# limiti esercizi

1) $\lim_{x \to \infty} \left(x - \sqrt{x^2 + \log x}\right) = x - \sqrt{x^2 + \log x}$

2) $\lim_{x \to 0^+} x^2 - x\sqrt{x^2 - \log x} = x^2\left(1 - \sqrt{1 - \frac{\log x}{x^2}}\right)$

$x-1=t;\ x=t-1$

3) $\lim_{x \to 1^+} \frac{x - \sqrt{x^2 + \log x}}{x-1} = \frac{0}{0} = \lim_{t \to 0^+} \frac{t+1-\sqrt{}}{}$

4) $\lim_{x \to 0^+} \frac{\arctan(\sqrt{x})}{\sqrt{\arctan(x)}} \sim \frac{\sqrt{x}}{\sqrt{x}} = 1$

5) $\lim_{x \to \infty} \frac{\arctan(x)}{\sqrt{x}} \leq \left|\frac{\pi/2}{\sqrt{x}}\right| = 0$

# studio convergenza inte

1) $\int_{\pi/2}^{\infty} \frac{\sin(x)}{x^2} dx \rightarrow \lim_{x\to\infty} \left|\frac{\sin(x)}{x^2}\right| \leq \left|\frac{1}{x^2}\right| =$

2) $\int_{0}^{+\infty} \frac{4x}{4x^8+1} dx \rightarrow \lim_{x\to 0} \frac{4x}{4x^8+1} = \frac{4x}{4x^8(1+$

3) $\int_{1}^{+\infty} \left(\frac{1}{x}\right)\left(\sqrt{\frac{x^2+2}{x^4-1}}\right) dx \rightarrow$ $\lim_{x\to 1} \underbrace{\left(\frac{1}{x}\right)}_{\to 1}\underbrace{\left(\sqrt{\frac{x^2+2}{x^4-1}}\right.}_{\infty}$

$\lim_{x\to\infty} \underbrace{\left(\frac{1}{x}\right)}_{\to 0}\underbrace{\left(\sqrt{\frac{x^2+2}{x^4-1}}\right.}_{\to 0}$

4) $\int_{0}^{\infty} e^{-x^2} dx \rightarrow$ non ci sono problem

5) $\int_{0}^{+\infty} \frac{dx}{\sqrt[3]{x} + 2\sqrt[9]{x} + x^3} \rightarrow$ Problemi in $x \to 0$ $\rightarrow \lim_{x\to 0}$

# Temi d'esame o

es 1.

a) $a_n \to \infty$ b) $\lim_{n\to\infty} (a_{n+1} - a_n) = 0$

1) $a \not\Rightarrow b \to$ controesempio : n ✓

2) $a \Rightarrow c$ vero, condizione convergenza ✗

$\left[\frac{1}{\infty} = 0\right]$

es 2

$$H(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{t^4 + \sqrt{t^2+1}}}$$

$$\lim_{x\to-\infty} \frac{1}{\sqrt{t^4+\sqrt{t^2+1}}} \sim \frac{1}{\sqrt{t^4+\sqrt{t^2}}} \sim \frac{1}{\sqrt{t^4}} = \frac{1}{t^2} \longrightarrow \text{in}$$

# Tema d'esame 22.

es 1

a) $(n \geq n_0) \Rightarrow (a_n < b_n)$ b) $\lim_{n \to \infty} \frac{a_n}{b_n} = 0$

x • $a \not\Rightarrow b$ - controesempio $\begin{cases} a_n = n \\ b_n = 2n \end{cases}$ $b \Rightarrow a$ → vera, converg limite.

• $a \not\Rightarrow c$ controesempio $\begin{cases} a_n = n \\ b_n = n^2 \end{cases}$ $b \not\Rightarrow c$ controese

es 2

$$M(x) = \int_{-\infty}^{x} e^{-\sqrt{t^2 - t + 1}}\, dt$$ → la funz pari e d

• $\lim_{t \to -\infty} \frac{1}{e^{\sqrt{t^2-t+1}}} \sim \frac{1}{e^{\sqrt{t^2}}} < \frac{1}{t^2} \to \frac{t^2}{e^{\sqrt{t^2}}} < 1$ é

la funzione $M(x)$ é definita su $\mathbb{R}$

$\lim M(x) = 0$

# Tema d'esame 27.4.

es 1

$$\lim_{x\to 0^+}\left(\frac{f(x)}{x}\right)=0 \quad (P$$

• $P \not\Rightarrow A$ controesempio: $f(x)=x^2$

• $P \Rightarrow D$ vera, per algebra dei limiti • $P \not\Rightarrow$

es 2

$$M(x)=\int_{-\infty}^{x} t^{30}e^{-t^{20}}\,dt \rightarrow$$ la conti

• $\lim_{t\to-\infty}\frac{t^{30}}{e^{t^{20}}}=0$ gerarchia infiniti

• $\lim_{x\to-\infty} M(x)=0^+$

• $\lim_{x\to\infty} M(x)=C^-(>0)$

# Tema d'esame 29.04.

es 1

a) $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$ b) $\lim_{x\to\infty} (f(x)-g(x)) = 0$

- $a \not\Rightarrow b$ : controesempio: $f(x)=x+1; g(x)=x-1$ • b
- $a \not\Rightarrow c$ : controesempio: $f(x)=x-1; g(x)=x-1$ • b

es 2

$$M(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{e^{\overbrace{t^2-t+1}^{>0}} + \underbrace{t^2-t+1}_{>0}}}$$

