ESERCITAZIONI

3 eq. e 3 inc.

$$\left(\begin{array}{ccc|c} k & 1 & 0 & 0 \\ 3 & k & -1 & k \\ 6 & -2 & 0 & 0 \end{array}\right) = (A|b)$$

$$A = \begin{pmatrix} k & 1 & 0 \\ 3 & k & -1 \\ 6 & -2 & 0 \end{pmatrix}$$

det A:
- $\neq 0 \to$ T. Comp. e det $\to \exists 1$ sol $\to$ Regola di Cramer $\dfrac{|\ \ |}{|A|}$
- $= 0 \to$ T.C. $\to$
  - $\nexists$ sol (impossibile)
  - $\exists_\infty$ sol (comp. e indeterminato) $\to$ R.C.

Rang A, Rang (A|b) $\Rightarrow$ (= $\exists\infty$ sol ; $\neq$ $\nexists$ sol)

$$\det A = +1\begin{vmatrix} k & 1 \\ 6 & -2 \end{vmatrix} = -2k-6 = 0 \Rightarrow +2k = -6 \Rightarrow k = -\frac{6}{2} \Rightarrow k = -3$$

$|A| \to -2(k+3)$

Se $k = -3$ avremo che il det $A = 0$.

Se $k \neq -3$ avremo che il det $A \neq 0 \to \exists 1$ sol $\to$

$$\begin{cases} x_1 = \dfrac{\begin{vmatrix} 0 & 1 & 0 \\ k & k & -1 \\ 0 & -2 & 0 \end{vmatrix}}{-2(k+3)} \\ x_2 = \dfrac{\begin{vmatrix} k & 0 & 0 \\ 3 & k & -1 \\ 6 & 0 & 0 \end{vmatrix}}{-2(k+3)} \\ x_3 = \dfrac{\begin{vmatrix} k & 1 & 0 \\ 3 & k & k \\ 6 & -2 & 0 \end{vmatrix}}{-2(k+3)} \end{cases}$$

$$\begin{cases} x_1 = \dfrac{-1\begin{vmatrix} k & -1 \\ 0 & 0 \end{vmatrix}}{-2(k+3)} = 0 \\ x_2 = \dfrac{k\begin{vmatrix} k & -1 \\ 0 & 0 \end{vmatrix}}{-2(k+3)} = 0 \\ x_3 = \dfrac{k\begin{vmatrix} k & k \\ -2 & 0 \end{vmatrix} - 1\begin{vmatrix} 3 & k \\ 6 & 0 \end{vmatrix}}{-2(k+3)} = \dfrac{k(+2k) - 1(6k)}{-2(k+3)} = \dfrac{2k^2+6k}{-2(k+3)} = \dfrac{\cancel{2}k\cancel{(k+3)}}{-\cancel{2}\cancel{(k+3)}} = -k \end{cases}$$

Se $k \neq -3$ allora $\exists 1$ sol.

$(0; 0; -k)$

Vediamo cosa succede se $k=-3$.

$$\left(\begin{array}{ccc|c} -3 & 1 & 0 & 0 \\ 3 & -3 & -1 & -3 \\ 6 & -2 & 0 & 0 \end{array}\right) = (A|b)$$

$$A = \begin{pmatrix} \boxed{\begin{matrix} -3 & 1 \\ 3 & -3 \end{matrix}} & \begin{matrix} 0 \\ -1 \end{matrix} \\ \begin{matrix} 6 & -2 \end{matrix} & 0 \end{pmatrix} = \det A = 0 \qquad \text{Rango } A = \cancel{1}, 2, \cancel{3}$$

$$D_{22} = \begin{vmatrix} -3 & 1 \\ 3 & -3 \end{vmatrix} = 9-3 = 6 \neq 0 \qquad \text{Rango } A = 2$$

$$\text{Rango } (A|b) = \cancel{1}, 2, \cancel{3} \qquad |D_{33}| = |A|$$

$$D'_{33} = \begin{vmatrix} -3 & 1 & 0 \\ \cancel{3} & \cancel{-3} & -3 \\ 6 & -2 & 0 \end{vmatrix} = +3 \begin{vmatrix} -3 & 1 \\ 6 & -2 \end{vmatrix} = +6-6 = 0 \qquad \text{Rango } (A|b) = 2$$

