

LIMITI  $\rightarrow$  UNICITÀ DEL LIMITE  $\rightarrow$  se  $\exists$  è unico  $f: A \subseteq \mathbb{R} \rightarrow \mathbb{R}$   $x_0 \in A \cap \mathbb{R}$

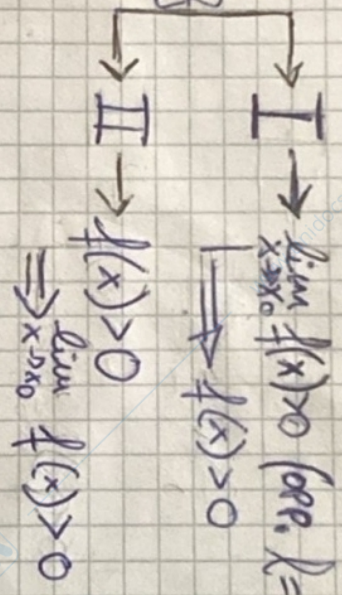
$\rightarrow$  CARATT. DELLA CONTINUITÀ  $\rightarrow f: I(x_0) \subseteq \mathbb{R} \rightarrow \mathbb{R}$   $f$  CONTINUA IN  $x_0 \Leftrightarrow f$  CONT. SIA DA SX CHE DA SX

$\rightarrow$  CARATT. DEL LIMITE  $\rightarrow f: A \subseteq \mathbb{R} \rightarrow \mathbb{R}$ ,  $x_0 \in A$  SIA DA DX CHE DA SX,  $l \in \mathbb{R}$

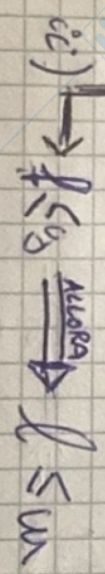
$$l = \lim_{x \rightarrow x_0} f(x) \Leftrightarrow \lim_{x \rightarrow x_0} f(x) = l$$

$\rightarrow$  PERM. DEL SEGNO  $\rightarrow f, g: A \subseteq \mathbb{R} \rightarrow \mathbb{R}$ ,  $x_0 \in A \cap \mathbb{R}$   $\rightarrow$  I  $\rightarrow \lim_{x \rightarrow x_0} f(x) > 0$  (opp.  $l = +\infty$ )

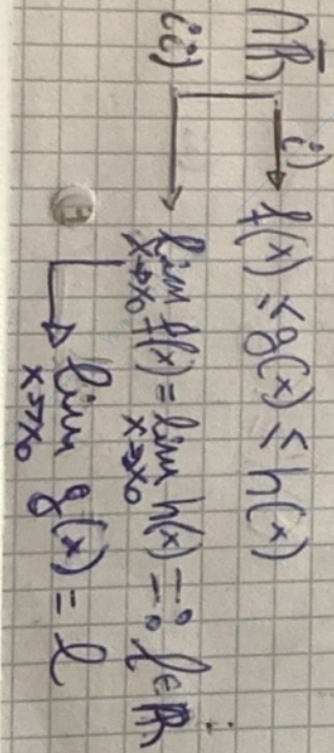
$\exists I(x_0): \forall x \in I(x_0) \cap A \setminus \{x_0\}$



$\rightarrow$  DEL CONFRONTO  $\rightarrow f, g: A \subseteq \mathbb{R} \rightarrow \mathbb{R}$ ,  $x_0 \in A \cap \mathbb{R}$ .  $\xrightarrow{ii)} \lim_{x \rightarrow x_0} f(x) = l$  e  $\lim_{x \rightarrow x_0} g(x) = w$



$\rightarrow$  DEI CARABINIERI  $\rightarrow f, g, h: A \subseteq \mathbb{R} \rightarrow \mathbb{R}$ ,  $x_0 \in A \cap \mathbb{R}$   $\xrightarrow{ii)} f(x) \leq g(x) \leq h(x)$



DERIVATE → TEOREMA →  $f: I \rightarrow \mathbb{R}, I \text{ INTERV.}, x_0 \in I^{\circ} \rightarrow f \text{ deriv. in } x_0 \Rightarrow f \text{ CONTINUA SU } I^{\circ}$

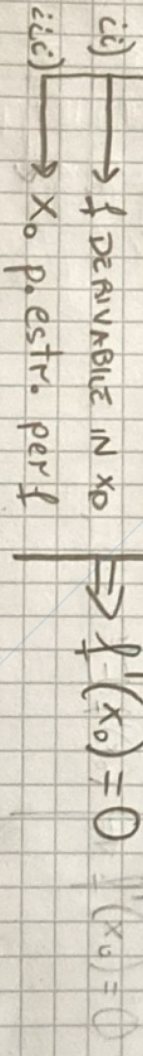
→ DER. DELLA FUNZ. INVERSA →  $f: I \subset \mathbb{R} \rightarrow \mathbb{R}, I \text{ intervallo}, x_0 \in I^{\circ} \rightarrow f \text{ CONTINUA E INVERT. SU } I(x_0)$

