

Formulario di Analisi

1 Limiti Notevoli

Goniometrici

1. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
2. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$
3. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$
4. $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$
5. $\lim_{x \rightarrow 0} \frac{\arcsin x}{x} = 1$
6. $\lim_{x \rightarrow 0} \frac{\arctan x}{x} = 1$
7. $\lim_{x \rightarrow 0} \frac{\sinh x}{x} = 1$
8. $\lim_{x \rightarrow 0} \frac{\cosh x - 1}{x^2} = \frac{1}{2}$
9. $\lim_{x \rightarrow 0} \frac{\tanh x}{x} = 1$

Esponenziali e Logaritmici

1. $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$
2. $\lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \frac{1}{\ln a}$
3. $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$
4. $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$
5. $\lim_{x \rightarrow 0} \frac{(1+x)^k - 1}{x} = k$
6. $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$
7. $\lim_{x \rightarrow \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$

2 Sviluppi di Taylor-McLaurin

- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$
- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$
- $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + o(x^8)$
- $\arcsin x = x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{3}{8} \cdot \frac{x^5}{5} + \dots + \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$
- $\arccos x = \frac{\pi}{2} - x - \frac{1}{2} \cdot \frac{x^3}{3} - \frac{3}{8} \cdot \frac{x^5}{5} - \dots - \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$
- $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$
- $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$
- $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$
- $\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + o(x^8)$
- $e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + o(x^n)$
- $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n+1} \frac{x^n}{n} + o(x^n)$
- $(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots + \binom{\alpha}{n} x^n + o(x^n)$

In Particolare:

- $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots + (-1)^n x^n \quad (\alpha = -1)$
- $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \dots + \frac{(-1)^{n+1}(2n-3)!!x^n}{(2n)!!} \quad (\alpha = \frac{1}{2})$
- $\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \dots + \frac{(-1)^n(2n-1)!!x^n}{(2n)!!} \quad (\alpha = -\frac{1}{2})$