Rango A = Rango$(A|b)$ $\exists \infty$ sol

Regola di Cramer

$$\left(\begin{array}{ccc|c} -3 & 1 & 0 & 0 \\ 3 & -3 & -1 & -3 \\ \cancel{6} & \cancel{-2} & \cancel{0} & \cancel{0} \end{array}\right)$$

Essendo Rango 2 solo le prime 2 eq. sono importanti.

$$\left(\begin{array}{cc|c} -3 & 1 & 0 \\ 3 & -3 & x_3-3 \end{array}\right)$$

$$\begin{cases} x_1 = \dfrac{\begin{vmatrix} 0 & 1 \\ x_3-3 & -3 \end{vmatrix}}{6} = \dfrac{-x_3+3}{6} = \dfrac{+\cancel{3}^{1} - x_3}{\cancel{6}_{2}} = \dfrac{+1-x_3}{2} \\[2ex] x_2 = \dfrac{\begin{vmatrix} -3 & 0 \\ 3 & x_3-3 \end{vmatrix}}{6} = \dfrac{-9+3x_3}{6} = \dfrac{-3+3x_3}{2} \end{cases}$$

Se $k=-3$ allora $\exists \infty$ sol:

$$\left(\frac{1-x_3}{2}, \frac{-3+3x_3}{2}, x_3\right) \forall x_3$$

Soluzione completa matrice

Se $k \neq -3$ allora $\exists$ 1 sol:

$(0; 0; -k)$

Se $k = -3$ allora $\exists \infty$ sol:

$\left(\frac{1-x_3}{2}; \frac{-3+3x_3}{2}; x_3\right) \forall x_3$

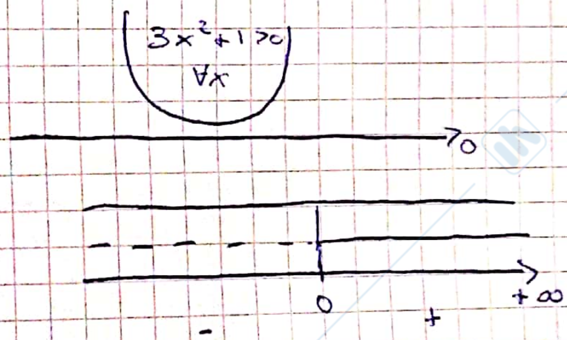
$f(x) = 1 - \log_{\frac{1}{2}}\left(\frac{x}{3x^2+1}\right)$

DOMINIO:

$3x^2+1 \neq 0 \Rightarrow (\Delta = b^2-4ac < 0 \Rightarrow \nexists \text{ sol}) \Rightarrow$ SEMPRE

$\frac{x}{3x^2+1} > 0 \Rightarrow$ $N(x) = x > 0$

$D(x) = 3x^2+1 > 0$



$D_f:]0; +\infty[$

LIMITI

$\lim_{x \to 0^+} f$ e $\lim_{x \to +\infty} f$

$\lim_{x \to 0^+} \frac{x}{3x^2+1} = \frac{0}{1} = 0$

$\lim_{x \to +\infty} 1 - \log_{\frac{1}{2}}\left(\frac{x}{3x^2+1}\right) = -\infty$

