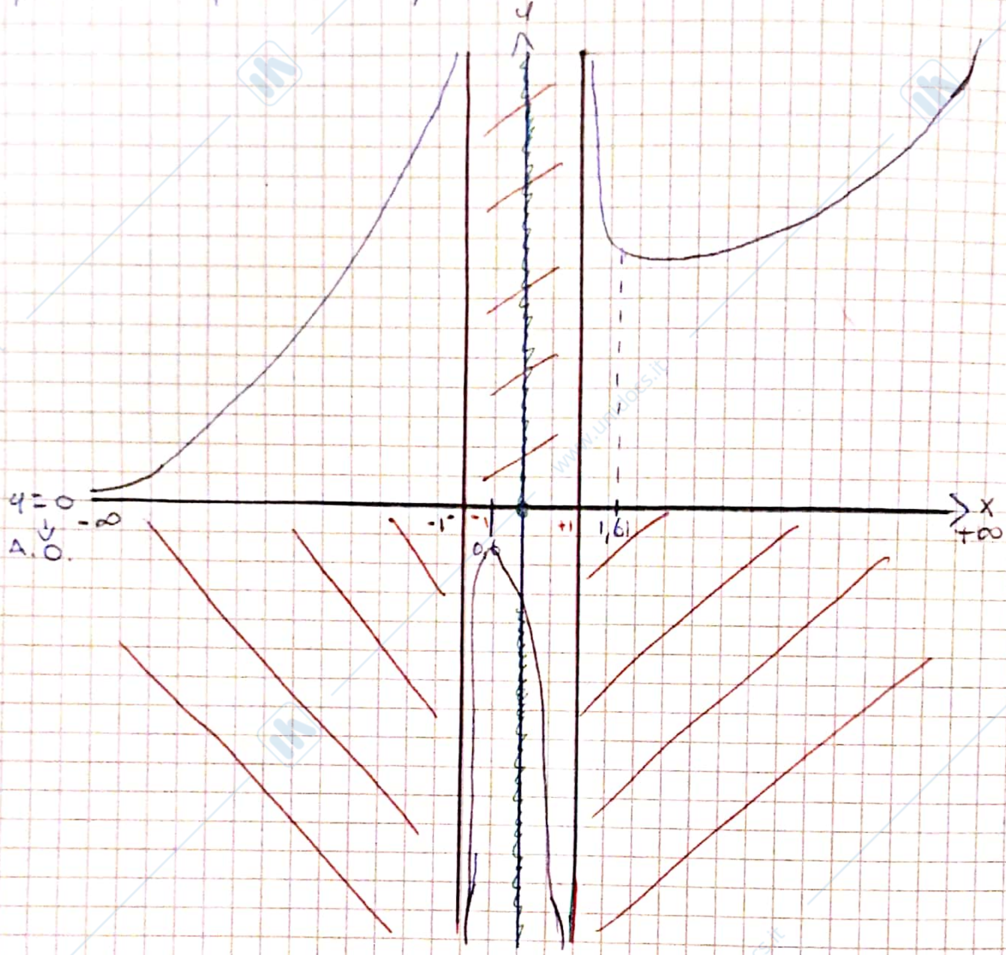
ESERCITAZIONE

$$f(x) = \frac{e^{2x-3}}{x^2-1}$$

$x^2 - 1 \neq 0 \Rightarrow x \neq \pm 1$ $\quad D_f = \mathbb{R} - \{\pm 1\}$

$D_f =]-\infty; -1[\cup]-1, +1[\cup]+1, +\infty[$

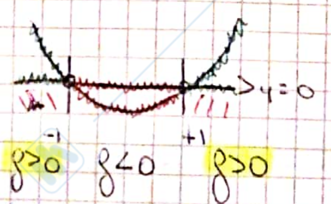


SEGNO: $f(x) \geq 0$

$$\frac{e^{2x-3}}{x^2-1} \geq 0 \quad \left(\frac{+}{?}\right) \Rightarrow$$

$\Rightarrow x^2 - 1 > 0$

$x^2 > \pm 1$



LIMITI E ASINTOTI

$$\lim_{x \to -\infty} \frac{e^{2x-3}}{x^2-1} = \frac{0}{+\infty} = 0 = y = 0$$

$$\lim_{x \to +\infty} \frac{e^{2x-3}}{x^2-1} = \frac{+\infty}{+\infty} (F.I.) = +\infty$$

(esponenziale più forte delle potenze)

Non è limitata superiormente.
sup $f(x) = +\infty$

$$\lim_{x\to -1^-} \frac{e^{2x-3}}{x^2-1} = \frac{e^{-5}}{0} \ (F.I) = +\infty$$ } Sia il N(x) che il D(x) sono positivi. $x=-1^-$ è un asintoto verticale

$$\lim_{x\to -1^+} \frac{e^{2x-3}}{x^2-1} = \frac{e^{-5}}{0} \ (F.I) = -\infty$$ } Avremo il N(x) negativo (regola dei segni) $x = -1^+$ è un asintoto verticale.

