown:

$$c_{W,HP} = \frac{c_1\left(\dot{E}x_1 - \dot{E}x_2 - \dot{E}x_3\right) + \dot{Z}_{HP}}{\dot{W}_{HP}} = 0.0667\frac{€}{kWh} \quad \rightarrow \quad c_{W,HP} = 66.75\frac{€}{MWh}$$

The cost of the outlet flows of the LP turbine can be derived once n auxiliary relations have been defined:

$$n_{aux,LP} = m_{flows} - l_{components} \rightarrow n_{aux,LP} = 3 - 1 = 2$$

Both the mechanical power and the steam are considered as useful product of the LP turbine.

$$\begin{cases} \dot{E}x_3 - \dot{E}x_4 - \dot{W}_{LP} - \dot{E}x_{D,LP} = 0 \\ \dot{C}_3 + \dot{Z}_{LP} = \dot{C}_{W,LP} + \dot{C}_4 \\ \dot{C} = c \cdot \dot{E}x \end{cases} \rightarrow \begin{aligned} & c_3\dot{E}x_3 + \dot{Z}_{LP} = c_{W,LP}\dot{W}_{LP} + c_4\dot{E}x_4 \\ & c_4 = c_{W,LP} = c \\ & c_3 = c_1 = 0.0378\,€/kWh \end{aligned}$$

$$c = \frac{c_1\dot{E}x_3 + \dot{Z}_{LP}}{\dot{W}_{LP} + \dot{E}x_4} = 0.0572\frac{€}{kWh} \quad \rightarrow \quad c = 57.16\frac{€}{MWh}$$

	Ex MW	c €/MWh	C €/h
0	0	-	-
1	39,7	37,80	1500
2	9,4	37,80	356
3	20,4	37,80	771
4	13,3	57,16	763
W_HP	8,7	66,75	583
W_LP	6,3	57,16	359

d. <u>Assuming that the system works at constant electric power production, derive and evaluate the structure of the cost of useful outlet flows for the two expanders, and the cost of exergy destructions and losses. Comment the obtained results.</u>

This system has no losses. All the outlet flows are products or by-products. Substituting the exergy balance into the cost balance, considering a demand driven model, the cost structure is derived.

For the HP turbine:

$$\begin{cases} c_1\left(\dot{E}x_1 - \dot{E}x_2 - \dot{E}x_3\right) + \dot{Z}_{HP} = c_{W,HP}\dot{W}_{HP} \\ \dot{E}x_1 - \dot{E}x_2 - \dot{E}x_3 = \dot{W}_{HP} + \dot{E}x_{D,HP} \end{cases} \rightarrow \quad c_1\left(\dot{W}_{HP} + \dot{E}x_{D,HP}\right) + \dot{Z}_{HP} = c_{W,HP}\dot{W}_{HP}$$

$$c_{W,HP} = c_1 + c_1\frac{\dot{E}x_{D,HP}}{\dot{W}_{HP}} + \frac{\dot{Z}_{HP}}{\dot{W}_{HP}} = \left(37.80 + 4.88 + 24.06\right)\frac{€}{MWh} = 66.75\frac{€}{MWh}$$

$$\dot{C}_{D,HP} = c_1\dot{E}x_{D,HP} = 42.6\,€/h$$

For the LP turbine:

$$\begin{cases} c_3\dot{E}x_3 + \dot{Z}_{LP} = c_{W,LP}\dot{W}_{LP} + c_4\dot{E}x_4 \\ \dot{E}x_3 = \dot{E}x_4 + \dot{W}_{LP} + \dot{E}x_{D,LP} = 0 \end{cases} \rightarrow \quad c_3\left(\dot{E}x_4 + \dot{W}_{LP} + \dot{E}x_{D,LP}\right) + \dot{Z}_{LP} = c\left(\dot{W}_{LP} + \dot{E}x_4\right)$$

$$c = c_3 + c_3\left(\frac{\dot{E}x_{D,LP}}{\dot{W}_{LP} + \dot{E}x_4}\right) + \frac{\dot{Z}_{LP}}{\dot{W}_{LP} + \dot{E}x_4} = \left(37.80 + 1.52 + 17.85\right)\frac{€}{MWh} = 57.16\frac{€}{MWh}$$

$$\dot{C}_{D,LP} = c_3\dot{E}x_{D,LP} = 29.7\,€/h$$

Exercise 3. (5pt)

The table shows the electricity balance for Germany in 2010. In the following years, 3 GW$_{el}$ of nuclear plants (NU, with load factor f of 0.9) will be disposed. The govern substitutes the electric energy produced by nuclear plants with 50% of energy produced by Combined Cycle power plants (CCPP, with η_I = 0.55, LHV$_{NG}$ = 50 MJ/kg, load factor f of 0.75), 20% of energy produced with solar photovoltaic plants (PV) and 30% by increasing the electricity imports (IMP).

Germany (2010)	Electric energy [GWh]
Coal	273456
Oil	8741
Natural Gas	90352
Biofuels and Waste	40658
Nuclear	140556
Hydro	27353
Solar PV	11729
Other renewables	40138
Imports	42962
Exports	57917
Energy industry own use	61630
Losses	23974

Considering a total population of 81.76 Millions, and with reference to the data in the table, it is required to:

a. (1pt) Evaluate the Total Final Consumption (TFC, in GWh) and the Final Consumption per capita (fc, in MWh/p);
b. (1pt) Evaluate the additional amount of electric energy produced by CCPP, PV and IMP after the disposal of the nuclear power plant;
c. (1pt) Considering a nominal electric power of 200 MW$_{el}$ for each one of the CCPP, evaluate the number of CCPP that need to be installed, and the amount of natural gas consumed by them in one year (in kton);
d. (2pt) Evaluate the differences in TFC and in TPES caused by the disposal of nuclear plants and the introduction of additional CCPP, PV and IMP (use the IEA conventions).

Exercise 3. Solution

a. Evaluate the Total Final Consumption (TFC, in GWh) and the Final Consumption per capita (fc, in MWh/p);

The TFC results as the following algebraic sum:

$$TFC = production + imports - (exports + own\,use + losses) = 532424\,GWh_{el}$$

The final consumption per capita results as:

$$fc = \frac{TFC}{p} = 532424\,GWh \cdot 10^3 \frac{MWh}{GWh} \cdot \frac{1}{81.76M \cdot 10^6} = 6.51 \frac{MWh}{p}$$

b. Evaluate the additional amount of electric energy produced by CCPP, PV and IMP after the disposal of the nuclear power plant;

The total electricity generated by 3 GW of nuclear plants is calculated as follows, together with the electricity produced by CCPP, PV and IMP:

$$EE_{NU} = P_{el,NU} \cdot 8760 \frac{h}{y} \cdot f_{NU} = 23652\,GWh \quad \rightarrow \quad \begin{cases} EE_{CCPP} = 0.5 \cdot E_{NU} = 11826\,GWh \\ EE_{PV} = 0.2 \cdot E_{NU} = 4730\,GWh \\ EE_{IMP} = 0.3 \cdot E_{NU} = 7096\,GWh \end{cases}$$

c. Considering a nominal electric power of 200 MWel for each one of the CCPP, evaluate the number of CCPP that need to be installed, and the amount of natural gas consumed by them in one year (in kton);

The total installed power required, and the number of power plants, are equal to:

$$P_{el,NGCC} = \frac{EE_{CCPP}}{8760 \cdot f_{NGCC}} \cdot 10^3 = 1800 MW_{el} \quad \rightarrow \quad n_{NGCC} = \frac{P_{el,NGCC}}{p_{NGCC}} = 9\, plants$$

$$m_{NG} = \frac{P_{el,NGCC} \cdot 8760 \cdot f_{NGCC}}{\eta_{NGCC} \cdot LHV_{NG}} = \frac{1800 MW_{el} \cdot 8760\, h/y \cdot 0.75}{0.55 \cdot 50\, MJ/kg} \cdot 3600 \frac{s}{h} \cdot 10^{-6} \frac{kg}{kton} = 1548.1 kton$$

d. Evaluate the differences in TFC and in TPES caused by the disposal of nuclear plants and the introduction of additional CCPP, PV and IMP (use the IEA conventions).

The difference in TFC is equal to zero, since no differences in final electricity consumption result by the substitution of the nuclear power.

$$\Delta TFC = 0$$

The difference in TPES can be evaluated according to the IEA conventions as:

$$\Delta TPES = \Delta NG + \Delta PV + \Delta IMP + \Delta NU = -3298 ktoe$$

$$\Delta NG = \frac{EE_{CCPP}}{\eta_{NGCC}} = \frac{11826 GWh}{0.55} \cdot 0.086 \frac{toe}{TWh} \cdot 10^3 \frac{TWh}{GWh} \cdot 10^{-3} \frac{ktoe}{toe} = 1849 ktoe$$

$$\Delta PV = EE_{PV} = 4730 GWh \cdot 0.086 \frac{toe}{TWh} \cdot 10^3 \frac{TWh}{GWh} \cdot 10^{-3} \frac{ktoe}{toe} = 407 ktoe$$

$$\Delta IMP = EE_{IMP} = 7096 GWh \cdot 0.086 \frac{toe}{TWh} \cdot 10^3 \frac{TWh}{GWh} \cdot 10^{-3} \frac{ktoe}{toe} = 610 ktoe$$

$$\Delta NU = \frac{EE_{NU}}{0.33} = \frac{-23652 GWh}{0.33} \cdot 0.086 \frac{toe}{TWh} \cdot 10^3 \frac{TWh}{GWh} \cdot 10^{-3} \frac{ktoe}{toe} = -6164 ktoe$$

Notice that energy produced by CCPP is converted into primary energy using efficiencies given by the text, while NU rely on IEA convention.

POLITECNICO DI MILANO

Milano – 8 October 2015

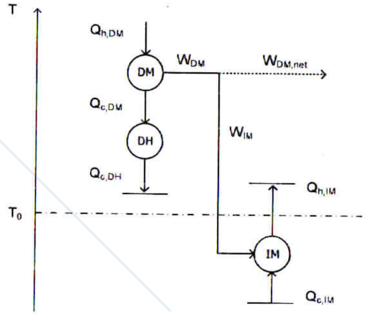
Final Test of 23-09-2015 – Problems

Exercise 1. (9pt)

Let's consider a system composed by a direct cycling machine (DM, with First Law efficiency of 80%) and an indirect cycling machine (IM, with a COP of 2.95). The DM produces electricity for national grid and heat for a district heating network (DH) through consumption of biomass ($ex_{ch} \sim LHV$). A portion of the whole electricity produced by the DM feed the IM, which is used to cool down a skating ring ($T_{C,IM} = -10°C$) and to heat up a swimming pool ($T_{H,IM} = 25°C$). The heat produced by the IM to heat up the pool is 450 MWh/y.

Additional data:

- The DM produces heat for the district heating network at a temperature of $T_{c,DM} = 355.5K$. The district heating network releases a net heat of 35 GWh/y at a temperature of $T_{DH} = 20°C$, with a 10% of heat losses;
- The ratio between total electricity production by the DM and the portion of that electricity absorbed by the IM is equal to 70;
- The COP of the IM is evaluated considering the heat produced for the swimming pool as the useful effect, while the First Law efficiency of the DM is evaluated considering both the electricity and the heat produced as useful effects;



Introducing proper hypotheses, and considering a reference temperature T₀ of 2°C, it is required to:

a. (1pt) Draw the scheme, define boundaries of the whole system and highlight the directions and the type (heat, work, bulk flow) of all the exergy flows. Then calculate the net electric energy delivered to the national grid and the heat absorbed by the DM through biomass consumption;
b. (2pt) Apply the exergy balance to the DM only, evaluate its exergy destructions and its functional exergy efficiency;
c. (2pt) Apply the exergy balance to the district heating network and evaluate its exergy destructions;
d. (2pt) Apply the exergy balance to the IM only, evaluate its exergy destructions and its functional exergy efficiency;
e. (2pt) Apply the exergy balance to the whole system, evaluate its exergy destructions and its functional exergy efficiency;

Exercise 2. Solution

a. **<u>Draw the scheme, define boundaries of the whole system and highlight the directions and the type (heat, work, bulk flow) of all the exergy flows. Then calculate the net electric energy delivered to the national grid and the heat absorbed by the DM through biomass consumption;</u>**

The heat produced by the DM and delivered to the district heating network is equal to the net heat released by the network, increased by its heat losses:

$$Q_{c,DM} = \frac{Q_{c,DH}}{1-\lambda_{loss}} = 38889 \frac{MWh}{y}$$ Gross heat produced by the DM

Notice that from now on subscript "c" and "h" stand for cold and hot.
The electrical work required by the IM can be derived considering the definition of COP:

$$COP_{PdC} = \frac{Q_{h,IM}}{W_{IM}} \quad \rightarrow \quad W_{IM} = \frac{Q_{h,IM}}{COP_{PdC}} = 153 \frac{MWh}{y}$$

The total electricity production of the DM, and the net electricity delivered by DM to national grid, are equal to:

$$\frac{W_{DM}}{W_{IM}} = 70 \quad \rightarrow \quad W_{DM} = 70 \cdot W_{IM} = 10678 \frac{MWh}{y} \quad \rightarrow \quad W_{DM,net} = W_{DM} - W_{IM} = 10525 \frac{MWh}{y}$$

The total heat absorbed by the DM through biomass is equal to:

$$\eta_{I,DM} = \frac{W_{DM} + Q_{c,DM}}{Q_{h,DM}} \quad \rightarrow \quad Q_{h,DM} = \frac{W_{DM} + Q_{c,DM}}{\eta_{I,DM}} = 61959 \frac{MWh}{y}$$

b. **<u>Apply the exergy balance to the DM only, evaluate its exergy destructions and its functional exergy efficiency;</u>**

Assuming that:

- DM operates at steady state;
- Exergy of biomass equals its lower heating value and can be assimilated to a heat flow interaction;
- Heat and work interactions occurs within the system;

The exergy balance can be simplified as follows:

$$\sum_i Ex_Q^{\leftarrow} + \sum_j Ex_W^{\leftarrow} - Ex_D = 0$$

$$Ex^{\leftarrow}_{Qh,DM} + Ex^{\leftarrow}_{Qc,DM} + Ex^{\leftarrow}_{W,DM} - Ex_{D,DM} = 0 \quad \rightarrow \begin{cases} Ex^{\leftarrow}_{Qh,DM} = Q_{h,DM} = 61959\, MWh/y \\ Ex^{\leftarrow}_{Qc,DM} = Q_{c,DM}\left(1 - T_0/T_{ml,DH}\right) = -8790\, MWh/y \\ Ex^{\leftarrow}_{W,DM} = W_{DM} = -10678\, MWh/y \end{cases}$$

$$Ex_{D,DM} = Ex^{\leftarrow}_{Qh,DM} + Ex^{\leftarrow}_{Qc,DM} + Ex^{\leftarrow}_{W,DM} = 42491\, MWh/y$$

$$\eta_{ex,F,DM} = \frac{Ex_{Qc,DM} + Ex_{W,DM}}{Ex_{Qh,DM}} = 0.31$$

c. **<u>Apply the exergy balance to the district heating network and evaluate its exergy destructions;</u>**

For the application of the exergy balance to the DH, the heat losses cross the boundaries at temperature equal to T0. The exergy balance for the DH can be thus written as follows:

$$Ex^{\leftarrow}_{Qc,DM} + Ex^{\leftarrow}_{Qc,DH} - Ex_{D,DH} = 0 \quad \rightarrow \quad \begin{cases} Ex^{\leftarrow}_{Qc,DM} = Q_{c,DM}\left(1 - T_0/T_{c,DM}\right) = 8790\, MWh/y \\ Ex^{\leftarrow}_{Qc,DH} = Q_{c,DH}\left(1 - T_0/T_{DH}\right) = -2149\, MWh/y \end{cases}$$

$$Ex_{D,DH} = Ex^{\leftarrow}_{Qc,DM} + Ex^{\leftarrow}_{Qc,DH} = 6641\, MWh/y$$

d. **Apply the exergy balance to the IM only, evaluate its exergy destructions and its functional exergy efficiency;**

The heat absorbed by the IM results from the application of the energy balance:

$$Q^{\leftarrow}_{c,IM} + Q^{\leftarrow}_{h,IM} + W^{\leftarrow}_{IM} = 0 \quad \rightarrow \quad Q^{\leftarrow}_{c,IM} = 297\,MWh/y$$

Under the same assumptions of the DM, the exergy balance can be written as follows:

$$Ex^{\leftarrow}_{Qh,IM} + Ex^{\leftarrow}_{Qc,IM} + Ex^{\leftarrow}_{W,IM} - Ex_{D,IM} = 0 \quad \rightarrow \quad \begin{cases} Ex^{\leftarrow}_{Qc,IM} = Q_{c,IM}\left(1 - T_0/T_{c,IM}\right) = -13.6\,MWh/y \\ Ex^{\leftarrow}_{Qh,IM} = Q_{h,IM}\left(1 - T_0/T_{h,IM}\right) = -34.7\,MWh/y \\ Ex^{\leftarrow}_{W,IM} = W^{\leftarrow}_{IM} = 153\,MWh/y \end{cases}$$

$$Ex_{D,IM} = Ex^{\leftarrow}_{Qh,IM} + Ex^{\leftarrow}_{Qc,IM} + Ex^{\leftarrow}_{W,IM} = 104\,MWh/y$$

$$\eta_{ex,F,IM} = \frac{Ex_{Qc,IM} + Ex_{Qh,IM}}{Ex_{W,IM}} = 0.32$$

e. **Apply the exergy balance to the whole system, evaluate its exergy destructions and its functional exergy efficiency;**

Considering system boundaries that encompasses the two cyclic machines and the district heating network, the exergy balance can be written as follows:

$$\sum_i Ex^{\leftarrow}_Q + \sum_j Ex^{\leftarrow}_W - Ex_D = 0$$

$$Ex^{\leftarrow}_{Qh,DM} + Ex^{\leftarrow}_{Qc,DH} + Ex^{\leftarrow}_{Qc,IM} + Ex^{\leftarrow}_{Qh,IM} - Ex_{D,tot} = 0 \quad \rightarrow \quad Ex_{D,tot} = 49236\,MWh/y$$

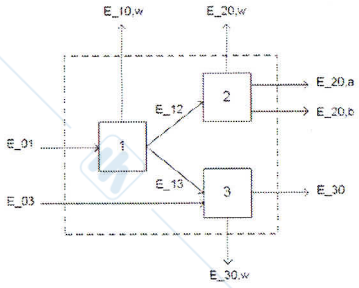
$$\eta_{ex,F,IM} = \frac{Ex^{\leftarrow}_{Qc,DH} + Ex^{\leftarrow}_{Qc,IM} + Ex^{\leftarrow}_{Qh,IM}}{Ex^{\leftarrow}_{Qh,DM}} = 0.21$$

Exercise 2. (7pt)

The system in the figure is composed by three processes connected to each other and with the environment through flows of energy and materials with a defined exergy content (see the table).
Process 1 receive a flow of exergy from the environment, producing chemical reactants for processes 2 and 3 and releasing residues in the environment.
Process 2 produces two useful products and one flow of residues. Process 3 receive a flow chemical reactants from process 2 and a flow of renewable resources from the environment, producing chemicals and residues.

Fluxes	symbol	Ex - MW
Materials / Energy	E_01	120,0
Renewable energy	E_03	25,0
Chemicals	E_12	55,0
Chemicals	E_13	10,0
Chemicals	E_20a	12,0
Chemicals	E_20b	18,0
Heat	E_30	8,0
Losses	E_10w	15,0
Losses	E_20w	8,0
Losses	E_30w	7,0

Processes	no	Z - €/h
process 1	1	1500
process 2	2	2340
process 3	3	750



Notes:

- Residues E_10w and E20w are released in the environment without any additional cost;
- Residue E_30w is release with an additional cost of 10 €/MWh;
- Specific cost of flow E_01 is equal to 20 €/MWh;
- Exergy flow E_03 is renewable and free;
- Investment costs of components are listed in the table;
- Nomenclature: flow E_01 means from the environment (0) to the process 1;
- The processes produce a constant amount of products;

Considering the data in the table, it is required to:

a. (2pt) Write the exergy balances for each component according to the Inlet-Outlet criterion, evaluating exergy destructions and rational exergy efficiencies for each component and for the whole system (use a table to collect the results);
b. (2pt) Introducing and motivating proper auxiliary relations, write the thermoeconomic system for each process according to the Inlet-Outlet criterion. For each component, derive specific average costs of Inlet and Outlet streams;
c. (3pt) Introducing proper hypotheses, evaluate the economic cost of exergy destructions in each component, considering the average specific cost of inlet or outlet streams as the cost of exergy destructions. Then apply the procedure of Design Evaluation and briefly discuss the obtained results.

Exercise 2. Solution

a. **Write the exergy balances for each component according to the Inlet-Outlet criterion, evaluating exergy destructions and rational exergy efficiencies for each component and for the whole system (use a table to collect the results);**

$$\dot{E}_I = \dot{E}_O + \dot{E}_D \quad \rightarrow \quad \eta_{ex,t} = \dot{E}_O / \dot{E}_I$$

Process	no	I - MW	O - MW	D - MW	eta,ex,R
process 1	1	120,0	80,0	40,0	0,67
process 2	2	55,0	38,0	17,0	0,69
process 3	3	35,0	15,0	20,0	0,43

Whole system	tot	145,0	68,0	77,0	0,47

b. Introducing and motivating proper auxiliary relations, write the thermoeconomic system for each process according to the Inlet-Outlet criterion. For each component, derive specific average costs of Inlet and Outlet streams;

For each component, thermoeconomic system can be defined as follows:

$$1:\begin{cases}\dot{E}_{01}=\dot{E}_{10W}+\dot{E}_{12}+\dot{E}_{13}+\dot{E}_{D,1}\\ \dot{C}_{01}+\dot{Z}_{1}=\dot{C}_{10W}+\dot{C}_{12}+\dot{C}_{13}\\ \dot{C}=c\cdot\dot{E}\end{cases}\quad\rightarrow\quad c_{01}\dot{E}_{01}+\dot{Z}_{1}=c_{10W}\dot{E}_{10W}+c_{12}\dot{E}_{12}+c_{13}\dot{E}_{13}$$

$$2:\begin{cases}\dot{E}_{12}=\dot{E}_{20W}+\dot{E}_{20a}+\dot{E}_{20b}+\dot{E}_{D,2}\\ \dot{C}_{12}+\dot{Z}_{2}=\dot{C}_{20W}+\dot{C}_{20a}+\dot{C}_{20b}\\ \dot{C}=c\cdot\dot{E}\end{cases}\quad\rightarrow\quad c_{12}\dot{E}_{12}+\dot{Z}_{2}=c_{20W}\dot{E}_{20W}+c_{20a}\dot{E}_{20a}+c_{20b}\dot{E}_{20b}$$

$$3:\begin{cases}\dot{E}_{13}+\dot{E}_{03}=\dot{E}_{30W}+\dot{E}_{30}+\dot{E}_{D,3}\\ \dot{C}_{13}+\dot{C}_{03}+\dot{Z}_{3}=\dot{C}_{30W}+\dot{C}_{30}\\ \dot{C}=c\cdot\dot{E}\end{cases}\quad\rightarrow\quad c_{13}\dot{E}_{13}+c_{03}\dot{E}_{03}+\dot{Z}_{3}=c_{30W}\dot{E}_{30W}+c_{30}\dot{E}_{30}$$

There are 10 flows and 3 components, thus 7 auxiliary relations are required.

$c_{01}=20\,€/MWh$	specific cost of flow E_01
$c_{10W}=0\,€/MWh$	cost of loss 1
$c_{20W}=0\,€/MWh$	cost of loss 2
$c_{30W}=-10\,€/MWh$	cost of loss 3 is negative
$c_{03}=0\,€/MWh$	cost of renewable stream is zero
$c_{12}=c_{13}=c_{1}$	specific costs of products out of 1 are equal
$c_{20a}=c_{20b}=c_{2}$	specific costs of products out of 2 are equal

With these equations, the system given by the three cost balances results as:

$$\begin{cases}c_{01}\dot{E}_{01}+\dot{Z}_{1}=c_{1}\left(\dot{E}_{12}+\dot{E}_{13}\right)\\ c_{1}\dot{E}_{12}+\dot{Z}_{2}=c_{2}\left(\dot{E}_{20a}+\dot{E}_{20b}\right)\\ c_{1}\dot{E}_{13}+\dot{Z}_{3}=c_{30}\dot{E}_{30}\end{cases}\quad\rightarrow\quad\begin{matrix}c_{1}=60\,€/MW\\ c_{2}=188\,€/MW\\ c_{3}=177.5\,€/MW\end{matrix}$$

Specific and total costs of streams are listed in the table.

	Ex MW	C €/MWh	C €/h
E_01	120,0	20	2400
E_03	25,0	0	0
E_12	55,0	60,0	3300
E_13	10,0	60,0	600
E_20a	12,0	188,0	2256
E_20b	18,0	188,0	3384
E_30	8,0	177,5	1420
E_10w	15,0	0	0
E_20w	8,0	0	0
E_30w	7,0	-10	-70

Specific costs of inlet and outlet streams are evaluate as the average costs:

$$c_{I/O,i}=\frac{\sum_{j}c_{j}\dot{E}_{j}}{\sum_{j}\dot{E}_{j}}$$

c_I €/MWh	c_O €/MWh
20,0	48,8
60,0	148,4
17,1	99,3

c. Introducing proper hypotheses, evaluate the economic cost of exergy destructions in each component, considering the average specific cost of inlet or outlet streams as the cost of exergy destructions. Then apply the procedure of Design Evaluation and briefly discuss the obtained results.

