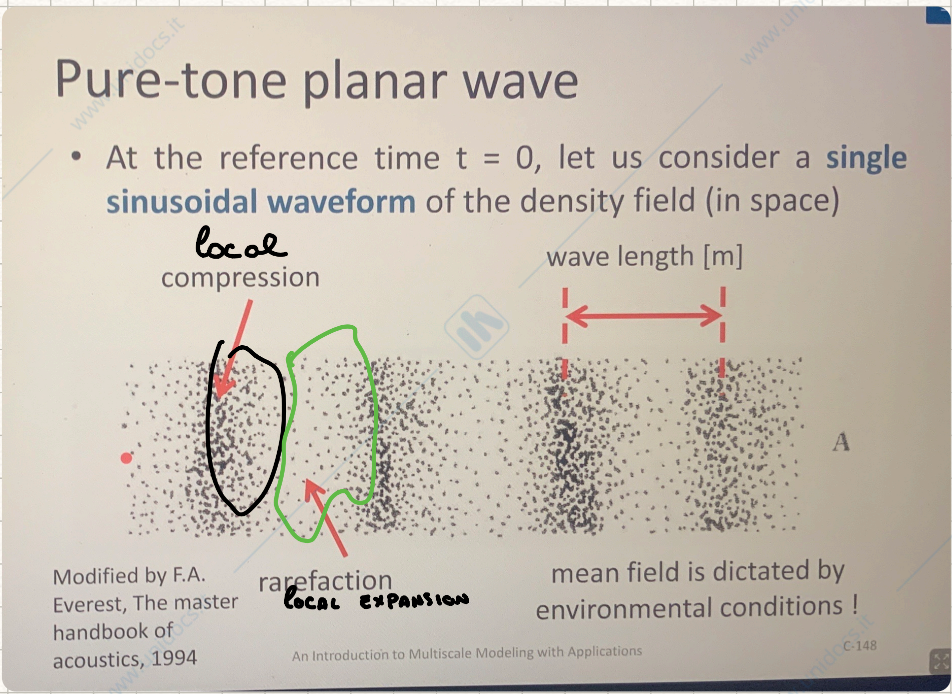
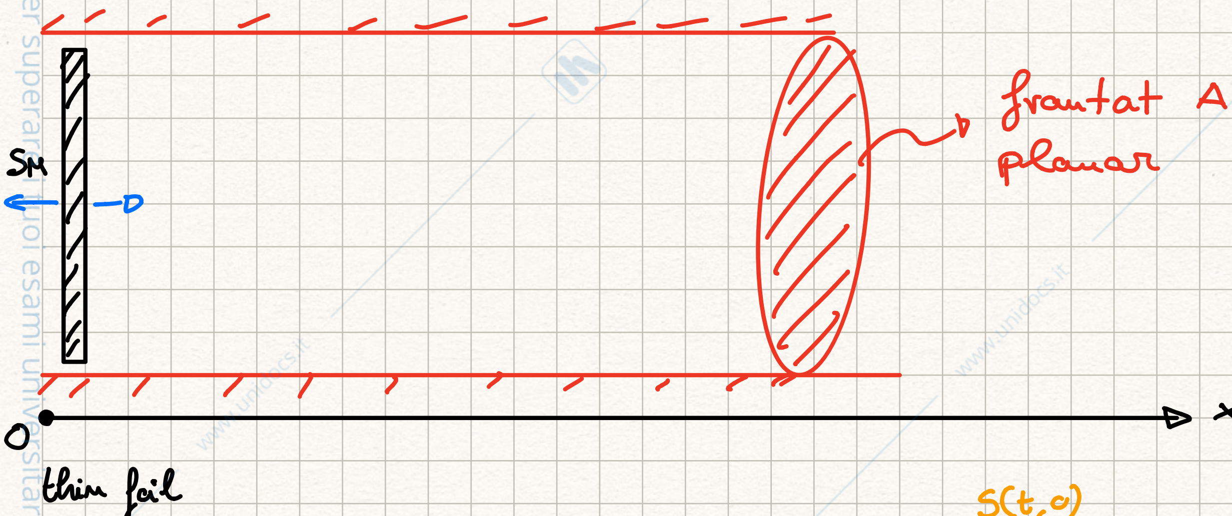
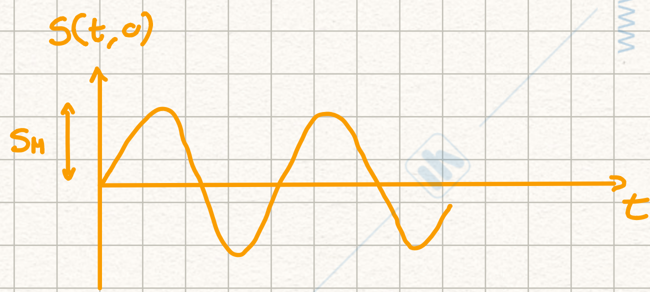


ACOUSTIC:
longitudinal waves
oscillation along same direction of propagation

PLANE WAVES



↳ oscillation around rest position



$x=0$ $\quad s(t,0) = S_M \sin(2\pi f t)$

↑ Amplitude

I impose (displacement propagation in tube)

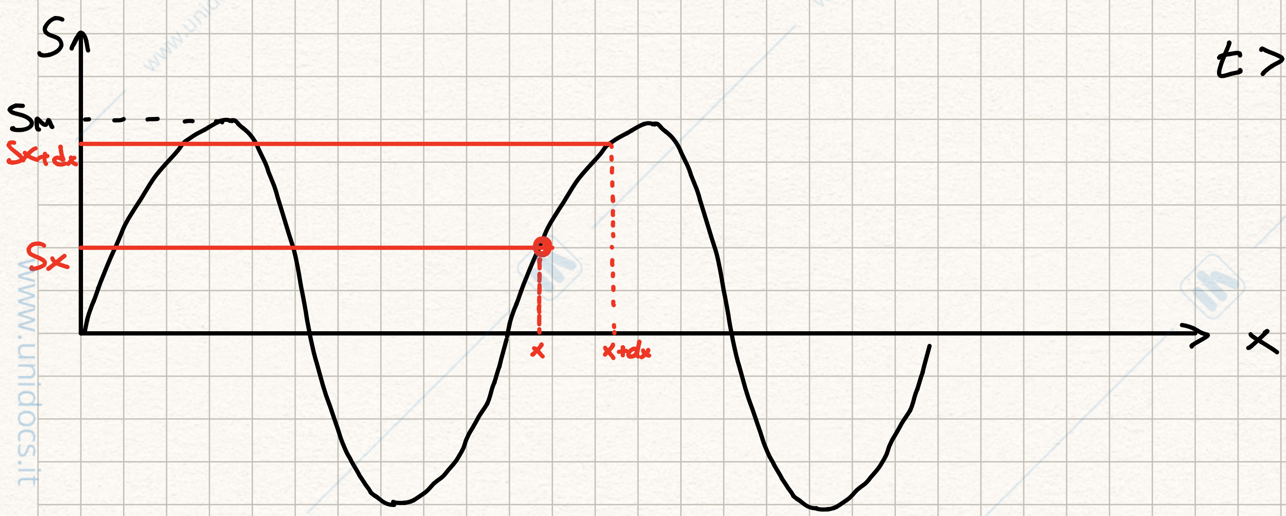
Perturbation travel at constant velocity assuming T = const

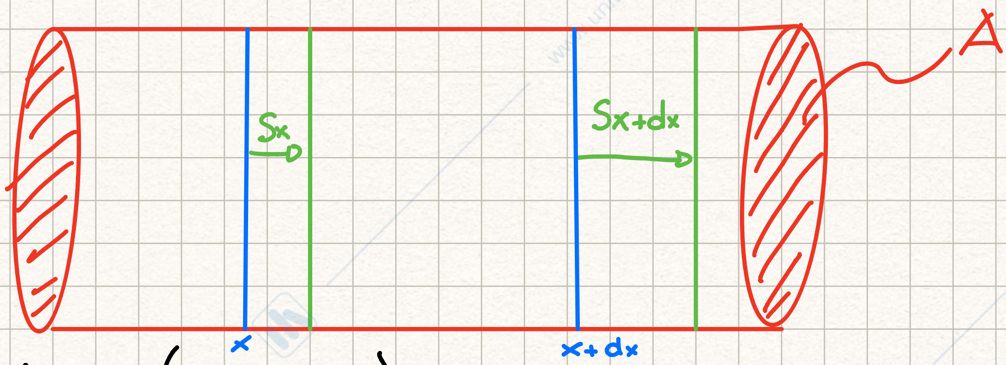
$$S(t,x) = S_M \sin\left[2\pi f \left(t - \overset{\text{TIME DELAY}}{\frac{x}{C_S}}\right)\right]$$

C_S: SOUND VEL.

DISPLACEMENT WAVE EQUATION

displacement at given time t along tube.





