(25) REGOLA DISCR. DI BAYES

$\pi_1 = P(x \in \Pi_1)$ e $\pi_2 = P(x \in \Pi_2)$ SONO LE PROB. A PRIORI

$$p^* = P[x \in R_1 \mid x \in \Pi_2] \cdot P[x \in \Pi_2] + P[x \in R_2 \mid x \in \Pi_1] \cdot P[x \in \Pi_1]$$
$$= p_{12}\pi_2 + p_{21}\pi_1$$

SE f_1 e f_2 DESCRIVONO Π_1 e Π_2:

$$p_{12} = \int_{R_1} f_2(x)\,dx \qquad p_{21} = \int_{R_2} f_1(x)\,dx$$

QUINDI:

$$p^* = \pi_2 \int_{R_1} f_2(x)\,dx + \pi_1 \int_{R_2} f_1(x)\,dx$$
$$= \pi_2 \int_{R_1} f_2(x)\,dx + \pi_1 \left(\int_R f_1(x)\,dx - \int_{R_1} f_1(x)\,dx\right)$$
$$= \pi_2 \int_{R_1} f_2(x)\,dx + \pi_1 \left(1 - \int_{R_1} f_1(x)\,dx\right)$$
$$= \pi_1 + \int_{R_1} \left(\pi_2 f_2(x) - \pi_1 f_1(x)\right) dx$$

$$R_1 = \left\{x \in R : \frac{f_1(x)}{f_2(x)} \geq \frac{\pi_2}{\pi_1}\right\} \qquad R_2 = \left\{x \in R : \frac{f_1(x)}{f_2(x)} < \frac{\pi_2}{\pi_1}\right\}$$

(SE $\pi_1 = \pi_2$ → REGOLA MAX VER.)

(26) REGOLA MINIMO COSTO ATTESO DI ERRATA CLASSIFICAZIONE

$\begin{cases} c_{ij} \text{ COSTO DERIVANTE DALL'ASSEGNAZIONE A } \Pi_i \text{ DI UN'OSS. DI } \Pi_j \\ c_{12} \text{ e } c_{21} \text{ COSTI DA ERRATA CLASSIFICAZIONE} \end{cases}$

$$c^* = c_{12}\,p_{12}\,\pi_2 + c_{21}\,p_{21}\,\pi_1 = c_{12}\,\pi_2 \int_{R_1} f_2(x)\,dx + c_{21}\,\pi_1 \int_{R_2} f_1(x)\,dx$$

$$R_1 = \left\{x \in R : \frac{f_1(x)}{f_2(x)} \geq \frac{c_{12}\pi_2}{c_{21}\pi_1}\right\} \qquad R_2 = \left\{x \in R ; \frac{f_1(x)}{f_2(x)} < \frac{c_{12}\pi_2}{c_{21}\pi_1}\right\}$$

SE $c_{12} = c_{21}$ → REGOLA DI BAYES

SE $c_{12} = c_{21}$ e $\pi_1 = \pi_2$ → REGOLA MAX VER.