Lezione 27-02-2023

Limite per funzioni di più variabili

Sia $A \subseteq \mathbb{R}^n$ $\quad x_0 \in \mathcal{D}(A)$

Sia $f : A \to \mathbb{R}$

ES

$$f(x,y) = x^2 + y^2 \qquad f : \mathbb{R}^2 \to \mathbb{R}$$

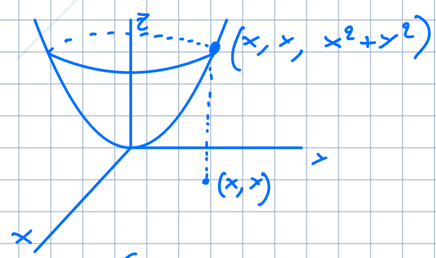


grafico di $f = \{ (x, y, f(x,y)), (x,y) \in A \}$

ES

$$f : \mathbb{R}^3 \setminus \{(0,0,0)\} \to \mathbb{R}$$

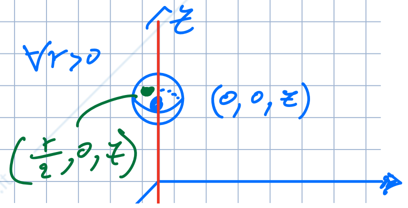
$$f(x,y,z) = \frac{x^2+y^2}{x^2+y^2+z^2}$$

ES

$$f(x,y,z) = \lg(x^2+y^2) + e^z$$

$$x^2 + y^2 > 0 \iff (x,y) \neq (0,0) \iff \mathbb{R}^3 \setminus \{(0,0,z)\ z \in \mathbb{R}\}$$

$$\mathcal{D}(\mathbb{R}^3 \setminus \{(0,0,z)\ z \in \mathbb{R}\}) = \mathbb{R}^3$$



LA ESCLUDO
PER IL DOMINIO

Limiti per funzioni di più variabili

Sia $A \subseteq \mathbb{R}^n$ $x_0 \in \mathcal{D}(A)$

Sia $f: A \to \mathbb{R}$

Def

Diciamo che $\lambda \in \mathbb{R}$ è il limite di $f(x)$ per x che tende a x_0 e scriviamo

$$\lim_{x \to x_0} f(x) = \lambda \qquad \text{oppure} \qquad f(x) \xrightarrow[x \to x_0]{} \lambda$$

se

$$\forall \varepsilon > 0 \quad \exists \delta_\varepsilon > 0 \quad t.c.$$

$$|f(x) - \lambda| < \varepsilon \quad \forall x \in B(x_0, \delta_\varepsilon) \cap A \setminus \{x_0\}$$

PALLA EUCLIDEA DI CENTRO x_0 e RAGGIO δ_ε

Es

$f(x,y) = x^2 + y^2$

$f : \mathbb{R}^2 \to \mathbb{R}$

$\lim_{(x,y)\to(0,0)} f(x,y) = 0,0$ INFATTI se io voglio che

$\forall \varepsilon > 0$ $\quad \left| (x^2 + y^2) - 0 \right| < \varepsilon$ è sufficiente

prendere $\delta_\varepsilon = \sqrt{\varepsilon}$

perché se $(x,y) \in B(0, \sqrt{\varepsilon}) \setminus \{(0,0)\} \Rightarrow$

$0 < x^2 + y^2 < \varepsilon$

$\Rightarrow \lim_{(x,y)\to(0,0)} f(x,y) = 0$

Teorema

UNICITÀ DEL LIMITE

Supponiamo $x_0 \in \mathcal{D}(A)$ e $f : A \to \mathbb{R}$

e supponiamo $\lim_{x\to x_0} f(x) = \lambda$ e $\lim_{x\to x_0} f(x) = \alpha \implies \lambda = \alpha$

DIMOSTRAZIONE

$\lim_{x \to x_0} f(x) = \lambda \Rightarrow \forall \varepsilon > 0 \quad \exists \delta_\varepsilon > 0$ t.c.