$dV = A\,dx$ (AT REST)

$\Delta(dV) = (S_{x+dx} - S_x)A \simeq (\cancel{S_x} + \frac{\partial S_x}{\partial x}dx - \cancel{S_x})A$

$\Delta(dV) = A\,dx\,\frac{\partial S_x}{\partial x}$

ELASTIC BEHAVIOUR OF AIR

Hook's law should hold

$\sigma = E\varepsilon$ $\quad\varepsilon = \frac{dl}{l}$ relative var. length

Stress, strain

$dP = -E_V \frac{dV}{V}$

ANALOGY

constant

if you expand dV is positive $\Rightarrow dP$ should be negative as compared to Rest

P_0 (Pressure at Rest)

$$P - P_0 = \Delta P = -E_v \frac{\Delta V}{V}$$

$$* \quad \Delta(dV) = A\,dx \frac{\partial S_x}{\partial x}$$

$$\Delta P = -E_v \cdot \frac{\Delta(dV)}{dV} = -E_v$$

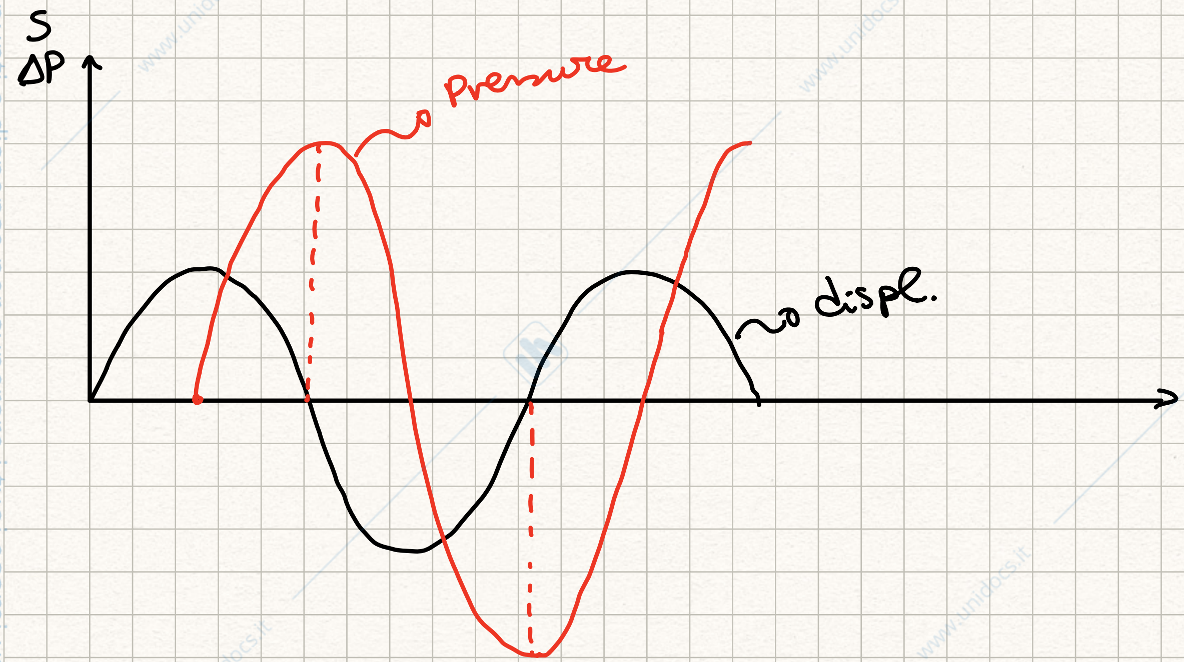
$$\Delta P = -E_v \frac{\cancel{A\,dx} \frac{\partial S_x}{\partial x}}{\cancel{A\,dx}}$$

$$\boxed{\Delta P = -E_v \frac{\partial S_x}{\partial x}}$$

Pressure wave

$$= E_v \frac{2\pi f S_M}{C_s} \cos\left[2\pi f\left(t - \frac{x}{C_s}\right)\right]$$

Pressure wave equation



WAVE LENGTH $\quad l = \dfrac{C_s}{f}$ [m]

FREQUENCY $\quad f = \dfrac{C_s}{l}$ [Hz]

ANGULAR FREQUENCY $\quad W = 2\pi f$

mechanical power transported by sound wave ?

$$I' = \frac{dW}{dA_{RIC}}(\underline{x}, t)$$

istantaneous intensity

Waves

PLANE WAVE (Tube)

work

$$W = \underbrace{F}_{} \cdot \boxed{\frac{\partial s_x}{\partial t}} = \underbrace{\Delta P \cdot A}_{} \cdot \underbrace{\frac{\partial s_x}{\partial t}}_{}$$

Displacement speed

LOCAL TOTAL (like our tube) ⇒ we have finite cross section

$$I'(x,t) = \overbrace{\frac{dW}{dA}}^{\text{LOCAL}} = \overbrace{\frac{W}{A}}^{\text{TOTAL}} = \Delta P(x,t) \cdot \underbrace{\frac{\partial s_x}{\partial t}(x,t)}_{}$$

$$\Delta P(x,t) = E_v \frac{2\pi f S_M}{C_s} \cos\left[2\pi f\left(t - \frac{x}{C_s}\right)\right]$$

$$\frac{\partial s_x}{\partial t} = S_M \, 2\pi f \cos\left[2\pi f\left(t - \frac{x}{C_s}\right)\right]$$

$$\Delta P(x,t) = \frac{E_v}{C_s} \frac{\partial s_x}{\partial t} \Rightarrow \frac{\partial s_x}{\partial t} = \frac{C_s}{E_v} \Delta P(x,t)$$

$$\underline{I'(x,t)} = \frac{C_s}{E_v} \underline{\Delta P^2(x,t)}$$

$$\boxed{E_v = \rho_0 \, C_s^2}$$

like Einstein famous eq.

$$\frac{J}{m^3} \quad \frac{kg}{m^3} \quad \frac{m^2}{s^2}$$

$$I'(x,t) = \frac{\cancel{C_s}}{\rho_0 C_s^{\cancel{2}}} \Delta P^2(x,t)$$

$$\boxed{I'(x,t) = \frac{\Delta P^2(x,t)}{\boxed{\rho_0 \, C_s}}}$$

$\rho_0 C_s$ → Acoustic Resistance

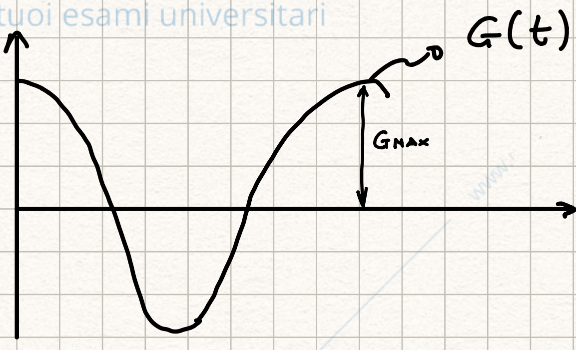
In application ⇒ we need:

TIME AVERAGE QUANTITIES:

$$\tau = \frac{1}{f} \qquad I(x) = \frac{1}{\tau}\int_0^\tau I'(x,t)\,dt = \frac{1}{\tau}\int_0^\tau \frac{1}{\rho_0 C_s} \Delta P^2(x,t)\,dt$$

$$\Delta P_{eff}^2(x) = \frac{1}{\tau}\int_0^{\tau} \Delta P^2(x,t)\,dt$$

$$I(x) = \frac{\Delta P_{eff}^2}{\rho_0 C_s}$$



$$G = G_{MAX} \cdot \cos(t)$$

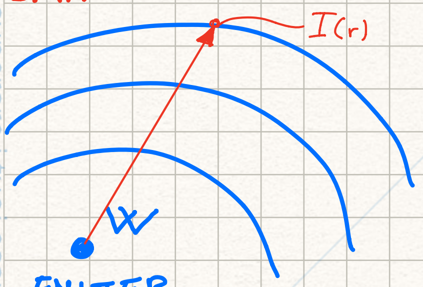
$$(G_{eff})^2 = \frac{1}{\tau}\int_{\tau} G_{MAX} \cos(t)\,dt \qquad G_{eff} = \frac{G_{MAX}}{\sqrt{2}}$$

$$\Delta P_{MAX} = \frac{E_V\, 2\pi f\, S_M}{C_s} \qquad \Delta P_{eff} = \frac{\Delta P_{MAX}}{\sqrt{2}} = \frac{2\pi \rho_0 C_s f S_M}{\sqrt{2}}$$

$$\boxed{I(x)} = \frac{\boxed{\Delta P_{eff}^2}}{\rho_0 C_s}$$

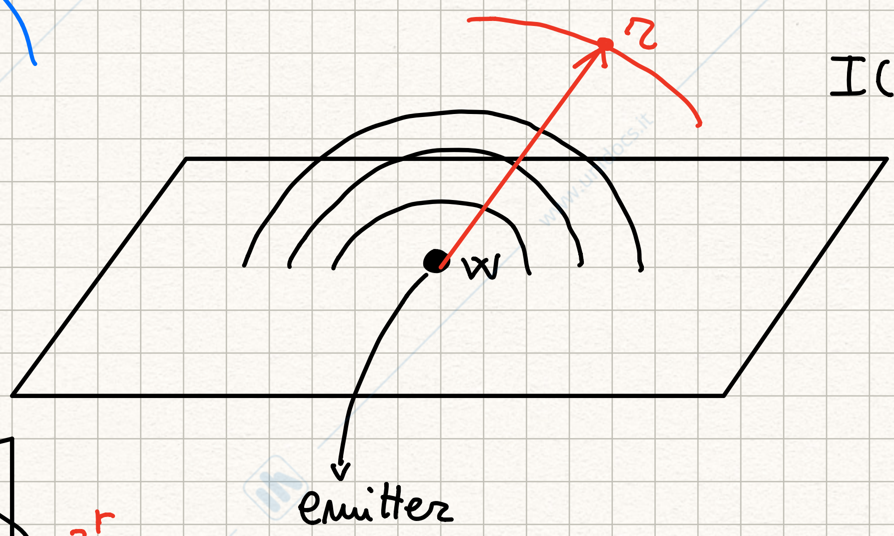
quantity measured by Acoustic instruments (PHONOMETER)