- $\lim_{t\to-\infty} \sim \frac{1}{\sqrt{e^{t^2}}} < \frac{1}{t^2}$ integrabile a $\pm\infty \longrightarrow$ d
- $\lim_{x\to-\infty} M(x) = 0^+$
- $\lim_{x\to\infty} M(x) = c \ (>0)$
- $M'(x) = \frac{1}{\underbrace{\sqrt{e^{x^2-x+1}+x^2-x+1}}_{(e^{f(x)}+g(x))^{-1/2}}} > 0; \ \forall x \in \mathbb{R}$
- $M''(x) = (-)\frac{1}{2}(e^{x^2-x+1}\cdot(2x-1)+2x-1)\cdot(e^{x^2-x+1}+x^2-x+1)$

# Tema d'esame 4/22

es 3.

$$I = \left(\frac{e^{\sqrt{x^2+x+1}}}{x^2}\right) \to I' = \frac{e^{\sqrt{x^2+x+1}} \cdot \frac{2x+1}{2\sqrt{x^2+x+1}} \cdot x^2 + e^{\sqrt{x^2+x+1}} \cdot 2x}{x^4}.$$

$$J = (x^2 \log(x) e^x) \to J' = 2x\log(x)e^x + \overbrace{x e^x}^{x^2 \cdot \frac{1}{x}} + x^2\log(x)e^x \to J'' = 2\log$$

$$K = xe^{x^2} \to K' = e^{x^2} + 2x^2e^{x^2} \to K'' = \underbrace{2xe^{x^2} + 4xe^{x^2}}_{6xe^{x^2}} + 4x^3e^{x^2} \to K''' = 6e^{x^2}$$

$6xe^{x^2} + 4x^3e^{x^2}$

es 5.

$$\int_0^1 \frac{x^3+2x^2+x+1}{x}\,dx \to \lim_{x\to 0} f(x) \sim \frac{1}{x} = +\infty \Big]\text{non é integrabile}$$

$$\int_1^2 \left(x^2 + \frac{1}{x}\right)\log(x)\,dx = \int_1^2 \frac{1}{x}\cdot\log(x)\,dx + \int_1^2 x^2\log(x)\,dx = \int_0^{\log(2)} t\,dt$$

$t = \log(x)$

$1/x\,dx = dt$

$$\int^{\infty} \frac{x^3+x^2+x^{3/2}+1}{\dots} = \int^{\infty}\left(x^{-2}+x^{-3}+x^{-9/2}+x^{-5}\right)dx = \left[-\frac{1}{\dots} - \frac{x^{-2}}{\dots} - \frac{2}{\dots}x^{-5/2} - \frac{x^{-4}}{\dots}\right.$$

# Tema d'esam



## es 1.

a) $\frac{d}{dx}\left[\log(f(x))\right] = \log\left(\frac{d}{dx} f(x)\right) \Rightarrow$ FA

b) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)}{g'(x)} \Rightarrow$ FALSA, controes

c) $\frac{d}{dx}\left[(f(x))^{g(x)}\right] = \left[\frac{d}{dx} f(x)\right]^{\frac{d}{dx} g(x)} \Rightarrow$ FALSA, contr

## es 2.

$$M(x) = \int_{-\infty}^{x} \frac{1}{\sqrt[4]{3-3t^3}}$$

1) La funzione é positiva, continua, defi

# Tema d'esame

## es 1

$\{a_n\}_{n=1}^{\infty}$ , $\forall n \geq 1, 0 < a_n <$

a) $\lim_{n\to\infty} a_n = 0$ , falso controesempio: $\lim_{n\to\infty} \frac{1}{4}$

d) $\sum_{n=1}^{\infty} e^{-a_n} = +\infty$, vero $e^{-a_n} > e^{-\frac{1}{2}} \Rightarrow \sum_{n=1}^{\infty} e^{-a_n} = +\infty$

## es 2.

$$M(x) = \int_{-\infty}^{x} \log\left(1 + \frac{1}{\sqrt{t^4 - t^2 +}}\right.$$

1) La funzione é positiva, com

2) $\lim_{x\to-\infty} \log\left(1 + \frac{1}{\sqrt{t^4-t^2+1}}\right) \sim \log\left(\frac{1}{\sqrt{t^4-t}}\right.$

# Tema d'esa

**Es 1.** $\int_1^{+\infty} f$

a) $\int_1^2 f(x)dx \geq \int_2^3 f(x)dx$ V, essendo f d

b) $\int_1^{\infty} f(x)dx < f(1)$ F, controesempio f(

c) $\sum_{k=1}^{\infty} f(k) < +\infty$ V, é il criterio integral

**Es 3.**

$$\lim_{n\to\infty} \left[n\left(\sqrt[n]{\pi} - 1\right)\right] = n\left[e^{\log\left(\frac{\pi}{n}\right)} - 1\right]$$

# Esercizi limit

1) $\lim_{x\to+\infty} x^x = e^{x\log(x)} = e^{\infty}$

2) $\lim_{x\to-\infty} \frac{\cos x}{3^x+10x^7} \sim \left|\frac{\cos(x)}{3^x}\right| \leq \left|\frac{1}{3}\right.$

3) $\lim_{x\to 0^+} x \ln x = \frac{\ln x}{1/x} \sim \frac{1}{1/x}$

più veloce

4) $\lim_{x\to 0^+} x^x = e^{\overbrace{x\log(x)}^{\to 0}} = 1$

$\lim \; x^x \quad x^{-\infty}$

# Tema d'esam

ES 1

$$f \in \mathcal{C}^1(\mathbb{R}$$

$P \Rightarrow A$, vero $f'(x+1) = \lim_{h \to 0} \frac{f(x+1+h) -}{h}$

$P \not\Rightarrow B$, falso per $f(1) = \int_0^1 1\,dx = 1$

$P \Rightarrow C$, vero perché $\int_0^1 f(x) \geq \int_1^2 f(x)$

ES 2.

$$\int^x e^{-(t^4-}$$