① $|f(x) - \lambda| < \varepsilon \qquad \forall x \in B(x_0, \delta_\varepsilon) \cap A \setminus \{x_0\}$

$\lim_{x \to x_0} f(x) = \alpha \Rightarrow \forall \varepsilon > 0 \quad \exists \delta'_\varepsilon > 0$ t.c.

② $|f(x) - \alpha| < \varepsilon \qquad \forall x \in B(x_0, \delta'_\varepsilon) \cap A \setminus \{x_0\}$

Sia $\delta''_\varepsilon = \min\{\delta_\varepsilon, \delta'_\varepsilon\} \Rightarrow \forall x \in B(x_0, \delta''_\varepsilon) \cap A \setminus \{x_0\}$

valgono sia ① che ②

$|\lambda - \alpha| = (\lambda - f(x)) + (f x - \alpha) \leq |\lambda - f(x)| + |f(x) - \alpha| < \varepsilon + \varepsilon = 2\varepsilon$

$\Rightarrow \forall \varepsilon > 0 \quad |\lambda - \alpha| < 2\varepsilon \Rightarrow \boxed{\lambda - \alpha = 0 \Rightarrow} \; \lambda = \alpha$

?

Definizione Restrizioni

$f : A \to \mathbb{R}$

$A \subseteq \mathbb{R}^n$

$x_0 \in \mathcal{D}(A)$

Sia $B \subset A$ Definisco la funzione $f_{|B} : B \to \mathbb{R}$

che opera così $f_{|B}(x) = f(x)$

Prop Restrizioni

$f : A \to \mathbb{R} \qquad A \subseteq \mathbb{R}^n \qquad x_0 \in \mathcal{D}(A)$

e $B \subset A \qquad x_0 \in \mathcal{D}(B)$

Se $\exists \lim_{x \to x_0} f(x) = \lambda \Rightarrow \exists \lim_{x \to x_0} f|_B (x) = \lambda$

segue dall'unicità del limite

Criterio in negativo

PUÒ ESSERE USATO SOLO IN negativo e non al contrario

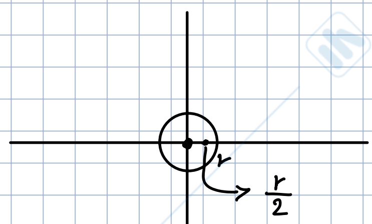
Sia $f : A \to \mathbb{R}$ e $x_0 \in \mathcal{D}(A)$

Siano B_1 e $B_2 \subset A$ t.c. $x_0 \in \mathcal{D}(B_1)$, $x_0 \in \mathcal{D}(B_2)$

e $\lim_{x \to x_0} f|_{B_1}(x) \neq \lim_{x \to x_0} f|_{B_2}(x) \Rightarrow \nexists \lim_{x \to x_0} f(x)$

Es

$f(x,y) = \frac{x \cdot y}{x^2 + y^2}$



$A = \mathbb{R}^2 \setminus \{(0,0)\}$ dominio naturale

$(0,0) \in \mathcal{D}(A)$

$\exists \lim_{(x,y) \to (0,0)} f(x,y)$?

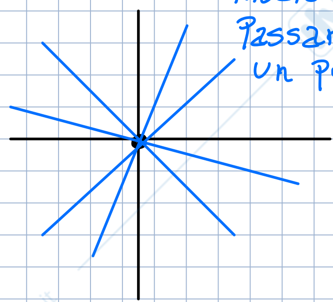
$B_1 = \{(x,0),\ x \neq 0\} \subset A$

$(0,0) \in \mathcal{D}(B_1)$

$f|_{B_1}(x,y) = f(x,0) = \frac{0}{x^2} = 0 \quad x \neq 0$

$\lim_{x \to (0,0)} f|_{B_1}(x,y) = 0$

FASCIO DI RETTE passanti per un punto



$C_m = \{(x, mx),\ x \neq 0\} \quad m \in \mathbb{Z}$

$f|\,(x,y) = f(x,mx) = \frac{m \cancel{x^2}}{(1+m^2)\cancel{x^2}} = \frac{m}{1+m^2}$