EAR

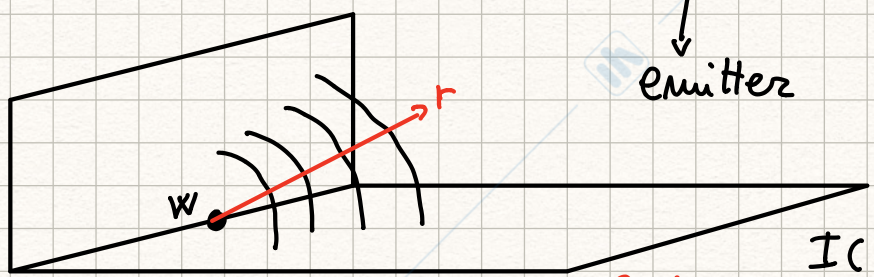


Spherical waves

$$I(r) = \frac{W}{4\pi r^2}$$



$$I(r) = \frac{W}{\frac{4\pi r^2}{2}} = \frac{4W}{2\pi r^2}$$



$$I(r) = \frac{W}{\frac{4\pi r^2}{4}}$$

Directional factor

$$I(r) = \frac{Q\, W}{4\pi r^2}$$

I , ΔP_{eff}) Physical quantities

$\left[\frac{W}{m^2}\right]$ $[Pa]$

What matters is the acoustic feelings

LOWER LIMIT OF AUDIBILITY — you start earing

you fix frequency ⇒ PURE TONE $f = 1000$ Hz (Reference frequency)

$$\Delta P_{eff} \geq 2 \cdot 10^{-5} [Pa] = \left[\frac{N}{m^2}\right]$$

$P_0 = 1$ bar $= 10^5$ Pa ⇒ 10 orders of magnitude

UPPER LIMIT OF HEARING

$\Delta P_{eff} = 20$ Pa ⟶ START FEELING PAIN !

$G \longrightarrow G_0$

generic Quantity — Reference value

Level of G $\quad L_G = 10 \cdot \log_{10} \frac{G}{G_0}$ ← deci Bell $[dB]$

1) SOUND Pressure level

$$L_p = 10 \cdot \log_{10} \frac{\Delta P_{eff}^2}{\Delta P_0^2} = 20 \log_{10} \frac{\Delta P_{eff}}{\Delta P_0} \quad [dB]$$

Characterizes Receiver

$$\Delta P_0 = 2 \cdot 10^{-5} \text{ Pa}$$

2) ACOUSTIC INTENSITY LEVEL

$$L_I = 10 \cdot \log_{10} \frac{I}{I_0} \qquad I_0 = 10^{-12} \frac{W}{m^2} \quad [dB]$$

3) ACOUSTIC POWER LEVEL:

$$L_W = 10 \cdot \log_{10} \frac{W}{W_0} \qquad W_0 = 10^{-12}\ W \quad [dB]$$

$$L_I = 10 \log_{10} \left(\frac{I}{I_0} \right) = 10 \log_{10} \left(\frac{1}{I_0} \frac{\Delta P_{eff}^2}{\rho_0 c_s} \cdot \frac{\Delta P_0^2}{\Delta P_0^2} \right) =$$

$$= 10 \cdot \log_{10} \left(\frac{\Delta P_{eff}^2}{\Delta P_0^2} \right) + \underbrace{10 \log_{10} \left(\frac{\Delta P_0^2}{\rho_0 c_s I_0} \right)}_{\text{CONST}}$$

$$= L_P + \text{Const}$$

Const $\simeq -0{,}2$ dB trascurable ⇒ lower than instruments accuracy

$$L_I \simeq L_P$$

$$L_I = 10 \cdot \log_{10} \frac{I}{I_0} = 10 \cdot \log_{10} \left(\frac{W}{S} \frac{S_0}{W_0} \right)$$

$$= 10 \cdot \log_{10} \left(\frac{W}{W_0} \right) - 10 \log_{10} \left(\frac{S}{S_0} \right) \qquad S_0 = 1\ m^2$$

$$\boxed{L_P \simeq L_I = L_W - 10 \log_{10} S}$$

LOWER LIMIT AUDIBILITY $\quad f = 1000$ Hz

$$L_P = 10 \log_{10} \frac{(2 \cdot 10^{-5})^2}{(2 \cdot 10^{-5})^2} = 0 \text{ dB}$$

UPPER LIMIT

$$L_p = 10 \log_{10}\left(\frac{20}{2\cdot 10^{-5}}\right)^2 = 120\ dB$$

$$0 \le L_p \le 120\ dB$$

ACOUSTIC FEELINGS

Weber and Fechner

$\boxed{f = 1000\ Hz}$ Fix

$L_p \longleftrightarrow$ Feelings

$\boxed{\text{Phisical QUANTITY}}$

$dS \to$ feeling measure

variation of feeling measure if relative variation in effec pressure

$$dS \propto \left.\frac{d\,\Delta P_{eff}}{\Delta P_{eff}}\right|_{W.F.}$$

$$dS = K_1 \frac{d(\Delta P_{eff})}{\Delta P_{eff}} \quad ; \quad S - S_0 = K_1 \ln \frac{\Delta P_{eff}}{\Delta P_0} = K_2 \log_{10}\left(\frac{\Delta P_{eff}}{\Delta P_0}\right)$$

$K_2 = 20$ $\quad S_0 = 0$ $\quad$ W.F. CHOICE

$$f = 1000\ Hz \qquad S = 20 \log_{10} \frac{\Delta P_{eff}}{\Delta P_0} = 10 \log_{10} \left(\frac{\Delta P_{eff}}{\Delta P_0}\right)^2 = L_p$$

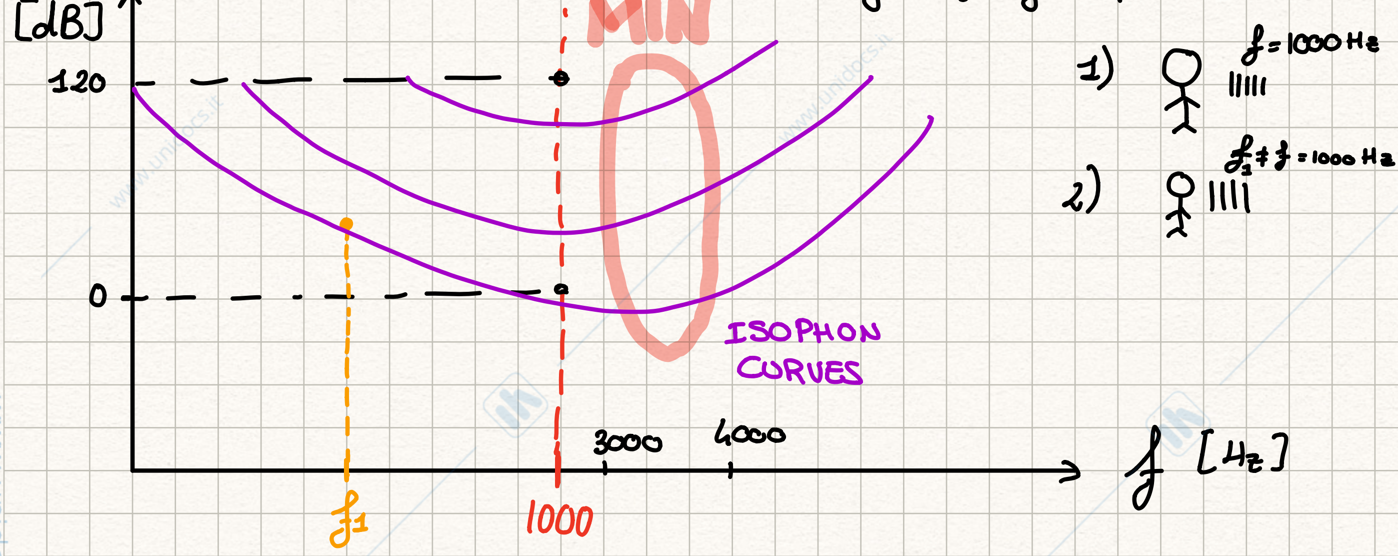
S → PHON

30 dB → 30 PHONS

50 dB → 50 PHONS

at $f = 1000\ Hz$

Set of Young People (18-25)



adjust f up
when person se
he/she has sa
feeling than pr
value (even if
≠ f)

