

Analisi Matematica 2

V esercitazione

1. CALCOLO DELLA SOLUZIONE PARTICOLARE

1. PROBLEMA Calcolare l'integrale generale delle seguenti equazioni

(1) $y'' + y' - 2y = 1$.

(2) $4y'' + y = 1$.

(3) $y'' - 2y' + 2y = x$.

(4) $y'' + 2y' + 3y = 2e^{3x}$.

(5) $y'' + 4y = 4 \cos x$.

(6) $y'' + 3y = 4x \cos 2x$.

2. PROBLEMA Calcolare l'integrale generale delle seguenti equazioni

(1) $y'' - y' = x^2$.

(2) $y'' + 4y = 4 \cos 2x$.

(3) $y'' - 3y' = 1$.

(4) $y'' - 3y' = x^2 + 1$.

(5) $y'' - 2y' + 5y = e^x \cos(2x)$.

(6) $y'' - y = \sinh x$.

3. PROBLEMA Calcolare l'integrale generale delle seguenti equazioni

(1) $y'' + 4y' + 4y = -e^{-2x}$.

(2) $y'' - 2y' + y = e^x + xe^x$.

(3) $y'' + 2y' + y = x^2 e^{-x}$.

(4) $y'' + 4y' + 4y = x + e^{-2x}$.

(5) $4y'' - 4y' + y = \sqrt{e^x}$.

(6) $y'' - 2y' + y = \cosh(x)$.

2. PROBLEMI DI CAUCHY

4. PROBLEMA Calcolare la soluzione dei seguenti problemi di Cauchy:

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$$(1) \begin{cases} y'' + y' - 2y = 1 \\ y(0) = 0 \\ y'(0) = 0 \end{cases} .$$

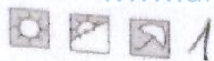
$$(2) \begin{cases} y'' - 5y' + 6y = 18x \\ y(0) = 1 \\ y'(0) = 0 \end{cases} .$$

$$(3) \begin{cases} y'' - 4y = 4e^{2x} \\ y(0) = 0 \\ y'(0) = 1 \end{cases} .$$

$$(4) \begin{cases} y'' + y = \cos x \\ y(\pi) = 0 \\ y'(\pi) = -1 \end{cases} .$$

$$(5) \begin{cases} y'' + 9y = 6 \sin 3x \\ y(\pi/6) = 1/3 \\ y'(\pi) = \pi/2 \end{cases} .$$

$$(6) \begin{cases} y'' - 6y' + 9y = 4e^{3x} \\ y(0) = -1 \\ y'(0) = 3 \end{cases} .$$



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1.1 $y'' + y' - 2y = 1$

1 Calcolo integrale generale della omogenea

$$y'' + y' - 2y = 0 \quad k^2 + k - 2 = 0$$

$$k_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} \quad \begin{matrix} -2 \\ 1 \end{matrix}$$

$$y(z) = A e^{-2x} + B e^x$$

2 Calcolo la soluzione particolare della non omogenea

forzante = costante

$$y_p = C \quad y_p' = 0 \quad y_p'' = 0$$

$$-2C = 1 \quad y_p = C = -\frac{1}{2}$$

$$y(x) = y(z) + y_p = A e^{-2x} + B e^x - \frac{1}{2}$$

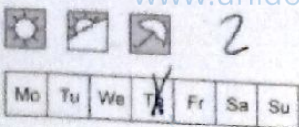
1.2 $4y'' + y = 1$

$$4k^2 + 1 = 0 \quad k = \pm \frac{i}{2}$$

$$y(z) = A \cos \frac{1}{2}x + B \sin \frac{1}{2}x$$

$$y_p = C \quad C = 1$$

$$y(x) = A \cos \frac{x}{2} + B \sin \frac{x}{2} + 1$$



$$1.6 \quad y'' + 3y = 4x \cos 2x$$

$$k^2 + 3 = 0 \quad k = \pm i\sqrt{3}$$

$$y(z) = A \cos \sqrt{3}x + B \sin \sqrt{3}x$$

$$y_p = (C_1 + C_2 x) \cos 2x + (D_1 + D_2 x) \sin 2x$$

$$y_p' = C_2 \cos 2x - 2(C_1 + C_2 x) \sin 2x + D_2 \sin 2x + 2(D_1 + D_2 x) \cos 2x$$