$x = 0^+$ è un asintoto verticale

$$\lim_{x \to +\infty} \frac{x}{3x^2+1} = \frac{\infty}{\infty} \text{ (F.I.)} = \lim_{x \to +\infty} \frac{\cancel{x}}{3x^{\cancel{2}}} = \lim_{x \to +\infty} \frac{1}{3x} = 0$$

$$\lim_{x \to 0} \log_{\frac{1}{2}} t = +\infty$$

$$\lim_{x \to +\infty} f(x) = -\infty \quad \text{NO AS. OR.}$$

MONOTONIA:

$$f'(x) = \frac{-1}{\frac{x}{3x^2+1} \cdot \ln\frac{1}{2}} \cdot \frac{1(3x^2+1) - x(6x)}{(3x^2+1)^2} \geq 0$$

$$= \frac{-1}{\frac{x}{3x^2+1} \cdot \left(\ln\frac{1}{2}\right)} \cdot \frac{3x^2+1-6x^2}{(3x^2+1)^2} \geq 0$$

$$= \frac{-1}{\frac{x}{3x^2+1} \cdot \left(\ln\frac{1}{2}\right)} \cdot \frac{1-3x^2}{(3x^2+1)^2} \geq 0 \Rightarrow 1-3x^2 \geq 0$$