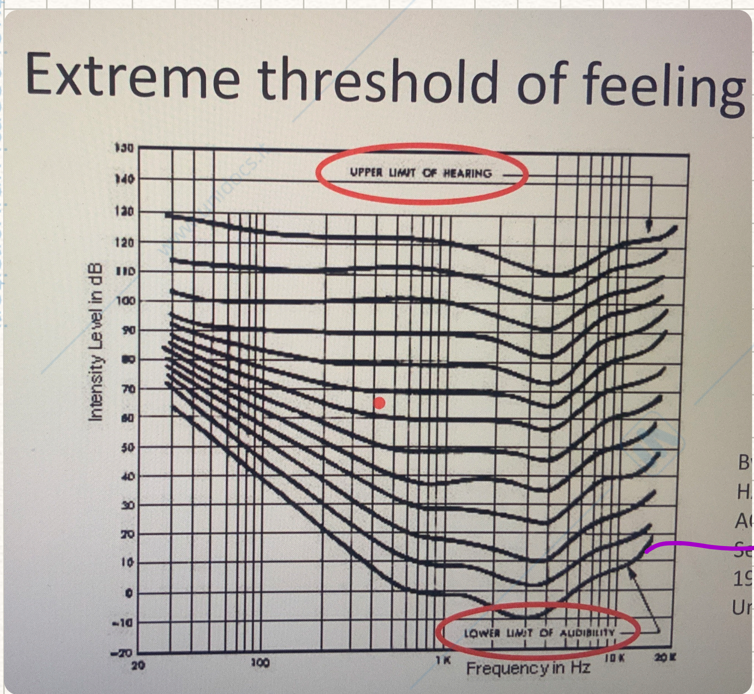
Diagram of normal acoustic Response

Equal - Loudness countour

Minimum in 3000 - 4000 ⇒ human ear ⇒ more sensible to these frequences

high and Low freq ⇒ high pressure variation ⇒ less sensible

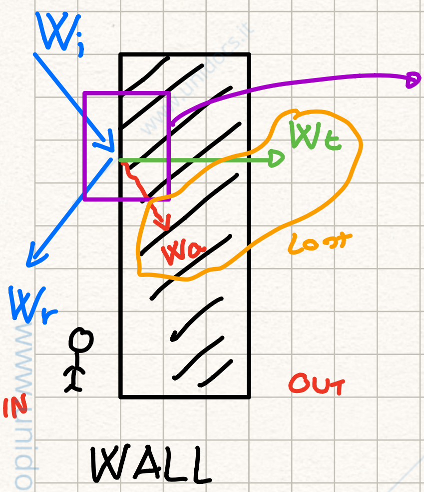
16 Hz ÷ 16 kHz   Range of values



As you approach upper limit
↓
not so high variation changing f

⇒ each curve ⇒ equal phon
⇒ they have the same feeling

SOUND IN ENCLOSED SPACES



r: reflected
t: transmitted
a: absorbed

$$\frac{W_i}{W_i} = \frac{W_r + W_a + W_t}{W_i}$$

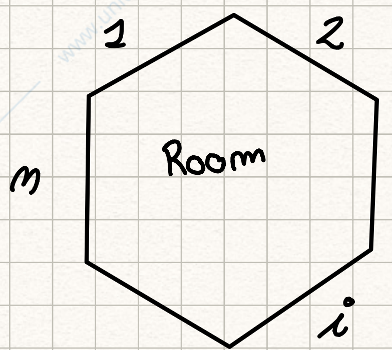
$$r = \frac{W_r}{W_i} \quad a = \frac{W_a}{W_i} \quad t = \frac{W_t}{W_i}$$

$$1 = r + a + t$$

$$1 = r + a'$$

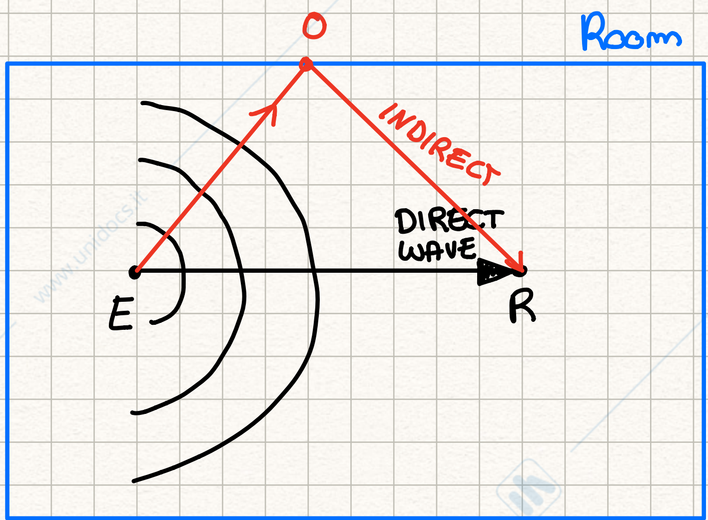
$$a' = a + t$$

→ apparent absorbtion coeff.



m walls

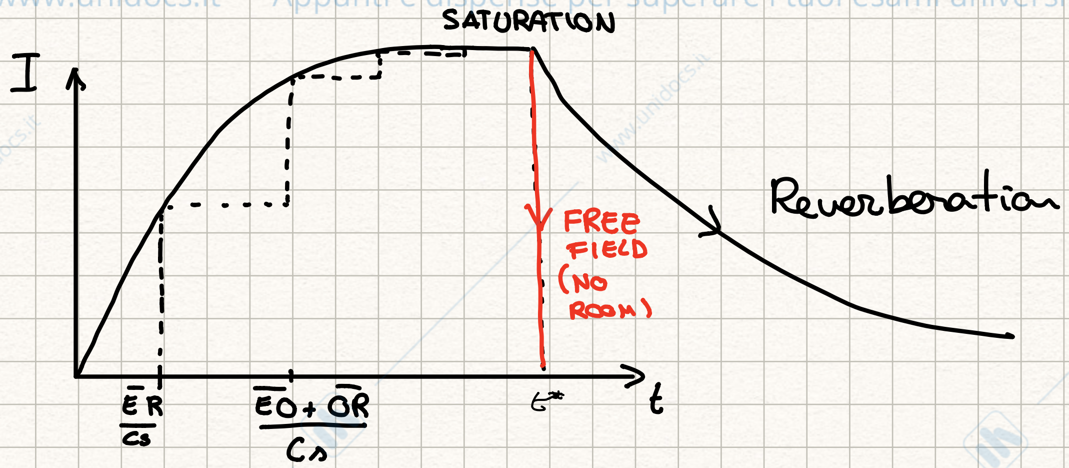
$$\bar{a}' = \frac{\sum_i a'_i}{\sum_i S_i} = \frac{\sum a'_i S_i}{S_{TOT}}$$



S: total absorbtion Area

E switched off ⇒ switch on ⇒ first wave to R is the direct one

STOP after t^* ⇒ you still hear ⇒ Reverberation
⇓
in free field
⇓
vertically



SABINE APPROX. APPROACH

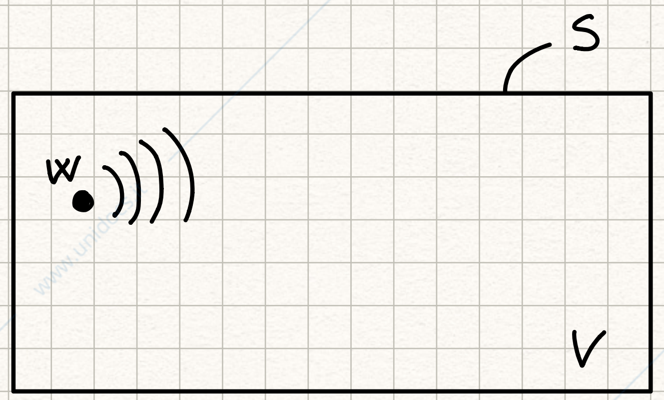
1) Big Environment (ROOM)

2) No space variation of acoustic Energy in the Room

$I = I(t)$

$J = J(t)$ ⟶ Volume density of acoustic energy $[J/m^3]$

$$\boxed{I = \frac{c_s}{4} J(t)}$$



ENERGY BALANCE

dt

$$W\,dt - I\,S\,dt\,\bar{a}' = d(J \cdot V) = V\,dJ$$

$$\frac{W}{V} - \frac{I S \bar{a}'}{V} = \frac{dJ}{dt}$$

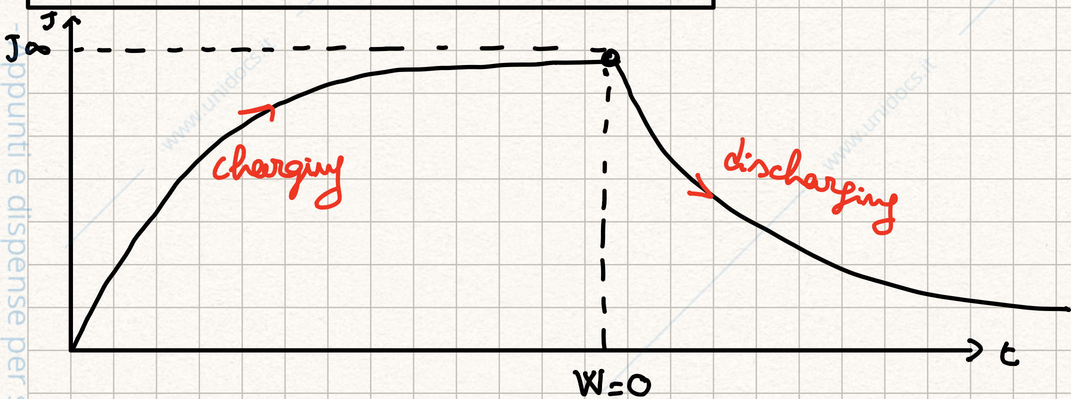
$$\boxed{\frac{W}{V} - \frac{\bar{a}' \, S \, C_s}{4V} J = \frac{dJ}{dt}}$$