$|_{C_m}$

$$\lim_{(x,y)\to(0,0)} f|_{C_m}(x,y) = \frac{m}{1+m^2} \quad \text{dipende da } m$$

$$\text{e } (0,0) \in \mathcal{D}(C_m) \Rightarrow \nexists \lim_{(x,y)\to(0,0)} f(x,y)$$

$$f(x,y) = \frac{x \cdot y^2}{x^2+y^4}$$

$$A = \mathbb{R}^2 \setminus \{(0,0)\}$$

$$(0,0) \in \mathcal{D}(A)$$

$$C_m = \{(x, mx), x \neq 0\} \qquad m \in \mathbb{Z}$$

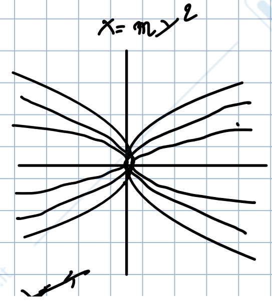
$$f|_{C_m}(x,y) = f(x, mx) = \frac{x\,(mx)^2}{x^2+(mx)^4} = \frac{m^2 x^{\cancel{3}}}{(1+m^4x^2)\,\cancel{x^2}} = \frac{m^2 x}{1+m^4x^2} \xrightarrow[x\to 0]{} 0$$

$$\exists \lim_{(x,y)\to(0,0)} f|_{C_m}(x,y) = 0$$

questo non permette di concludere l'esistenza del limite di f

Intatti

$$P_m = \{(my^2, y), y \neq 0\} \qquad m \in \mathbb{Z}$$



$$f|_{P_m}(x,y) = f(my^2, y) = \frac{my^2 \cdot y^2}{(my^2)^2 + y^4} = \frac{m\cancel{y^4}}{(m^2+1)\cancel{y^4}} = \frac{m}{m^2+1}$$

$$f|_{P_m}(x,y) = \frac{m}{m^2+1} : \text{ dipende da } m$$

$$\exists \lim_{(x,y)\to(0,0)} f|_{P_m}(x,y) = \frac{m}{m^2+1} \text{ dipende da } m \in \mathbb{Z}$$

$$\Rightarrow \nexists \lim_{(x,y)\to(0,0)} f(x,y)$$

Def

Sia $f: A \to \mathbb{R}$, $A \subseteq \mathbb{R}$, $x_0 \in \mathcal{D}(A)$

Diciamo che:

$$\lim_{x\to x_0} f(x) = +\infty$$

se $\forall M>0 \ \exists \delta>0$ t.c. $f(x) > M$

$$\forall x \in B(x_0,\delta) \cap A \setminus \{x_0\}$$

NON ESSENDOCI UNA RELAZ. D'ORDINE NON POSSIAMO FARE IL LIMITE PER $x \to \pm\infty$

Es

$$f(x,y) = \frac{1}{x^2+y^2}$$

$A = \mathbb{R}^2 \setminus \{(0,0)\}$

$(0,0) \in \mathcal{D}(A)$

$$\lim_{(x,y)\to(0,0)} f(x,y) = +\infty \ ?$$

Se voglio che $\frac{1}{x^2+y^2} > M \iff 0 < x^2+y^2 < \frac{1}{M} \iff \sqrt{x^2+y^2} = \frac{1}{\sqrt{M}}$ $\overset{\delta}{\shortparallel}$

$\forall M>0$ se $(x,y)\in B\left(0,\frac{1}{\sqrt{M}}\right)\setminus\{(0,0)\} \Rightarrow f(x,y)>M$

