Tables for Group Theory

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This provides the essential tables (character tables, direct products, descent in symmetry and subgroups) required for those using group theory, together with general formulae, examples, and other relevant information.

Character Tables:

Direct Products:

Notes and Illustrations:

Character Tables

Notes:

(1) Schönflies symbols are given for all point groups. Hermann–Maugin symbols are given for the 32 crystaliographic point groups.

(2) In the groups containing the operation C_5 the following relations are useful:

$$\eta^{+} = \tfrac{1}{2}(1+5^{\frac{1}{2}}) = 1{\cdot}61803\cdots = -2\cos 144^{\circ}$$

$$\eta^{-} = \tfrac{1}{2}(1-5^{\frac{1}{2}}) = -0{\cdot}61803\cdots = -2\cos 72^{\circ}$$

$$\eta^{+}\eta^{+} = 1+\eta^{+} \qquad \eta^{-}\eta^{-} = 1+\eta^{-} \qquad \eta^{+}\eta^{-} = -1$$

$$\eta^{+}+\eta^{-} = 1 \qquad 2\cos 72^{\circ} + 2\cos 144^{\circ} = -1$$

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1. The Groups C_1, C_s, C_i

C_1 (1)	E
A	1

$C_s = C_h$ (m)	E	σ_h		
A′	1	1	x, y, R_z	x^2, y^2, z^2, xy
A″	1	–1	z, R_x, R_y	yz, xz

$C_i = S_2$ ($\bar{1}$)	E	i		
A_g	1	1	R_x, R_y, R_z	$x^2, y^2, z^2, xy, xz, yz$
A_u	1	–1	x, y, z	

2. The Groups C_n (n = 2, 3,…,8)

C_2 (2)	E	C_2		
A	1	1	z, R_z	x^2, y^2, z^2, xy
B	1	–1	x, y, R_x, R_y	yz, xz

C_3 (3)	E	C_3	C_3^2		$\varepsilon = \exp(2\pi i/3)$
A	1	1	1	z, R_z	$x^2 + y^2, z^2$
E	1	ε	ε^*	$(x, y)(R_x, R_y)$	$(x^2 - y^2, 2xy)(yz, xz)$
	1	ε^*	ε		

C_4 (4)	E	C_4	C_2	C_4^3		
A	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
B	1	–1	1	–1		$x^2 - y^2, 2xy$
E	1	i	–1	–i	$(x, y)(R_x, R_y)$	(yz, xz)
	1	–i	–1	i		

C_5	E	C_5	C_5^2	C_5^3	C_5^4		$\varepsilon = \exp(2\pi i/5)$
A	1	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
E_1	1	ε	ε^2	ε^{*2}	ε^*	$(x,y)(R_x, R_y)$	(yz, xz)
	1	ε^*	ε^{*2}	ε^2	ε		
E_2	1	ε^2	ε^*	ε	ε^{*2}		$(x^2 - y^2, 2xy)$
	1	ε^{*2}	ε	ε^*	ε^2		

C_6 (6)	E	C_6	C_3	C_2	C_3^2	C_6^5		$\varepsilon = \exp(2\pi i/6)$
A	1	1	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
B	1	–1	1	–1	1	–1		
E_1	1	ε	$-\varepsilon^*$	–1	$-\varepsilon$	ε^*	(x, y) (R_z, R_y)	(xy, yz)
	1	ε^*	$-\varepsilon$	–1	$-\varepsilon^*$	ε		
E_2	1	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$		$(x^2 - y^2, 2xy)$
	1	$-\varepsilon$	$-\varepsilon^*$	1	$-\varepsilon$	$-\varepsilon^*$		