ODE Rules $\rightarrow$ acousting charging / discharging Process

$$J^* = -\frac{W}{V} + \beta J \qquad \beta = \left[\frac{\bar{a}' \, S \, C_s}{4V}\right] \rightarrow \text{inverse of a time}$$

$$\frac{1}{\beta}\frac{dJ^*}{dt} + J^* = 0 \Rightarrow \text{separation of variables}$$

$$\boxed{J(t) = \frac{W}{\beta V}\left(1 - e^{-\beta t}\right)} \quad \text{energy density}$$

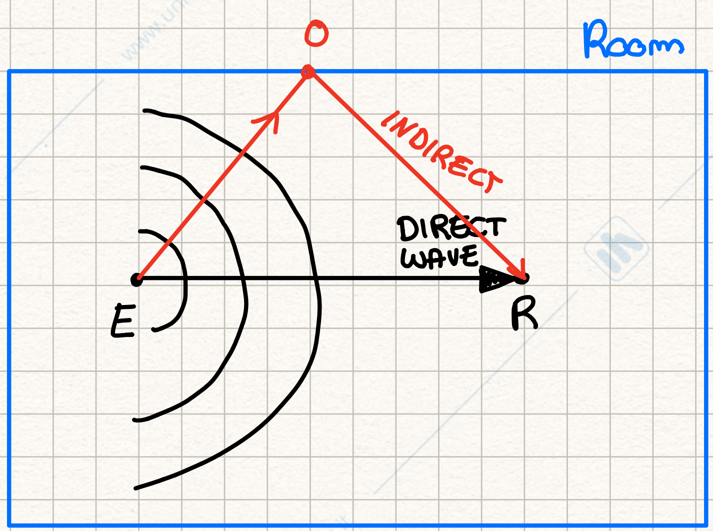


W=0
↓
fully charged
↓
turn off emettitore

$$J_\infty = \frac{4W}{\bar{a}' \cdot S \cdot C_s}$$

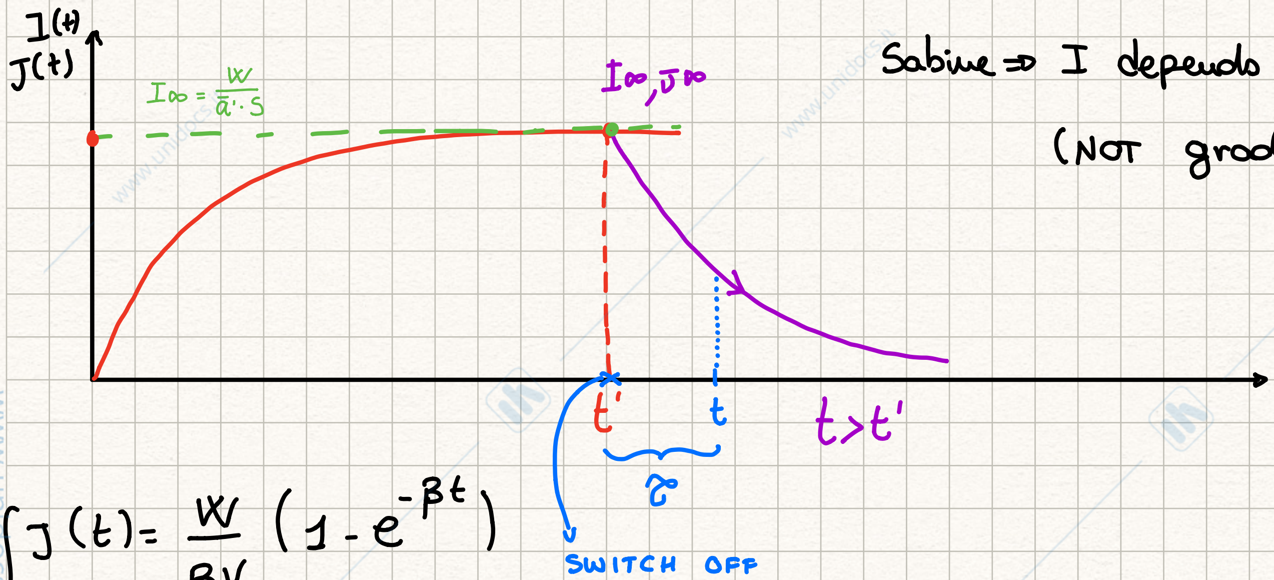
$$\boxed{W = \bar{a}' \cdot S \cdot I_\infty} \quad t \to \infty$$

Lesson 19/05



$$W\,dt - S\,\bar{a}'\,I\,dt = V\,dJ$$ variation of acustic density in the volume

SOURCE TERM



Sabine ⇒ I depends only in time (NOT gradient of spacial ex.)

$$\begin{cases} J(t) = \dfrac{W}{\beta V}\left(1 - e^{-\beta t}\right) \\ \beta = \dfrac{\bar{a}' \cdot S \cdot c_0}{4V} \quad \text{inverse of a time} \end{cases}$$

$t \geq t' \qquad W = 0$

$$0 - \bar{a}' S I \, dt = V dJ$$

$$\Downarrow$$

$$\frac{dJ}{dt} = -\beta J \longrightarrow \int_{J_\infty}^{J} \frac{dJ}{J} = -\beta \int_{t'}^{t} dt$$

PURPLE EVOLUTION

$$J(t) = J_\infty \, e^{-\beta(t - t')} \qquad t - t' = \tau$$

$$\boxed{J(t) = J_\infty \cdot e^{-\beta \tau}}$$

CONVENTIONAL REVERBERATION TIME (characteristic of room)

Time needed ⇒ from $I_\infty, J_\infty \xrightarrow[\tau]{} I, J$ $\quad \boxed{\tau_{60}}$

$$\frac{I_\infty}{I} = \frac{J_\infty}{J} = 10^6$$

change in 6 ORDER OF MAGNITUDE

τ_{20} $\quad \frac{J_\infty}{J} = 10^2$ $\qquad \tau_{10} \to \frac{J_\infty}{J} = 10$

$$J(t) = J_\infty \, e^{-\beta\tau}$$

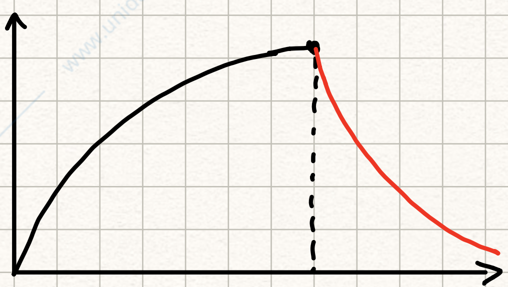
$$\frac{J_\infty}{J(t)} = \frac{\cancel{J_\infty}}{\cancel{J_\infty} e^{-\beta \tau_{60}}} = 10^6$$

$$\ln 10^6 = \beta \tau_{60} \qquad \tau_{60} = \frac{2.3 \cdot 6}{\boxed{\beta}} \Rightarrow$$

SABINE FORMULA

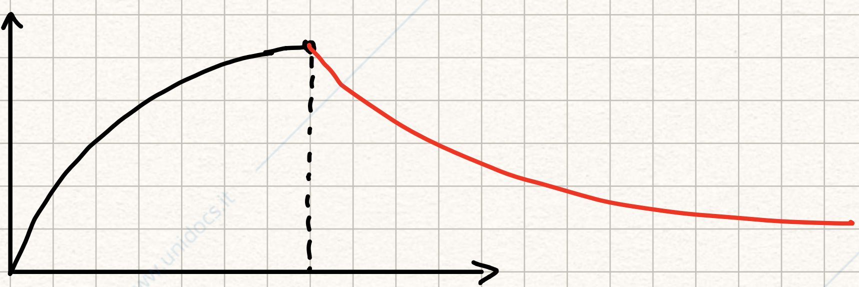
$$\boxed{\tau_{60} = \frac{0,16 \cdot V}{\bar{a}' \cdot S}}$$

τ_{60} SHOULD NOT BE TOO SMALL !



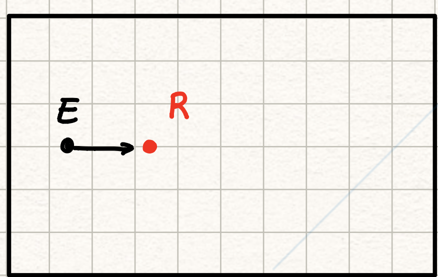
too small not good because sound disappea too fast manner

τ_{60} SHOULD NOT BE TOO BIG !