\+ \+ \+

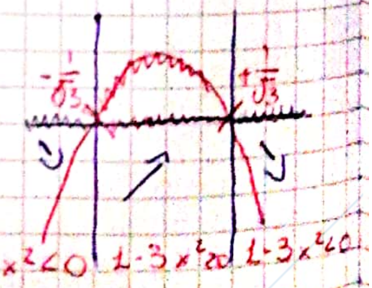
Essendo primo di 1 è negativo

$1-3x^2=0$

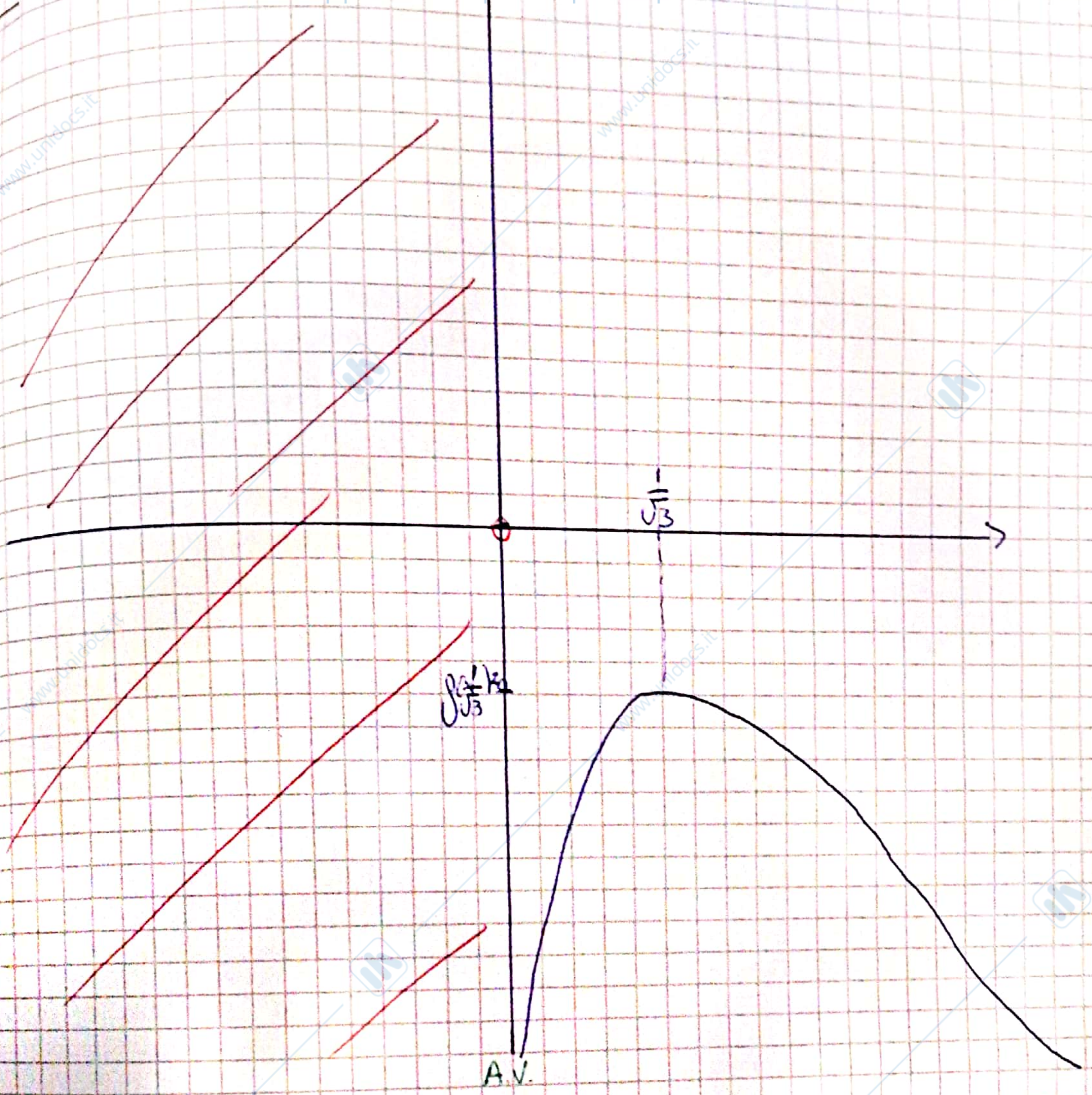
$3x^2=1$

$x^2=+\frac{1}{\sqrt{3}}$

$x^2=-\frac{1}{\sqrt{3}}$



$x=\frac{1}{\sqrt{3}}$ punto di Max assoluto



SEGNO DELLA FUNZIONE:

$$f(x) = 1 - \log_{\frac{1}{2}} \frac{x}{3x^2+1} \geq 0$$

$$-\log_{\frac{1}{2}} \frac{x}{3x^2+1} \geq -1$$

$$\log_{\frac{1}{2}} \frac{x}{3x^2+1} \leq +1$$

$$\left(\frac{1}{2}\right)^{\log_{\frac{1}{2}} \frac{x}{3x^2+1}} \geq \left(\frac{1}{2}\right)^1$$

$$\frac{x}{3x^2+1} \geq \frac{1}{2} \Rightarrow \frac{x}{3x^2+1} - \frac{1}{2} \geq 0 \Rightarrow \frac{2x-(3x^2+1)}{2(3x^2+1)} \geq 0 \Rightarrow \frac{2x-3x^2-1}{2(3x^2+1)} \geq 0$$

$$\frac{-3x^2+2x-1}{2(3x^2+1)} \geq 0 \Rightarrow f \geq 0 \Rightarrow -3x^2+2x-1 \text{ MAI}$$

$\nexists$ sol

3

$$\left(\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 2 & k & 0 & 0 \\ -1 & 9 & k & 0 \end{array}\right)$$

Un sistema omogeneo ha sempre solu... non è mai impossibile, ma sempre com...

$x = 0 \ \forall x$

è sempre una soluzione , soluzion...

$\exists$ 1 sol → Determinato $(0;0;0)$

$\exists \infty$ sol → Indeterminat...

b=0 OMOGENEO.

8'

$$|A| = -2\begin{vmatrix} 2 & k \\ -1 & 9 \end{vmatrix} k \begin{vmatrix} 1 & -1 \\ 2 & k \end{vmatrix} = -2(18+k)\,k(k+2) = -36 - \cancel{2k} - k^2 + \cancel{2k}$$

$|A| = -k^2 - 36 \qquad |A| = 0 \Rightarrow k = \pm 6$

$$\left(\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 2 & \pm 6 & 0 & 0 \\ \cancel{-1} & \cancel{9} & \cancel{\pm 6} & \cancel{0} \end{array}\right) \quad \left(\begin{array}{cc|c} 1 & -1 & +2x_3 \\ 2 & \pm 6 & 0 \end{array}\right)$$

∞ sol

Se $k \neq \pm 6$ allora $|A| \neq 0$ e abbiamo 1 sol.
$(0;0;0)$

Se $k = \pm 6$ allora $|A| = 0$ e abbiamo ∞ sol.

Scansionato con CamScanner