The plant works with constant product, so the demand driven model is selected and the irreversibilities within the components cause extra consumption of inlet streams. For any ith component:

$$\dot{C}_{D,i}=c_{I,i}\cdot\dot{E}_{D,i}$$

Design evaluation procedure requires the calculation of the following indicators:

$\dot{C}_{D,i}=c_{I,i}\cdot\dot{E}_{D,i}$	Cost of exergy destructions;
$\dot{Z}_{i}+\dot{C}_{D,i}$	Cost of exergy destructions plus investment costs;
$r_{i}=\dfrac{c_{p}-c_{f}}{c_{f}}$	Relative cost difference;
$f_{i}=\dfrac{\dot{Z}_{i}}{\dot{Z}_{i}+\dot{C}_{D,i}}$	Exergoeconomic Factor;

Results:

Process	C_D €/h	Z+C_D,L €/h	r -	f -
process 1	800	2300	1,44	0,652
process 2	1020	3360	1,47	0,696
process 3	343	1093	4,79	0,686

Process 1 is the most inefficient from a thermodynamic viewpoint, followed by process 3. However, the economic cost of irreversibilities of process 2 is larger than processes 1 and 3: this process should be improved to reduce the global cost of products.

Exercise 3. (5pt)

For the next year, the Italian electricity production must be increased by 30 TWh/y through the installation of nuclear (NU) and off-shore wind (W) power plants. Nuclear power plants will produce 80% of the required electric energy.
Considering a reference natural gas power plant (NG) fueled with pure methane (LHV = 50 MJ/kg) with an energy efficiency of 0.55, a load factor of 0.7, it is required to:

a. (1pt) Evaluate the installed power for NU and for W (in GW);
b. (1pt) Evaluate the avoided CO2 emissions for NU and for W (in kton/y);
c. (1pt) Evaluate the primary energy required by NU and by W (in ktoe/y), according to both the BP and IEA conventions;
d. (2pt) Evaluate the cost of CO2 emissions (in €/ton) required to make the cost of electricity production by NU equal to the cost of electricity production by NG. Repeat the calculation for W;

Additional data:

- Nuclear and wind power plants respectively have a load factor of 0.9 and 0.15;
- Economic costs of electricity: 61 €/MWh for NG, 73 €/MWh for NU and 117 €/MWh for W;

Exercise 2. Solution

a. **Evaluate the installed power for NU and for W (in GW);**

The electric energy production for the two plants are respectively equal to:

$$E_{el,NU} = E_{el,TOT} \cdot 0.8 = 24000\,GWh/y$$
$$E_{el,W} = E_{el,TOT} \cdot 0.2 = 6000\,GWh/y$$

The installed power for the two plants are respectively equal to:

$$P_{el,NU} = \frac{E_{el,NU}}{8760\,h/y \cdot f_{NU}} = 3GW$$
$$P_{el,W} = \frac{E_{el,W}}{8760\,h/y \cdot f_{W}} = 4.6GW$$

b. **Evaluate the avoided CO2 emissions for NU and for W (in kton/y);**

To account for the avoided emissions, the reference NG plant is considered:

$$m_{CO2,i} = E_{el,i}\frac{GJh}{s\cdot y}\cdot\frac{1}{\eta_{NG}}\cdot\frac{1}{LHV_{ng}}\frac{kg_{ng}}{MJ}\cdot 3600\frac{s}{h}\cdot 10^3\frac{MJ}{GJ}\cdot\frac{44}{16}\frac{kg_{CO2}}{kg_{ng}}\cdot\frac{1}{10^6}\frac{kton_{CO2}}{kg_{CO2}} \rightarrow \begin{cases} m_{CO2,NU} = 8640\,kton_{CO2}/y \\ m_{CO2,W} = 2160\,kton_{CO2}/y \end{cases}$$

c. **Evaluate the primary energy required by NU and by W (in ktoe/y), according to both the BP and IEA conventions;**

According to the BP conventions, primary energy is evaluated as the avoided consumption of fossil fuels caused by a plant with efficiency of 0.38. IEA uses different conventions: 0.33 efficiency for nuclear, and no conversion for wind.

$$BP:\begin{cases} E_{p,NU} = \frac{E_{el,NU}}{0.38}\cdot 3600\frac{s}{h}\cdot\frac{1}{41860}\frac{ktoe}{GJ} = 5432\frac{ktoe}{y} \\ E_{p,W} = \frac{E_{el,W}}{0.38}\cdot 3600\frac{s}{h}\cdot\frac{1}{41860}\frac{ktoe}{GJ} = 1358\frac{ktoe}{y} \end{cases}$$

$$IEA:\begin{cases} E_{p,NU} = \frac{E_{el,NU}}{0.33}\cdot 3600\frac{s}{h}\cdot\frac{1}{41860}\frac{ktoe}{GJ} = 6255\frac{ktoe}{y} \\ E_{p,W} = E_{el,W}\cdot 3600\frac{s}{h}\cdot\frac{1}{41860}\frac{ktoe}{GJ} = 516\frac{ktoe}{y} \end{cases}$$

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d. **Evaluate the cost of CO2 emissions (in €/ton) required to make the cost of electricity production by NU equal to the cost of electricity production by NG. Repeat the calculation for W;**

The following economic balance must be written for both the technologies:

$$c_{el,i}E_{el,i} = c_{el,NG}E_{el,i} + c_{CO2,i}m_{CO2,i} \rightarrow \quad c_{CO2,i} = \frac{E_{el,i}\left(c_{el,i} - c_{el,NG}\right)}{m_{CO2,i}} \quad \begin{cases} c_{CO2,NU} = 33.3\,€/ton_{CO2} \\ c_{CO2,W} = 155.6\,€/ton_{CO2} \end{cases}$$

Politecnico di Milano
Dipartimento di Energia

POLITECNICO DI MILANO

Milano –7 September 2015

Final Test of 08-09-2015 – Problems

Exercise 1. (8pt)

An industrial system produces cyclohexane (C_6H_{12}) through the reaction of benzene (C_6H_6) hydrogenation:

$$C_6H_6 + 3H_2 \rightarrow C_6H_{12}$$

Liquid benzene flows through a pump (P), where it is taken from 38°C and 1 bar to 39.9 and 22.5 bar. After compression, liquid benzene enters the hydrogenation reactor (R). At the same time, gaseous hydrogen (H_2) enters the reactor at 49°C and 22.5 bar. The reactor produces liquid C_6H_{12} at 200°C and 21 bar. After the reactor, the flow of C_6H_{12} pass through a heat exchanger and a lamination valve, where it is cooled and flashed till 49°C and 1 bar.

			C6H6	H2	C6H12
molar weight	MW	kg/kmol	78	2	84
isobarich heat capacity	cp	kJ/kg-K	1.74	14.31	2.31
density	rho	kg/m3	874	0,081	688
enthalpy of formation	h_f (ideal gas)	kJ/kmol		0	
	h_f (ideal liquid)	kJ/kmol	49040		-156400
chemical exergy	ex_ch (ideal gas)	kJ/kmol		238490	
	ex_ch (ideal liquid)	kJ/kmol	3305350		3922990

With reference to the data provided in the table, it is required to:

a. (1pt) Draw the plant scheme, highlighting bulk flows and energy flows exchanged by the components;
b. (1pt) Introduce proper hypotheses and evaluate mechanical power absorbed by the pump per kg of C_6H_6;
c. (2pt) Introduce proper hypotheses and apply the energy balance to the reactor, and determine whether the chemical reaction is endothermic or exothermic, evaluating the amount of heat produced/absorbed per both kmol and kg of C_6H_{12};
d. (2pt) Apply the exergy balance to the chemical reactor only, evaluating the exergy destructions and the functional exergy efficiency. Notice that:
    - For endothermic reaction: the heat is absorbed by a heat source at 550 K;
    - For exothermic reaction: the heat is used to produce a flow of saturated steam starting from liquid water at 38°C and environmental pressure;
e. (2pt) Apply the exergy balance to the heat exchanger and the lamination valve, evaluating the exergy destructions per kg of C_6H_{12} produced.

Exercise 1. Solution

a. <u>Drawn the plant scheme, highlighting bulk flows and energy flows exchanged by the components;</u>

b. <u>(1pt) Introducing proper hypotheses, evaluate mechanical power absorbed by the pump per kg of C6H6;</u>

The pump is assumed as:
- Operating in steady state;
- Perfectly insulated;
- With rigid boundaries;
- With no large differences in velocity and height of the fluid that cross it;

- Water is assumed as perfect liquid;

The energy balance applied to the pump only then results as:

$$\dot{W}_P^{\leftarrow} + \dot{m}_{C6H6}\left(h_{in} - h_{out}\right)_{C6H6} = 0$$
$$\left(h_{in} - h_{out}\right)_{C6H6} = \left[c\left(T_{in} - T_{out}\right) + v\left(p_{in} - p_{out}\right)\right]_{C6H6}$$

$$\frac{\dot{W}_P^{\leftarrow}}{\dot{m}_{C6H6}} = w_P^{\leftarrow} = \left[c\left(T_{out} - T_{in}\right) + v\left(p_{out} - p_{in}\right)\right]_{C6H6} = 4.72\,kJ/kg_{C6H6}$$

Notice that both the effects of temperature and pressure changes have been taken into account.

c. **(2pt) Introducing proper hypotheses, apply the energy balance to the reactor, and determine whether the chemical reaction is endothermic or exothermic, evaluating the amount of heat produced/absorbed per both kmol and kg of C6H12;**

The energy balance applied on the reactor – expressed in molar units – results as:

$$\dot{n}_{C6H6}\bar{h}_{C6H6} + \dot{n}_{H2}\bar{h}_{H2} - \dot{n}_{C6H12}\bar{h}_{C6H12} + \dot{Q}_R^{\leftarrow} = 0$$

Dividing by molar flow rate of C6H6, the molar heat flow can be evaluated as follows. Notice that molar flow rates of C6H6 and C6H12 are equal, and molar flow rate of H2 is three times that C6H6.

$$q_R^{\leftarrow} = \bar{h}_{C6H12} - \left(\bar{h}_{C6H6} + 3\cdot\bar{h}_{H2}\right)$$

The molar enthalpies can be evaluated as follows:

$$\bar{h}_i = \bar{h}_{f,i} + \Delta\bar{h}\left(T,p\right) \qquad \rightarrow \begin{cases} \Delta\bar{h}_{C6H6} = \left[c\left(T - T_0\right) + v\left(p - p_0\right)\right]MW_{C6H6} = 2130\,kJ/kmol_{C6H6} \\ \Delta\bar{h}_{H2} = c_p\left(T - T_0\right)MW_{H2} = 692\,kJ/kmol_{H2} \\ \Delta\bar{h}_{C6H12} = \left[c\left(T - T_0\right) + v\left(p - p_0\right)\right]MW_{C6H12} = 34224\,kJ/kmol_{C6H12} \end{cases}$$

Therefore:

$$\begin{cases} \bar{h}_{C6H6} = 51170\,kJ/kmol_{C6H6} \\ \bar{h}_{H2} = 692\,kJ/kmol_{H2} \\ \bar{h}_{C6H12} = -122176\,kJ/kmol_{C6H12} \end{cases} \qquad \rightarrow \qquad q_R^{\leftarrow} = -175423\,kJ/kmol_{C6H12}\left(-2084\,kJ/kg_{C6H12}\right)$$

The heat is released by the reaction.

d. **(2pt) Apply the exergy balance to the chemical reactor only, evaluating the exergy destructions and the functional exergy efficiency;**

Since the reaction is exothermic, a flow of saturated water is produced by the reactor. The amount of water produced is equal to:

$$\begin{cases} q_R^{\leftarrow} = \dfrac{\dot{m}_w}{\dot{n}_{C6H12}}\left(h_{w,in} - h_{w,out}\right) \\ h_{w,in} - h_{w,out} = c_{p,w}\left(T_{w,in} - T_{w,out}\right) \end{cases} \qquad \rightarrow \qquad \frac{\dot{m}_w}{\dot{n}_{C6H12}} = \frac{q_R^{\leftarrow}}{c_{p,w}\left(T_{w,in} - T_{w,out}\right)} = 675.9\frac{kg_w}{kmol_{C6H12}}\left(37.6\frac{kmol_w}{kmol_{C6H12}}\right)$$

The exergy balance applied to the reactor, crossed by a flow of water, is simplified as follows. Values are expressed in molar terms. Each bulk flow exergy is assumed as the sum of physical and chemical components only.

$$\overline{ex}_{C6H6}+3\frac{kmol_{H2}}{kmol_{C6H12}}\cdot\overline{ex}_{H2}+37.6\frac{kmol_{w}}{kmol_{C6H12}}\left(\overline{ex}_{w,in}-\overline{ex}_{w,out}\right)-\overline{ex}_{C6H12}-\overline{ex}_{D,R}=0$$

$$\overline{ex}_{i}=\overline{ex}_{i,ch}+\overline{ex}_{i,ph}\qquad\rightarrow\qquad\begin{cases}C6H6,C6H12:\overline{ex}_{i,ph}=c\cdot\Delta T+v\cdot\Delta p-T_0R\ln\left(T/T_0\right)\\ H2:\overline{ex}_{i,ph}=c_p\cdot\Delta T-T_0\left[c_p\ln\left(T/T_0\right)-R\ln\left(p/p_0\right)\right]\\ w:\overline{ex}_{i,ph}=c\cdot\Delta T-T_0R\ln\left(T/T_0\right)\end{cases}$$

	ex_ch kJ/kmol	ex_ph kJ/kmol	ex kJ/kmol
C6H6	3305350	237,3	3305587
H2	238490	7744,3	246234
C6H12	3922990	7488,9	3930479
w (in-out)	-	-589,6	-590

$$\overline{ex}_{D,R}=91672\,kJ/kmol_{C6H12}$$

$$\eta_{ex,F,R}=\frac{37.6\frac{kmol_{w}}{kmol_{C6H12}}\left(\overline{ex}_{w,out}-\overline{ex}_{w,in}\right)+\overline{ex}_{C6H12}}{\overline{ex}_{C6H6}+3\frac{kmol_{H2}}{kmol_{C6H12}}\cdot\overline{ex}_{H2}}=0.977$$

e. <u>(2pt) Apply the exergy balance to the heat exchanger and the lamination valve, evaluating the exergy destructions per kg of C6H12 produced.</u>

The exergy balance for the last stages of the plant can be written as follows, assuming that the heat is released in the environment at reference temperature.

$$\overline{ex}_{C6H12,in}-\overline{ex}_{C6H12,out}-\overline{ex}_{D,\text{last}}=0\qquad\rightarrow\qquad\overline{ex}_{D,\text{last}}=\Delta\overline{ex}_{C6H12}$$

The change in exergy of C6H12 can be evaluated as follows:

$$\overline{ex}_{D,\text{last}}=\Delta\overline{ex}_{C6H12}=\left(\Delta\overline{h}-T_0\Delta\overline{s}\right)_{C6H12}\qquad\rightarrow\qquad\overline{ex}_{D,\text{last}}=c\cdot\left(T-T_0\right)+v\cdot\left(p-p_0\right)-T_0R\ln\left(T/T_0\right)=7311\,kJ/kmol_{C6H12}$$

Exercise 2. (8pt)

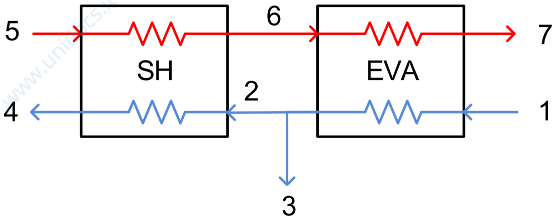
An industrial process reject two material streams as losses:

- A mass flow rate of flue gases (fg) equal to 34.7 kg/s at 0,2 MPa and 780 K, with cp = 1.1 kJ/kg-K and R*=0.287 kJ/kg-K;
- A mass flow rate of saturated water (w) equal to 4.1 kg/s at 4 MPa and 524 K;

In order to recover heat from flue gases and produce two streams of steam at different conditions, a heat recovery steam generator (in the figure) is then added:

- Stream 3 is equal to 0.21 kg/s and it is assumed as a by-product of the evaporator;
- Stream 4 is equal to 3.90 kg/s;

The system is formed by two heat exchangers: the evaporator (EVA, with Z_{eva} = 750 €/h) and the super-heater (SH, with Z_{sh} = 430 €/h).



	p	T	q	h-h0	s-s0	ex
	MPa	K	-	kJ/kg	kJ/kg-K	kJ/kg
0	0,1	298,15	-	-	-	-
1	4,0	524	0	983	2,4296	
2	4,0	524	1	2696	5,7024	
3	4,0	524	1	2696	5,7024	
4	4,0	773	-	3341	6,7245	
5	0,2	780	-			
6	0,2	714	-			
7	0,2	530	-			

Considering the properties of the streams in the table, and introducing proper hypotheses, it is required to:

a. (2pt) Draw the thermodynamic transformations in the T-s and in the T-Q diagrams. Then calculate specific enthalpies and entropies of flue gases and specific and total exergy of water and flue gases streams. Notice that enthalpies and entropies of water streams are already evaluated with respect to the reference conditions;

b. (2pt) Apply the exergy balance to both the SH and the EVA, evaluating the exergy destructions and the rational exergy efficiency of the heat exchangers. Briefly discuss the obtained results;

c. (2pt) Define the thermoeconomic systems for both the SH and the EVA, according to the Inlet-Outlet paradigm. Then define the required auxiliary relations and justify them. Finally, evaluate the economic costs of system products;

d. (2pt) Evaluate the economic cost of exergy destructions, the relative cost difference and the exergoeconomic factor, assuming that both the heat exchangers pay the thermodynamic inefficiencies at the average cost of outlet streams. Briefly discuss the obtained results;

Exercise 2. Solution

a. **Draw the thermodynamic transformations in the T-s and in the T-Q diagrams. Then calculate specific enthalpies and entropies of flue gases and specific and total exergy of water and flue gases streams. Notice that enthalpies and entropies of water streams are already evaluated with respect to the reference conditions;**

Specific enthalpy and entropy of the flue gases stream can be evaluated according to the perfect gas model.

$$h_i = c_{p,i}\left(T_i - T_0\right)$$

$$s_i = c_{p,i}\ln\frac{T_i}{T_0} - R^*\ln\frac{p_i}{p_0}$$

Specific and total exergy values for both flue gases and water streams can be evaluated as follows. Notice that all the enthalpy and entropy values are already evaluated with respect to the reference conditions, so it is not required to subtract reference enthalpy and entropy values.

$$ex_i = h_i - T_0 s_i \qquad \rightarrow \qquad \dot{E}x_i = \dot{m}_i \cdot ex_i$$

The final results are here collected:

	p	T	q	h	s	ex	Ex
	MPa	K	-	kJ/kg	kJ/kg-K	kJ/kg	kW
ref.	0,1	298,15	-	-	-	-	-
1	4,0	524	0	983	2,4296	258	1059
2	4,0	524	1	2696	5,7024	996	3878
3	4,0	524	1	2696	5,7024	996	204
4	4,0	773	-	3341	6,7245	1336	5203
5	0,2	780	-	530	0,8620	273	9468
6	0,2	714	-	458	0,7650	230	7959
7	0,2	530	-	255	0,4369	125	4326

b. Apply the exergy balance to both the SH and the EVA, evaluating the exergy destructions and the rational exergy efficiency of the heat exchangers. Briefly discuss the obtained results;

The exergy balance can be simplified according to the following assumptions for both the heat exchangers:

- Steady state is assume throughout;
- No work and heat interactions occurs across the boundaries, only bulk flows;
- No relevant changes in elevations of velocity of the fluids occurs;
- Chemical exergy of the fluids is accounted;

$$\frac{dEx}{dt} = \sum \dot{E}x_W^{\leftarrow} + \sum \dot{E}x_Q^{\leftarrow} + \sum \dot{m}_i^{\leftarrow} ex_i - \dot{E}x_D = 0 \qquad \rightarrow \qquad \sum \dot{m}_i^{\leftarrow} ex_i - \dot{E}x_D = 0$$

Therefore:

$$EVA: \quad \dot{m}_6 ex_6 + \dot{m}_1 ex_1 - \left(\dot{m}_2 ex_2 + \dot{m}_3 ex_3 + \dot{m}_7 ex_7\right) - \dot{E}x_{D,EVA} = 0 \qquad \eta_{ex,R,EVA} = \frac{\dot{m}_2 ex_2 + \dot{m}_3 ex_3 + \dot{m}_7 ex_7}{\dot{m}_6 ex_6 + \dot{m}_1 ex_1}$$

$$SH: \quad \dot{m}_5 ex_5 + \dot{m}_2 ex_2 - \left(\dot{m}_6 ex_6 + \dot{m}_4 ex_4\right) - \dot{E}x_{D,SH} = 0 \qquad \eta_{ex,R,SH} = \frac{\dot{m}_5 ex_5 + \dot{m}_2 ex_2}{\dot{m}_6 ex_6 + \dot{m}_4 ex_4}$$

	eta_ex_r	Ex_D
	-	kW
EVA	0,932	609
SH	0,986	184

The evaporator results as the worst component from a purely thermodynamic viewpoint.

c. Define the thermoeconomic systems for both the SH and the EVA, according to the Inlet-Outlet paradigm. Then define the required auxiliary relations and justify them. Finally, evaluate the economic costs of system products;

For the evaporator:

$$EVA: \begin{cases} \dot{E}x_6 + \dot{E}x_1 - \dot{E}x_7 - \dot{E}x_2 - \dot{E}x_3 - \dot{E}x_{D,EVA} = 0 \\ \dot{C}_6 + \dot{C}_1 + \dot{Z}_{EVA} = \dot{C}_7 + \dot{C}_2 + \dot{C}_3 \\ \dot{C}_i = c_i \dot{E}x_i \end{cases} \rightarrow \quad c_6 \dot{E}x_6 + c_1 \dot{E}x_1 + \dot{Z}_{EVA} = c_7 \dot{E}x_7 + c_2 \dot{E}x_2 + c_3 \dot{E}x_3$$

$$SH: \begin{cases} \dot{E}x_5 + \dot{E}x_2 - \dot{E}x_6 - \dot{E}x_4 - \dot{E}x_{D,SH} = 0 \\ \dot{C}_5 + \dot{C}_2 + \dot{Z}_{SH} = \dot{C}_6 + \dot{C}_4 \\ \dot{C}_i = c_i \dot{E}x_i \end{cases} \rightarrow \quad c_5 \dot{E}x_5 + c_2 \dot{E}x_2 + \dot{Z}_{SH} = c_6 \dot{E}x_6 + c_4 \dot{E}x_4$$

The required auxiliary relations are:

$$n_{aux} = m_{fluxes} - l_{components} = 7 - 2 = 5$$

$c_5 = 0$ flue gases are losses of other processes;
$c_1 = 0$ saturated water is a loss of other processes;

$c_3 = c_1 = 0$ steam bleed is a by-product of the evaporator, then its specific cost is equal to the steam 1;
$c_6 = c_5 = 0$ flue gases are by-products of the super heater, then its specific cost is equal to the steam 5;
$c_7 = 0$ flue gases are losses of the evaporator, then its specific cost is equal to zero;

With these assumptions, the economic cost balances become:

$$\dot{Z}_{EVA} = c_2 \dot{E}x_2 \qquad c_2 = \frac{\dot{Z}_{EVA}}{\dot{E}x_2} = 0.193 \frac{\text{€}}{kWh} \qquad \dot{C}_2 = c_2 \dot{E}x_2 = 750 \frac{\text{€}}{h}$$

$$c_2 \dot{E}x_2 + \dot{Z}_{SH} = c_4 \dot{E}x_4 \qquad c_4 = \frac{c_2 \dot{E}x_2 + \dot{Z}_{SH}}{\dot{E}x_4} = 0.227 \frac{\text{€}}{kWh} \qquad \dot{C}_4 = c_4 \dot{E}x_4 = 1180 \frac{\text{€}}{h}$$

d. <u>Evaluate the economic cost of exergy destructions, the relative cost difference and the exergoeconomic factor, assuming that both the heat exchangers pay the thermodynamic inefficiencies at the average cost of outlet streams. Briefly discuss the obtained results;</u>

The average costs of inlet and outlet streams are defined as follows:

$$EVA: \begin{cases} c_{IN} = \dfrac{\dot{C}_6 + \dot{C}_1}{\dot{E}x_6 + \dot{E}x_1} = 0 \dfrac{\text{€}}{kWh} \\ c_{OUT} = \dfrac{\dot{C}_7 + \dot{C}_2 + \dot{C}_3}{\dot{E}x_7 + \dot{E}x_2 + \dot{E}x_3} = 0.089 \dfrac{\text{€}}{kWh} \end{cases} \qquad SH: \begin{cases} c_{IN} = \dfrac{\dot{C}_5 + \dot{C}_2}{\dot{E}x_5 + \dot{E}x_2} = 0.056 \dfrac{\text{€}}{kWh} \\ c_{OUT} = \dfrac{\dot{C}_6 + \dot{C}_4}{\dot{E}x_6 + \dot{E}x_4} = 0.090 \dfrac{\text{€}}{kWh} \end{cases}$$

Thermoeconomic parameters can be then evaluated as follows:

$$EVA: \begin{cases} \dot{C}_{ex,D,EVA} = c_{OUT,EVA} \dot{E}x_{D,EVA} = 54 \dfrac{\text{€}}{h} \\ r_{EVA} = \dfrac{c_{OUT} - c_{IN}}{c_{IN}} = 1 \\ f_{EVA} = \dfrac{\dot{Z}_{EVA}}{\dot{Z}_{EVA} + \dot{C}_{ex,D,EVA}} = 0.932 \end{cases} \qquad SH: \begin{cases} \dot{C}_{ex,D,\mathrm{SH}} = c_{OUT,SH} \dot{E}x_{D,SH} = 16 \dfrac{\text{€}}{h} \\ r_{SH} = \dfrac{c_{OUT} - c_{IN}}{c_{IN}} = 0.37 \\ f_{SH} = \dfrac{\dot{Z}_{SH}}{\dot{Z}_{SH} + \dot{C}_{ex,D,\mathrm{SH}}} = 0.963 \end{cases}$$

Exercise 3. (5pt)

The table shows the electricity production for Canada in 2012, together with the energy efficiency of thermoelectric power generation technologies.
Additional data:

- The electric energy imports are equal to 10887 GWh;
- The electric energy exports are equal to 57865 GWh;
- Electric energy requirements of power plant auxiliaries are equal to 40754 GWh;
- Electric energy losses are equal to 44796 GWh.