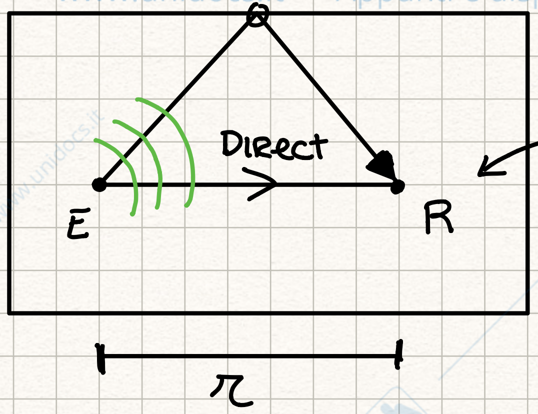


when sounds stops ⇒ we h overlapping of different sound contribution (acoustic signal)

— FREE - FIELD CONDITIONS



major contribution is the direct one



$$I = I_d + I_r$$

$$I_d = \frac{QW}{4\pi r^2}$$

Q : fraction

$$I_r \propto (1-\bar{a}')\,W$$

$$I_r = \frac{4(1-\bar{a}')W}{\bar{a}'\,S} \longrightarrow \text{Reverberated intensity}$$

$$(*)\; I = \frac{QW}{4\pi r^2} + \frac{4(1-\bar{a}')}{\bar{a}'\,S}W = W\left[\frac{Q}{4\pi r^2} + 4\,\frac{1-\bar{a}'}{\bar{a}'\,S}\right]$$

$$\frac{I}{I_0} = \frac{W}{W_0}\,S_0\left[\frac{Q}{4\pi r^2} + 4\,\frac{1-\bar{a}'}{\bar{a}'\cdot S}\right]$$

$$I = \frac{W}{A}$$

$$I_0 = 10^{-12}\,\frac{W}{m^2}$$

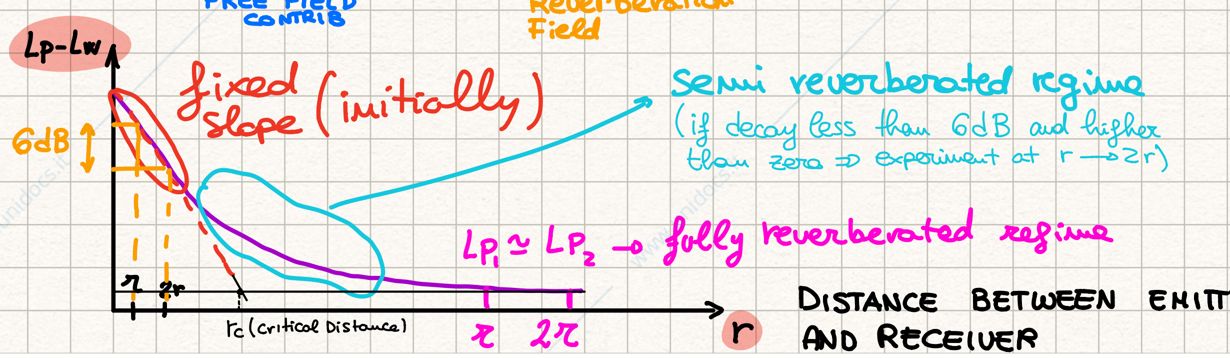
$$10\log_{10}\left(\frac{I}{I_0}\right) = 10\log_{10}\left(\frac{W}{W_0}\right) + \cancel{\log_{10}(S_0)} + \log_{10}\left[\frac{Q}{4\pi r^2} + 4\,\frac{1-\bar{a}'}{\bar{a}'\,S}\right]\cdot 10$$

$$L_I = L_W + 10\log_{10}\left[\frac{Q}{4\pi r^2} + 4\,\frac{1-\bar{a}'}{\bar{a}'\cdot S}\right]$$

$$(L_I - L_W) = 10\cdot\log_{10}\left[\underbrace{\frac{Q}{4\pi r^2}}_{\text{FREE FIELD CONTRIB}} + 4\,\underbrace{\frac{1-\bar{a}'}{\bar{a}'\cdot S}}_{\text{Reverberation Field}}\right]$$

$$\parallel$$

$$(L_p - L_W)$$



E is close to R

free field contribution

r

$$(L_I - L_W)_1 = 10 \log_{10}\left(\frac{Q}{4\pi r^2}\right)$$

$2r$

$$(L_I - L_W)_2 = 10 \log_{10}\left(\frac{Q}{4\pi (2r)^2}\right)$$

SAME

$$(L_I - \cancel{L_W})_1 - (L_I - \cancel{L_W})_2 \simeq \underbrace{6\ dB}_{\text{Free Field Condition}}$$

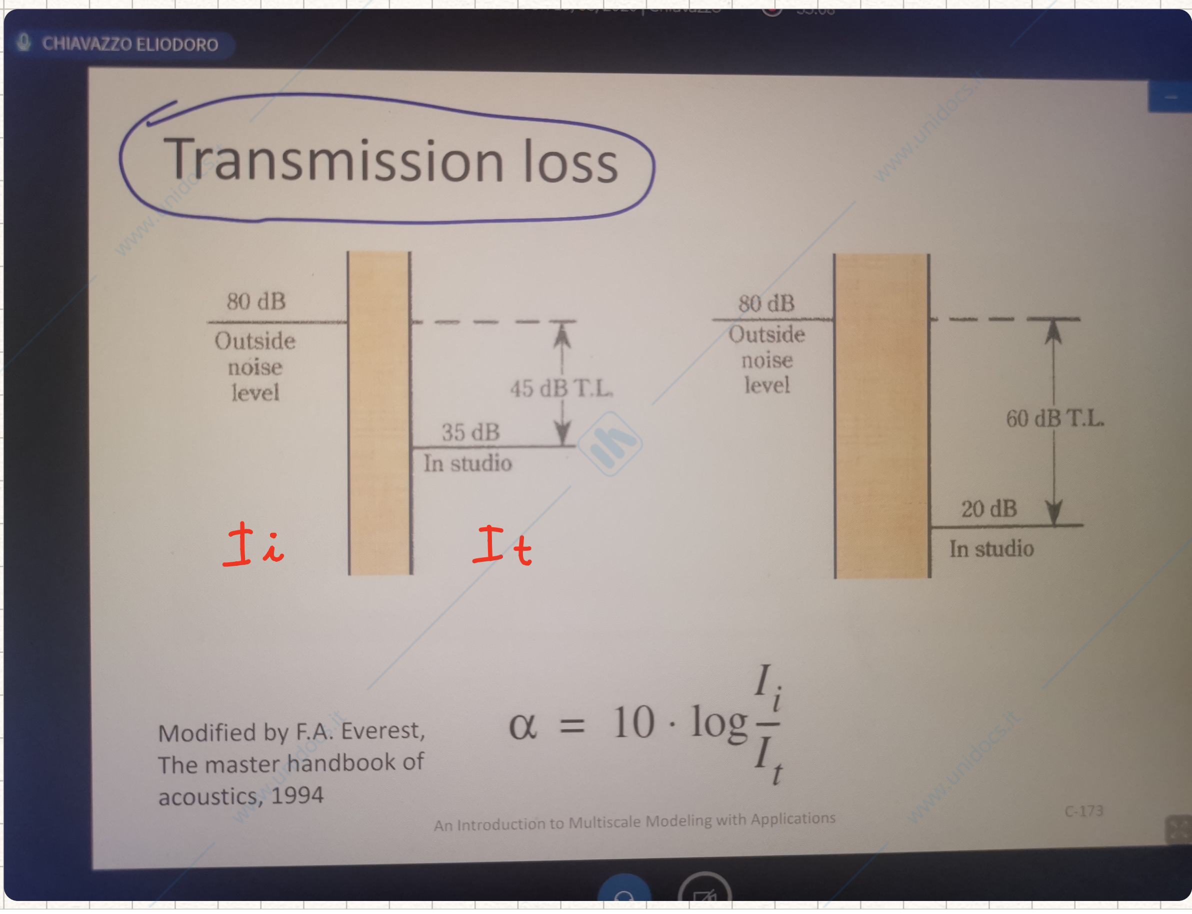
$$(L_I)_1 - (L_I)_2$$

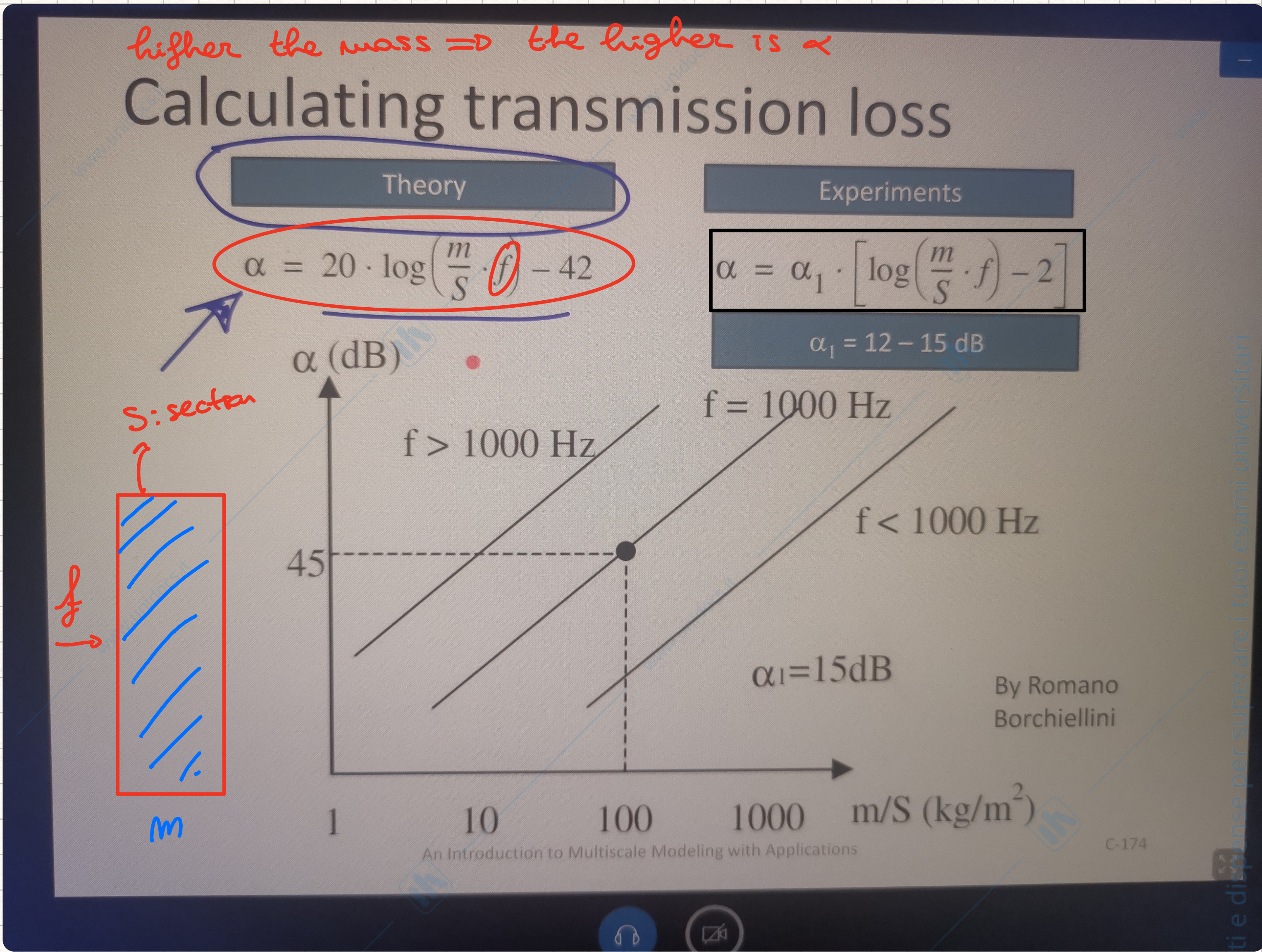
level of power depends only on the source and source is the same