$$y_p'' = -2C_2 \sin 2x - 2C_2 \sin 2x - 4(C_1 + C_2 x) \cos 2x + 2D_2 \cos 2x +$$

$$2D_2 \cos 2x - 4(D_1 + D_2 x) \sin 2x =$$

$$(C_1 + D_2) \cos 2x - 4C_2 \sin 2x + 4D_2 \cos 2x - 4(C_1 + C_2 x) \cos 2x - 4(D_1 + D_2 x) \sin 2x$$

$$-4C_2 \sin 2x + 4D_2 \cos 2x - 4(C_1 + C_2 x) \cos 2x - 4(D_1 + D_2 x) \sin 2x +$$

$$+ 3(C_1 + C_2 x) \cos 2x + 3(D_1 + D_2 x) \sin 2x = 4x \cos 2x$$

$$-(4C_1 - 4D_2 - 3C_1) \cos 2x - (4C_2 + 4D_2 - 3D_1) \sin 2x$$

$$+ (-4C_2 + 3C_2)x \cos 2x + (-4D_2 + 3D_2)x \sin 2x = 4x \cos 2x$$

$$\begin{cases} 4D_2 - C_1 = 0 & C_1 = 0 \\ 4C_2 + D_1 = 0 & D_2 = 0 \\ -C_2 = 4 & C_2 = -4 \\ D_2 = 0 & D_1 = 16 \end{cases}$$

$$y(x) = A \cos \sqrt{3}x + B \sin \sqrt{3}x - 4 \cos 2x + 16 \sin 2x$$

1.3

$$y'' - 2y' + 2y = x$$

$$k^2 - 2k + 2 = 0 \quad k_{1,2} = 1 \pm \sqrt{1-2} = 1 \pm i$$

$$y(x) = A e^x \cos x + B e^x \sin x$$

$$y_p = C + Dx \quad y' = D \quad y'' = 0$$

$$-2D + 2C + 2Dx = x$$

$$\begin{cases} D = \frac{1}{2} & C = \frac{1}{2} \\ -D + C = 0 \end{cases}$$

$$y(x) = A e^x \cos x + B e^x \sin x + \frac{1}{2} + \frac{x}{2}$$

1.4

$$y'' + 2y' + 3y = 2e^{3x}$$

$$k^2 + 2k + 3 = 0 \quad k_{1,2} = -1 \pm \sqrt{1-3} = -1 \pm \sqrt{2}i$$

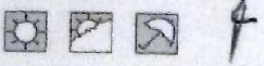
$$y(x) = A e^{-x} \cos \sqrt{2}x + B e^{-x} \sin \sqrt{2}x$$

$$y_p = C e^{3x} \quad y_p' = 3C e^{3x} \quad y_p'' = 9C e^{3x}$$

$$C e^{3x} (9 + 6 + 3) = 2 e^{3x}$$

$$18C = 2 \quad C = \frac{1}{9}$$

$$y(x) = A e^{-x} \cos \sqrt{2}x + B e^{-x} \sin \sqrt{2}x + \frac{e^{3x}}{9}$$



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2.1

$$x'' - y' = x^2$$

$$k^2 - k \begin{array}{l} \swarrow 0 \\ \searrow 1 \end{array}$$

$$y(x) = A + B e^x$$

$$y_p = C + Dx + Ex^2$$

è anche sol. dell'omogenea

$$\rightarrow \text{Proviamo } y_p = x y_p = Cx + Dx^2 + Ex^3$$

$$y_p' = C + 2Dx + 3Ex^2$$

$$y_p'' = 2D + 6Ex$$

$$2D + 6Ex - C - 2Dx + 3Ex^2 = x^2$$

$$\begin{cases} 3E = 1 & E = \frac{1}{3} \\ 2D - C = 0 & D = -1 \\ 6E + 2D = 0 & C = -2 \end{cases}$$