	Gross electric energy production GWh	Energy efficiency -
Coal	63678	0,38
Oil	6984	0,34
Natural Gas	67536	0,45
Biomass and Waste	9098	0,38
Nuclear	94862	
Hydro	380622	
Solar PV	332	
Wind	11310	
Geothermal	27	

It is required to:

a. (1pt) Evaluate the net final consumption of electric energy in GWh and in ktoe;
b. (2pt) Evaluate primary energy absorbed by all the technologies in ktoe, according to the IEA conventions;
c. (2pt) In 2013, Canadian govern decide to substitute the nuclear energy production with Combined Cycle Power Plants (CCPP, with $\eta_I = 0.55$, $LHV_{NG} = 50$ MJ/kg). Evaluate the change in CO2 production (in Mton) and the change in total primary energy requirements (in ktoe) of the electric energy generation, assuming natural gas as pure methane. Comment the obtained results.
d.

Exercise 3. Solution

a. <u>Evaluate the net final consumption of electric energy in GWh and in ktoe;</u>

The net final consumption of electricity results as the following algebraic sum:

$$EE_{net} = \sum EE_{gross\ production} + imports - (exports + own\,use + losses) = 501921\,GWh_{el}$$

$$EE_{net} = 501921\,GWh_{el} \cdot 0.086\frac{ktoe}{GWh} = 43157\,ktoe$$

b. <u>Evaluate primary energy absorbed by all the technologies in ktoe, according to the IEA conventions;</u>

Primary energy requirements can be generally evaluated as:

$$PE = \frac{EE_{gross}}{\eta_I} \cdot 0.086\frac{ktoe}{GWh}$$

For thermoelectric technologies, the efficiency of conversion is given. For other technologies, the efficiency is given by the IEA conventions:

- For Idro, Solar PV and Wind the efficiency is 1;
- For Nuclear is 0.33;
- For Geothermal electric plant is equal to 0.1;

Therefore:

	Primary energy requirements ktoe
Coal	14409
Oil	1766
Natural Gas	12905
Biomass and Waste	2059
Nuclear	24717
Hydro	32728
Solar PV	29
Wind	972
Geothermal	23
Total	**89607**

c. **In 2013, Canadian govern decide to substitute the nuclear energy production with Combined Cycle Power Plants (CCPP, with ηI = 0.55, LHV_{NG} = 50 MJ/kg). Evaluate the change in CO2 production (in Mton) and the change in total primary energy requirements (in ktoe) of the electric energy generation, assuming natural gas as pure methane. Comment the obtained results.**

Considering the efficiency of the CCPP and the properties of the natural gas given in the text, the primary energy requirements of the CCPP is equal to:

$$PE_{CCPP} = \frac{EE_{NU}}{\eta_{I,CCPP}} = 172476\,GWh_{el}\,(14830ktoe)$$

The new value of the primary energy requirement of electric power generation is thus equal to:

$$PE_{new} = PE - PE_{NU} + PE_{CCPP} = 79720ktoe$$

The change in primary energy requirement of electric power generation is thus equal to:

$$\Delta PE = PE_{new} - PE = -9887ktoe$$

The change in production of CO2 is equal to:

$$m_{CH4} = \frac{PE_{CCPP}}{LHV_{NG}} = 172476\,GWh \cdot \frac{1}{50}\frac{kg}{MJ} \cdot 1000\frac{MJ}{GJ} \cdot 3600\frac{s}{h} \cdot 10^{-6}\frac{kton}{kg} = 12418kton_{CH4}$$

$$m_{CO2} = m_{CH4} \cdot \frac{44}{16}\frac{kton_{CO2}}{kton_{CH4}} = 34150kton_{CO2}$$

Department of Energy
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Milan, 8th Sept 2016

Final Test of 08-09-2016 – Problems

Exercise 1. (9pt)

Let us consider a thermoelectric power plant fueled by coal operating with a regenerative Rankine cycle with double superheater. A fraction y of the steam out of the high pressure turbine HP (4) goes in the regenerator R, and it is mixed with the pressurized water (1’) out of the low pressure pump P1. The mass flow rate of saturated water (2) exits the regenerator and goes through the high pressure pump P2. The water out of the pump (2’) goes through the steam generator with superheater (EVA+SH1) and is converted into superheated steam (3). After the expansion in the HP turbine, the fraction of steam 1-y goes in the second superheater (SH2) till conditions (5) and then it is expanded in the low pressure turbine (LP) till condition (6). The steam is finally condensed in the condenser (C) and exits as saturated liquid (1) directed to the low pressure pump P1.

Notice that:

- The two pumps LP and HP are isentropic;
- The Szargut factor for the coal is equal to 1.05;

properties	point	T [C]	p [bar]	v [m3/kg]	h [kJ/kg]	s [kJ/kgK]	q [-]	h_ls [kJ/kg]	h_vs [kJ/kg]	s_ls [kJ/kgK]	s_vs [kJ/kgK]
environment	0	25	1.013	-	104.9	0.3672	-				
IN P1	1	45.8	0.1	0.001	191.8	0.6493	0				
IN R	1'	45.8	10	0.001	?	?	-				
IN P2	2	179.9	10	0.001	762.6	2.1382	0				
IN SG+SH1	2'	179.9	60	0.001	?	?	0				
IN HP	3	500	60	-	3422.2	6.8818	-				
IN R	4	240.6	10	-	2921.1	6.9518	-				
IN LP	5	500	10	-	3478	7.7627	-				
IN C	6	45.8	0.1	-	?	?	0.97	191.8	2584.8	0.6493	8.1511

With reference to the properties collected in the table, and introducing proper assumptions, it is required to:

a. (1pt) Draw the plant scheme and the qualitative shape of the Thermodynamic cycle in the T-s diagram;
b. (2pt) Apply the energy balance to the pumps and to the regenerator, deriving the mass fraction of steam y and the missing properties in the table. Finally calculate the exergy of all the points of the cycle;
c. (2pt) Apply the energy and the exergy balance to the whole plant, deriving the analytical expressions for the energy efficiency, rational exergy efficiency and exergy destructions of the plant;
d. (2pt) Apply the exergy balance to the regenerator R, the steam generator (including EVA, SH1 and SH2) and the steam turbine (HP+LP): derive the analytical expressions for the functional exergy efficiency and the exergy destructions of such components;
e. (2pt) Derive the numerical results for questions c and d. Comment the obtained results;

Exercise 1. Solution

a. **Draw the plant scheme and the qualitative shape of the Thermodynamic cycle in the T-s diagram;**

b. **Apply the energy balance to the pumps and to the regenerator, deriving the mass fraction of steam y and the missing properties in the table. Finally calculate the exergy of all the points of the cycle;**

Application of the energy balance to the pumps:

- Steady state;
- Adiabatic and rigid component;
- No relevant changes in elevation and speed of fluids;
- Perfect liquid;

$$\sum_i \dot{W}_i^{\leftarrow} + \sum_j \left(\dot{m}^{\leftarrow} h\right)_j = 0 \quad \rightarrow \quad \begin{cases} \dot{W}_{P1}^{\leftarrow} = \dot{m}_s \left(1-\lambda\right)\left(h_{1'} - h_1\right) = \dot{m}_s \left(1-\lambda\right)\left[\cancel{c_s\left(T_{1'} - T_1\right)} + v_s\left(p_{1'} - p_1\right)\right] \\ \dot{W}_{P2}^{\leftarrow} = \dot{m}_s\left(h_{2'} - h_2\right) = \dot{m}_s\left[\cancel{c_s\left(T_{2'} - T_2\right)} + v_s\left(p_{2'} - p_2\right)\right] \end{cases}$$

The enthalpies of water out of the pumps can be derived:

$$h_{1'} - h_1 = v_s\left(p_{1'} - p_1\right) = 0.99\,kJ/kg \quad \rightarrow \quad h_{1'} = 192.79\,kJ/kg$$
$$h_{2'} - h_2 = v_s\left(p_{2'} - p_2\right) = 5\,kJ/kg \quad \rightarrow \quad h_{2'} = 767.6\,kJ/kg$$

The entropies of inlet and outlet sections of the pumps are equal:

$$s_{1'} = s_1 = 0.6493\,kJ/kgK$$
$$s_{2'} = s_2 = 2.1382\,kJ/kgK$$

The steam mass fraction lambda is derived by applying the energy balance to the regenerator:

- Steady state;
- Adiabatic and rigid component;
- No relevant changes in elevation and speed of fluids;

$$\sum_j \left(\dot{m}^{\leftarrow} h\right)_j = 0 \quad \rightarrow \quad \dot{m}_s\left(1-\lambda\right)\left(h_{1'} - h_0\right) + \dot{m}_s\lambda\left(h_4 - h_0\right) - \dot{m}_s\left(h_2 - h_0\right) = 0 \quad \rightarrow \quad \lambda = \frac{h_2 - h_{1'}}{h_4 - h_{1'}} = 0.209$$

The enthalpy and entropy out of the LP turbine can be derived by recalling the definition of quality:

$$q_6 = \frac{h_6 - h_{6ls}}{h_{6vs} - h_{6ls}} = \frac{s_6 - s_{6ls}}{s_{6vs} - s_{6ls}} = 0.97 \quad \rightarrow \quad \begin{cases} h_6 = h_{6ls} + q_6\left(h_{6vs} - h_{6ls}\right) = 2513.01\,kJ/kg \\ s_6 = s_{6ls} + q_6\left(s_{6vs} - s_{6ls}\right) = 7.9260\,kJ/kgK \end{cases}$$

The exergy of bulk flows can be computed as follows:

$$ex_i = h_i - h_0 - T_0\left(s_i - s_0\right)$$
$$ex_1 = 2.8\,kJ/kg$$
$$ex_{1'} = 3.8\,kJ/kg$$
$$ex_2 = 129.7\,kJ/kg$$
$$ex_{2'} = 134.7\,kJ/kg$$
$$ex_3 = 1375.0\,kJ/kg$$
$$ex_4 = 853.0\,kJ/kg$$
$$ex_5 = 1168.1\,kJ/kg$$
$$ex_6 = 154.4\,kJ/kg$$

c. Apply the energy and the exergy balance to the whole plant, deriving the analytical expressions for the energy efficiency, rational exergy efficiency and exergy destructions of the plant;

Application of the energy balance to the whole plant:

- Steady state;
- No relevant changes in elevation and speed of fluids;
- No mass exchanges across the cv: the heat provided by the coal combustion is equal to the change in enthalpy of the steam;

$$\sum_i \dot{Q}_i^{\leftarrow} + \sum_i \dot{W}_i^{\leftarrow} = 0$$

$$\begin{aligned}
\dot{Q}_{GV+SH1}^{\leftarrow} &= \dot{m}_s\left(h_3 - h_{2'}\right) & & & q_{GV+SH1}^{\leftarrow} &= h_3 - h_{2'}\\
\dot{Q}_{SH2}^{\leftarrow} &= \dot{m}_s\left(1-\lambda\right)\left(h_5 - h_4\right) & & & q_{SH2}^{\leftarrow} &= \left(1-\lambda\right)\left(h_5 - h_4\right)\\
\dot{W}_{HP}^{\rightarrow} &= \dot{m}_s\left(h_4 - h_3\right) & & & w_{HP}^{\rightarrow} &= h_4 - h_3\\
\dot{W}_{LP}^{\rightarrow} &= \dot{m}_s\left(1-\lambda\right)\left(h_5 - h_6\right) & &\rightarrow & w_{LP}^{\rightarrow} &= \left(1-\lambda\right)\left(h_5 - h_6\right)\\
\dot{W}_{P1}^{\leftarrow} &= \dot{m}_s\left(1-\lambda\right)\left(h_{1'} - h_1\right) & & & w_{P1}^{\leftarrow} &= \left(1-\lambda\right)\left(h_{1'} - h_1\right)\\
\dot{W}_{P2}^{\leftarrow} &= \dot{m}_s\left(h_{2'} - h_2\right) & & & w_{P2}^{\leftarrow} &= h_{2'} - h_2
\end{aligned}$$

The energy efficiency is defined as:

$$\eta_{en} = \frac{\dot{W}_{net}^{\rightarrow}}{\dot{Q}_{coal}^{\leftarrow}} = \frac{\dot{W}_{HP}^{\rightarrow} + \dot{W}_{LP}^{\rightarrow} - \dot{W}_{P1}^{\leftarrow} - \dot{W}_{P2}^{\leftarrow}}{\dot{Q}_{GC+SH1}^{\leftarrow} + \dot{Q}_{SH2}^{\leftarrow}}$$

Simplifying mass flow rate of steam:

$$\eta_{en} = \frac{w_{net}^{\rightarrow}}{q_{coal}^{\leftarrow}} = \frac{\left(h_4 - h_3\right) + \left(1-\lambda\right)\left(h_5 - h_6\right) - \left(1-\lambda\right)\left(h_{1'} - h_1\right) - \left(h_{2'} - h_2\right)}{\left(h_3 - h_{2'}\right) + \left(1-\lambda\right)\left(h_5 - h_4\right)}$$

Application of the exergy balance to the whole plant:

- Steady state;
- No relevant changes in elevation and speed of fluids;
- No mass exchanges across the cv: the heat provided by the coal combustion is converted into its exergy equivalent thanks to the Szargut factor. The exergy of the coal is assumed as heat exergy entering the control volume at very high temperature;

$$\sum_i \dot{E}x_{Q,i}^{\leftarrow} + \sum_i \dot{E}x_{W,i}^{\leftarrow} - \dot{E}x_{D,tot} = 0$$

$$\dot{E}x_{Q,coal}^{\leftarrow} = \left(\dot{Q}_{GV+SH1}^{\leftarrow} + \dot{Q}_{SH2}^{\leftarrow}\right)\beta_{coal} = \dot{m}_s\left[\left(h_3 - h_{2'}\right) + \left(1-\lambda\right)\left(h_5 - h_4\right)\right]\beta_{coal}$$

$$\dot{E}x_{W,net}^{\rightarrow} = \left(\dot{W}_{HP}^{\rightarrow} + \dot{W}_{LP}^{\rightarrow} - \dot{W}_{P1}^{\leftarrow} - \dot{W}_{P2}^{\leftarrow}\right) = \dot{m}_s\left[\left(h_4 - h_3\right) + \left(1-\lambda\right)\left(h_5 - h_6\right) - \left(1-\lambda\right)\left(h_{1'} - h_1\right) - \left(h_{2'} - h_2\right)\right]$$

The rational exergy efficiency is defined as:

$$\eta_{ex} = \frac{\dot{E}x_{W,net}^{\rightarrow}}{\dot{E}x_{Q,coal}^{\leftarrow}} = \frac{\dot{W}_{HP}^{\rightarrow} + \dot{W}_{LP}^{\rightarrow} - \dot{W}_{P1}^{\leftarrow} - \dot{W}_{P2}^{\leftarrow}}{\left(\dot{Q}_{GV+SH1}^{\leftarrow} + \dot{Q}_{SH2}^{\leftarrow}\right)\beta_{coal}}$$

Simplifying mass flow rate of steam:

$$\eta_{ex} = \frac{ex^{\rightarrow}_{W,net}}{ex^{\leftarrow}_{Q,coal}} = \frac{(h_4-h_3)+(1-\lambda)(h_5-h_6)-(1-\lambda)(h_{1'}-h_1)-(h_{2'}-h_2)}{\left[(h_3-h_{2'})+(1-\lambda)(h_5-h_4)\right]\beta_{coal}} = \frac{\eta_{en}}{\beta_{coal}}$$

$$ex_D = \left[(h_3-h_{2'})+(1-\lambda)(h_5-h_4)\right]\beta_{coal} - \left[(h_4-h_3)+(1-\lambda)(h_5-h_6)-(1-\lambda)(h_{1'}-h_1)-(h_{2'}-h_2)\right]$$

d. Apply the exergy balance to the regenerator R, the steam generator (including EVA, SH1 and SH2) and the steam turbine (HP+LP): derive the analytical expressions for the functional exergy efficiency and the exergy destructions of such components

Regenerator R:
- Steady state;
- Adiabatic and rigid component;
- No relevant changes in elevation and speed of fluids;

$$\sum_k \left(\dot m^{\leftarrow} ex\right)_k - \dot{E}x_{D,\mathrm{R}} = 0 \quad\rightarrow\quad \dot m_s \lambda ex_4 + \dot m_s (1-\lambda) ex_{1'} - \dot m_s ex_2 - \dot{E}x_{D,\mathrm{R}} = 0 \quad\rightarrow\quad \lambda ex_4 + (1-\lambda) ex_{1'} - ex_2 - ex_{D,\mathrm{R}} = 0$$

$$\eta_{ex,R} = \frac{ex_2}{\lambda ex_4 + (1-\lambda) ex_{1'}}$$

$$ex_{D,\mathrm{R}} = \lambda ex_4 + (1-\lambda) ex_{1'} - ex_2$$

Steam generator SG+SH1+SH2:
- Steady state;
- Adiabatic and rigid component;
- No relevant changes in elevation and speed of fluids;
- No mass exchanges across the cv: the heat provided by the coal combustion is converted into its exergy equivalent thanks to the Szargut factor. The exergy of the coal is assumed as heat exergy entering the control volume at very high temperature;

$$\sum_i \dot{E}x^{\leftarrow}_{Q,i} + \sum_k \left(\dot m^{\leftarrow} ex\right)_k - \dot{E}x_{D,SG} = 0$$

$$\dot m_s \left[(h_3-h_{2'})+(1-\lambda)(h_5-h_4)\right]\beta_{coal} - \dot m_s (ex_3 - ex_{2'}) - \dot m_s (1-\lambda)(ex_5-ex_4) - \dot{E}x_{D,SG} = 0$$

$$\eta_{ex,SG} = \frac{\dot m_s (ex_3-ex_{2'}) + \dot m_s (1-\lambda)(ex_5-ex_4)}{\dot m_s \left[(h_3-h_{2'})+(1-\lambda)(h_5-h_4)\right]\beta_{coal}}$$

$$ex_{D,SG} = \left[(h_3-h_{2'})+(1-\lambda)(h_5-h_4)\right]\beta_{coal} - (ex_3-ex_{2'}) - (1-\lambda)(ex_5-ex_4)$$

Steam turbine HP+LP:
- Steady state;
- Adiabatic and rigid component;
- No relevant changes in elevation and speed of fluids;

$$\sum_i \dot{E}x^{\leftarrow}_{W,i} + \sum_k \left(\dot m^{\leftarrow} ex\right)_k - \dot{E}x_{D,ST} = 0$$

$$\dot m_s ex_3 - \dot m_s \lambda ex_4 - \dot m_s (1-\lambda) ex_4 + \dot m_s (1-\lambda) ex_5 - \dot m_s (1-\lambda) ex_6 - \dot W_{HP} - \dot W_{LP} - \dot{E}x_{D,ST} = 0$$

$$\dot m_s ex_3 - \dot m_s \lambda ex_4 - \dot m_s (1-\lambda) ex_4 + \dot m_s (1-\lambda) ex_5 - \dot m_s (1-\lambda) ex_6 - \dot m_s (h_4-h_3) - \dot m_s (1-\lambda)(h_5-h_6) - \dot{E}x_{D,ST} = 0$$

$$\eta_{ex,ST} = \frac{\lambda ex_4 + (1-\lambda) ex_4 + (1-\lambda) ex_6 + (h_4-h_3) + (1-\lambda)(h_5-h_6)}{ex_3 + (1-\lambda) ex_5}$$

$$ex_{D,ST} = ex_3 - \lambda ex_4 - (1-\lambda) ex_4 + (1-\lambda) ex_5 - (1-\lambda) ex_6 - (h_4-h_3) - (1-\lambda)(h_5-h_6)$$

e. Derive the numerical results for questions c and d. Comment the obtained results;

$$\eta_{en} = \frac{(h_4 - h_3) + (1-\lambda)(h_5 - h_6) - (1-\lambda)(h_{1'} - h_1) - (h_{2'} - h_2)}{(h_3 - h_{2'}) + (1-\lambda)(h_5 - h_4)} = \frac{1258.8\,kJ/kg}{3095.2\,kJ/kg} = 0.407$$

$$\eta_{ex} = \frac{\eta_{en}}{\beta_{coal}} = 0.387 \qquad ex_D = 1991.2\,kJ/kg$$

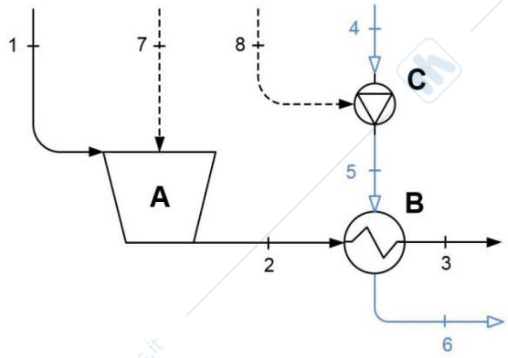
Comments:

- There is not a relevant difference between the numerical values of the two efficiency indicators;
- Exergy efficiency does not provide more information with respect to the energy efficiency at the highest aggregation level;

	eta_en	eta_ex	ex_d
	-		kJ/kg
R		0.716	51.5
SG		0.458	1760.3
ST		0.974	59.4
TOT	0.407	0.387	1991.2

Exercise 2. (7pt)

The system in the figure is used to provide compressed air and hot water for heating purposes to a larger industrial process. The system is composed by an air compressor A, an heat exchanger B and a water pump C. An air mass flow rate of 10 kg/s is brought by the compressor from environmental conditions (point 1) to pressure and temperature of 2 MPa and 60°C (point 2). The compressed air enters the heat exchanger, where it is cooled by heating a flow of liquid water that circulates thanks to the pump. Both the compressor and the pump absorb electricity from the grid. The system operates as demand driven (constant products).



	m	Ex
	kg/s	kW
1	10.0	0.0
2	10.0	5003.2
3	10.0	2540.7
4	35.8	154.3
5	35.8	178.3
6	35.8	835.2
7	-	5824.1
8	-	28.0

operative life	n	years	5
availability factor	f	-	0.5
interest rate on capital	i	-	0.06
from PEC to TCI	PtT	-	6.32
cost of electricity	c_el	€/kWh	0.15
compressor A	PEC_A	€	112031
heat exchanger B	PEC_B	€	69437
pump C	PEC_C	€	9472

With reference to the data provided in the tables, it is required to:

a. (1pt) Calculate the investment costs for the three components in €/h. The O&M costs are enclosed in the PEC given by the table;

b. (2pt) Write the thermoeconomic systems of equations for each component of the system. Appropriately set the required auxiliary relations, justifying each assumption with a brief comment, and determine the analytical expression for the cost of products of each component. Notice that the environmental air 1 and the liquid water 4 are provided at zero economic cost;

c. (2pt) Solve the defined thermoeconomic systems, calculating specific and total economic costs of useful products for all the components. Derive the cost structure for each product;

d. (2pt) Calculate the costs of exergy destructions and losses for all the components. Apply the first iteration of the Design Evaluation process: comment the obtained results and propose one possible way to increase the economic efficiency of the system.

Exercise 2. Solution

a. **Calculate the investment costs for the three components in €/h. The O&M costs are enclosed in the PEC given by the table;**

The plant operates at nominal conditions for only a part of the year:

$$f = \frac{h_{op,n}}{h_y} \rightarrow \qquad h_{op,n} = f \cdot h_y = 4380\,h$$

The investment cost of each component can be calculated thanks to the capital recovery factor:

$$PEC_{\%,i} = \frac{PEC_i}{\sum(PEC_i)}$$

$$\dot{Z}_i = \sum(PEC) \cdot crf \cdot 6{,}32 \cdot \frac{1}{h_{op}} \cdot PEC_{\%,i}$$

		PEC €	PEC_% %	Z_y €/y	Z €/h
Compressor	A	112031	0,587	168085	38,38
Heat exchanger	B	69437	0,364	104179	23,79
Pump	C	9472	0,050	14211	3,24
Total	A+B+C	190940	1,000	286476	65,41

b. **Write the thermoeconomic systems of equations for each component of the system. Appropriately set the required auxiliary relations, justifying each assumption with a brief comment, and determine the analytical expression for the cost of products of each component. Notice that the environmental air 1 and the liquid water 4 are provided at zero economic cost;**

The required auxiliary relations are: $n_{aux} = n_{flows} - n_{components} = 8 - 3 = 5$

$c_1 = 0$ the air is provided at zero cost

$c_4 = 0$ the water is provided at zero cost

$c_7 = 0.15\,€/kWh$ the electricity is provided at positive cost

$c_8 = 0.15\,€/kWh$ the electricity is provided at positive cost

$c_3 = c_6 = c_{P,B}$ the compressed air and the hot water are both products of the CHP system.