In free field decay is 6dB

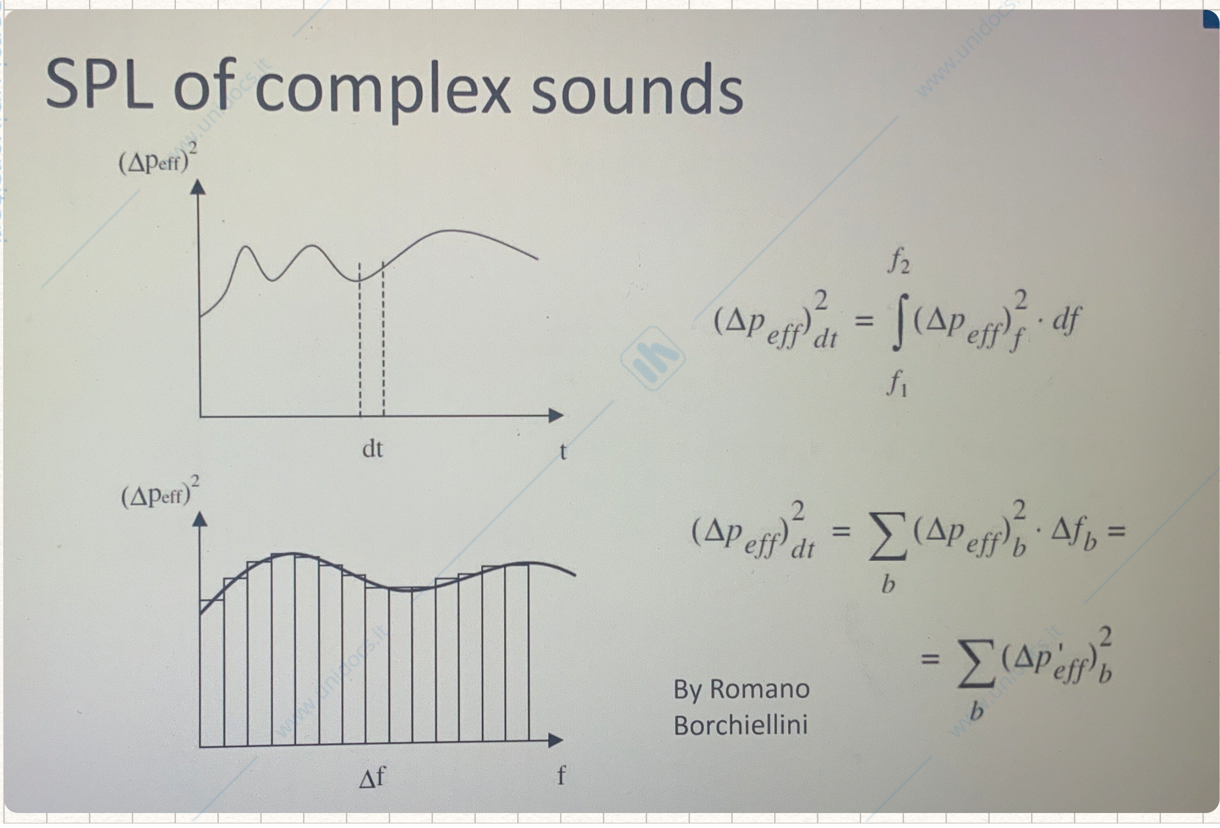
r_c: direct intensity = reflected one

SOUND INSULATION

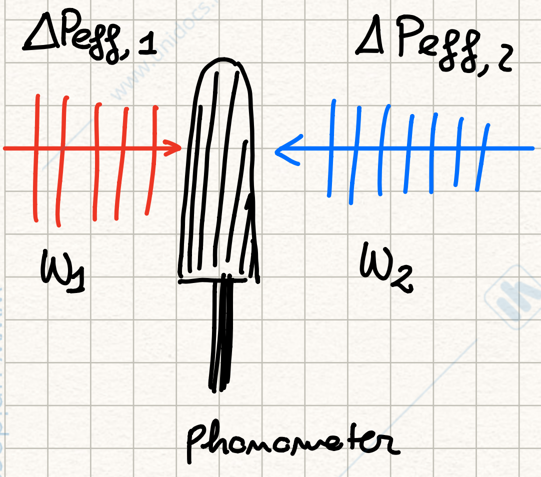




higher f => transmission loss increases



COMPLEX SOUNDS



$$\Delta P_{eff} = \sqrt{(\Delta P_{eff_1})^2 + (\Delta P_{eff_2})^2}$$

$$\boxed{\Delta P_{eff}^2 = \sum_i \Delta P_{eff,i}^2}$$

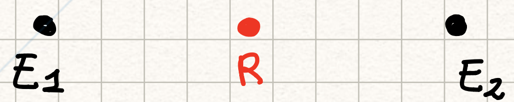
$$I = \frac{\Delta P_{eff}^2}{\rho_0 \, c_s}$$

$$\begin{cases} \Delta P_{eff}^2 = \Delta P_{eff,1}^2 + \Delta P_{eff,2}^2 \\ \Downarrow \\ I = I_1 + I_2 \end{cases}$$

ex.

$L_{p_1} = 70\,dB$

$L_{p_2} = 70\,dB$



Switch off one at time and measure 70dB

$\simeq 73\,dB$

$$L_p \neq L_{p1} + L_{p2} = 140\,dB$$

$$L_p = 10 \log_{10} \frac{\Delta P_{eff_1}^2}{\Delta P_0^2} \qquad L_{p_2} = 10 \log_{10} \frac{\Delta P_{eff_2}^2}{\Delta P_0^2}$$

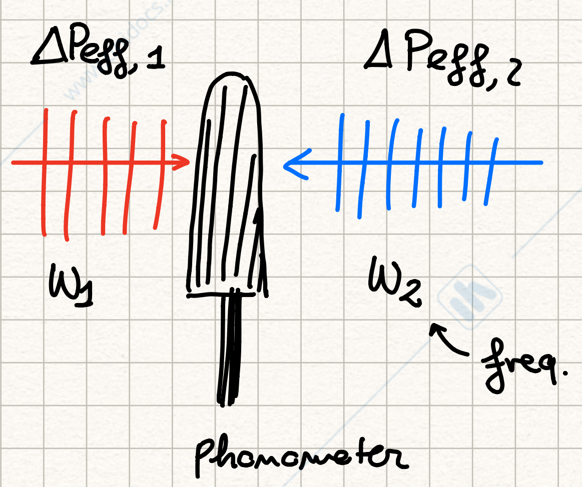
$$\Delta P_{eff}^2 = \Delta P_{eff,1}^2 + \Delta P_{eff_2}^2$$