→ Non è limitata inferiormente · $\inf f(x) = -\infty$

$$\lim_{x\to +1^-} \frac{x^{2x-3}}{x^2-1} = \frac{x^{-1}}{0} = -\infty$$ } $x=+1^-$ è un asintoto verticale

$$\lim_{x\to +1^+} \frac{e^{2x-3}}{x^2-1} = \frac{x^{-1}}{0} = +\infty$$ } $x=+1^+$ è un asintoto verticale

MONOTONIA:

$$f'(x) = \frac{e^{2x-3}\cdot(2)\cdot(x^2-1) - e^{2x-3}(2x)}{(x^2-1)^2}$$

$$f'(x) = \frac{e^{2x-3}(2x^2-2-2x)}{(x^2-1)^2} \cdot \underbrace{(x^2-x-1)}_{?} \geq 0$$

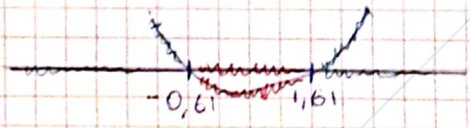
↓ +

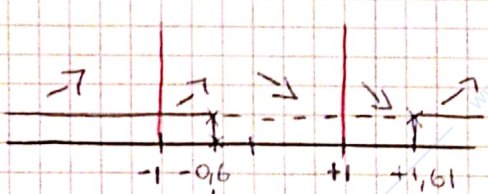
$x^2-x-1 \geq 0$

$x = \frac{1 \pm \sqrt{1+4}}{2}$

$x_1 = \frac{1-\sqrt{5}}{2} = \frac{-1,23}{2} = -0,61$

$x_2 = \frac{1+\sqrt{5}}{2} = \frac{3,23}{2} = 1,61$





Punto di massimo RELATIVO

Punto di minimo RELATIVO

CODOMINIO:

$D_f:]-\infty; -1[\cup]-1; \frac{1-\sqrt{5}}{2}] \cup [\frac{1-\sqrt{5}}{2}; +1[\cup]1; \frac{1+\sqrt{5}}{2}] \cup [\frac{1+\sqrt{5}}{2}; +\infty[$

$f(x):]0; +\infty[\cup]-\infty; f(\frac{1-\sqrt{5}}{2})] \cup]+\infty; f(\frac{1-\sqrt{5}}{5})[\cup [f(\frac{1+\sqrt{5}}{2}); +\infty[\cup [f(\frac{1+\sqrt{5}}{2}); +\infty[$

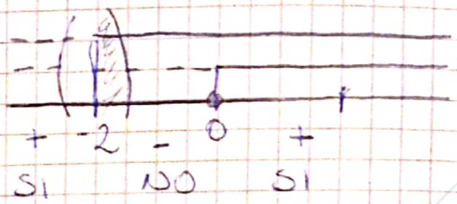
$f(x) =]-\infty; f(\frac{1-\sqrt{5}}{2})] \cup]0; +\infty[$ } Questo sarà il nostro codominio.

Scansionato con CamScanner

ESERCITAZIONE:

$$f(x) = \log_{\frac{1}{2}} \frac{x}{x+2}$$

$$\begin{cases} x+2 \neq 0 \Rightarrow x \neq -2 \\ \frac{x}{x+2} > 0 \Rightarrow \begin{matrix} x > 0 \\ x+2 > 0 \Rightarrow x > -2 \end{matrix} \end{cases}$$



$D_f =]-\infty; 2[\cup]0; +\infty[$

SEGNO: $f(x) \geq 0$

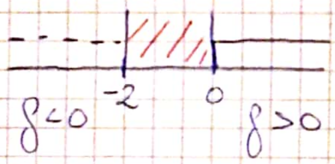
$$\log_{\frac{1}{2}} \frac{x}{x+2} \geq 0 \Rightarrow \left(\frac{1}{2}\right)^{\log_{\frac{1}{2}} \frac{x}{x+2}} \leq \left(\frac{1}{2}\right)^0$$