Thermoeconomic systems at the lower aggregation level:

Compressor A:

$$\begin{cases} \dot{E}_1 + \dot{E}_7 = \dot{E}_2 + \dot{E}_{D,A} \\ \dot{C}_1 + \dot{C}_7 + \dot{Z}_A = \dot{C}_2 \\ \dot{C} = c \cdot \dot{E} \end{cases} \qquad \rightarrow \qquad c_1\dot{E}_1 + c_7\dot{E}_7 + \dot{Z}_A = c_2\dot{E}_2 \qquad c_2 = \frac{c_1\dot{E}_1 + c_7\dot{E}_7 + \dot{Z}_A}{\dot{E}_2}$$

Heat exchanger B:

$$\begin{cases}\dot{E}_2+\dot{E}_5=\dot{E}_3+\dot{E}_6+\dot{E}_{D,\mathrm{B}}\\ \dot{C}_2+\dot{C}_5+\dot{Z}_B=\dot{C}_3+\dot{C}_6\\ \dot{C}=c\cdot\dot{E}\end{cases}\rightarrow\qquad c_2\dot{E}_2+c_5\dot{E}_5+\dot{Z}_B=c_3\dot{E}_3+c_6\dot{E}_6\qquad c_{P,B}=\frac{c_2\dot{E}_2+c_5\dot{E}_5+\dot{Z}_B}{\dot{E}_3+\dot{E}_6}$$

Pump C:

$$\begin{cases}\dot{E}_8+\dot{E}_4=\dot{E}_5+\dot{E}_{D,\mathrm{C}}\\ \dot{C}_8+\dot{C}_4+\dot{Z}_C=\dot{C}_5\\ \dot{C}=c\cdot\dot{E}\end{cases}\rightarrow\qquad c_8\dot{E}_8+c_4\dot{E}_4+\dot{Z}_C=c_5\dot{E}_5\qquad c_5=\frac{c_8\dot{E}_8+c_4\dot{E}_4+\dot{Z}_C}{\dot{E}_5}$$

c. **<u>Solve the defined thermoeconomic systems, calculating specific and total economic costs of useful products for all the components. Derive the cost structure for each product;</u>**

Cost structure is obtained by substituting the exergy balance of each component in the expressions of products cost.

$$c_2=c_7+c_7\frac{\dot{E}_{D,A}}{\dot{E}_2}+\frac{\dot{Z}_A}{\dot{E}_2}=\left(0.150+0.025+0.008\right)\frac{€}{kWh}$$

$$c_{P,B}=\frac{\overline{c}_{IN,B}\left(\dot{E}_2+\dot{E}_5\right)+\dot{Z}_B}{\dot{E}_3+\dot{E}_6}=\overline{c}_{IN,B}+\overline{c}_{IN,B}\frac{\dot{E}_{D,\mathrm{B}}}{\dot{E}_3+\dot{E}_6}+\frac{\dot{Z}_B}{\dot{E}_3+\dot{E}_6}=\left(0.177+0.095+0.007\right)\frac{€}{kWh}$$

$$c_5=\overline{c}_{IN,\mathrm{C}}+\overline{c}_{IN,\mathrm{C}}\frac{\dot{E}_{D,\mathrm{C}}}{\dot{E}_5}+\frac{\dot{Z}_C}{\dot{E}_5}=\left(0.023+0.001+0.018\right)\frac{€}{kWh}$$

flow no.	c_i €/kWh	C_i €/h
2	0,18	912
3 , 6	0,28	943
5	0,04	7

d. **<u>Calculate the costs of exergy destructions and losses for all the components. Apply the first iteration of the Design Evaluation process: comment the obtained results and propose one possible way to increase the economic efficiency of the system.</u>**

The exergy destructions are paid as the average cost of inlet flows:

$$c_{DL,A}=\frac{c_1\dot{E}_1+c_7\dot{E}_7}{\dot{E}_1+\dot{E}_7}=c_7=0.15\frac{€}{kWh}\rightarrow\qquad \dot{C}_{D,A}=c_{D,A}\dot{E}_{D,A}=123.1\frac{€}{h}$$

$$c_{DL,\mathrm{B}}=\frac{c_2\dot{E}_2+c_5\dot{E}_5}{\dot{E}_2+\dot{E}_5}=0.18\frac{€}{kWh}\qquad\rightarrow\qquad \dot{C}_{D,\mathrm{B}}=c_{D,\mathrm{B}}\dot{E}_{D,\mathrm{B}}=320.4\frac{€}{h}$$

$$c_{DL,\mathrm{C}}=\frac{c_8\dot{E}_8+c_4\dot{E}_4}{\dot{E}_4+\dot{E}_8}=0.0008\frac{€}{kWh}\qquad\rightarrow\qquad \dot{C}_{D,\mathrm{C}}=c_{D,\mathrm{C}}\dot{E}_{D,\mathrm{C}}=0\frac{€}{h}$$

Application of DE consists in the evaluation of the following indexes for each one of the plant components:

$\dot{Z}_i+\dot{C}_{DL,i}$	Sum of cost of exergy destruction and losses, and investment cost;
$r_i=\left(c_{out}-c_{in}\right)/c_{in}$	Relative difference of average cost of outlet and inlet flows;

$$f_i = \frac{\dot{Z}_i}{\dot{Z}_i + \dot{C}_{DL,i}}$$ Exergoeconomic Factor;

		c_in	c_out	C_DL	Z	C_DL+Z	r	f
		€/kWh	€/kWh	€/h	€/h	€/h	-	-
Compressor	A	0.15	0.18	123.1	38.38	161.5	0.22	0.238
Heat exchanger	B	0.18	0.28	320.4	23.79	344.2	0.57	0.069
Pump	C	0.0008	0.04	0.00	3.24	3.2	49.74	0.999

Comments:

- The Heat exchanger B is the most critical component, followed by the compressor and the pump;
- Irreversibilities occurring in the pump are not relevant with respect to the cost efficiency of the CHP;
- r suggests that there are more margins of improvements in the heat exchanger rather than in the compressor;
- f suggests that lowest cost of products may be obtained by increasing the efficiency of the heat exchanger, even if this imply higher investment costs.

Exercise 3. (6pt)

Let us consider the energy statistics of Argentina in 2013 in the table (IEA).

Argentina 2013		coal	oil	natural gas	nuclear	hydro	biomass	electricity	total
Production	ktoe	49	30239	32387	1626	2672	4454	0	71427
Import	ktoe	1166	5706	9602	0	0	0	1264	17738
Export	ktoe	53	4193	68	0	0	1232	26	5572
Primary energy devoted to electricity prod.	ktoe	821	4632	14020	1626	2672	906	1238	25915
Gross electricity production	GWh	3294	19844	75774	?	?	2955	-	?
Own energy uses of the energy sector	ktoe	123	1525	6019	0	0	3	-	7670

In addition to the data provided by the table, the electric energy consumptions of auxiliaries and the transmission losses are equal to 32630 GWh. It is required to calculate:

a. (2pt) The gross electricity production and the net electricity production (both in GWh);
b. (2pt) The Total Final Consumption (TFC, in ktoe) and the electricity penetration (EP);

In the next year, the whole Nuclear energy production will be substituted by Natural Gas Combined Cycle plants (NGCC, with η_I=0.55 and LHV_{ng}=50 MJ/kg of pure methane). It is then required to calculate:

c. (2pt) The additional mass of natural gas required and the change in the CO2 emissions (both in Mton).

Exercise 3. Solution

a. <u>The gross electricity production and the net electricity production (both in GWh);</u>

The gross electricity production is the sum of the corresponding line in the table. Notice that the electricity production from nuclear and hydro can be accounted thanks to the IEA conventions:

$$EE_{gross} = \left(3294 + 19844 + 75774 + \frac{1626\,ktoe \cdot 0.33}{0.086\,ktoe/GWh} + \frac{2672\,ktoe \cdot 1}{0.086\,ktoe/GWh} + 2955\right) GWh = 139176\,GWh$$

$$EE_{net} = EE_{gross} - EE_{own-uses} = \left(139176 - \frac{7670}{0.086}\right) GWh = 49990 GWh$$

b. The Total Final Consumption (TFC, in ktoe) and the electricity penetration (EP);

The TPES for Argentina in 2013 is evaluated through the sum of the last column of the given table (pay attention to the signs!).

$$TPES = (71427+17738\text{-}5572)\,ktoe = 83593\,ktoe$$

With the available data, the TFC cannot be calculated. Indeed, TFC is equal to:

$$TFC = TFC = TPES - \sum\left(\frac{energy}{conversions} + \frac{energy}{own\,uses} + losses\right)$$

Primary energy devote to electricity production (25915ktoe) is only a part of the primary energy converted into final uses (heat plants, gas works, oil refineries, coal transformations, etc. are missing).

The electric penetration is defined as the patio between the primary energy devoted to electric energy production [ktoe] and the TPES [ktoe]:

$$EP = \frac{\sum_i E_{P,i}}{TPES} = \frac{25915 ktoe}{83593\,ktoe} = 0.31$$

c. The additional mass of natural gas required and the change in the CO2 emissions (both in Mton);

Electricity production from NGCC:

$$EE_{NGCC} = E_{primary,NU} \cdot 0.33 = \frac{1626 \cdot 0.33}{0.086} GWh = 6239 GWh$$

Natural gas requirements and CO2 emissions:

$$E_{NGCC} = \frac{EE_{NGCC}}{0.55} = 11344 GWh \qquad \rightarrow \qquad m_{NG} = \frac{E_{NGCC} \cdot 3600}{LHV_{NG}} = \frac{11344 \cdot 3600 \cdot 10^3}{50 \cdot 10^9} = 0.82 Mton_{NG}$$

$$m_{CO2} = m_{CO2}\frac{44}{16} = 0.82 Mton \frac{44}{16} = 2.25 Mton_{CO2}$$

Department of Energy
Politecnico di Milano

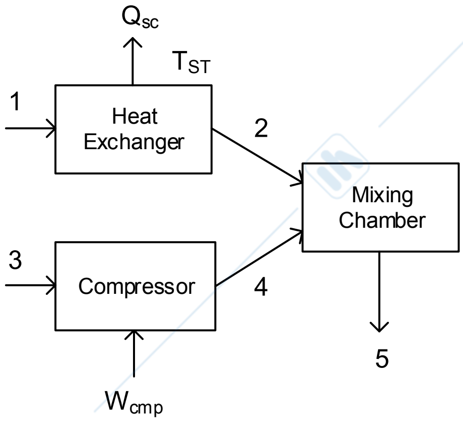
Milan, 27th Sept 2016

Final Test of 29-09-2016 – Problems

Exercise 1. (8pt)

A molar flow rate of propylene ($C3H6$) at conditions 1 flows through the Heat Exchanger (A) providing heat to one exogenous process and decreasing its pressure till conditions 2. A molar flow rate of nitrogen ($N2$) at conditions 3 goes through a Compressor (B) till conditions 4. The two flows are finally mixed in the Mixing Chamber (C). The environmental reference pressure and temperature are respectively equal to 25°C and 101325 Pa.

no	formula	Molar flow rate mol/s	Temperature °C	Pressure MPa	Isobaric Heat Capacity kJ/kmolK
1	C3H6	3	150.0	1.000	80.0
2	C3H6	3	95.0	0.700	85.7
3	N2	7	75.0	0.101	29.2
4	N2	7	394.4	0.700	29.2



With reference to the properties collected in the table, and introducing proper assumptions, it is required to:

a. (2pt) Apply the energy balance to the Heat Exchanger and the Compressor, deriving heat loss by C3H6 and the mechanical power absorbed by the Compressor. Then apply the energy balance to the Mixing Chamber, deriving the temperature of the mixed flow 5;

b. (2pt) Apply the exergy balance to the Heat Exchanger, assuming that the heat cross the boundaries at the logarithmic average temperature among the flows 1 and 2. Derive the analytical expressions for the exergy destructions and the functional exergy efficiency of the system, considering the heat flow released by the Heat Exchanger as its unique useful effect. Then derive the numerical results;

c. (2pt) Calculate the (specific and total) chemical exergy of the gas flows 2, 4 and 5, assuming a composition for the reference environmental air equal to 75,67% N2, 20,35% O2, 3,95% H2O e 0,03% CO2;

d. (2pt) Apply the exergy balance to the Mixing Chamber, deriving its exergy destruction and its functional exergy efficiency;

Exercise 1. Solution

a. <u>Apply the energy balance to the Heat Exchanger and the Compressor, deriving heat loss by C3H6 and the mechanical power absorbed by the Compressor. Then apply the energy balance to the Mixing Chamber, deriving the temperature of the mixed flow 5;</u>

<u>Heat Exchanger</u>

- Steady state operation;
- Rigid component;

- No changes in elevation and velocity of fluids;

$$\frac{dE^A}{dt} = \sum_i \dot{W}_i^{A\leftarrow} + \sum_j \dot{Q}_j^{A\leftarrow} + \sum_k \left(\dot{m}_k^{A\leftarrow} \cdot e_k\right) \quad \rightarrow \quad \dot{Q}_{sc}^{\leftarrow} + \dot{N}_1 h_1 - \dot{N}_2 h_2 = 0$$

The fluid can be seen as a perfect gas and the molar flow rate is constant. The isobaric heat capacity is calculated as the average among inlet and outlet sections:

$$\Delta h = \bar{c}_p \Delta T \; ; \; \dot{N}_1 = \dot{N}_2 = \dot{N}_{C3H6}$$

$$\dot{Q}_{sc}^{\rightarrow} = \dot{N}_{C3H6} \cdot c_{p,C3H6} \left(T_1 - T_2\right) = 0.003 \frac{kmol}{s} 82.9 \frac{kJ}{kmolK} \left(423.2 - 368.2\right) K = 13.6 \; kW$$

<u>Compressor:</u>

- Steady state operation;
- Rigid and adiabatic component;
- No changes in elevation and velocity of fluids;

$$\frac{dE^A}{dt} = \sum_i \dot{W}_i^{A\leftarrow} + \sum_j \dot{Q}_j^{A\leftarrow} + \sum_k \left(\dot{m}_k^{A\leftarrow} \cdot e_k\right) \quad \rightarrow \quad \dot{W}_{cmp}^{\leftarrow} + \dot{N}_3 h_3 - \dot{N}_4 h_4 = 0$$

The fluid can be seen as a perfect gas and the molar flow rate is constant. The molar flow rate and the isobaric heat capacity are constant between the inlet and outlet sections:

$$\Delta h = c_p \Delta T \; ; \; \dot{N}_3 = \dot{N}_4 = \dot{N}_{N2}$$

$$\dot{W}_{cmp}^{\leftarrow} = \dot{N}_{N2} \cdot c_{p,N2} \left(T_4 - T_3\right) = 0.007 \frac{kmol}{s} 29.2 \frac{kJ}{kmolK} \left(667.5 - 348.2\right) K = 65.3 kW$$

<u>Mixing Chamber:</u>

- Steady state operation;
- Rigid and adiabatic component;
- No changes in elevation and velocity of fluids;

$$\frac{dE^A}{dt} = \sum_i \dot{W}_i^{A\leftarrow} + \sum_j \dot{Q}_j^{A\leftarrow} + \sum_k \left(\dot{m}_k^{A\leftarrow} \cdot e_k\right) \quad \rightarrow \quad \dot{N}_2 h_2 + \dot{N}_4 h_4 - \dot{N}_5 h_5 = 0$$

The molar flow rate 5 can be obtained as the sum of the molar flows 2 and 4, while the isobaric heat capacity of 5 is the weighted average among flows 2 and 4. The fluid can be seen as a perfect gas, and the enthalpies are calculated with respect to the environmental conditions.

$$\dot{N}_2 + \dot{N}_4 = \dot{N}_5 \quad \rightarrow \quad \dot{N}_5 = \left(0.003 + 0.007\right) \frac{kmol}{s} = 0.01 \frac{kmol}{s}$$

$$c_{p,mix} = c_{p,C3H6} x_{C3H6} + c_{p,N2} x_{N2} = 85.7 \cdot \frac{0.003}{0.01} + 29.2 \cdot \frac{0.007}{0.01} = 45.304 \frac{kJ}{kmolK}$$

$$h_i = c_p \left(T_i - T_0\right)$$

$$\dot{N}_{C3H6} c_{p,C3H6} \left(T_2 - T_0\right) + \dot{N}_{N2} c_{p,N2} \left(T_4 - T_0\right) - \dot{N}_{mix} c_{p,mix} \left(T_5 - T_0\right) = 0 \quad \rightarrow \quad T_5 = 503.5K \left(230.4°C\right)$$

b. <u>**Apply the exergy balance to the Heat Exchanger, assuming that the heat cross the boundaries at the logarithmic average temperature among the flows 1 and 2. Derive the analytical expressions for the exergy destructions and the functional exergy efficiency of the system, considering the heat flow released by the Heat Exchanger as its unique useful effect. Then derive the numerical results;**</u>

Heat Exchanger

- Steady state operation;
- Rigid component;
- No changes in elevation and velocity of fluids;

$$\frac{dEx_A}{dt} = \sum_i \dot{E}x_{W,i}^{A\leftarrow} + \sum_j \dot{E}x_{Q,j}^{A\leftarrow} + \sum_k \dot{E}x_{\mathrm{BF},j}^{A\leftarrow} - \dot{E}x_D \qquad \rightarrow \qquad \dot{E}x_Q^{\leftarrow} + \dot{N}_1 ex_1 - \dot{N}_2 ex_2 - \dot{E}x_{D,he} = 0$$

$$T_{ml,1-2} = \frac{T_1 - T_2}{\ln\left(T_1/T_2\right)} = 395.0\,\mathrm{K}\left(121.9°C\right)$$

$$\dot{E}x_Q^{\leftarrow} = \dot{Q}_{sc}^{\leftarrow} \cdot \left(1 - \frac{T_0}{T_{ml,1-2}}\right) = -13.6\,kW \cdot 0.245 = -3.35kW$$

exiting the system

$$ex_1 - ex_2 = h_1 - h_2 - T_0\left(s_1 - s_2\right) = c_{p,C3H6}\left(T_1 - T_2\right) - T_0\left(c_{p,C3H6}\ln\frac{T_1}{T_2} - R\ln\frac{p_1}{p_2}\right) = 1997.1\frac{kJ}{kmol}$$

$$\dot{N}_{C3H6}\left(ex_1 - ex_2\right) = 5.99\,kW$$

$$\dot{E}x_{D,he} = \dot{E}x_Q^{\leftarrow} + \dot{N}_1 ex_1 - \dot{N}_2 ex_2 = \left(-3.34 + 5.99\right)kW = 2.65\ kW$$

$$\eta_{\mathrm{ex,F},he} = \frac{P}{F} = \frac{\dot{E}x_Q}{\dot{E}x_1 - \dot{E}x_2} = 0.56$$

c. **Calculate the (specific and total) chemical exergy of the gas flows 2, 4 and 5, assuming a composition for the reference environmental air equal to 75,67% N2, 20,35% O2, 3,95% H2O e 0,03% CO2;**

It is assumed the following chemical relation for the combustion of the C3H6:

$$C_3H_6 + 4,5O_2 \rightarrow 3CO_2 + 3H_2O$$

$$ex_{ch,\mathrm{C3H6}} = -\Delta g_{C3H6}\left(T_0, p_0\right) - n_{O2}ex_{ch,O2} + \left(\sum_i n_i ex_{ch,i} + T_0 R \sum_i n_i \ln x_i\right)_P = 1987915\frac{kJ}{kmol}\left(5963.7kW\right)$$

$$-\Delta G_{C3H6}\left(T_0, p_0\right) = \sum_{R-P} \dot{N}_i g_{f,i} - \sum_P \dot{N}_j g_{f,j} = \left[(1\cdot 62720 + 4{,}5\cdot 0) - (3\cdot -394380 + 3\cdot -228590)\right] = 1931630\frac{kJ}{kmol}$$

$$-n_{O2}ex_{ch,O2} = -n_{O2}T_0 R\ln\frac{p_{0,O2}}{p_{00,O2}} = 288K \cdot 8{,}314\frac{kJ}{kmolK} \cdot 4{,}5\frac{kmol_{O2}}{kmol_{C3H6}} \cdot \ln\frac{1}{0{,}2035} = -17759{,}3\frac{kJ}{kmol}$$

$$\left(\sum_i n_i ex_{ch,i} + T_0 R \sum_i n_i \ln x_i\right)_P = 74044\frac{kJ}{kmol}$$

	CO2	H2O	tot
$n_i ex_{ch,i}$	60323	24031	84353
$T_0 R \sum_i n_i \ln x_i$	-5155	-5155	-10309
tot	55168	18876	**74044**

Chemical exergy of the N2 can be calculated as the expansion work of an ideal gas till the dead state:

$$ex_{ch,\mathrm{N2}} = T_0 R\ln\frac{p_{N2,0}}{p_{N2,00}} = 288K \cdot 8.314\frac{kJ}{kmolK} \cdot \ln\frac{1}{0.7576} = 691.1\frac{kJ}{kmol}\left(4.8kW\right)$$

The chemical exergy (total and specific) for the gas mixture can be calculated as:

$$ex_{ch,mix} = \sum_i x_i ex_{ch,i} + T_0 R \sum_i x_i \ln x_i$$

$$ex_{ch,mix} = \left(\frac{3}{10}1987914 + \frac{7}{10}691.1\right)\frac{kJ}{kmol} + 288.15K \cdot 8.314\frac{kJ}{kmolK}\left(\frac{3}{10}\ln\frac{3}{10} + \frac{7}{10}\ln\frac{7}{10}\right) = 595343.9\frac{kJ}{kmol}$$

$$\dot{Ex}_{ch,mix} = \dot{N}_{mix} ex_{ch,mix} = 5953.4\,kW$$

To sum up:

	ex_ch kJ/kmol	Ex_ch kW
2	1987915	5964
4	691	5
5	496103	5953

d. <u>Apply the exergy balance to the Mixing Chamber, deriving its exergy destruction and its functional exergy efficiency;</u>

<u>Mixing Chamber:</u>

- Steady state operation;
- Rigid and adiabatic component;
- No changes in elevation and velocity of fluids;

The exergy of bulk flow must take into account both the physical and the chemical components. Physical exergy of the molar flows can be computed with respect to the environmental conditions, approximating fluids as perfect gases:

$$\frac{dEx_A}{dt} = \sum_i \dot{Ex}^{A\leftarrow}_{W,i} + \sum_j \dot{Ex}^{A\leftarrow}_{Q,j} + \sum_k \dot{Ex}^{A\leftarrow}_{\mathrm{BF},j} - \dot{Ex}_D \qquad \rightarrow \qquad \dot{N}_2 ex_2 + \dot{N}_4 ex_4 - \dot{N}_5 ex_5 - \dot{Ex}_{D,mc} = 0$$

$$ex_i = h_i - h_0 - T_0\left(s_i - s_0\right) = c_{p,\mathrm{i}}\left(T_i - T_0\right) - T_0\left(c_{p,\mathrm{i}} \ln\frac{T_i}{T_0} - R\ln\frac{p_i}{p_0}\right)$$

	n kmol/s	h kJ/kmol	s kJ/kmolK	ex_ph kJ/kmol	ex_ch kJ/kmol	ex_tot kJ/kmol	Ex_ph kW	Ex_ch kW	Ex_tot kW
2	0.003	5776.4	1.33	5379	1987915	1993293	16.14	5963.74	5979.88
4	0.007	10784.6	7.46	8559	691	9251	59.92	4.84	64.75
5	0.01	9282.2	7.62	7012	595344	602356	70.12	5953.44	6023.56

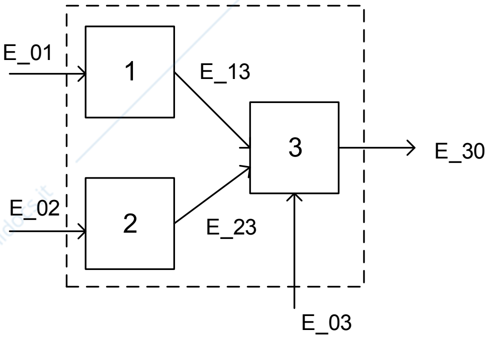
$$\dot{Ex}_{D,\mathrm{mc}} = \dot{N}_2 ex_2 + \dot{N}_4 ex_4 - \dot{N}_5 ex_5 = 21.1\,kW$$

$$\eta_{\mathrm{ex,F,sc}} = \frac{P}{F} = \frac{\dot{Ex}_5}{\dot{Ex}_2 + \dot{Ex}_4} = 0{,}997$$

Exercise 2. (9pt)

The system in the figure is composed by three processes connected to each other and with the environment by flows of energy and materials quantified by means of their exergy equivalents. Rational exergy efficiencies and economic costs of investment per hour for all the processes are listed in the table.
Process 3 must deliver a constant quantity of product (E_30, 100 kW) by consuming exergy from different sources: 25% from outside system boundaries (E_03), 35% from process 1 (E_13) and 40% from process 2 (E_23). The economic cost of flows E_01, E_02 and E_03 are respectively equal to 0.04 €/kWh, 0.014 €/kWh and 0.01 €/kWh.