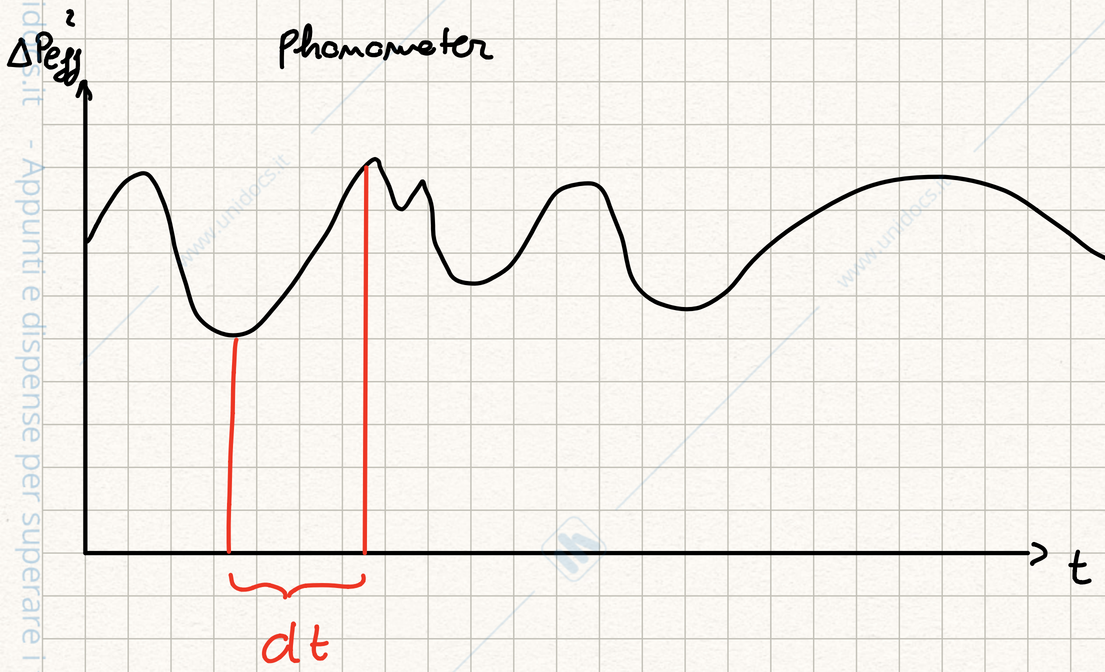
$$\hookrightarrow L_p = 10 \log_{10} \frac{\Delta P_{eff}^2}{\Delta P} \simeq 73\,dB$$

So we use L_{p1} and L_{p2} to take off $\Delta P_{eff_1}^2$ and $\Delta P_{eff_2}^2$

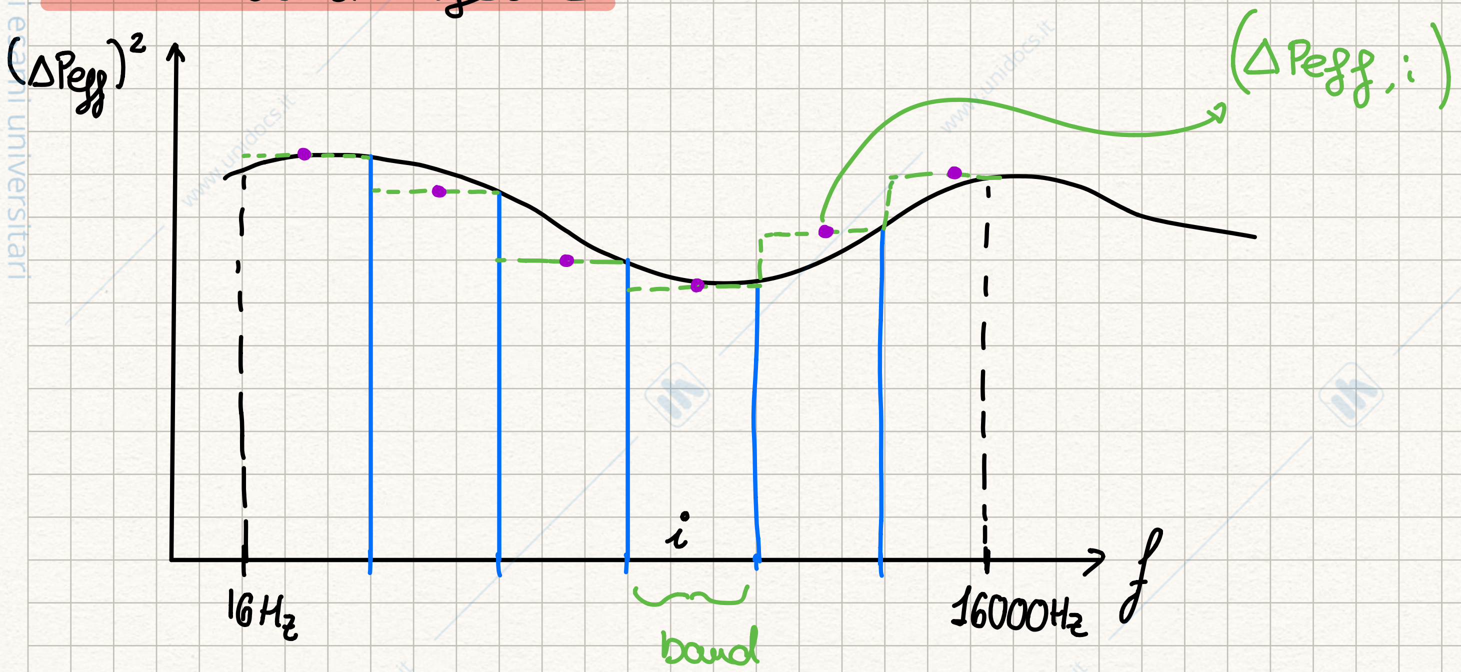
COMPLEX SOUND

$$\Delta P_{eff}^2 = \Delta P_{eff,1}^2 + \Delta P_{eff,2}^2$$





Fast Fourier transformed



$$\boxed{\left(\Delta P_{eff}\right)^2_{dt}} = \frac{\sum_b \left(\Delta P_{eff,i}\right)^2 \delta f_{b_i}}{f_{max} - f_{min}}$$

NUMBER YOU GET FROM PHONOMETER

OCTAVE BANDS

16 - 16 K Hz

you start from $f_0 = 1\ kHz$ — you divide and multipl[y] by 2

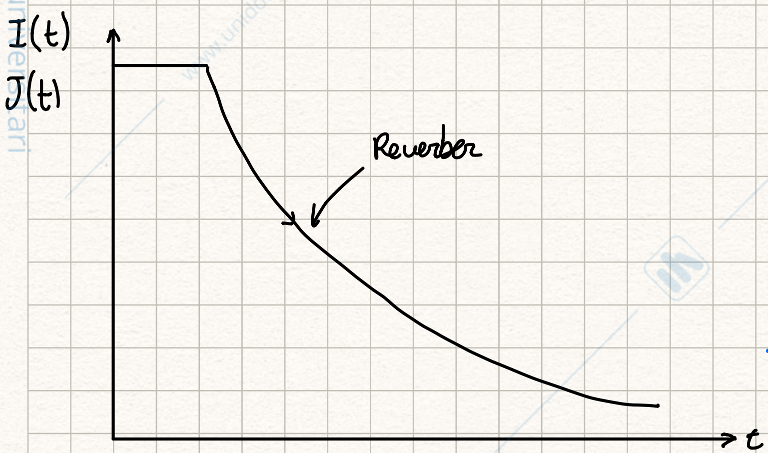
$$63 \leftarrow 125 \leftarrow 250 \leftarrow 1\,kHz \rightarrow 2k \rightarrow 4k \rightarrow 8k \rightarrow 16k$$

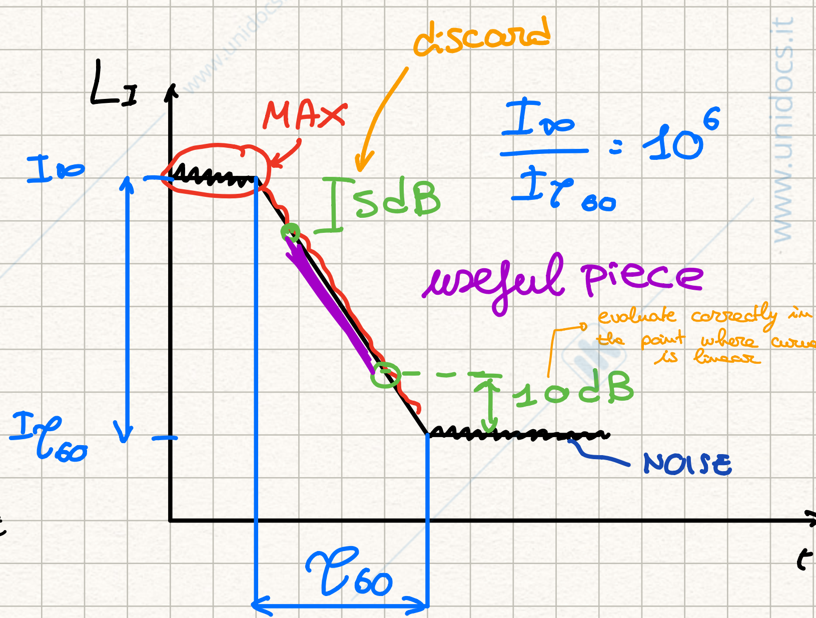
$$\cdot \xleftarrow{:2} \cdot \xleftarrow{:2} \cdot \xleftarrow{:2} f_0 \xrightarrow{\times 2} \xrightarrow{\times 2} \xrightarrow{\times 2} \xrightarrow{\times 2}$$

f_i — f_{i+1} $\longrightarrow \sqrt{f_i \cdot f_{i+1}}$

Subdivide frequency band ⇒ easier mod[el]

Reverberation phenomenon





$$\frac{I_\infty}{I\tau_{60}} = 10^6$$

tipically what is measured is τ_{30}, τ_{20} → Not enough useful piece

$$\frac{I_{\infty}}{I_{\tau_{20}}} = 10^2 \; ; \quad \frac{I_{\infty}}{I_{\tau_{30}}} = 10^3$$

most of time → set → phisically measure τ_{20}, τ_{30} measuring useful piece of Reverberation ⇒ then interpolate and find τ_{60}