$\Rightarrow \lim_{(x,y)\to(0,0)} f(x) = +\infty$

Def

$f: A\to\mathbb{R}$, $A\subseteq\mathbb{R}^n$, $x_0\in \mathcal{D}(A)$

Diciamo che $\lim_{x\to x_0} f(x) = -\infty$ se

$\forall M>0\ \exists\delta>0$ t.c. $f(x)<-M\ \forall x\in B(x_0,\delta)\cap A\setminus\{x_0\}$

Es

$f(x,y) = \lg(x^2+y^2)$

$f:\mathbb{R}^2\setminus\{(0,0)\}\to\mathbb{R}$

$\lim_{(x,y)\to(0,0)} \lg(x^2+y^2) = -\infty$?

$\delta = e^{-M/2}$

Perché

$\forall M>0$ $\quad \lg(x^2+y^2)<-M \iff 0<x^2+y^2<\delta=e^{-M/2}$

$\forall (x,y)\in B(0,e^{-M/2})\setminus\{(0,0)\}\quad f(x,y)<-M$

$f(x)\xrightarrow[x\to x_0]{}\alpha$ e $g(x)\xrightarrow[x\to x_0]{}\beta\neq 0$

$\Rightarrow \frac{f(x)}{g(x)}\xrightarrow[x\to x_0]{}\frac{\alpha}{\beta}$

Le FORME INDETERMINATE sono

$\infty - \infty$

$\frac{\infty}{\infty}, \frac{0}{0}, \frac{1}{0}, 1^{\infty}, 1^{-\infty}$

Teorema del confronto

$f,g,h : A \to \mathbb{R}, A \subseteq \mathbb{R}^n, x_0 \in \mathcal{D}(A)$

$\exists \delta > 0$ t.c. (1) $f(x) \le g(x) \le h(x) \quad \forall x \in B(x_0, \delta) \cap A$

se $\exists \lim_{x \to x_0} f(x) = \lim_{x \to x_0} h(x) = \lambda \in \mathbb{R} \cup \{\pm\infty\} \Rightarrow \exists \lim_{x \to x_0} g(x) = \lambda$

DIMOSTRAZIONE

$\lim_{x \to x_0} f(x) = \lambda \in \mathbb{R} \Rightarrow \forall \varepsilon > 0 \ \exists \delta_\varepsilon > 0$ t.c.

(2) $|f(x) - \lambda| < \varepsilon \quad \forall x \in B(x_0, \delta_\varepsilon) \cap A \setminus \{x_0\}$

$\lim_{x \to x_0} h(x) = \lambda \in \mathbb{R} \Rightarrow \forall \varepsilon > 0 \ \exists \delta'_\varepsilon > 0$ t.c.

(3) $|h(x) - \lambda| < \varepsilon \quad \forall x \in B(x_0, \delta'_\varepsilon) \cap A \setminus \{x_0\}$

sia $\delta''_\varepsilon = \min\{\delta, \delta_\varepsilon, \delta'_\varepsilon\} \ \forall x \in B(x_0, \delta''_\varepsilon) \cap A \setminus \{x_0\}$

$g(x) - \lambda \overset{(1)}{\le} h(x) - \lambda \le |h(x) - \lambda| \overset{(3)}{<} \varepsilon$

$\vee$ (1)

$\underbrace{f(x) - \lambda}$

$\vee$ (2)