Process	eta_ex -	Z_inv €/h
process 1	0.45	8
process 2	0.35	2
process 3	0.65	3



It is required to:

a. (2pt) Determine the amount of exergy absorbed and produced by each process, and their exergy destructions. Then write the thermoeconomic system of equations for each process, define and briefly comment the required auxiliary relations;

b. (2pt) Derive the analytical expressions for the structures of the cost of each product, and the expression of the economic cost of exergy destructions.

c. (2pt) Calculate the numerical values for questions a and b, and briefly comment the cost structure of each product. Calculate the Thermoeconomic indicators and comment the obtained values;

d. (3pt) Two options are available to improve the economic efficiency of the system. Based on the Thermoeconomic indicators calculated in question c, select one of the given option and check whether its application allows to reduce the specific cost of flow E_30.
    a. Change process 1 by reducing its economic investment cost (Z_inv = 4.5 €/h) and its exergy efficiency (eta_ex = 0.35);
    b. Change process 3 by increasing its economic cost (Z_inv = 7.5 €/h) and its exergy efficiency (eta_ex = 0.85);

Exercise 2. Solution

a. **Determine the amount of exergy absorbed and produced by each process, and their exergy destructions. Then write the thermoeconomic system of equations for each process, define and briefly comment the required auxiliary relations;**

Based on the definition of rational exergy efficiency, the inlet and outlet flows of each process can be calculated:

$$\dot{E}_I = \dot{E}_O + \dot{E}_D \qquad \rightarrow \qquad \eta_{ex,r} = \dot{E}_O / \dot{E}_I$$

Process	I - kW	O - kW	D - kW	eta,ex,R
process 1	119.7	53.8	65.8	0.45
process 2	175.8	61.5	114.3	0.35
process 3	153.8	100.0	53.8	0..65

The inlet flows in process 3 can be calculated as:

$$\dot{E}_{I,3} = \dot{E}_{13} + \dot{E}_{23} + \dot{E}_{03} = 0.35\dot{E}_I + 0.40\dot{E}_I + 0.25\dot{E}_I = (53.8 + 61.5 + 38.5)kW$$

Thermoeconomic systems of equations:

$$1:\begin{cases}\dot E_{01}=\dot E_{13}+\dot E_{D,1}\\ \dot C_{01}+\dot Z_1=\dot C_{13}\\ \dot C=c\cdot\dot E\end{cases}\quad\rightarrow\quad c_{01}\dot E_{01}+\dot Z_1=c_{13}\dot E_{13}$$

$$2:\begin{cases}\dot E_{02}=\dot E_{23}+\dot E_{D,2}\\ \dot C_{02}+\dot Z_2=\dot C_{23}\\ \dot C=c\cdot\dot E\end{cases}\quad\rightarrow\quad c_{02}\dot E_{02}+\dot Z_2=c_{23}\dot E_{23}$$

$$3:\begin{cases}\dot E_{03}+\dot E_{13}+\dot E_{23}=\dot E_{30}+\dot E_{D,3}\\ \dot C_{13}+\dot C_{23}+\dot C_{03}+\dot Z_3=\dot C_{30}\\ \dot C=c\cdot\dot E\end{cases}\quad\rightarrow\quad c_{13}\dot E_{13}+c_{23}\dot E_{23}+c_{03}\dot E_{03}+\dot Z_3=c_{30}\dot E_{30}$$

There are 6 flows and 3 components, thus 3 auxiliary relations are required:

$c_{01}=0.04€/kWh$	specific cost of flow E_01
$c_{02}=0.014€/kWh$	specific cost of flow E_02
$c_{03}=0.01€/kWh$	specific cost of flow E_03

With these equations, the system given by the three cost balances results as:

$$\begin{cases}c_{01}\dot E_{01}+\dot Z_1=c_{13}\dot E_{13}\\ c_{02}\dot E_{02}+\dot Z_2=c_{23}\dot E_{23}\\ c_{13}\dot E_{13}+c_{23}\dot E_{23}+c_{03}\dot E_{03}+\dot Z_3=c_{30}\dot E_{30}\end{cases}\quad\rightarrow\quad\begin{aligned}c_{13}&=\frac{c_{01}\dot E_{01}+\dot Z_1}{\dot E_{13}}\quad\left(\dot C_{13}=c_{13}\dot E_{13}\right)\\ c_{23}&=\frac{c_{02}\dot E_{02}+\dot Z_2}{\dot E_{23}}\quad\left(\dot C_{23}=c_{23}\dot E_{23}\right)\\ c_{30}&=\frac{c_{13}\dot E_{13}+c_{23}\dot E_{23}+c_{03}\dot E_{03}+\dot Z_3}{\dot E_{30}}\quad\left(\dot C_{30}=c_{30}\dot E_{30}\right)\end{aligned}$$

b. <u>Derive the analytical expressions for the structures of the cost of each product, and the expression of the economic cost of exergy destructions.</u>

Substituting the exergy balance in the cost balance for each process, the cost structure for each product can be obtained:

$$1:\begin{cases}\dot E_{01}=\dot E_{13}+\dot E_{D,1}\\ c_{01}\dot E_{01}+\dot Z_1=c_{13}\dot E_{13}\end{cases}\quad\rightarrow\quad c_{13}=c_{01}+c_{01}\frac{\dot E_{D,1}}{\dot E_{13}}+\frac{\dot Z_1}{\dot E_{13}}\quad;\quad\dot C_{D,1}=c_{01}\dot E_{D,1}$$

$$2:\begin{cases}\dot E_{02}=\dot E_{23}+\dot E_{D,2}\\ c_{02}\dot E_{02}+\dot Z_2=c_{23}\dot E_{23}\end{cases}\quad\rightarrow\quad c_{23}=c_{02}+c_{02}\frac{\dot E_{D,2}}{\dot E_{23}}+\frac{\dot Z_2}{\dot E_{23}}\quad;\quad\dot C_{D,2}=c_{02}\dot E_{D,2}$$

To derive the cost structure of the product of process 3, the average cost of inlet flows is defined:

$$\overline{c}_{3,in}=\frac{c_{13}\dot E_{13}+c_{23}\dot E_{23}+c_{03}\dot E_{03}}{\dot E_{13}+\dot E_{23}+\dot E_{03}}$$

$$3:\begin{cases}\dot{E}_{03}+\dot{E}_{13}+\dot{E}_{23}=\dot{E}_{30}+\dot{E}_{D,3}\\ \bar{c}_{3,in}\left(\dot{E}_{13}+\dot{E}_{23}+\dot{E}_{03}\right)+\dot{Z}_3=c_{30}\dot{E}_{30}\end{cases}\rightarrow\quad c_{30}=\bar{c}_{3,in}+\bar{c}_{3,in}\frac{\dot{E}_{D,3}}{\dot{E}_{30}}+\frac{\dot{Z}_3}{\dot{E}_{30}}\ ;\qquad \dot{C}_{D,3}=\bar{c}_{3,in}\dot{E}_{D,3}$$

c. <u>Calculate the numerical values for questions a and b, and briefly comment the cost structure of each product. Calculate the Thermoeconomic indicators and comment the obtained values;</u>

	Ex kW	c €/kWh	C €/h
E_01	119.7	0.040	4.79
E_02	175.8	0.014	2.46
E_03	38.5	0.010	0.38
E_13	53.8	0.237	12.79
E_23	61.5	0.073	4.46
E_30	100.0	0.206	20.60

Cost structure of products:

	fuel €/kWh	destruction €/kWh	investment €/kWh
E_01	0.040	0.049	0.149
E_02	0.014	0.026	0.033
E_03	0.115	0.062	0.030

Design evaluation procedure requires the calculation of the following indicators:

$\dot{C}_{D,i}=c_{I,i}\cdot\dot{E}_{D,i}$ Cost of exergy destructions;

$\dot{Z}_i+\dot{C}_{D,i}$ Cost of exergy destructions plus investment costs;

$r_i=\dfrac{c_p-c_f}{c_f}$ Relative cost difference;

$f_i=\dfrac{\dot{Z}_i}{\dot{Z}_i+\dot{C}_{D,i}}$ Exergoeconomic Factor;

Process	E_D kWh	C_D €/h	Z_inv €/h	Z_inv+C_D €/h	r -	f -
process 1	65.8	2.63	8	10.63	4.94	0.752
process 2	114.3	1.60	2	3.60	4.18	0.556
process 3	53.8	6.17	3	9.17	0.80	0.327

Comments:

- The cost of investment and exergy destructions is greater for process 1, followed by process 3 and process 2. Therefore analysts should focus on process 1 in order to increase the cost efficiency of the system;
- Relative cost difference for process 1 is greater than other processes, revealing more margins of economic improvement by acting on thermodynamic parameters;
- Exergoeconomic factor reveal that process 1 should be improved by reducing its investment cost, while process 3 should be improved by increasing its thermodynamic efficiency.

d. <u>Two options are available to improve the economic efficiency of the system. Based on the Thermoeconomic indicators calculated in question c, select one of the given option and check whether its application allows to reduce the specific cost of flow E_30.</u>

   a. <u>Change process 1 by reducing its economic investment cost (Z_inv = 4.5 €/h) and its exergy efficiency (eta_ex = 0.35);</u>

b. **Change process 3 by increasing its economic cost (Z_inv = 7.5 €/h) and its exergy efficiency (eta_ex = 0.85);**

Based on the results of TE design evaluation, both the interventions may be suited to increase the efficiency of the process. On the other hand, process 1 presents larger values of both Z_inv+C_D and r indicators with respect to process 3. Therefore, the best choice results as intervention a, which lead to a decrease in economic cost of product E_30.

	Ex kW	c €/kWh	C €/h
E_01	153.8	0.040	4.79
E_02	175.8	0.014	2.46
E_03	38.5	0.010	0.38
E_13	53.8	0.198	10.65
E_23	61.5	0.073	4.46
E_30	**100.0**	**0.185**	**18.50**

Intervention b would increase the economic cost of product E_30:

	Ex kW	c €/kWh	C €/h
E_01	103.7	0.040	4.79
E_02	152.4	0.014	2.46
E_03	33.3	0.010	0.38
E_13	46.7	0.260	12.15
E_23	53.3	0.078	4.13
E_30	**100.0**	**0.241**	**24.1**

Exercise 3. (6pt)

Let us consider the energy statistics of Kazakhstan in 2013 in the table (IEA data).

Primary energy (ktoe)	Coal	Oil	Natural Gas	Hydro	Biomass	Electricity	Heat
Production	52439	85207	30693	665	66	-	-
Import	598	10237	4322	-	-	185	-
Export	14827	77142	8561	-	-	258	-
Inventory changes	-731	-137	-1046	-	-	-	-

Consider also the data related to the primary energy devoted to electricity and heat production, the gross electricity and the gross heat productions in the table below.

	Coal	Oil	Natural Gas	Hydro
Primary energy devoted to electricity and heat production (ktoe)	20918	169	2337	665
Gross electricity production (GWh)	77515	601	9515	-
Gross heat production (TJ)	392882	3807	0	-

Notice that the heat is produced only by CHP plants, and that the primary energy devoted to heat production is conventionally assumed as equal to the energy absorbed by conventional boilers with an energy efficiency of 90%.
It is required to calculate:

a. (2pt) The primary energy devoted to heat production;
b. (2pt) The primary energy devoted to electricity production and the electric penetration;
c. (2pt) The average efficiencies of electric energy production from coal, oil and natural gas.

Exercise 3. Solution

a. <u>The primary energy devoted to heat production;</u>

The primary energy devoted to heat production can be derived considering the definition of energy efficiency for the conventional boiler:

$$\eta_{I,\mathrm{Q}}=\frac{E_{heat}}{E_{primary,heat}} \rightarrow \quad E_{primary,heat}=\frac{E_{heat}}{\eta_{I,\mathrm{Q}}}=\begin{cases}E_{primary,heat,coal}=10426ktoe\\ E_{primary,heat,\mathrm{oil}}=101ktoe\\ E_{primary,heat,\mathrm{natgas}}=0ktoe\end{cases} \rightarrow \quad E_{primary,heat}=10527ktoe$$

Notice that to pass from TJ to ktoe a suited conversion is required.

b. <u>The primary energy devoted to electricity production and the electric penetration;</u>

The primary energy devoted to electricity production can be derived by difference:

$$E_{primary,electricity}=E_{primary,\mathrm{total}}-E_{primary,heat}=\begin{cases}10492ktoe\\ 68ktoe\\ 2337ktoe\\ 665ktoe\end{cases} \rightarrow \quad E_{primary,electricity}=13562ktoe$$

Notice that the hydro source produces electricity only.

The electric penetration is defined as the ratio between the primary energy devoted to electric energy production [ktoe] and the TPES [ktoe].
The TPES is evaluated as the sum of total production plus imports minus exports plus invent changes (pay attention to the signs!).

$$TPES = 81710\,ktoe$$

$$EP = \frac{\sum_i E_{P,i}}{TPES} = \frac{13562 ktoe}{81710\,ktoe} = 0.17$$

c. **The average efficiencies of electric energy production from coal, oil and natural gas.**

Average efficiencies are computed as the ratio between the electricity production and the primary energy devoted to electricity production:

$$\eta_{ee,\mathrm{i}} = \frac{E_{electricity,i}}{E_{primary,\mathrm{electricity,i}}} \quad \rightarrow \quad \begin{cases} \eta_{ee,\mathrm{coal}} = 0.64 ktoe \\ \eta_{ee,oil} = 0.76 ktoe \\ \eta_{ee,natgas} = 0.35 ktoe \end{cases}$$

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Exam – Advanced Thermodynamic and Thermoeconomics

Session of 21-06-2018

Exercise 1. (8pt)

Consider **a process** made by the following sequence: 1. an adiabatic **combustion chamber** where a complete and isobaric combustion of *acetylene* with *excess air* takes place. 2. after combustion, *flue gases* enter a **counter-flow heat exchanger** and are cooled at constant pressure heating a *flow of water* that enter as liquid and exit as *superheated steam*

The **main hypotheses and data** are below described:

- Acetylene , C_2H_2 has LHV=1256460 kJ/kmol and ex_{ch}=1269310 kJ/Kmole;
- Air and Acetylene enter the combustion chamber @ 353 K and environmental pressure.
- The adiabatic flame temperature results 2250 K and is associated to the given excess air.
- Flue gasses in the heat exchanger are cooled down @ 673 K,
- Water enter the heat exchanger @ 323 K and 500 kPa (h=209.3 kJ/kg, s=0.7027 kJ/kg-K)
- The superheated steam exits then @ 873 K and 500 kPa (h=3701 kJ/kg, s=8.3513 kJ/kg-K).
- Environmental temperature and Pressure are assumed to be T_0=298 K and p_0=101.3 kPa,
- For products P and reactants R useful data are below given:

	Δh_R (353 K) kJ/kmol	Δh_P (2250 K) kJ/kmol	Δh_P (673 K) kJ/kmol	$ex_{ph,R}$ (353 K) kJ/kmol	$ex_{ph,P}$ (2250 K) kJ/kmol	$ex_{ph,P}$ (673 K) kJ/kmol
C2H2	3101			260		
N2	1600	65205	11121	132	45905	3944
CO2		106640	16450		76029	5995
H2O		86162	13164		61636	4697
O2	1622	68771	11647	134	48453	4164

a. (2pt) Write the analytical formulation of the energy balance for the combustion process and derive the analytical expression for the excess air λ. Then calculate it. Do you expect any change in the excess air with the variation of the pressure within the combustion chamber? Please justify your answer [in case you are not able to proceed with this question, use λ=1,478]

b. (2pt) Write the exergy balance (per kmol of C_2H_2) for the combustion process, identifying all the necessary hypothesis and simplify it accordingly. Derive the analytical formulation for the rational exergy efficiency and then calculate it. Are these values functions of the reactants' temperature and pressure? Neglect the chemical exergy of products of combustion.

c. (2pt) Write the exergy balance to the heat exchanger, evaluate the exergy destructions (per kmol of C_2H_2) and the functional exergy efficiency considering water the useful product. And evaluate the mass flow rate of acetylene required to produce 180 kg/h of superheated steam. Neglect the chemical exergy of products of combustion.

d. (2pt) In case they were not given, please explain how you could calculate theoretically the chemical exergy for C2H2.

Exercise 1. Solution

a. Write the analytical formulation of the energy balance for the combustion process and derive the analytical expression for the excess air λ. Then calculate it. Do you expect any change in the excess air with the variation of the pressure within the combustion chamber? Please justify your answer? [in case you are not able to proceed with this question, use λ=1,478]

The combustion reaction can be written as follows:

$$C_2H_2 + \lambda \cdot 2.5\left(O_2 + 3.76 N_2\right) \rightarrow 2CO_2 + H_2O + \lambda \cdot 2.5 \cdot 3.76 N_2 + \left(\lambda - 1\right) \cdot 2.5 \cdot O_2$$

Assumptions for the application of the energy balance:

- The combustion chamber is adiabatic and rigid;
- Steady state operation is assumed throughout;
- The balance is written per unit of kmol of acetylene;

$$\frac{dE}{dt} = \sum \dot{W}^{\leftarrow} + \sum \dot{Q}^{\leftarrow} + \sum \dot{N}_R \bar{h}_R - \sum \dot{N}_P \bar{h}_P$$

The energy balance states that

$$\bar{h}_R = \bar{h}_P \qquad \text{[eq1]}$$

Enthalpies of reactants and products can be evaluated as follows:

$$\bar{h}_R\left(T_R, p_0\right) = 1\left(\bar{h}_f + \Delta\bar{h}\right)_{C2H2} + \lambda \cdot 2.5 \cdot \left[1\left(\cancel{\bar{h}_f} + \Delta\bar{h}\right)_{O2} + 3.76\left(\cancel{\bar{h}_f} + \Delta\bar{h}\right)_{N2}\right]$$

$$\bar{h}_P\left(T_P, p_0\right) = 2\left(\bar{h}_f + \Delta\bar{h}\right)_{CO2} + 1\left(\bar{h}_f + \Delta\bar{h}\right)_{H2O} + \lambda \cdot 2.5 \cdot 3.76\left(\cancel{\bar{h}_f} + \Delta\bar{h}\right)_{N2} + \left(\lambda - 1\right) \cdot 2.5 \cdot \left(\cancel{\bar{h}_f} + \Delta\bar{h}\right)_{O2}$$

So we obtain

$$\left(\bar{h}_f + \Delta\bar{h}\right)_{C2H2} + \lambda \cdot 2.5 \cdot \left[1\left(\Delta\bar{h}\right)_{O2} + 3.76\left(\Delta\bar{h}\right)_{N2}\right] = 2\left(\bar{h}_f + \Delta\bar{h}\right)_{CO2} + \left(\bar{h}_f + \Delta\bar{h}\right)_{H2O} + \lambda \cdot 2.5 \cdot 3.76\left(\Delta\bar{h}\right)_{N2} + \left(\lambda - 1\right) \cdot 2.5 \cdot \left(\Delta\bar{h}\right)_{O2}$$

By aggregating the remaining heat of formation we recognize the LHV

$$\underbrace{\bar{h}_{f,C2H2} - 2 \cdot \bar{h}_{f,CO2} - \bar{h}_{f,H2O}}_{LHV_{C2H2}} + \left(\Delta\bar{h}\right)_{C2H2} + \lambda \cdot 2.5 \cdot \left[1\left(\Delta\bar{h}\right)_{O2} + 3.76\left(\Delta\bar{h}\right)_{N2}\right] =$$
$$= 2\left(\Delta\bar{h}\right)_{CO2} + \left(\Delta\bar{h}\right)_{H2O} + \lambda \cdot 2.5 \cdot 3.76\left(\Delta\bar{h}\right)_{N2} + \left(\lambda - 1\right) \cdot 2.5 \cdot \left(\Delta\bar{h}\right)_{O2}$$

$$LHV_{C2H2} + \Delta\bar{h}_{C2H2}\left(T_R\right) + \lambda \cdot 2.5 \cdot \left[\Delta\bar{h}_{O2}\left(T_R\right) + 3.76 \cdot \Delta\bar{h}_{N2}\left(T_R\right)\right] =$$
$$= 2 \cdot \Delta\bar{h}_{CO2}\left(T_P\right) + \Delta\bar{h}_{H2O}\left(T_P\right) + \lambda \cdot 2.5 \cdot 3.76 \cdot \Delta\bar{h}_{N2}\left(T_P\right) + \left(\lambda - 1\right) \cdot 2.5 \cdot \Delta\bar{h}_{O2}\left(T_P\right)$$

$$LHV_{C2H2} + \Delta\bar{h}_{C2H2}\left(T_R\right) - 2 \cdot \Delta\bar{h}_{CO2}\left(T_P\right) - \Delta\bar{h}_{H2O}\left(T_P\right) + 2.5 \cdot \Delta\bar{h}_{O2}\left(T_P\right) =$$
$$= \lambda \cdot \left[2.5 + 3.76 \cdot \Delta\bar{h}_{N2}\left(T_P\right) + 2.5 \cdot \Delta\bar{h}_{O2}\left(T_P\right) - 2.5 \cdot \Delta\bar{h}_{O2}\left(T_R\right) - 2.5 \cdot 3.76 \cdot \Delta\bar{h}_{N2}\left(T_R\right)\right]$$

In the last equation, all the enthalpy differences can be derived from the table, and the only unknown is the excess air:

$$\lambda = \frac{LHV_{C2H2} + \Delta\bar{h}_{C2H2} + 2.5 \cdot \Delta\bar{h}_{O2}\left(T_P\right) - 2 \cdot \Delta\bar{h}_{CO2}\left(T_P\right) - \Delta\bar{h}_{H2O}\left(T_P\right)}{2.5 \cdot 3.76 \cdot \Delta\bar{h}_{N2}\left(T_P\right) + 2.5 \cdot \Delta\bar{h}_{O2}\left(T_P\right) - 2.5 \cdot \Delta\bar{h}_{O2}\left(T_R\right) - 2.5 \cdot 3.76 \cdot \Delta\bar{h}_{N2}\left(T_R\right)}$$

$$\lambda = \frac{1256460 + 3101 + 2.5\cdot 68771 - 2\cdot 106640 - 86162}{2.5\cdot 3.76\cdot 65205 + 2.5\cdot 68771 - 2.5\cdot 1622 - 2.5\cdot 3.76\cdot 1600} = 1.478$$

An increase in temperature of reactants results in an increase in the enthalpies of them: if the adiabatic flame temperature is kept constant, the excess air will decrease and the composition of the products will change.
A change in pressure do not affect the combustion process and the composition of the products.

As a reminder, recalling the definition of Lower Heating Value for a stoichiometric combustion reaction in air:

$$C_2H_2 + 2.5(O_2 + 3.76N_2) \rightarrow 2CO_2 + H_2O + 2.5\cdot 3.76N_2$$

$$LHV = \sum_R \nu_i \bar{h}_{f,i} - \sum_P \nu_j \bar{h}_{f,j} \rightarrow \begin{array}{l} LHV_{C2H2} = \bar{h}_{f,C2H2} + 2.5\left(\nu_{O2}\bar{h}_{f,O2} + \nu_{N2}\bar{h}_{f,N2}\right) - \left(\nu_{CO2}\bar{h}_{f,CO2} + \nu_{H2O}\bar{h}_{f,H2O} + 2.5\cdot\nu_{N2}\bar{h}_{f,N2}\right) \\ LHV_{C2H2} = \bar{h}_{f,C2H2} - \nu_{CO2}\bar{h}_{f,CO2} - \nu_{H2O}\bar{h}_{f,H2O} \\ LHV_{C2H2} = \bar{h}_{f,C2H2} - 2\cdot\bar{h}_{f,CO2} - 1\cdot\bar{h}_{f,H2O} \end{array}$$

b. Write the exergy balance ((per kmol of C_2H_2) for the combustion process, identifying all the necessary hypothesis and simplify it accordingly. Derive the analytical formulation for the rational exergy efficiency and then calculate it. Are these values functions of the reactants' temperature and pressure? Neglect the chemical exergy of products of combustion.