$f \text{ DERIV. IN } x_0 \text{ E } f'(x_0) \neq 0 \Rightarrow f^{-1} \text{ DERIV. IN } y_0 = f(x_0) \text{ E INOLTRE}$

$$[f^{-1}]'(y_0) = \frac{1}{f'(f^{-1}(y_0))} \rightarrow D f^{-1}(x_0) = \frac{1}{f'(x_0)}$$

→ DER. DELLA FUNZ. COMPOSTA →  $g \circ f: \text{DOM } f \rightarrow \mathbb{R} \text{ E DERIVABILE IN } x_0 \rightarrow D(g \circ f)(x) = g'(f(x))$

→ FERMAT →  $f: I \subset \mathbb{R} \rightarrow \mathbb{R}, I \text{ intervallo}, x_0 \in I^{\circ} = ]a, b[$

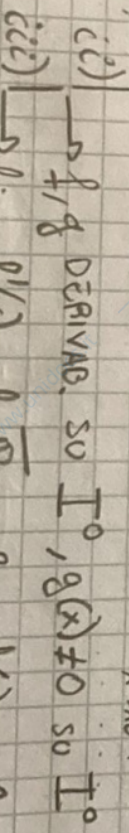


→ ROLLE →  $f$  CONTINUA →  $f$  DERIV. →  $f(b) = f(a) \rightarrow \exists c \in ]a, b[ \rightarrow f'(c) = 0$

→ LAGRANGE →  $f$  SU  $]a, b[$  →  $f$  SU  $]a, b[ \rightarrow f'(c) = \frac{f(b) - f(a)}{b - a}$

→ CARATT. DELLE FUNZ. COST. →  $f: I \subseteq \mathbb{R}, I \text{ INTERV.} \rightarrow f \text{ DERIV. SU } I \rightarrow f \text{ COST.} \Leftrightarrow f'(x) = 0$

→ DE L'HOPITAL →  $f, g: I \rightarrow \mathbb{R}, I \text{ INTERV.}, x_0 \in I^{\circ} \rightarrow \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = 0 = \lim_{x \rightarrow x_0} g(x) \text{ OPPURE } \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = +\infty = \lim_{x \rightarrow x_0} g(x)$



$f(x)$  CONTINUE  $\rightarrow$  ZERI  $\rightarrow f: [a,b] \rightarrow \mathbb{R}$

$f$  CONTINUA SU  $[a,b]$   
 $f(a)f(b) < 0 \Rightarrow \exists c \in ]a,b[ : f(c) = 0$

$\rightarrow$  VALORI INTERMEDI  $\rightarrow f: [a,b] \rightarrow \mathbb{R} \xrightarrow{f}$   $f$  CONT. SU  $[a,b] \Rightarrow f$  assume tutti i valori tra  $f(a)$  e  $f(b)$

$\rightarrow$  WEIERSTRASS  $\rightarrow f: [a,b] \rightarrow \mathbb{R} \xrightarrow{f}$

$f$  CONT. SU  $[a,b] \Rightarrow f$  ammette MAX e MIN ASSOLUTI  
 $\Leftrightarrow \exists M, m \in \mathbb{R} : M = \max f(a,b)$  e  $m = \min f(a,b)$   
 $\Leftrightarrow \exists x_M, x_m \in [a,b] : m = f(x_m) \leq f(x) \leq f(x_M) = M, \forall x \in [a,b]$

$\rightarrow$  MONOTONIA  $\rightarrow f: I \subseteq \mathbb{R} \rightarrow \mathbb{R}, I$  inter.

$f$  CONTINUA SU  $I$   $\xrightarrow{f}$   $f$  DIETIVA  $\xrightarrow{f}$   $f$  STRETTAMENTE MONOTONA

# INTEGRALI

MEDIA INTEGRALE  $\rightarrow \mathbb{R}$  CONTINUA  $\Rightarrow$

$$\Rightarrow \exists c \in [a, b] : f(c) = M(f; a, b) := \frac{\int_a^b f(x) dx}{b-a}$$

AREA ALG. RETTANGOLO  $(b-a) f(c) = \int_a^b f(x) dx$  AREA ALG. REGIONE SOTTO  $f$

TORRICELLI  $\rightarrow f: [a, b] \rightarrow \mathbb{R}$  CONTINUA  $\Rightarrow F$  è DERIV. su  $[a, b]$  e  $F'(x) = f(x)$

CALCOLO INTEGRALE  $\rightarrow f: [a, b] \rightarrow \mathbb{R}$  CONTINUA  $\Rightarrow \int_a^b f(x) dx = G(b) - G(a)$ ,  
ove  $G \in \int f(x) dx = D^{-1}(f)$   
 $\forall x \in [a, b]$

INTEGRAB. DELLE  $f(x)$  CONTINUE  $\rightarrow f: [a, b] \rightarrow \mathbb{R}$  CONTINUA  $\Rightarrow f$  INTEGRABILE

INTEGRAB. DELLE  $f(x)$  MONOTONE  $\rightarrow f: [a, b] \rightarrow \mathbb{R}$  MONOTONA  $\Rightarrow f$  INTEGRABILE