$-\varepsilon$

perchè $|f(x) - \lambda| < \varepsilon$

$\Rightarrow |g(x) - \lambda| < \varepsilon \iff -\varepsilon < g(x) - \lambda < \varepsilon$

$$\Rightarrow \lim_{x \to x_0} g(x) = \lambda$$

Corollario

se $f: A \to \mathbb{R}$ è limitata (cioè $\exists M > 0$ tc $|f(x)| < M$)

e $g: A \to \mathbb{R}$ $x_0 \in \mathcal{D}(A)$ e $\lim_{x \to x_0} g(x) = 0$

$$\Rightarrow f(x) \cdot g(x) \xrightarrow[x \to x_0]{} 0$$

Dimostrazione

$$\underset{\substack{\downarrow x \to x_0 \\ 0}}{0} \leq \underbrace{|f(x) \cdot g(x)|}_{\substack{\downarrow x \to x_0 \\ 0}} = |f(x)|\,|g(x)| \leq \underbrace{M\,|g(x)|}_{\substack{\downarrow x \to x_0 \\ 0}}$$

Perché $\underset{\substack{\downarrow \\ 0}}{-M|g(x)|} \leq \underbrace{f(x)\,g(x)}_{\substack{\downarrow x \to x_0 \\ 0}} \leq \underset{\substack{\downarrow \\ 0}}{M|g(x)|}$

ES

$$f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$$

$$f: \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$$

$$|x| = \sqrt{x^2} \leq \sqrt{x^2+y^2}$$

$$\Downarrow$$

$$\frac{|x|}{\sqrt{x^2+y^2}} < 1 \quad \forall (x,y) \neq (0,0)$$

$\mathcal{D}(\mathbb{R}^2 \setminus \{...\})$

$(0,0) \in \mathcal{D}(\mathbb{R} \setminus \{(0,0)\})$

$$|f(x,y)| = \frac{|x|}{\sqrt{x^2+y^2}} \cdot |y| \leq |y|$$

Disuguaglianza utili per applicare il teorema del confronto

1) $|y| \leq \sqrt{x^2+y^2} \quad \forall (x,y) \neq (0,0)$

2) $|x \cdot y| \leq \frac{1}{2}(x^2+y^2) \quad \forall (x,y) \neq (0,0)$

DIM 2)

$0 \leq (x+y)^2 = x^2+y^2+2xy \Rightarrow -2xy \leq x^2+y^2 \quad \forall (x,y) \in \mathbb{R}^2$

$0 \leq (x-y)^2 = x^2+y^2-2xy \Rightarrow 2xy \leq x^2+y^2 \quad \forall (x,y) \in \mathbb{R}^2$

$$\frac{\cancel{2}|xy|}{\cancel{2}} \leq \frac{x^2+y^2}{2} \quad \forall (x,y) \in \mathbb{R}^2$$

$\Rightarrow$ se $(x,y) \neq (0,0)$ $\quad \frac{|x \cdot y|}{x^2+y^2} \leq \frac{1}{2}$ è limitata per $(x,y) \neq (0,0)$

Lezione 28-02-2023

CAMBIAMENTO DI VARIABILE

$f : A \to \mathbb{R}$

$x_0 \in \mathcal{D}(A)$

$$\lim_{x \to x_0} f(x) = \lim_{y \to 0} f(y + x_0)$$

C.V. $\quad x - x_0 = y \xrightarrow[x \to x_0]{} 0$

$x = y + x_0$

Es

Calcolare $\lim_{x \to 0} f(x)$ nei seguenti casi

1) $f(x,y) = \dfrac{x^3 - y^2}{x^2 - xy + y^2}$

2) $f(x,y) = \dfrac{x^3 y^2}{x^6 + y^4}$

3) $f(x,y) \quad \dfrac{x^2 \sin(y)}{x^2 + y^2}$

1

$x^2 - xy + y^2 \neq 0$

$|xy| \leq \dfrac{x^2 + y^2}{2}$

$0 = x^2 - xy + y^2 \geq x^2 + y^2 - \left(\dfrac{x^2 + y^2}{2}\right) =$

$= \dfrac{1}{2}(x^2 + y^2)$

$(0,0) \in \mathcal{D}\left(\mathbb{R}^2 \setminus \{0,0\}\right.$

$C_m = \{(x, mx) \quad x \neq 0\}$

$$f\Big|_{C_m}(x,y) = f(x, mx) = \frac{x^3 - m^2x^2}{x^2 - mx^2 + m^2x^2}$$

$$= \frac{(x - m^2)\cancel{x^2}}{(1-m+m^2)\cancel{x^2}} \xrightarrow{x \to 0} \frac{-m^2}{1-m+m^2}$$