Assumptions for the application of the exergy balance:

- The combustion chamber is adiabatic and rigid;
- Steady state operation is assumed throughout;
- The balance is written per unit of kmol of acetylene;

$$\frac{dEx_A}{dt} = \sum_j \dot{E}x_{W,j}^{A\leftarrow} + \sum_k \dot{E}x_{Q,k}^{A\leftarrow} + \sum_i \left(\dot{m}^{A\leftarrow} ex\right)_i - \dot{E}x_{des} \quad \rightarrow \quad \overline{ex}_R\left(T_R, p_0\right) - \overline{ex}_P\left(T_P, p_0\right) - \overline{ex}_{D,comb} = 0$$

$$\overline{ex}_R\left(T_R, p_0\right) = \left[\overline{ex}_{ch} + \overline{ex}_{ph}\left(T_R, p_0\right)\right]_{C2H2} + \lambda\cdot 2.5\cdot\left[\overline{ex}_{ph,O2}\left(T_R, p_0\right) + 3.76\cdot\overline{ex}_{ph,N2}\left(T_R, p_0\right)\right] = 1271821.5\, kJ/kmol_{C2H2}$$

$$\overline{ex}_P\left(T_P, p_0\right) = \sum_P\left[\nu_i \overline{ex}_{ph,i}\left(T_P, p_0\right)\right] = 874781.1\, kJ/kmol_{C2H2}$$

$$\overline{ex}_{D,\mathrm{comb}} = 397040.4\, kJ/kmol_{C2H2} \quad \rightarrow \quad \eta_{ex,r} = \frac{\overline{ex}_P}{\overline{ex}_R} = 0.6878$$

Differently from the previous question, both temperature and pressure of reactants affects values of exergy destruction and exergy efficiency, since physical exergy of ideal gases is a function of both temperature and pressure.
Specifically, exergy destructions increase if temperature and pressure increase.

c. Write the exergy balance to the heat exchanger, evaluating the exergy destructions (per kmol of C_2H_2) and the functional exergy efficiency. Neglect the chemical exergy of products of combustion;

Before applying exergy balance, it is required to derive the ratio between the mass flow rate of water and the molar flow rate of flue gases. The heat released per molar unit of fuel is equal to:

$$\Delta\bar{h}_{fg}\left(2250K \rightarrow 673K, p_0\right) = \sum_P \nu_i\cdot\left[\Delta\bar{h}\left(2250K\right) - \Delta\bar{h}\left(673K\right)\right]_i = 1073257\, kJ/kmol_{C2H2}$$

The heat absorbed by a mass unit of water in the heat exchanger is equal to:

$$\Delta h_w\left(873K \rightarrow 323K, p_w\right) = h_{w,OUT} - h_{w,\mathrm{IN}} = 3491.9\, kJ/kg_w$$

Therefore by applying the energy balance

$$N\cdot\Delta\bar{h}_{fg}\left(2250K\rightarrow 673K, p_0\right) = M\cdot\Delta h_w\left(873K\rightarrow 323K, p_w\right)$$

We do not need to calculate both M and N since, by defining $r=\frac{M}{N}$ we get

$$r=\frac{\Delta\bar{h}_{fg}\left(2250K\rightarrow 673K, p_0\right)}{\Delta h_w\left(873K\rightarrow 323K, p_w\right)}=307.4kg_w/kmol_{C2H2}$$

Assumptions for the application of the exergy balance:

- The heat exchanger is adiabatic and rigid;
- Steady state operation is assumed throughout;
- The balance is written per unit of kmol of acetylene;

$$N\cdot\Delta ex_{fg}\left(2250K\rightarrow 673\,\mathrm{K}, p_0\right)-M\cdot\Delta ex_w\left(873K\rightarrow 323\,\mathrm{K}, p_w\right)-N\cdot ex_{D,\mathrm{HE}}=0$$

$$\Delta ex_{fg}\left(2250K\rightarrow 673\,\mathrm{K}, p_0\right)-r\cdot\Delta ex_w\left(873K\rightarrow 323\,\mathrm{K}, p_w\right)-\overline{ex}_{D,\mathrm{HE}}=0$$

$$\Delta ex_{fg}\left(2250K\rightarrow 673\,\mathrm{K}, p_0\right)=\sum_P \nu_i\cdot\left[ex\left(2250K, p_0\right)-ex\left(673K, p_0\right)\right]_i=833072.5\,kJ/kmol_{C2H2}$$

$$\Delta ex_w\left(873K\rightarrow 323\,\mathrm{K}, p_w\right)=\left[h_{w,\mathrm{OUT}}-h_{w,\mathrm{IN}}-T_0\left(s_{w,\mathrm{OUT}}-s_{w,\mathrm{IN}}\right)\right]kJ/kg_w=1211.5\,kJ/kg_w$$

$$\Delta ex_w\cdot r=372352.7\,kJ/kmol_{C2H2}$$

$$ex_{D,\mathrm{HE}}=460719.7\,kJ/kmol_{C2H2}$$

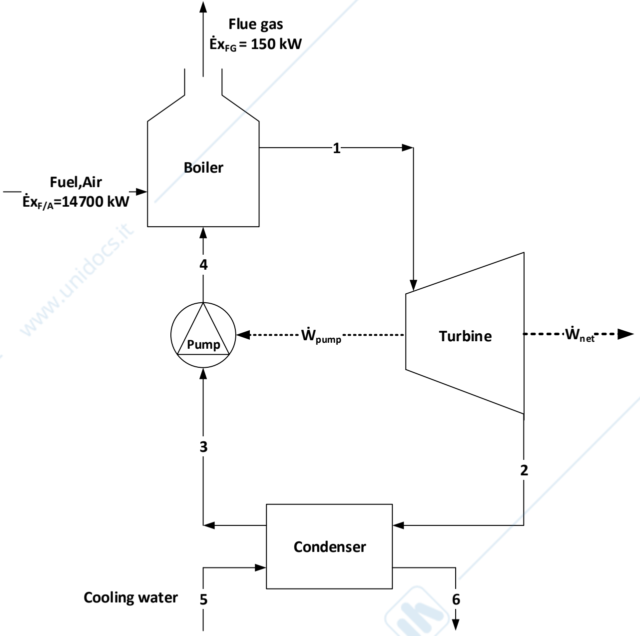
$$\eta_{ex,\mathrm{f}}=\frac{\overline{ex}_P}{\overline{ex}_F}=\frac{r\cdot\Delta ex_w\left(873K\rightarrow 323\,\mathrm{K}, p_w\right)}{\Delta ex_{fg}\left(2250K\rightarrow 673\,\mathrm{K}, p_0\right)}=0.4470$$

Evaluate the mass flow rate of acetylene required to produce 180 kg/h of superheated steam.

$$\dot{m}_{C2H2}=\dot{m}_w\cdot\frac{1}{3600}\cdot\frac{1}{r}\cdot MW_{C2H2}=180\frac{kg_w}{h}\cdot\frac{1}{3600}\frac{h}{s}\cdot\frac{1}{307}\frac{kmol_{C2H2}}{kg_w}\cdot 26\frac{kg_{C2H2}}{kmol_{C2H2}}=0.0042\frac{kg_{C2H2}}{s}$$

Exercise 2. (7pt)

Consider the power plant depicted in the figure with a **boiler,** a **turbine** and a **condenser**. The Boiler is fed by fuel and air and it produces steam which enter in the turbine. The turbine produces power for the grid and gives power to the pump. The condenser condenses the steam by using a second stream of cooling water. The pump close the cycle and allow the water to enter the boiler again.
Flue gases water (flow number 6) are released to the environment without any added cost.
Thermodynamic data, Physical data and Economic data are given in the below tables



#	T ºC	P bar	h kJ/kg	s kJ/kgK
1	520	100	3426.37	6.66
2	0.9	0.08	2335.97	7.46
3	0	0.08	173.84	0.59
4	43	100	188.83	0.61
5	20	-	83.91	0.30
6	35	-	146.63	0.51

Fuel,Air mass flow rate [kg/s]	1
Fuel,Air Exergy rate [kW]	14700
Flue gas Exergy rate [kW]	150
Net Power [Kw]	4208
T_0 [ºC]	20
P_0 [atm]	1

unit cost of Fuel,Air (cF/A)[€/GJ]	1
Z_boiler (Investment, O&M) [€/s]	0.25
Z_turbine (Investment, O&M) [€/s]	0.5
Z_condenser (Investment, O&M) [€/s]	0.15
Z_pump (Investment, O&M) [€/s]	0.05

It is requested to:

a. (1pt) Calculate the mass flow rate of steam and cooling water.
b. (1pt) Write the exergy balance for each component of the plant and for the whole system, and compute analytically, the exergy destruction rate, and fraction of exergy destruction, the rational exergy efficiency and the functional exergy efficiency (and make comment on their differences)
c. (3pt) Write the Thermoeconomic system of equations for each component and by making the proper assumption for auxiliary equations, derive the analytical cost structure for the product of each components. Then derive the analytical expression for the relative cost difference and the exergoeconomic factor for all the components.
d. (3 pt) Now you can calculate the specific cost of the product for each component, the relative cost difference and the exergoeconomic factors making some comments on the obtained results.

Exercise 2. (Solution)

a. Calculate the mass flow rate of steam and cooling water.

These can be derived by applying energy balance to turbine and condenser.

$$\dot{W}_{net} = \dot{m}_{st}\left[(h_1 - h_2) - (h_4 - h_3)\right]$$

$$\dot{m}_{st} = 3.91 \text{ kg/s}$$

$$\dot{m}_{st}(h_2 - h_3) = \dot{m}_{cw}(h_6 - h_5)$$

$$\dot{m}_{cw} = 134.89 \text{ kg/s}$$

b. Write the exergy balance for each component of the plant and for the whole system, and compute analytically, the exergy destruction rate, and fraction of exergy destruction, the rational exergy efficiency and the functional exergy efficiency (and make comment on their differences)

Boiler

$$\left\{\begin{array}{l} \dot{E}x_{F/A} + \dot{E}x_4 = \dot{E}x_1 + \dot{E}x_{FG} + \dot{E}x_{D,boiler} \;\rightarrow\; \eta_{ex,r,boiler} = \dfrac{\dot{E}x_1 + \dot{E}x_{FG}}{\dot{E}x_{F/A} + \dot{E}x_4} = 0.4015 \rightarrow \eta_{ex,f,boiler} = \dfrac{\dot{E}x_1 - \dot{E}x_4}{\dot{E}x_{F/A}} = 0.3892 \\ \dot{E}x_{D,boiler} = \dot{E}x_{F/A} + (\dot{E}x_4 - \dot{E}x_1) - \dot{E}x_{FG} \\ \dot{E}x_{D,boiler} = \dot{E}x_{F/A} + \dot{m}_{st}\left[(h_4 - h_1) - T_0(s_4 - s_1)\right] - \dot{E}x_{FG} = 8829.3kW \end{array}\right.$$

Turbine

$$\left\{\begin{array}{l} \dot{E}x_1 = \dot{E}x_2 + \dot{W}_{net} + \dot{W}_{pump} + \dot{E}x_{D,turbine} \;\rightarrow\; \eta_{ex,r,turbine} = \dfrac{\dot{E}x_2 + \dot{W}_{net} + \dot{W}_{pump}}{\dot{E}x_1} = 0.8413 \rightarrow\; \eta_{ex,f,turbine} = \dfrac{\dot{W}_{net}}{\dot{E}x_1 - \dot{E}x_2} = 0.8119 \\ \dot{E}x_{D,turbine} = (\dot{E}x_1 - \dot{E}x_2) - \dot{W}_{net} - \dot{W}_{pump} \\ \dot{E}x_{D,turbine} = \dot{m}_{st}\left[(h_1 - h_2) - T_0(s_1 - s_2)\right] - \dot{W}_{net} - \dot{m}_{st}(h_4 - h_3) = 916.44kW \end{array}\right.$$

Condenser

$$\left\{\begin{array}{l} \dot{E}x_2 + \dot{E}x_5 = \dot{E}x_3 + \dot{E}x_6 + \dot{E}x_{D,condenser} \;\rightarrow\; \eta_{ex,r,condenser} = \dfrac{\dot{E}x_3 + \dot{E}x_6}{\dot{E}x_2 + \dot{E}x_5} = 0.376 \rightarrow\; \eta_{ex,f,condenser} = \dfrac{\dot{E}x_3}{\dot{E}x_2} = 0.02 \\ \dot{E}x_{D,condenser} = (\dot{E}x_2 - \dot{E}x_3) + (\dot{E}x_5 - \dot{E}x_6) \\ \dot{E}x_{D,condenser} = \dot{m}_{st}\left[(h_2 - h_3) - T_0(s_2 - s_3)\right] + \dot{m}_{cw}\left[(h_5 - h_6) - T_0(s_5 - s_6)\right] = 368.58kW \end{array}\right.$$

Pump:

$$\left\{\begin{array}{l} \dot{E}x_3 + \dot{W}_{pump} = \dot{E}x_4 + \dot{E}x_{D,pump} \;\rightarrow\; \eta_{ex,r,pump} = \dfrac{\dot{E}x_4}{\dot{E}x_3 + \dot{W}_{pump}} = 0.7472 \rightarrow\; \eta_{ex,f,pump} = \dfrac{\dot{E}x_4 - \dot{E}x_3}{\dot{W}_{pump}} = 0.6940 \\ \dot{E}x_{D,pump} = (\dot{E}x_3 - \dot{E}x_4) + \dot{W}_{pump} \\ \dot{E}x_{D,pump} = \dot{m}_{st}\left[(h_3 - h_4) - T_0(s_3 - s_4)\right] + \dot{m}_{st}(h_4 - h_3) = 17.95kW \end{array}\right.$$

Whole plant

$$\begin{cases} \dot{E}x_{F/A} + \dot{E}x_5 = \dot{E}x_{FG} + \dot{E}x_6 + \dot{W}_{net} + \dot{E}x_{D,plant} \rightarrow \eta_{ex,r,plant} = \dfrac{\dot{E}x_{FG} + \dot{E}x_6 + \dot{W}_{net}}{\dot{E}x_{F/A} + \dot{E}x_5} = 0.3107 \rightarrow \eta_{ex,f,plant} = \dfrac{\dot{W}_{net}}{\dot{E}x_{F/A}} = 0.2863 \\ \dot{E}x_{D,plant} = \dot{E}x_{F/A} + (\dot{E}x_5 - \dot{E}x_6) - \dot{E}x_{FG} - \dot{W}_{net} \\ \dot{E}x_{D,plant} = \dot{E}x_{F/A} + \dot{m}_{cw}\left[(h_5 - h_6) - T_0(s_5 - s_6)\right] - \dot{E}x_{FG} - \dot{W}_{net} = 10132.27kW \end{cases}$$

Component	ηex,r	ηex,f	ExD	yD
Boiler	40.15	38.92	8829.30	0.87
Turbine	84.13	81.19	916.44	0.09
Condenser	37.60	36.27***	368.58	0.04
Pump	74.72	69.40	17.95	0.00
Plant	31.07	0.29	10132.27	1

*** not sure of the definition of functional efficiency for a condenser..

c. Write the Thermoeconomic system of equations for each component and by making the proper assumption for auxiliary equations, derive the analytical cost structure for the product of each components. Then derive the analytical expression for the relative cost difference and the exergoeconomic factor for all the components.

For each component it is required to derive the analytical cost structure for the product ;

Boiler:

$$\begin{cases} Ex_{F/A} + \dot{E}x_4 = \dot{E}x_1 + \dot{E}x_{FG} + \dot{E}x_{D,boiler} \\ \dot{C}_{F/A} + \dot{C}_4 + \dot{Z}_{boiler} = \dot{C}_1 + \dot{C}_{FG} \\ \dot{C} = c \cdot \dot{E}x \end{cases}$$

$$c_{F/A} \cdot \dot{E}x_{F/A} + c_4 \cdot \dot{E}x_4 + \dot{Z}_{boiler} = c \cdot \dot{E}x_1 + \cancel{c_{FG}} \cdot \dot{E}x_{FG}$$

$$\rightarrow \quad \bar{c}_{in,B} = \frac{c_4 \dot{E}x_4 + c_{F/A} \dot{E}x_{F/A}}{\dot{E}x_{F/A} + \dot{E}x_4}$$

$$c = \frac{c_{F/A} \dot{E}x_{F/A} + c_4 \dot{E}x_4 + \dot{Z}_{boiler} - c_{FG} \cdot \dot{E}x_{FG}}{\dot{E}x_1}$$

$$\boxed{c = \bar{c}_{in,B} + \bar{c}_{in,B} \cdot \left(\frac{\dot{E}x_{FG} + \dot{E}x_{D,boiler}}{\dot{E}x_1} \right) + \frac{\dot{Z}_{boiler}}{\dot{E}x_1}} \; ($$

$$\boxed{r_{boiler} = \frac{c - \bar{c}_{in,B}}{\bar{c}_{in,B}} = \left(\frac{\dot{E}x_{FG} + \dot{E}x_{D,boiler}}{\dot{E}x_1} \right) + \frac{\dot{Z}_{boiler}}{\bar{c}_{in,B} \cdot \dot{E}x_1}}$$

$$\boxed{f_{turbine} = \frac{\dot{Z}_{boiler}}{\dot{E}x_1} \Bigg/ \left(\frac{\bar{c}_{in,B} \left(\dot{E}x_{FG} + \dot{E}x_{D,boiler} \right)}{\dot{E}x_1} + \frac{\dot{Z}_{boiler}}{\dot{E}x_1} \right)}$$

Turbine

$$\begin{cases} \dot{E}x_1 = \dot{E}x_2 + \dot{W}_{net} + \dot{W}_{pump} + \dot{E}x_{D,turbine} \\ \dot{C}_1 + \dot{Z}_{turbine} = \dot{C}_2 + \dot{C}_{elec} \\ \dot{C} = c \cdot \dot{E}x \end{cases}$$

$$c \cdot \dot{E}x_1 + \dot{Z}_{turbine} = c \cdot \dot{E}x_2 + c_{elec} \cdot \dot{W}_{net} + c_{elec} \cdot \dot{W}_{pump}$$

$$\rightarrow \quad c_{elec} = \frac{c\left(\dot{E}x_1 - \dot{E}x_2\right) + \dot{Z}_{turbine}}{\dot{W}_{net} + \dot{W}_{pump}}$$

$$\boxed{c_{elec} = c + c\left(\frac{\dot{E}x_{D,turbine}}{\dot{W}_{net} + \dot{W}_{pump}}\right) + \frac{\dot{Z}_{turbine}}{\dot{W}_{net} + \dot{W}_{pump}}}$$

$$\boxed{r_{turbine} = \frac{c_{elec} - c}{c} = \left(\frac{\dot{E}x_{D,turbine}}{\dot{W}_{net} + \dot{W}_{pump}}\right) + \frac{\dot{Z}_{turbine}}{c\left(\dot{W}_{net} + \dot{W}_{pump}\right)}}$$

$$\boxed{f_{turbine} = \frac{\dot{Z}_{turbine}}{\left(\dot{W}_{net} + \dot{W}_{pump}\right)} \Bigg/ \left(\frac{c \cdot \dot{E}x_{D,turbine}}{\dot{W}_{net} + \dot{W}_{pump}} + \frac{\dot{Z}_{turbine}}{\left(\dot{W}_{net} + \dot{W}_{pump}\right)}\right)}$$

Condenser:

$$\begin{cases} \dot{E}x_2 + \dot{E}x_5 = \dot{E}x_3 + \dot{E}x_6 + \dot{E}x_{D,condenser} \\ \dot{C}_2 + \dot{C}_5 + \dot{Z}_{condenser} = \dot{C}_3 + \dot{C}_6 \\ C = c \cdot \dot{E}x \end{cases}$$

$$c \cdot \dot{E}x_2 + \cancel{c_5} \cdot \dot{E}x_5 + \dot{Z}_{condenser} = c \cdot \dot{E}x_3 + \cancel{c_6}^{\,0} \cdot \dot{E}x_6$$

$$\rightarrow \quad c_3 = \frac{c\dot{E}x_2 + \dot{Z}_{condenser}}{\dot{E}x_3}$$

$$\boxed{c_3 = c + c\frac{\dot{E}x_{D,condenser} + \dot{E}x_6}{\dot{E}x_3} + \frac{\dot{Z}_{condenser}}{\dot{E}x_3}}$$

$$\boxed{r_{condenser} = \frac{c_3 - c}{c} = \frac{\dot{E}x_{D,condenser} + \dot{E}x_6}{\dot{E}x_3} + \frac{\dot{Z}_{condenser}}{c \cdot \dot{E}x_3}}$$

$$\boxed{f_{condenser} = \frac{\dot{Z}_{condenser}}{\dot{E}x_3} \Bigg/ \left(\frac{c \cdot \left(\dot{E}x_{D,condenser} + \dot{E}x_6\right)}{\dot{E}x_3} + \frac{\dot{Z}_{condenser}}{\dot{E}x_3}\right)}$$

Pump:

$$\begin{cases} \dot{E}x_3 + \dot{W}_{pump} = \dot{E}x_4 + \dot{E}x_{D,pump} \\ \dot{C}_3 + \dot{C}_{elec} + \dot{Z}_{pump} = \dot{C}_4 \\ \dot{C} = c \cdot \dot{E}x \end{cases}$$

$$c_3 \cdot \dot{E}x_3 + c_{elec} \cdot \dot{W}_{pump} + \dot{Z}_{pump} = c_4 \cdot \dot{E}x_4$$

$$\rightarrow \quad \bar{c}_{in,pump} = \frac{c_3 \dot{E}x_3 + c_{elec} \dot{W}_{pump}}{\dot{E}x_3 + \dot{W}_{pump}}$$

$$\boxed{c_4 = \bar{c}_{in,pump} + \bar{c}_{in,pump} \frac{\dot{E}x_{D,pump}}{\dot{E}x_4} + \frac{\dot{Z}_{pump}}{\dot{E}x_4}}$$

$$\boxed{r_{pump} = \frac{c_4 - \bar{c}_{in,pump}}{\bar{c}_{in,pump}} = \frac{\dot{E}x_{D,pump}}{\dot{E}x_4} + \frac{\dot{Z}_{pump}}{\bar{c}_{in,pump} \cdot \dot{E}x_4}}$$

$$\boxed{f_{pump} = \frac{\dot{Z}_{pump}}{\dot{E}x_4} \Bigg/ \left(\bar{c}_{in,pump} \frac{\dot{E}x_{D,pump}}{\dot{E}x_4} + \frac{\dot{Z}_{pump}}{\dot{E}x_4} \right)}$$

d. Now you can calculate the specific cost of the product for each component, the relative cost difference and the exergoeconomic factors making some comments on the obtained results.

We can write the 4 exergy cost balances for the 4 components:

$$\begin{cases} c_{F/A} \cdot \dot{E}x_{F/A} + c_4 \cdot \dot{E}x_4 + \dot{Z}_{boiler} = c_1 \cdot \dot{E}x_1 + c_{FG} \cdot \dot{E}x_{FG} \\ c_1 \cdot \dot{E}x_1 + \dot{Z}_{turbine} = c_2 \cdot \dot{E}x_2 + c_{net} \cdot \dot{W}_{net} + c_{pump} \cdot \dot{W}_{pump} \\ c_2 \cdot \dot{E}x_2 + c_5 \cdot \dot{E}x_5 + \dot{Z}_{condenser} = c_3 \cdot \dot{E}x_3 + c_6 \cdot \dot{E}x_6 \\ c_3 \cdot \dot{E}x_3 + c_{pump} \cdot \dot{W}_{pump} + \dot{Z}_{pump} = c_4 \cdot \dot{E}x_4 \end{cases}$$

Exergy cost balance composed by 4 equation and 10 unknowns. Therefore, in order to close the system of equations, (*n-m*) numbers of auxiliary relations are required:
n: Number of exergy flows = 10
m: number of components = 4
n-m = 10 - 4 = 6

$$\text{Auxiliary relations} \begin{cases} 1)\ c_{FG} = 0 \\ 2)\ c_{F/A} = given \\ 3)\ c_{net} = c_{pump} = c_{elec.} \\ 4)\ c_5 = 0 \\ 5)\ c_6 = 0 \\ 6)\ c_1 = c_2 = c \end{cases}$$

Now substituting auxiliary relations in the economic cost balance:

$$\begin{cases} c_{F/A}\cdot \dot{E}x_{F/A} + c_4\cdot \dot{E}x_4 + \dot{Z}_{boiler} = c\cdot \dot{E}x_1 \\ c\cdot\dot{E}x_1 + \dot{Z}_{turbine} = c\cdot \dot{E}x_2 + c_{elec.}\cdot(\dot{W}_{net}+\dot{W}_{pump}) \\ c\cdot\dot{E}x_2+\dot{Z}_{condenser} = c_3\cdot\dot{E}x_3 \\ c_3\cdot\dot{E}x_3 + c_{elec.}\cdot\dot{W}_{pump}+\dot{Z}_{pump} = c_4\cdot\dot{E}x_4 \end{cases}$$

we have 4 equations and 4 unknowns (c,c_4, c_3, and $c_{elec.}$).
By coupling the third and the fourth equation we get

$$c\cdot\dot{E}x_2+\dot{Z}_{condenser}+c_{elec.}\cdot\dot{W}_{pump}+\dot{Z}_{pump}=c_4\cdot\dot{E}x_4$$

By inserting this last equation into the first equation in the system we get

$$c_{F/A}\cdot\dot{E}x_{F/A}+c\cdot\dot{E}x_2+\dot{Z}_{condenser}+c_{elec.}\cdot\dot{W}_{pump}+\dot{Z}_{pump}+\dot{Z}_{boiler}=c\cdot\dot{E}x_1$$

By using the second equation

$$c_{F/A}\cdot\dot{E}x_{F/A}+\dot{Z}_{condenser}+c_{elec.}\cdot\dot{W}_{pump}+\dot{Z}_{pump}+\dot{Z}_{boiler}+\dot{Z}_{turbine}=c_{elec.}\cdot(\dot{W}_{net}+\dot{W}_{pump})$$

$$c_{elec.}\dot{W}_{net}=c_{F/A}\cdot\dot{E}x_{F/A}+\dot{Z}_{pump}+\dot{Z}_{boiler}+\dot{Z}_{condenser}+\dot{Z}_{turbine}$$

$$c_{elec.}=\frac{c_{F/A}\cdot\dot{E}x_{F/A}+\dot{Z}_{pump}+\dot{Z}_{boiler}+\dot{Z}_{condenser}+\dot{Z}_{turbine}}{\dot{W}_{net}}$$

Please note that This equation is also coherent with the TE system written for the whole aggregate plants and would also lead to

$$c_{elec.}=c_{F/A}+c_{F/A}\frac{\dot{E}x_{DTOT}+\dot{E}x_{FG}+\dot{E}x_6}{\dot{W}_{net}}+\frac{\dot{Z}_{pump}+\dot{Z}_{boiler}+\dot{Z}_{condenser}+\dot{Z}_{turbine}}{\dot{W}_{net}}$$

e. Compute the values making some comments on the obtained results

$c_{elec.}$ [€/kWh]	0.82531
c [€/kWh]	0.33211
c_3 [€/kWh]	59.70660
c_4 [€/kWh]	18.18325