2. The Groups C_n (n = 2, 3,…,8) (cont..)

C_7	E	C_7	C_7^2	C_7^3	C_7^4	C_7^5	C_7^6		$\varepsilon = \exp(2\pi i/7)$
A	1	1	1	1	1	1	1	z, R_z	x^2+y^2, z^2
E_1	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\varepsilon\\\varepsilon^*\end{matrix}$	$\begin{matrix}\varepsilon^2\\\varepsilon^{*2}\end{matrix}$	$\begin{matrix}\varepsilon^3\\\varepsilon^{*3}\end{matrix}$	$\begin{matrix}\varepsilon^{*3}\\\varepsilon^3\end{matrix}$	$\begin{matrix}\varepsilon^{*2}\\\varepsilon^2\end{matrix}$	$\left.\begin{matrix}\varepsilon^*\\\varepsilon\end{matrix}\right\}$	(x, y) (R_x, R_y)	(xz, yz)
E_2	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\varepsilon^2\\\varepsilon^{*2}\end{matrix}$	$\begin{matrix}\varepsilon^{*3}\\\varepsilon^3\end{matrix}$	$\begin{matrix}\varepsilon^*\\\varepsilon\end{matrix}$	$\begin{matrix}\varepsilon\\\varepsilon^*\end{matrix}$	$\begin{matrix}\varepsilon^3\\\varepsilon^{*3}\end{matrix}$	$\left.\begin{matrix}\varepsilon^{*2}\\\varepsilon^2\end{matrix}\right\}$		$(x^2-y^2, 2xy)$
E_3	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\varepsilon^3\\\varepsilon^{*3}\end{matrix}$	$\begin{matrix}\varepsilon^*\\\varepsilon\end{matrix}$	$\begin{matrix}\varepsilon^2\\\varepsilon^{*2}\end{matrix}$	$\begin{matrix}\varepsilon^{*2}\\\varepsilon^2\end{matrix}$	$\begin{matrix}\varepsilon\\\varepsilon^*\end{matrix}$	$\left.\begin{matrix}\varepsilon^{*3}\\\varepsilon^3\end{matrix}\right\}$		

C_8	E	C_8	C_4	C_2	C_4^3	C_8^3	C_8^5	C_8^7		$\varepsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	z, R_z	x^2+y^2, z^2
B	1	−1	1	1	1	−1	−1	−1		
E_1	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\varepsilon\\\varepsilon^*\end{matrix}$	$\begin{matrix}i\\-i\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-i\\i\end{matrix}$	$\begin{matrix}-\varepsilon^*\\-\varepsilon\end{matrix}$	$\begin{matrix}-\varepsilon\\-\varepsilon^*\end{matrix}$	$\left.\begin{matrix}\varepsilon^*\\\varepsilon\end{matrix}\right\}$	(x, y) (R_x, R_y)	(xz, yz)
E_2	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}i\\-i\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-i\\i\end{matrix}$	$\begin{matrix}i\\-i\end{matrix}$	$\left.\begin{matrix}-i\\i\end{matrix}\right\}$		$(x^2-y^2, 2xy)$
E_3	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}-\varepsilon\\-\varepsilon^*\end{matrix}$	$\begin{matrix}i\\-i\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-i\\i\end{matrix}$	$\begin{matrix}\varepsilon^*\\\varepsilon\end{matrix}$	$\begin{matrix}\varepsilon\\\varepsilon^*\end{matrix}$	$\left.\begin{matrix}-\varepsilon^*\\-\varepsilon\end{matrix}\right\}$		

3. The Groups D_n (n = 2, 3, 4, 5, 6)

D_2 (222)	E	$C_2(z)$	$C_2(y)$	$C_2(x)$		
A	1	1	1	1		x^2, y^2, z^2
B_1	1	1	–1	–1	z, R_z	xy
B_2	1	–1	1	–1	y, R_y	xz
B_3	1	–1	–1	1	x, R_x	yz

D_3 (32)	E	$2C_3$	$3C_2$		
A_1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	–1	z, R_z	
E	2	–1	0	$(x, y)(R_x, R_y)$	$(x^2 - y^2, 2xy)$ (xz, yz)

D_4 (422)	E	$2C_4$	$C_2(=C_4^2)$	$2C_2'$	$2C_2''$		
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	–1	–1	z, R_z	
B_1	1	–1	1	1	–1		$x^2 - y^2$
B_2	1	–1	1	–1	1		xy
E	2	0	–2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)

D_5	E	$2C_5$	$2C_5^2$	$5C_2$		
A_1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	–1	z, R_z	
E_1	2	2 cos 72°	2 cos 144°	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, 2xy)$

D_6 (622)	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$		
A_1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	–1	–1	z, R_z	
B_1	1	–1	1	–1	1	–1		
B_2	1	–1	1	–1	–1	1		
E_1	2	1	–1	–2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	–1	–1	2	0	0		$(x^2 - y^2, 2xy)$