Dipende da $m \in \mathbb{Z}$

$$\Rightarrow \nexists \lim_{(x,y)\to(0,0)} f(x,y)$$

2

$$f(x,y) = \frac{x^3y^2}{x^6+y^4}$$

$A = \mathbb{R}^2 \setminus \{(0,0)\}$ $\quad (0,0) \in \mathcal{D}(\mathbb{R}^2 \setminus \{(0,0)\})$

$$C_m = \{(x, mx^3),\ x \neq 0\}$$

$$f\Big|_{C_m}(x,y) = f(x, mx^3) = \frac{x^3 m^2 x^6}{x^6 + m^4 x^{12}} = \frac{m^2 x^{\cancel{9}\,7}}{(1+m^4x^6)\cancel{x^6}} = \frac{\cancel{m^2}x^3}{1+m^{\cancel{4}\,2}x^6}$$

$$\frac{\cancel{m^2}x^3}{1+m^2x^6} \xrightarrow{x\to 0} 0$$

$P_m \{(t^2, mt^3)\ \ t \neq 0\}$ $\quad m \in \mathbb{Z}$

$(0,0) \in \mathcal{D}(P_m)$

Perchè $(t^2, mt^3) \xrightarrow[t \to 0]{} 0$

$$f|_{P_m}(x,y) = f(t^2, mt^3) = \frac{t^6 m^2 t^6}{t^{12} + m^4 t^{12}} = \frac{\cancel{t^{12}} m^2}{(1+m^4)\cancel{t^{12}}} = \frac{m^2}{1+m^4}$$

$\frac{m^2}{1+m^4}$ dipende da $m \in \mathbb{Z}$

$\Rightarrow \nexists \lim_{(x,y)\to(0,0)} f(x)$

③

$$f(x,y) = \frac{x^2 \sin(y)}{x^2+y^2}$$

$A = \mathbb{R}^2 \setminus \{(0,0)\}$, $(0,0) \in \mathcal{D}(A)$

$f(x,y)$ $\quad \frac{x^2}{x^2+y^2}$ $\quad \sin(y) \xrightarrow[y \to 0]{} 0$

LIMITATA

corrollario

Per il teorema del confronto $\exists \lim_{(x,y)\to(0,0)} f(x,y) = 0$

$\frac{x^2}{x^2+y^2} \le \frac{x^2+y^2}{x^2+y^2}$ se $(x,y) \neq (0,0)$

$$\frac{x^2}{x^2+y^2} \leq 1$$

Es

4

$$f(x,y) = \frac{x^4}{x^2+y^2}$$

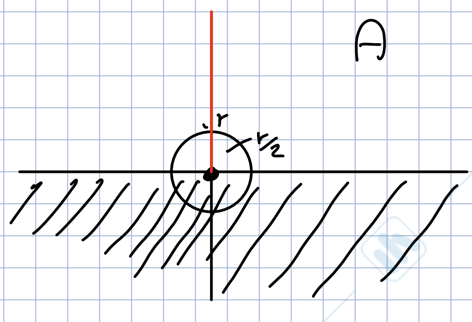
$A = \mathbb{R}^2 \setminus \{(0,0)\}$ $\qquad (0,0) \in \mathcal{D}(A)$

$$f(x,y) = \underbrace{\frac{x^2}{x^2+y^2}}_{\text{LIMITATA}} \; \underbrace{x^2}_{\substack{\to 0 \\ x \to 0}} \quad \Rightarrow \exists \lim_{(x,y)\to(0,0)} f(x,y) = 0$$