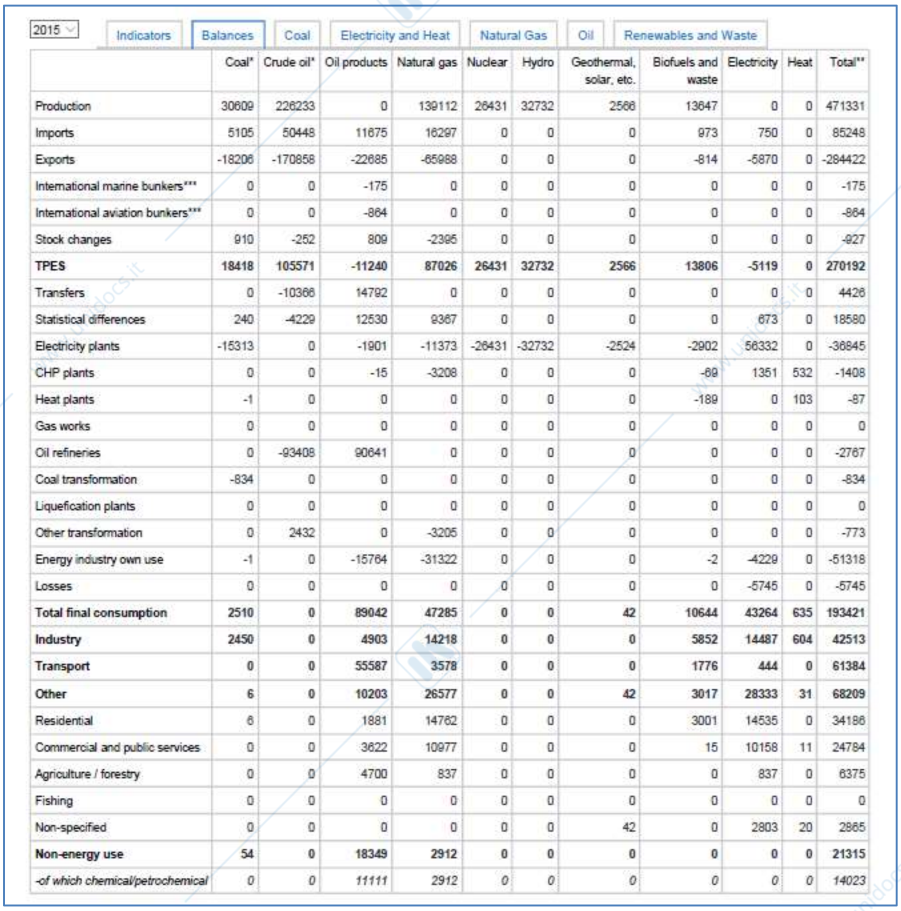
Cost structure for each component:

	Specific cost of the product (a)	Cost of fuel for the given component (b)	Cost of destruction and Losess (c)	Cost of Investment (d)
$c_{elec.}$ [€/kWh] TOT	0.82531	0.0036	0.00898	0.81274
$c_{elec.}$ [€/kWh]	0.82531	0.33211	0.07133	0.42187
c [€/kWh]	0.33211	0.06897	0.10726	0.15588
c_3 [€/kWh]	59,70660	0.33211	15.57724	43.79726
c_4 [€/kWh]	18.18325	11.05050	3.73956	3.39319

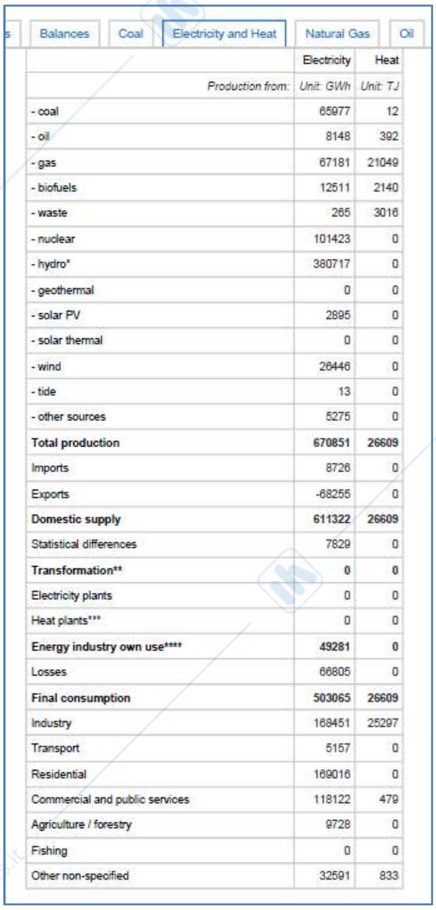
	Relative cost difference [(a)-(b)]/(b) Or [(c)+(d)]/(b)	Exergoeconomic factor (d)/[(c)+(d)]
TOT	228,2538	0,98908
Boiler	3,81533	0,59238
Turbine	1.48509	0.85537
Condenser	178.78182	0.73764
Pump	0.64547	0.47572

Exercise 3. (5pt)

Consider the energy statistics of Canada for 2015 in the tables below (IEA). The right end side table tis the overall energy balance , while the left end side is the electricity balance



	Coal*	Crude oil*	Oil products	Natural gas	Nuclear	Hydro	Geothermal, solar, etc.	Biofuels and waste	Electricity	Heat	Total**
Production	30609	226233	0	139112	26431	32732	2566	13647	0	0	471331
Imports	5105	50448	11675	16297	0	0	0	973	750	0	85248
Exports	-18206	-170858	-22685	-65988	0	0	0	-814	-5870	0	-284422
International marine bunkers***	0	0	-175	0	0	0	0	0	0	0	-175
International aviation bunkers***	0	0	-864	0	0	0	0	0	0	0	-864
Stock changes	910	-252	809	-2395	0	0	0	0	0	0	-927
TPES	**18418**	**105571**	**-11240**	**87026**	**26431**	**32732**	**2566**	**13806**	**-5119**	**0**	**270192**
Transfers	0	-10366	14792	0	0	0	0	0	0	0	4426
Statistical differences	240	-4229	12530	9367	0	0	0	0	673	0	18580
Electricity plants	-15313	0	-1901	-11373	-26431	-32732	-2524	-2902	56332	0	-36845
CHP plants	0	0	-15	-3208	0	0	0	-69	1351	532	-1408
Heat plants	-1	0	0	0	0	0	0	-189	0	103	-87
Gas works	0	0	0	0	0	0	0	0	0	0	0
Oil refineries	0	-93408	90641	0	0	0	0	0	0	0	-2767
Coal transformation	-834	0	0	0	0	0	0	0	0	0	-834
Liquefication plants	0	0	0	0	0	0	0	0	0	0	0
Other transformation	0	2432	0	-3205	0	0	0	0	0	0	-773
Energy industry own use	-1	0	-15764	-31322	0	0	0	-2	-4229	0	-51318
Losses	0	0	0	0	0	0	0	0	-5745	0	-5745
Total final consumption	**2510**	**0**	**89042**	**47285**	**0**	**0**	**42**	**10644**	**43264**	**635**	**193421**
Industry	**2450**	**0**	**4903**	**14218**	**0**	**0**	**0**	**5852**	**14487**	**604**	**42513**
Transport	**0**	**0**	**55587**	**3578**	**0**	**0**	**0**	**1776**	**444**	**0**	**61384**
Other	**6**	**0**	**10203**	**26577**	**0**	**0**	**42**	**3017**	**28333**	**31**	**68209**
Residential	6	0	1881	14762	0	0	0	3001	14535	0	34186
Commercial and public services	0	0	3622	10977	0	0	0	15	10158	11	24784
Agriculture / forestry	0	0	4700	837	0	0	0	0	837	0	6375
Fishing	0	0	0	0	0	0	0	0	0	0	0
Non-specified	0	0	0	0	0	0	42	0	2803	20	2865
Non-energy use	**54**	**0**	**18349**	**2912**	**0**	**0**	**0**	**0**	**0**	**0**	**21315**
-of which chemical/petrochemical	*0*	*0*	*11111*	*2912*	*0*	*0*	*0*	*0*	*0*	*0*	*14023*



	Electricity	Heat
Production from:	*Unit: GWh*	*Unit: TJ*
- coal	65977	12
- oil	8148	392
- gas	67181	21049
- biofuels	12511	2140
- waste	265	3016
- nuclear	101423	0
- hydro*	380717	0
- geothermal	0	0
- solar PV	2895	0
- solar thermal	0	0
- wind	26446	0
- tide	13	0
- other sources	5275	0
Total production	**670851**	**26609**
Imports	8726	0
Exports	-68255	0
Domestic supply	**611322**	**26609**
Statistical differences	7829	0
Transformation**	**0**	**0**
Electricity plants	0	0
Heat plants***	0	0
Energy industry own use******	**49281**	**0**
Losses	66805	0
Final consumption	**503065**	**26609**
Industry	168451	25297
Transport	5157	0
Residential	169016	0
Commercial and public services	118122	479
Agriculture / forestry	9728	0
Fishing	0	0
Other non-specified	32591	833

Please calculate the

a. (1pt) The share of renewable energy in both the primary mix and in the electricity production mix
b. (2 pt) primary energy devoted to electricity production and the electric penetration.
c. (2 pt) The average efficiency of the electric system in Canada

Exercise 3. Solution

a. (1pt) The share of renewable energy in both the primary mix and in the electricity production mix

Share in the primary mix $= \frac{\text{Renewable Primary Energy}}{TPES} = 0.18$

Share in the electricity production mix

question 1	prod [GWh]	share
coal	65977	0.10
oil	8148	0.01
gas	67181	0.10
biofuel	12511	0.02
waste	265	0.00
nuclear	101423	0.15
hydro	380717	0.57
geothermal	0	0.00
solar pv	2895	0.00
solar thermal	0	0.00
wind	26446	0.04
tide	13	0.00
other sources	5275	0.01
total fossil fuels	242729	0.36
total RES	428122	0.64
total	670851	1.00

Share of renewable in the energy production mix = 0.638

b. (2 pt) primary energy devoted to electricity production and the electric penetration

Primary energy devoted to electricity production: 96468 ktoe

TPES = 270192 ktoe

Electric penetration: the ratio between the primary energy devoted to electric energy production and the TPES

$$EP = \frac{\sum_i E_{P,i}}{TPES} = 0.357$$

c. (2 pt) The average efficiency of the electric system in Canada

Average efficiency of the electric system: the ration between the electricity production in ktoe and the primary energy devoted to electricity in ktoe.

Average efficiency = 0.6

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Milan, 1st August 2018

Exam – Advanced Thermodynamic and Thermoeconomics

Session of 24-07-2018

Exercise 1.

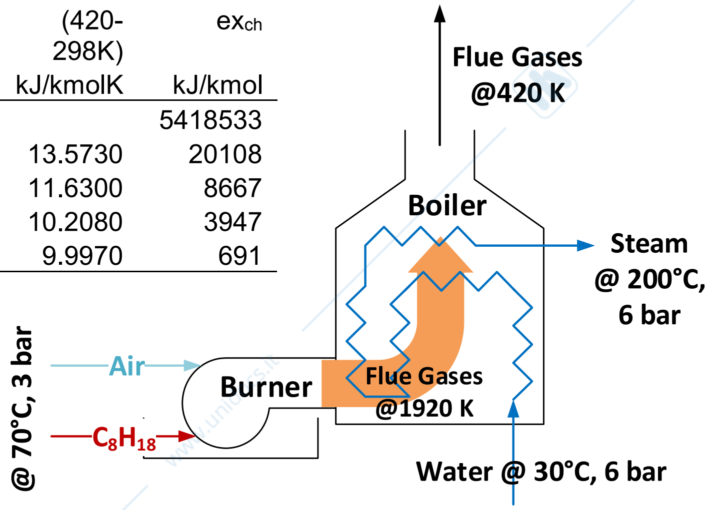
The system depicted in the figure is composed by a Burner and a Boiler. The Burner produces flue gases at 1920 K and 3 bar by burning gaseous propane with excess air λ equal to 1.446. Notice that fuel and air enter separately in the burner at the conditions of 70°C and 3 bar. Flue gases exit the Boiler at 420 K (pressure drop is negligible), and the Boiler produces 90 t/h of superheated steam. Properties of inlet/outlet bulk flows in the two components are reported in table 1 and 2.

Table 1

	h_f kJ/kmol	Δh (1920-298K) kJ/kmol	Δs (1920-298K) kJ/kmolK	Δh (420-298K) kJ/kmol	Δs (420-298K) kJ/kmolK	ex_{ch} kJ/kmol
C8H18	-208450					5418533
CO2	-393520	86631	93.0660	4842	13.5730	20108
H2O	-241820	68623	73.7770	4139	11.6300	8667
O2	0	56186	62.0820	3632	10.2080	3947
N2	0	53267	59.0000	3556	9.9970	691

Table 2

	Unit	Water	Steam
T	°C	30	200
p	bar	6	6
h	kJ/kg	126	2851
s	kJ/kgK	0.4366	6.9683



Isobaric heat capacities of the fuel and the air are respectively equal to 2.401 kJ/kg-K and 29.17 kJ/kmol-K. Considering the data provided in tables 1 and 2, and the environmental reference conditions of 25°C and 1 bar, it is required to:

a. Write the chemical combustion reaction. Then write and simplify the exergy balance for the burner and write the analytical expressions for the exergy of its reactants and products streams, including both the physical and the chemical components.
b. Derive numerical results (per unit of molar flow of C8H18) for point a.
c. Write the analytical expressions for the exergy destruction of the burner and for the rational exergy efficiency of the combustion process. Briefly discuss the possible ways to simplify the calculation of the rational exergy efficiency of the combustion process.
d. Derive numerical results (per unit of molar flow of C8H18) for point c.
e. Apply the energy balance to the boiler and determine the molar flow rate of fuel required to produce the mass flow rate of steam.

f. Write and simplify the exergy balance of the boiler. Then write the analytical expressions for the exergy destructions and for the functional exergy efficiency of the boiler (the exergy of flue gases and the change in exergy of the steam are considered as useful products). Finally derive numerical results.

Exercise 1. (Solution)

a. **Write the chemical combustion reaction. Then write and simplify the exergy balance for the burner and write the analytical expressions for the exergy of its reactants and products streams, including both the physical and the chemical components.**

The general formulation for the chemical reaction with excess air results as follows:

$$C_aH_b+\lambda\left(a+\frac{b}{4}\right)(O_2+3.76N_2)\rightarrow aCO_2+\frac{b}{2}H_2O+\lambda\left(a+\frac{b}{4}\right)3.76N_2+(\lambda-1)\left(a+\frac{b}{4}\right)O_2$$

The combustion reaction of Octane can be written as follows:

$$C_8H_{18}+\lambda\cdot 12.5(O_2+3.76N_2)\rightarrow 8CO_2+9H_2O+\lambda\cdot(12.5\cdot 3.76)N_2+(\lambda-1)\cdot 12.5\cdot O_2$$
$$C_8H_{18}+18.06(O_2+3.76N_2)\rightarrow 8CO_2+9H_2O+67.915N_2+5.563O_2$$

Assumptions for the application of the exergy balance:

- Adiabatic and steady state combustion process;
- No work flow exergy;
- The balance is written per unit of kmol of C8H18;

$$\frac{dEx_A}{dt}=\sum_j \dot{E}x_{W,j}^{A\leftarrow}+\sum_k \dot{E}x_{Q,k}^{A\leftarrow}+\sum_i\left(\dot{N}^{A\leftarrow}\overline{ex}\right)_i-\dot{E}x_{des}\qquad\rightarrow\qquad \overline{ex}_R(T_R,p_R)-\overline{ex}_P(T_P,p_P)-\overline{ex}_{D,comb}=0$$

According to the text, both physical and chemical exergies must be computed for reactants and products:

Analytical Formulation

$$\overline{ex}_R(T_R,p_R)=\overline{ex}_{ph}+\overline{ex}_{ch}$$

$$\overline{ex}_{ph}=\sum_i \nu_i\left[\overline{c}_{p,i}(T_1-T_0)-T_0\left(\overline{c}_{p,i}\ln\frac{T_1}{T_0}-R\ln\frac{p_1}{p_0}\right)\right]$$

$$\overline{ex}_{ch}=\underbrace{\nu_{C8H18}\overline{ex}_{ch,\mathrm{C8H18}}}_{tables}+\cancel{\nu_{air}\overline{ex}_{ch,air}}$$

$$\overline{ex}_P(T_P,p_P)=\overline{ex}_{ph}+\overline{ex}_{ch}$$

$$\overline{ex}_{ph}=\sum_i \nu_i\left\{\Delta\overline{h}(1920-298)-T_0\Delta\overline{s}(1920-298)+T_0R\ln\frac{p_1}{p_0}\right\}$$

$$\overline{ex}_{ch}=\nu_{tot,P}\cdot\left(\sum_i x_i ex_{ch,i}+T_0R\sum_i x_i\ln x_i\right)$$

b. Derive numerical results (per unit of molar flow of C8H18) for point a.

$$\overline{ex}_R\left(T_R,p_R\right)=\overline{ex}_{ph}+\overline{ex}_{ch}$$

$$\overline{ex}_{ph}=\sum_i \nu_i\left[\overline{c}_{p,i}\left(T_1-T_0\right)-T_0\left(\overline{c}_{p,i}\ln\frac{T_1}{T_0}-R\ln\frac{p_1}{p_0}\right)\right]=\left(1\cdot 3565+12.5\cdot\lambda\cdot 4.76\cdot 2813\right)kJ/kmol_{C3H8}=245818\,kJ/kmol_{C8H18}$$

$$\overline{ex}_{ch}=\underbrace{\nu_{C8H18}\overline{ex}_{ch,C8H18}}_{tables}+\cancel{\nu_{air}\overline{ex}_{ch,air}}=1\cdot 5418533=5418533\ kJ/kmol_{C8H18}$$

$$\rightarrow\quad \overline{ex}_R\left(T_R,p_R\right)=5664351\ kJ/kmol_{C8H18}$$

$$\overline{ex}_P\left(T_P,p_P\right)=\overline{ex}_{ph}+\overline{ex}_{ch}$$

$$\overline{ex}_{ph}=\sum_i \nu_i\left\{\Delta\overline{h}\left(1920-298\right)-T_0\Delta\overline{s}\left(1920-298\right)+T_0R\ln\frac{p_1}{p_0}\right\}=3897135\ kJ/kmol_{C8H18}$$

$$\overline{ex}_{ch}=\nu_{tot,P}\cdot\left(\sum_i x_i ex_{ch,i}+T_0R\sum_i x_i\ln x_i\right)=121372\ kJ/kmol_{C8H18}$$

$$\rightarrow \overline{ex}_P\left(T_P,p_P\right)=4018508\ kJ/kmol_{C8H18}$$

c. Write the analytical expression for the exergy destruction of the burner, and the rational exergy efficiency of the combuston process. Briefly discuss the possible ways to simplify the calculation of the rational exergy efficiency of the combustion process.

Following the exergy balance written in previous section, exergy destruction and rational exergy can be computed as follows:

$$\overline{ex}_{D,comb}=\overline{ex}_R\left(T_R,p_R\right)-\overline{ex}_P\left(T_P,p_P\right)\quad,\quad \eta_{ex,r}=\frac{\overline{ex}_P}{\overline{ex}_R}$$

One possible way of simplification is to ignore physical exergy of the reactants and chemical exergy of the products. Let see how the rational exergy efficiency varies:

$$\overline{ex}_R\left(T_R,p_R\right)=\cancel{\overline{ex}_{ph}}+\overline{ex}_{ch}$$

$$\overline{ex}_{ch}=\underbrace{\nu_{C8H18}\overline{ex}_{ch,C8H18}}_{tables}+\cancel{\nu_{air}\overline{ex}_{ch,air}}$$

$$\overline{ex}_P\left(T_P,p_P\right)=\overline{ex}_{ph}+\cancel{\overline{ex}_{ch}}$$

$$\overline{ex}_{ph}=\sum_i \nu_i\left\{\Delta\overline{h}\left(1920-298\right)-T_0\Delta\overline{s}\left(1920-298\right)+T_0R\ln\frac{p_1}{p_0}\right\}$$

d. Derive numerical results (per unit of molar flow of C8H18) for point c.

Results for obtained considering both physical and chemical exergy for products and reactants:

$$\overline{ex}_{D,comb}=1645844\ kJ/kmol_{C8H18}\quad,\quad \eta_{ex,r}=0.709$$

Simplify calculation in the abovementioned way it is possible to obtain a new value for exergy of products and reactants thanks to which a new value for exergy destruction and rational efficiency is computed.

$$\overline{ex}_R\left(T_R,p_R\right)=5418533\ kJ/kmol_{C8H18}$$

$$\overline{ex}_P\left(T_P,p_P\right)=3897135\ kJ/kmol_{C8H18}$$

$$\overline{ex}_{D,\text{comb}} = 1521398\ kJ/kmol_{C3H8} \qquad \rightarrow \qquad \eta_{ex,r} = \frac{\overline{ex}_P}{\overline{ex}_R} = 0.719$$

Neglecting those contributions results in slightly overestimation of rational efficiency. In this specific case (type of fuel, inlet and outlet condition…) the two terms can be neglected.

e. <u>**Apply the energy balance to the boiler, and determine the molar flow rate of fuel required to produce the mass flow rate of steam;**</u>

Assumptions for the application of the energy balance:

- Adiabatic and steady state combustion process;
- No work flow

$$\frac{dE}{dt} = \sum \dot{W}^{\leftarrow} + \sum \dot{Q}^{\leftarrow} + \Delta\dot{H}_{fg} + \Delta\dot{H}_w \quad \rightarrow \qquad \Delta\dot{H}_{fg} = \Delta\dot{H}_w$$

$$\dot{N}_{C8H18}\Delta h_{fg} - \dot{m}_{water}\Delta h_{water} = 0$$

$$\Delta\bar{h}_{fg} = \sum_i \nu_i\left[\Delta\bar{h}_i(1920-298) - \Delta\bar{h}_i(420-298)\right] = 5084036\ kJ/kmol_{C8H18}$$

$$\Delta h_{water} = h_{water,\text{out}} - h_{water,\text{in}} = 2724\ kJ/kg_{water}$$

$$\dot{N}_{C8H18} = \frac{\dot{m}_w\Delta h_w}{\Delta\bar{h}_{fg}} = \frac{25\,kg/s\cdot 2725\,kJ/kg}{5080860\,kJ/kmol_{C8H18}} = 0.013\,kmol_{C8H18}/s$$

f. <u>**Write and simplify the exergy balance of the boiler. Then the analytical expressions for the exergy destructions and the functional exergy efficiency of the boiler (the exergy of flue gases and the change in exergy of the steam are considered as useful products). Finally derive numerical results.**</u>

Assumptions for the application of the exergy balance:

- Steady state and adiabatic process is assumed for the boiler;

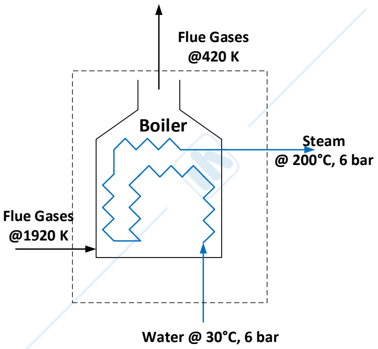
$$\dot{N}_{fg}\left[\overline{ex}_{fg}(1920K,3bar) - \overline{ex}_{fg}(420K,3bar)\right] - \dot{m}_{water}\Delta ex_{water}(200-30K,5bar) - \dot{E}x_{D,boiler} = 0$$

$$\overline{ex}(ph)_{fg,in}(1920K,3bar) = 3897135\,kJ/kmol_{C8H18} \qquad (already\ computed)$$

$$\rightarrow \dot{E}x(ph)_{fg,in} = \dot{N}_{C8H18}\cdot\overline{ex}(ph)_{fg}(1920K,3bar) = 52221kW$$

$$\overline{ex}_{fg,out}(420K,3bar) = \overline{ex}_{ph} + \overline{ex}_{ch}$$

$$\overline{ex}_{ph} = \sum_i \nu_i\left\{\Delta\bar{h}(420-298) - T_0\Delta\bar{s}(420-298) + T_0 R\ln\frac{p_1}{p_0}\right\} = 301328\,kJ/kmol_{C8H18}$$



$$\rightarrow \dot{E}x(ph)_{fg,out} = \dot{N}_{fg}\cdot\overline{ex}(ph)_{fg}(420K,3bar) = 4038\,kW$$

$$ex_{water}(200-30K,6bar) = h_{water,\text{out}} - h_{water,\text{in}} - T_0\left(s_{water,\text{out}} - s_{water,\text{in}}\right) = 776.89\,kJ/kg_{water}$$

$$\rightarrow \dot{E}x_{water} = 19439\,kW$$

$$\dot{E}x_{D,boiler} = 28743kW \qquad \rightarrow \qquad \eta_{ex,f} = \frac{\dot{E}x_P}{\dot{E}x_R} = \frac{\dot{E}x_{water} + \dot{E}x_{fg,out}}{\dot{E}x_{fg,in}} = 0.45$$

Exercise 2.

Let’s consider a simple Energy Conversion System (ECS) enclosed in the control volume depicted in the figure (dashed line). The system composed by 2 micro-CHP systems (a and b), and one absorption chiller (c), exchanging mass and energy flows between themselves and environment. Two micro-CHP systems receive primary energy from outside of the ECS’s boundary, producing heat and electricity. Electricity is used for the residential buildings, while Heat is used for the absorption chiller to provide cooling load for the buildings. Streams number 8 (flue gases) and 9 (steam leakage) are released to the environment without any additional cost.