$$y(x) = A + B e^x - 2x - x^2 - \frac{x^3}{3}$$



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2.2

$$y'' + 4y = 4\cos 2x$$

$$k^2 + 4 = 0 \quad k = \pm 2i$$

$$y(z) = A \cos 2x + B \sin 2x$$

$$y_p = C_1 \cos 2x + C_2 \sin 2x \rightarrow x y_p = y_{p1} = C_1 x \cos 2x + C_2 x \sin 2x$$

$$y_{p1}' = -C_1 \sin 2x - 2C_1 x \cos 2x + C_2 \sin 2x + 2C_2 x \cos 2x$$

$$y_{p1}'' = -2C_1 \cos 2x - 2C_1 \sin 2x - 4C_1 x \sin 2x + 2C_2 \cos 2x + 2C_2 \sin 2x - 4C_2 x \cos 2x$$

$$\sin 2x (-4C_1) + x \sin 2x (2C_2) + \cos 2x (4C_2) + 0 = 4\cos 2x$$

$$\begin{cases} 4C_2 = 4 & \rightarrow C_2 = 1 \\ -4C_1 = 0 & C_1 = 0 \end{cases}$$

$$y(x) = A \cos 2x + B \sin 2x + x \sin 2x$$

2.6 $y'' - y = \sinh x = \frac{e^x - e^{-x}}{2}$

$$k^2 - 1 = 0 \quad k = \pm 1$$

$$y(t) = A e^x + B e^{-x}$$

prima la caso per $\frac{e^x}{2}$ poi per $-\frac{e^{-x}}{2}$
e poi sommo le 2 particolari

1° caso $\left(\frac{e^x}{2}\right)$: $y_1 = C e^x \rightarrow y_{p1} = C x e^x$

$$y_{p1}' = C e^x + C x e^x$$

$$y_{p1}'' = C e^x + C e^x + C x e^x$$

$$2 C e^x = \frac{e^x}{2}$$

$$\rightarrow C = \frac{1}{4}$$

2° caso $\left(-\frac{e^{-x}}{2}\right)$ $y_1 = D e^{-x} \rightarrow y_{p1} = D x e^{-x}$

$$y_{p1}' = D e^{-x} - D x e^{-x}$$

$$y_{p1}'' = -D e^{-x} - D e^{-x} + D x e^{-x}$$

$$-2 D e^{-x} = -\frac{e^{-x}}{2}$$

$$\rightarrow D = \frac{1}{4}$$

$$y_{part} = \frac{x e^x}{4} + \frac{x e^{-x}}{4}$$

$$y(x) = A e^x + B e^{-x} + \frac{x e^x}{4} + \frac{x e^{-x}}{4}$$



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$$3.1 \quad y'' + 4y' + 4y = -e^{-2x}$$

$$k^2 + 4k + 4 = 0 \quad k_{1,2} = -2 \pm \sqrt{4-4} = -2$$

$$y(x) = A e^{-2x} + B x e^{-2x}$$

$$y_p = x^2 (C e^{-2x}) \quad y_p' = 2x C e^{-2x} + 2x^2 C e^{-2x}$$

$$y_p'' = 2C e^{-2x} - 4x C e^{-2x} - 4x C e^{-2x} + 4x^2 C e^{-2x}$$

$$x^2 e^{-2x} (4C - 8C) + x e^{-2x} (-8C + 8C) + 2C e^{-2x} = -e^{-2x}$$

$$2C = -1 \quad C = -\frac{1}{2}$$

$$y(x) = A e^{-2x} + B x e^{-2x} - \frac{x^2}{2} e^{-2x}$$

Problema di Cauchy

$$\textcircled{1} \begin{cases} y'' + y' - 2y = 1 \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

$$k^2 + k - 2 = 0 \quad k_{1,2} = -1, 2$$

$$y(x) = A e^x + B e^{-2x}$$

$$y_p = C \quad y_p' = 0 \quad y_p'' = 0$$

$$\hookrightarrow C = -\frac{1}{2}$$

$$y(x) = A e^x + B e^{-2x} - \frac{1}{2}$$

$$\begin{cases} A + B = \frac{1}{2} \\ 3B = \frac{1}{2} \Rightarrow B = \frac{1}{6} \end{cases} \Rightarrow y(x) = \frac{e^x}{3} + \frac{e^{-2x}}{6} - \frac{1}{2}$$

$$A - 2B = 0 \quad A = \frac{1}{3}$$