5

$$f(x,y) = x \lg y$$

$A = \{(x,y) \in \mathbb{R} \times \mathbb{R}^+\} = \{y > 0\}$



$(0,0) \in \mathcal{D}(A)$

1^a Restrizione

$C = \{(0,y) \quad y > 0\}$

$(0,0) \in \mathcal{D}(C)$

$f|_C(x,y) = f(0,y) = 0 \lg y \xrightarrow[y \to 0^+]{} 0$

$P = \{ (x, e^{-1/x}) \; x > 0 \}$

$e^{-\frac{1}{x}} \xrightarrow[x \to 0^+]{e^{-\infty}} 0 \Rightarrow (0,0) \in \mathcal{D}(P)$

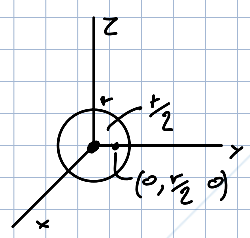
$f|_P(x,y) = f(x, e^{-\frac{1}{x}}) = x \lg(e^{-\frac{1}{x}}) = \cancel{x}\left(-\frac{1}{\cancel{x}}\right) = -1$

$\Rightarrow \nexists \lim_{(x,y) \to (0,0)} f(x,y)$

Perché $f|_P(x,y) \xrightarrow[(x,y) \to (0,0)]{} -1 \neq f|_C(x,y) \xrightarrow[(x,y) \to (0,0)]{} 0$

6

$f(x,y,z) = \frac{xyz}{x^2+y^2+z^2}$



$f : \mathbb{R}^3 \setminus \{(0,0,0)\} \longrightarrow \mathbb{R}$

$(0,0,0) \in \mathcal{D}\left(\mathbb{R}^3 \setminus \{(0,0,0)\}\right)$

$g(x,y,z) = xyz$

$g(\lambda x, \lambda y, \lambda z) = \lambda^3 (xyz)$ g omogenea di grado 3

$D(x,y,z) = x^2+y^2+z^2$

$D(\lambda x, \lambda y, \lambda z) = \lambda^2 (x^2+y^2+z^2)$ D omogenea di grado 2

$\forall x,y,z \in \mathbb{R}$

$|xy| \le \frac{1}{2}(x^2+y^2) \le \frac{1}{2}(x^2+y^2+z^2)$

se divido per $x^2+y^2+z^2$ con $x,y,z \neq (0,0,0)$

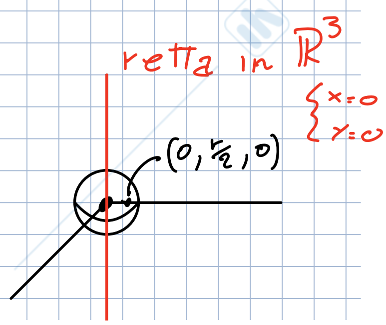
$$\frac{|xy|}{x^2+y^2+z^2} \le \frac{1}{2}$$

$$\frac{xyz}{x^2+y^2+z^2} = \underbrace{\frac{xy}{x^2+y^2+z^2}}_{\text{LIMITATA}} \cdot \underset{z \to 0}{z \to 0}$$

$$\Rightarrow \exists \lim_{(x,y,z)\to(0,0,0)} f(x,y,z) = 0$$

(7)

$$f(x,y,z) = \frac{z}{x^2+y^2}$$



$(0,0,0) \in \mathcal{D}(\mathbb{R} \setminus \{(0,0,0)\})$

$P_m = \{(x,y, m(x^2+y^2)), (x,y) \neq (0,0)\}$

Fascio di paraboloidi con vertice in $(0,0,0)$

$(0,0,0) \in \mathcal{D}(P_m)$

$$f|_{P_m}(x,y,z) = f(x,y,m(x^2+y^2)) = \frac{m\cancel{(x^2+y^2)}}{\cancel{x^2+y^2}} = m$$