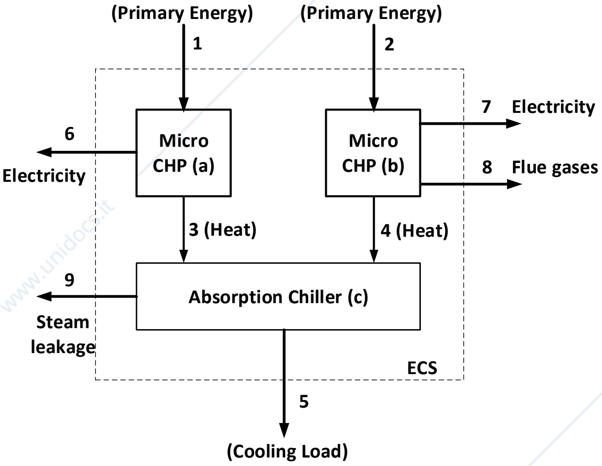


Table 1

State		Ex [MW]
1	Primary Energy	100
2	Primary Energy	100
3	Heat	30
4	Heat	25
5	Cooling Load	25
6	Electricity	40
7	Electricity	20
8	Flue gases	5
9	Steam leakage	5

Table 2

unit cost of Fuel (c_{1},c_{2}) [€/GJ]	8.333
Z_CHP a (Investment, O&M) [€/s]	0.2
Z_CHP b (Investment, O&M) [€/s]	0.13
Z_Absorption Chiller (Investment, O&M) [€/s]	0.1

With reference to the data provided by the tables, it is required to:

a. Write the exergy balance and compute exergy destruction of each component.
b. Derive the number and the type of auxiliary relations required to solve the Thermoeconomic system of equations, properly justifying your choices.
c. Write one Thermoeconomic system of equations for each component and derive the cost structure for each of them
d. Calculate the numerical values of the unit economic costs of each material/energy flow. Collect numerical results in one table.
e. Based on the cost structure in point c. compute the economic cost of the exergy destructions, Exergoeconomic factor and relative cost difference for each component. Collect numerical results in one table and comment on the obtained results.
f. Solve the system at the highest level of aggregation (1 unique component having different products) and calculate the new structure of the cost. Based on the analytical formulation obtained for the products in point c. please make some comments on their difference.

Exercise 2. (Solution)

a. <u>Write the exergy balance and compute exergy destruction of each component.</u>

Exergy Balance for each component:

$$\begin{cases} a)\ \dot{E}x_1 = \dot{E}x_3 + \dot{E}x_6 + \dot{E}x_{D,a} & \rightarrow \quad \dot{E}x_{D,a} = 30\ MW \\ b)\ \dot{E}x_2 = \dot{E}x_4 + \dot{E}x_7 + \dot{E}x_8 + \dot{E}x_{D,b} & \rightarrow \quad \dot{E}x_{D,b} = 50\ MW \\ c)\ \dot{E}x_3 + \dot{E}x_4 = \dot{E}x_5 + \dot{E}x_9 + \dot{E}x_{D,c} & \rightarrow \quad \dot{E}x_{D,c} = 25\ MW \end{cases}$$

b. Derive the number and the type of auxiliary relations required to solve the Thermoeconomic system of equations, properly justifying your choices.

The system composed by 9 streams and 3 components, therefore, in order to define and close Thermoeconomic system of equations, (*n-m*) numbers of auxiliary relations are required:
n: Number of exergy flows = 9, *m*: number of components = 3 : *n-m = 9 - 3 = 6*

Auxiliary relations:
$$\begin{array}{lll} 1)\ c_1 = given, & 3)\ c_3 = c_6 = c_a, & 5)\ c_8 = 0 \\ 2)\ c_2 = given, & 4)\ c_4 = c_7 = c_b, & 6)\ c_9 = 0 \end{array}$$

c. Write one Thermoeconomic system of equations for each component and derive the cost structure for each of them.

For each component it is required to write economic cost balance based on Inlet/Outlet paradigm and substituting exergy cost relation as follows;

$$\begin{cases} \sum_{in} \dot{C}_{in} + \dot{Z}_{component} = \sum_{out} \dot{C}_{out} & ;Economic\ cost\ balance \\ \dot{C} = c \cdot \dot{E}x & ;Exergy\ cost\ relation \end{cases}$$

CHP 1:

$$\begin{cases} \dot{E}x_1 = \dot{E}x_3 + \dot{E}x_6 + \dot{E}x_{D,a} \\ c_1 \cdot \dot{E}x_1 + \dot{Z}_a = c_3 \cdot \dot{E}x_3 + c_6 \cdot \dot{E}x_6 \to c_1 \cdot \dot{E}x_1 + \dot{Z}_a = c_a \cdot \left(\dot{E}x_3 + \dot{E}x_6\right) \end{cases}$$

In order to compute cost structure of the outlet flow of CHP1 ($\mathbf{c_a}$), it is enough to substitute ($\dot{E}x_1$) in the above formula for ($\mathbf{c_a}$), with its equivalent expression derived from exergy balance. Therefore, the cost structure of the electric power produced by CHP1 can be expressed as follows

$$c_a = \frac{c_1 \cdot \dot{E}x_1 + \dot{Z}_a}{\dot{E}x_3 + \dot{E}x_6} = \frac{c_1 \cdot (\dot{E}x_3 + \dot{E}x_6 + \dot{E}x_{D,a}) + \dot{Z}_a}{(\dot{E}x_3 + \dot{E}x_6)}$$

$$\to \boxed{c_a = \underset{inlet}{c_1} + c_1 \cdot \underbrace{\frac{\dot{E}x_{D,a}}{(\dot{E}x_3 + \dot{E}x_6)}}_{ExergyDestruction} + \underbrace{\frac{\dot{Z}_a}{(\dot{E}x_3 + \dot{E}x_6)}}_{Investment\ Cost}}$$

CHP 2: same procedure of CHP 1

$$\begin{cases} \dot{E}x_2 = \dot{E}x_4 + \dot{E}x_7 + \dot{E}x_8 + \dot{E}x_{D,b} \\ c_2 \cdot \dot{E}x_2 + \dot{Z}_b = c_4 \cdot \dot{E}x_4 + c_7 \cdot \dot{E}x_7 + \cancel{c_8} \cdot \dot{E}x_8 \to c_2 \cdot \dot{E}x_2 + \dot{Z}_b = c_b \cdot \left(\dot{E}x_4 + \dot{E}x_7\right) \end{cases}$$

$$c_b = \frac{c_2 \cdot \dot{E}x_2 + \dot{Z}_b}{\dot{E}x_4 + \dot{E}x_7} = \frac{c_2 \cdot (\dot{E}x_4 + \dot{E}x_7 + \dot{E}x_8 + \dot{E}x_{D,b}) + \dot{Z}_b}{(\dot{E}x_4 + \dot{E}x_7)}$$

$$\to \boxed{c_b = \underset{inlet}{c_2} + c_2 \cdot \underbrace{\frac{\dot{E}x_8 + \dot{E}x_{D,b}}{(\dot{E}x_4 + \dot{E}x_7)}}_{ExergyDestruction} + \underbrace{\frac{\dot{Z}_b}{(\dot{E}x_4 + \dot{E}x_7)}}_{Investment\ Cost}}$$

Absorption Chiller:

$$\begin{cases} \dot{E}x_3 + \dot{E}x_4 = \dot{E}x_5 + \dot{E}x_9 + \dot{E}x_{D,c} \\ c_3 \cdot \dot{E}x_3 + c_4 \cdot \dot{E}x_4 + \dot{Z}_c = c_5 \cdot \dot{E}x_5 + \cancel{c_9} \cdot \dot{E}x_9 \to c_a \cdot \dot{E}x_3 + c_b \cdot \dot{E}x_4 + \dot{Z}_c = c_5 \cdot \dot{E}x_5 \end{cases}$$

since there are two inlet streams, a weighted average of the economic cost needs to be computed:

$$c_{in,c} = \frac{c_a \cdot \dot{E}x_3 + c_b \cdot \dot{E}x_4}{\dot{E}x_3 + \dot{E}x_4}$$

Then, the formula related to the computation of c_5 can be rearranged as follows:

$$c_5 = \frac{c_a \cdot \dot{E}x_3 + c_b \cdot \dot{E}x_4 + \dot{Z}_c}{\dot{E}x_5} = \frac{c_{in,c} \cdot (\dot{E}x_3 + \dot{E}x_4) + \dot{Z}_c}{\dot{E}x_5}$$

Now, $(\dot{E}x_3 + \dot{E}x_4)$ in the above formula should be substituted with its equivalent expression derived from exergy balance. Therefore, the cost structure of the cooling energy produced by absorption chiller can be expressed as follows:

$$c_5 = \frac{c_{in,c} \cdot (\dot{E}x_3 + \dot{E}x_4) + \dot{Z}_c}{\dot{E}x_5} \rightarrow \frac{c_{in,c} \cdot (\dot{E}x_5 + \dot{E}x_9 + \dot{E}x_{D,c}) + \dot{Z}_c}{\dot{E}x_5}$$

$$\rightarrow \boxed{c_5 = \underbrace{c_{in,c}}_{inlet} + \underbrace{c_{in,c} \cdot \frac{\dot{E}x_9 + \dot{E}x_{D,c}}{\dot{E}x_5}}_{ExergyDestruction} + \underbrace{\frac{\dot{Z}_c}{\dot{E}x_5}}_{Investment\ Cost}}$$

d. **Calculate the numerical values of the unit economic costs of each material/energy flow. Collect numerical results in one table.**

Starting from the cost structure of each component it is possible to compute the cost of outlet for each component

Component	c_in	c_dest	c_inv	c_out
CHPa	30.00	12.86	10.29	53.14
CHPb	30.00	36.67	10.40	77.06
Absorption Chiller	64.02	76.82	14.40	155.23

It is now possible to determine the cost of each flow.

c_1 [€/MWh]	c_2 [€/MWh]	c_3 [€/MWh]	c_4 [€/MWh]	c_5 [€/MWh]	c_6 [€/MWh]	c_7 [€/MWh]	$c_{8,9}$ [€/MWh]
30	30	53.14	77.06	155.23	53.14	77.06	0

e. **Based on the cost structure in point c. compute the economic cost of the exergy destructions, Exergoeconomic factor and relative cost difference for each component. Collect numerical results in one table and comment on the obtained results.**

The economic cost of the exergy destruction is computed as follows:

$$\dot{C}_{D,j} = \frac{\bar{c}_{in,j}\left(\dot{E}x_{loss,j} + \dot{E}x_{D,j}\right)}{\dot{E}x_j}$$

Exergoeconomic factor is computed as:

$$f_j = \frac{\dot{Z}_j}{\dot{Z}_j + \dot{C}_{D,j}} = f_j = \frac{\dot{Z}_j}{\dot{E}x_j} \Big/ \left(\frac{\bar{c}_{in,j}\left(\dot{E}x_{loss,j} + \dot{E}x_{D,j}\right)}{\dot{E}x_j} + \frac{\dot{Z}_j}{\dot{E}x_j} \right) =$$

Relative cost difference is computed as:

$$r_j = \frac{c_{out,j} - c_{in,j}}{c_{in,j}} = \left(\frac{\dot{E}x_{loss,j} + \dot{E}x_{D,j}}{\dot{E}x_{in,j}} \right) + \frac{\dot{Z}_j}{\bar{c}_{in,j} \cdot \dot{E}x_{in,j}}$$

Component	Ex_D	Cost of Ex_D	f	r
a	30	900	0.44	0.77
b	55	1650	0.22	1.31
c	25	1725	0.17	1.42

Comments:

- According to the exergy destruction, component b is most responsible component in total exergy destruction of the system, then component a and c respectively. It is interesting that the order of components according to the economic cost of the exergy destruction is changing: component c has the highest rank and needs to be improved prior to the other components. Economic cost of the exergy destruction indicates the economic expenses that caused by irreversibilities within the considered component.
- Exergoeconomic factor indicates whether the capital and O&M expenses ($\dot{Z}$) is the major source of economic cost increase, or it is the economic cost of exergy destruction ($\dot{C}_D$). It takes values between zero and one: high values indicate ($\dot{Z}$) is the major cost source and the primary aim is to reduce cost of the products by reduction of investment and O&M costs. While low values indicate exergy destruction and losses are primary sources of product cost increment. Therefore, increasing the component efficiency even if it requires higher capital investment is worthy. Base on such explanation, component c has le low value of Exergoeconomic factor and needs to be improved by increasing its efficiency.
- Relative cost difference: High value of this parameter means there is more room for improvement. Based on that, component c is again the first candidate to be improved, followed by component b and a.

f. **Solve the system at the highest level of aggregation (1 unique component having different products) and calculate the new structure of the cost. Based on the analytical formulation obtained for the products in point c. please make some comments on their difference.**

$$\begin{cases} \dot{E}x_1 + \dot{E}x_2 = \dot{E}x_5 + \dot{E}x_6 + \dot{E}x_7 + \dot{E}x_8 + \dot{E}x_9 + \dot{E}x_{D,tot} \\ c_1 \cdot \dot{E}x_1 + c_2 \cdot \dot{E}x_2 + \dot{Z}_{tot} = c_5 \cdot \dot{E}x_5 + c_6 \cdot \dot{E}x_6 + c_7 \cdot \dot{E}x_7 + \cancel{c_8} \cdot \dot{E}x_8 + \cancel{c_9} \cdot \dot{E}x_9 \end{cases}$$

$$c_1 \cdot \left(\dot{E}x_1 + \dot{E}x_2 \right) + \dot{Z}_{tot} = c_{products} \cdot \left(\dot{E}x_5 + \dot{E}x_6 + \dot{E}x_7 \right)$$

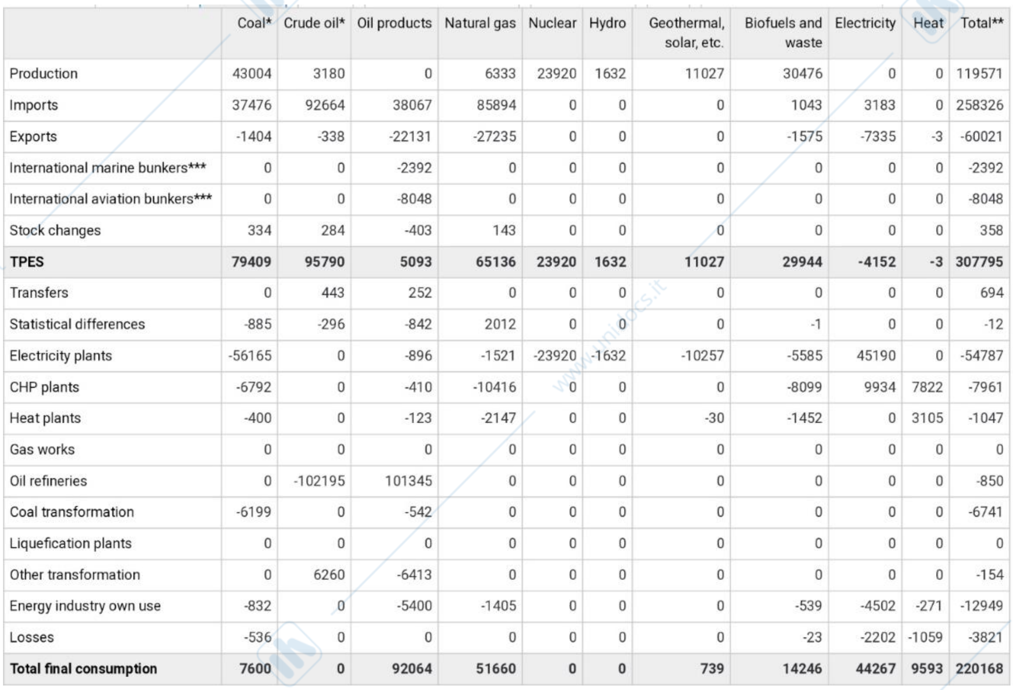
$$c_1 \cdot \left(\dot{E}x_5 + \dot{E}x_6 + \dot{E}x_7 + \dot{E}x_8 + \dot{E}x_9 + \dot{E}x_{D,tot} \right) + \dot{Z}_{tot} = c_{products} \cdot \left(\dot{E}x_5 + \dot{E}x_6 + \dot{E}x_7 \right)$$

$$c_{products} = c_1 + \frac{\dot{E}x_8 + \dot{E}x_9 + \dot{E}x_{D,tot}}{\dot{E}x_5 + \dot{E}x_6 + \dot{E}x_7} + \frac{\dot{Z}_a + \dot{Z}_b + \dot{Z}_c}{\dot{E}x_5 + \dot{E}x_6 + \dot{E}x_7}$$

Exercise 3.

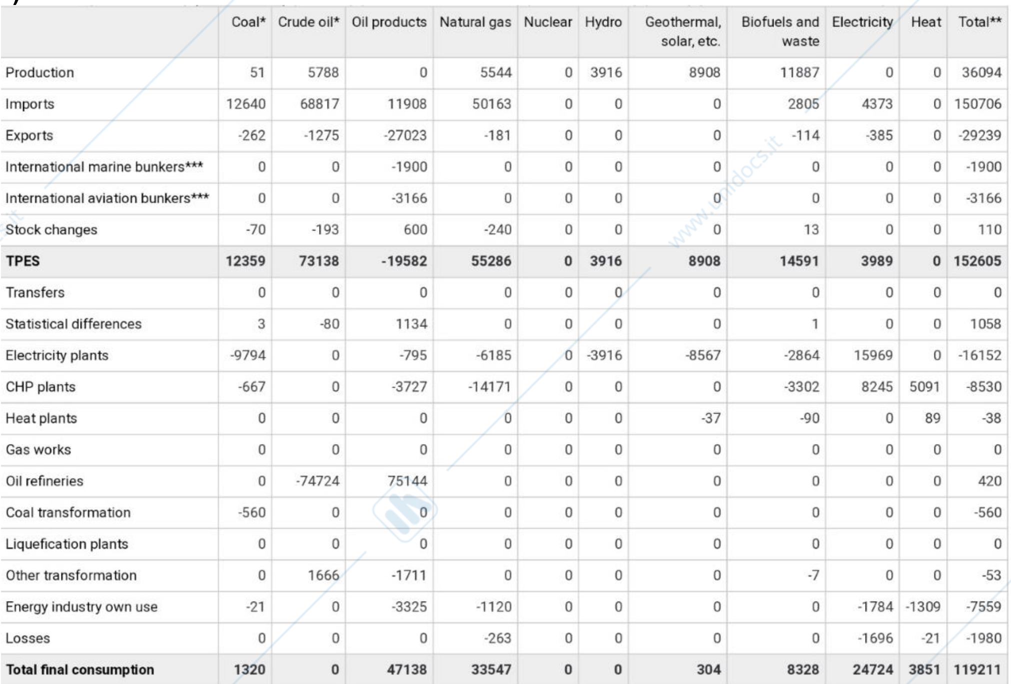
Consider the energy statistics of Germany and Italy for 2015 in the tables below (IEA). Please consider also that Emissions are 730 Mton for Germany and 331 for Italy, while GDP are 3697 billion USD for Germany and 2060 for Italy).

GERMANY (ktoe)



	Coal*	Crude oil*	Oil products	Natural gas	Nuclear	Hydro	Geothermal, solar, etc.	Biofuels and waste	Electricity	Heat	Total**
Production	43004	3180	0	6333	23920	1632	11027	30476	0	0	119571
Imports	37476	92664	38067	85894	0	0	0	1043	3183	0	258326
Exports	-1404	-338	-22131	-27235	0	0	0	-1575	-7335	-3	-60021
International marine bunkers***	0	0	-2392	0	0	0	0	0	0	0	-2392
International aviation bunkers***	0	0	-8048	0	0	0	0	0	0	0	-8048
Stock changes	334	284	-403	143	0	0	0	0	0	0	358
TPES	**79409**	**95790**	**5093**	**65136**	**23920**	**1632**	**11027**	**29944**	**-4152**	**-3**	**307795**
Transfers	0	443	252	0	0	0	0	0	0	0	694
Statistical differences	-885	-296	-842	2012	0	0	0	-1	0	0	-12
Electricity plants	-56165	0	-896	-1521	-23920	-1632	-10257	-5585	45190	0	-54787
CHP plants	-6792	0	-410	-10416	0	0	0	-8099	9934	7822	-7961
Heat plants	-400	0	-123	-2147	0	0	-30	-1452	0	3105	-1047
Gas works	0	0	0	0	0	0	0	0	0	0	0
Oil refineries	0	-102195	101345	0	0	0	0	0	0	0	-850
Coal transformation	-6199	0	-542	0	0	0	0	0	0	0	-6741
Liquefication plants	0	0	0	0	0	0	0	0	0	0	0
Other transformation	0	6260	-6413	0	0	0	0	0	0	0	-154
Energy industry own use	-832	0	-5400	-1405	0	0	0	-539	-4502	-271	-12949
Losses	-536	0	0	0	0	0	0	-23	-2202	-1059	-3821
Total final consumption	**7600**	**0**	**92064**	**51660**	**0**	**0**	**739**	**14246**	**44267**	**9593**	**220168**

ITALY (ktoe)



	Coal*	Crude oil*	Oil products	Natural gas	Nuclear	Hydro	Geothermal, solar, etc.	Biofuels and waste	Electricity	Heat	Total**
Production	51	5788	0	5544	0	3916	8908	11887	0	0	36094
Imports	12640	68817	11908	50163	0	0	0	2805	4373	0	150706
Exports	-262	-1275	-27023	-181	0	0	0	-114	-385	0	-29239
International marine bunkers***	0	0	-1900	0	0	0	0	0	0	0	-1900
International aviation bunkers***	0	0	-3166	0	0	0	0	0	0	0	-3166
Stock changes	-70	-193	600	-240	0	0	0	13	0	0	110
TPES	**12359**	**73138**	**-19582**	**55286**	**0**	**3916**	**8908**	**14591**	**3989**	**0**	**152605**
Transfers	0	0	0	0	0	0	0	0	0	0	0
Statistical differences	3	-80	1134	0	0	0	0	1	0	0	1058
Electricity plants	-9794	0	-795	-6185	0	-3916	-8567	-2864	15969	0	-16152
CHP plants	-667	0	-3727	-14171	0	0	0	-3302	8245	5091	-8530
Heat plants	0	0	0	0	0	0	-37	-90	0	89	-38
Gas works	0	0	0	0	0	0	0	0	0	0	0
Oil refineries	0	-74724	75144	0	0	0	0	0	0	0	420
Coal transformation	-560	0	0	0	0	0	0	0	0	0	-560
Liquefication plants	0	0	0	0	0	0	0	0	0	0	0
Other transformation	0	1666	-1711	0	0	0	0	-7	0	0	-53
Energy industry own use	-21	0	-3325	-1120	0	0	0	0	-1784	-1309	-7559
Losses	0	0	0	-263	0	0	0	0	-1696	-21	-1980
Total final consumption	**1320**	**0**	**47138**	**33547**	**0**	**0**	**304**	**8328**	**24724**	**3851**	**119211**

Please comment on the following phrases: Are they true or false. Your judgment needs to be supported by quantitative data

a. "Germany is more advanced than Italy in renewable energy penetration"
b. "Energy & CO2 intensity are better in Germany than Italy" [toe/thousand of USD and kg C02/USD]

c. “Overall efficiency of the Electricity Sector is higher in Italy than Germany”

Exercise 3. Solution

a. “Germany is more advanced than Italy in renewable energy penetration”

This is not true since the share of the RES is greater in Italy

Share in the primary mix for italy $= \dfrac{\text{Renewable Primary Energy}}{TPES} = \dfrac{3916+8908+14591}{152605} = 0.18$

Share in the primary mix for Germany $= \dfrac{\text{Renewable Primary Energy}}{TPES} = \dfrac{1632+11027+29944}{307795} = 0.14$

b. “Energy and CO2 intensity are better in Germany than Italy”

This is not true since the two indicators are smaller for Italy. This means that Italy is able to produce a unit of GDP using less primary energy and emitting less CO2 with respect to Germany.

Energy intensity for Italy $= \dfrac{\text{TPES}}{GDP} = \dfrac{152605*10\hat{}3}{2060*10\hat{}6} = 0.074\left[toe/kUSD\right]$

Energy Intensity for Germany $= \dfrac{\text{TPES}}{GDP} = \dfrac{307795*10\hat{}3}{3697*10\hat{}6} = 0.083\left[toe/kUSD\right]$

Co2 intensity for Italy $= \dfrac{\text{CO2}}{GDP} = \dfrac{331*10\hat{}9}{2060*10\hat{}9} = 0.161\left[kg/USD\right]$

Co2 intensity for Germany $= \dfrac{\text{CO2}}{GDP} = \dfrac{730*10\hat{}9}{3697*10\hat{}9} = 0.198\left[kg/USD\right]$

c. Overall efficiency of the Electricity Sector is higher in Italy than Germany

Average efficiency of Italy:

$$\eta = \frac{\text{Electricity \& CHP production in ktoe - own use - losses}}{Electricity\_tranformation\_withCHP} =$$

$$= \frac{15969+8245-1784-1696}{9794+795+6185+3916+8567+2864+667+3727+14171+3302} = 0.38$$

Average efficiency of Germany:

$$\eta = \frac{45190+9934-4502-2202}{56165+896+1521+23920+1632+10257+5585+6792+410+10416+8099} = 0.39$$