$$\Rightarrow \frac{x}{x+2} \leq 1 \Rightarrow \frac{x}{x+2} - 1 \leq 0$$

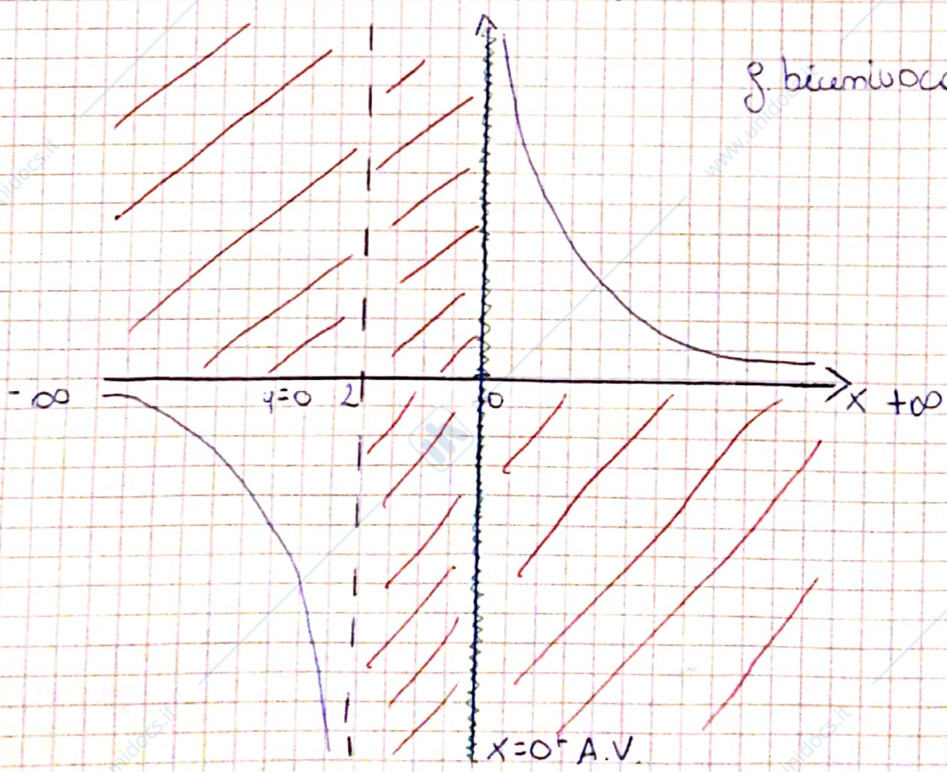
$$\frac{x-x-2}{x+2} \leq 0 \Rightarrow \frac{2}{x+2} \geq 0$$

$$\Rightarrow f(x) > 0 \Rightarrow x+2 > 0 \Rightarrow x > -2$$

$f(x) = 0$ mai



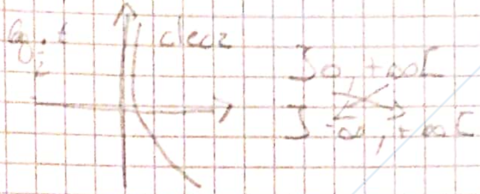
f biunivoca



LIMITI E ASINTOTI.

$\lim_{x \to \pm\infty} \log_{\frac{1}{2}} \frac{x}{x+2} = \lim_{t \to +1} \log_{\frac{1}{2}} t = \log_{\frac{1}{2}} 1 = 0$

$y = 0$ ↳ As. Oriz. $(\pm\infty)$



$\lim_{x \to \pm\infty} \frac{x}{x+2} = \frac{\infty}{\infty}$ (F.I.) $= +1$

$\lim_{x \to -2^-} \log_{\frac{1}{2}} \frac{x}{x+2} = \lim_{t \to +\infty} \log_{\frac{1}{2}} t = -\infty$

$\lim_{x \to -2^-} \frac{x}{x+2} = \frac{-2}{0}$ (F.I.) $= +\infty$

$x = -2^-$ A.V.

f non è limitata inferiormente $\inf f(x) = -\infty$

$\lim_{x \to 0^+} \log_{\frac{1}{2}} \frac{x}{x+2} = \lim_{t \to 0} \log_{\frac{1}{2}} t = +\infty$

$\lim_{x \to 0^+} \frac{x}{x+2} = \frac{0}{2} = 0$

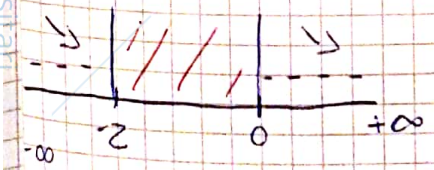
$x = 0^+$ A.V. f non è limitata superiormente. $\sup f = +\infty$

MONOTONIA

$f(x) = \log_{\frac{1}{2}} \frac{x}{x+2}$

$f'(x) = \frac{1}{\frac{x}{x+2} \cdot (\ln \frac{1}{2})} \cdot \frac{\cancel{x}+2-\cancel{x}}{(x+2)^2} = \frac{2}{(\frac{x}{x+2})(\ln\frac{1}{2})(x+2)^2} \geq 0$

$f'(x) \geq 0$ mai



CODOMINIO

$D_f:]-\infty, -2[\cup]0, +\infty[$

$f(x):]-\infty; 0[\cup]0, +\infty[= \mathbb{R} - \{0\}$