dipende da $m \in \mathbb{Z}$

$$\Rightarrow \nexists \lim_{(x,y,z)\to(0,0,0)} f(x,y,z)$$

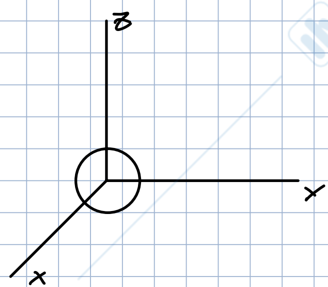
8

$$f(x,y,z) = \frac{x \cos z}{\sqrt{x^2+y^2+z^2}}$$

$A = \mathbb{R}^3 \setminus \{(0,0,0)\}$ $\quad (0,0,0) \in \mathcal{D}(A)$

Restrizione a un piano

$C = \{(0,y,z), (y,z) \neq (0,0)\}$



$(0,0,0) \in \mathcal{D}(C)$

$$f_{|_C}(x,y,z) = f(0,y,z) = \frac{0 \cdot \cos z}{\sqrt{x^2+z^2}} = 0$$

$R_m \; \{(x, mx, 0),\; x \neq 0\}$

$(0,0,0) \in \mathcal{D}(R_m)$

$$f_{|_{R_m}}(x,y,z) = f(x, mx, 0) = \frac{x\cos(0)}{\sqrt{x^2+m^2x^2}} = \frac{x}{\sqrt{1+m^2}\,|x|}$$

$$\frac{x}{\sqrt{1+m^2}\,|x|} \begin{cases} \xrightarrow{x\to 0^+} \dfrac{1}{\sqrt{1+m^2}} \\ \xrightarrow{x\to 0^-} -\dfrac{1}{\sqrt{1+m^2}} \end{cases} \Rightarrow \nexists \lim_{(x,y,z)\to(0,0,0)} f(x,y,z)$$

Funzioni Vettoriali

$f : A \longrightarrow \mathbb{R}^m$

$A \subseteq \mathbb{R}^n \qquad m, n \in \mathbb{N}$

$\forall x \in A \quad f(x) = (f_1(x), f_{11}(x), \ldots, f_m(x))$

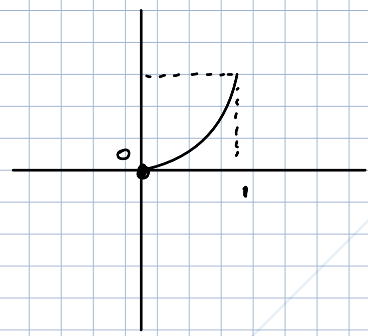
$f_j : A \to \mathbb{R}$

ES

$f(t) = (t, t^2)$

$t \in [0, 1]$

$f : [0, 1] \to \mathbb{R}^2$



Es

$f(r, \theta) = (r \cos \theta, r \sin \theta)$

$f : [0, 1] \times [0 \ 2\pi] \longrightarrow \mathbb{R}^2$

$f : A \longrightarrow \mathbb{R}^m \qquad x_0 \in \mathcal{D}(A) \qquad A \subseteq \mathbb{R}^n$

Def

Diciamo che $\lim_{x \to x_0} f(x) = \lambda = (\lambda_1, \lambda_2, \ldots \lambda_m) \in \mathbb{R}^m$

se $\forall \varepsilon > 0 \quad \exists \delta_\varepsilon > 0$ t.c. $d(f_x, \lambda) < \varepsilon$

$\forall x \in B(x_0, \delta_\varepsilon) \cap A \setminus \{x_0\}$

dove $d(fx, \lambda) = \sqrt{\sum_{j=1}^{m} |f_j(x) - \lambda_i|^2}$

$\lim_{x \to x_0} f(x) = \lambda \iff \lim_{x \to x_0} f_j(x) = \lambda_j \quad \forall j = 1, \dots, m$