4. The Groups C_{nv} (n = 2, 3, 4, 5, 6)

C_{2v} (2*mm*)	E	C_2	$\sigma_v(xz)$	$\sigma'_v(yz)$		
A_1	1	1	1	1	z	x^2, y^2, z^2
A_2	1	1	–1	–1	R_z	xy
B_1	1	–1	1	–1	x, R_y	xz
B_2	1	–1	–1	1	y, R_x	yz

C_{3v} (3*m*)	E	$2C_3$	$3\sigma_v$		
A_1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	–1	R_z	
E	2	–1	0	$(x, y)(R_x, R_y)$	$(x^2 - y^2, 2xy)(xz, yz)$

C_{4v} (4*mm*)	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$		
A_1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	–1	–1	R_z	
B_1	1	–1	1	1	–1		$x^2 - y^2$
B_2	1	–1	1	–1	1		xy
E	2	0	–2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)

C_{5v}	E	$2C_5$	$2C_5^2$	$5\sigma_v$		
A_1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	–1	R_z	
E_1	2	2 cos 72°	2 cos 144°	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, 2xy)$

C_{6v} (6*mm*)	E	$2C_6$	$2C_3$	C_2	$3\sigma_v$	$3\sigma_d$		
A_1	1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	1	–1	–1	R_z	
B_1	1	–1	1	–1	1	–1		
B_2	1	–1	1	–1	–1	1		
E_1	2	1	–1	–2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	–1	–1	2	0	0		$(x^2 - y^2, 2xy)$

5. The Groups C_{nh} (n = 2, 3, 4, 5, 6)

C_{2h} (2/m)	E	C_2	I	σ_h		
A_g	1	1	1	1	R_z	x^2, y^2, z^2, xy
B_g	1	–1	1	–1	R_x, R_y	xz, yz
A_u	1	1	–1	–1	z	
B_u	1	–1	–1	1	x, y	

C_{3h} ($\bar{6}$)	E	C_3	C_3^2	σ_h	S_3	S_3^5		$\varepsilon = \exp(2\pi i/3)$
A'	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
E'	$\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$	$\begin{matrix} \varepsilon \\ \varepsilon^* \end{matrix}$	$\begin{matrix} \varepsilon^* \\ \varepsilon \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$	$\begin{matrix} \varepsilon \\ \varepsilon^* \end{matrix}$	$\begin{matrix} \varepsilon^* \\ \varepsilon \end{matrix}$	(x, y)	$(x^2 - y^2, 2xy)$
A''	1	1	1	–1	–1	–1	z	
E''	$\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$	$\begin{matrix} \varepsilon \\ \varepsilon^* \end{matrix}$	$\begin{matrix} \varepsilon^* \\ \varepsilon \end{matrix}$	$\begin{matrix} -1 \\ -1 \end{matrix}$	$\begin{matrix} -\varepsilon \\ -\varepsilon^* \end{matrix}$	$\begin{matrix} -\varepsilon^* \\ -\varepsilon \end{matrix}$	(R_x, R_y)	(xz, yz)

C_{4h} (4/m)	E	C_4	C_2	C_4^3	i	S_4^3	σ_h	S_4		
A_g	1	1	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
B_g	1	–1	1	–1	1	–1	1	–1		$(x^2 - y^2, 2xy)$
E_g	$\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$	$\begin{matrix} i \\ -i \end{matrix}$	$\begin{matrix} -1 \\ -1 \end{matrix}$	$\begin{matrix} -i \\ i \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$	$\begin{matrix} i \\ -i \end{matrix}$	$\begin{matrix} -1 \\ -1 \end{matrix}$	$\begin{matrix} -i \\ i \end{matrix}$	(R_x, R_y)	(xz, yz)
A_u	1	1	1	1	–1	–1	–1	–1	z	
B_u	1	–1	1	–1	–1	1	–1	1		
E_u	$\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$	$\begin{matrix} i \\ -i \end{matrix}$	$\begin{matrix} -1 \\ -1 \end{matrix}$	$\begin{matrix} -i \\ i \end{matrix}$	$\begin{matrix} -1 \\ -1 \end{matrix}$	$\begin{matrix} -i \\ i \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$	$\begin{matrix} i \\ -i \end{matrix}$	(x, y)	

5. The Groups C_{nh} (n = 2, 3, 4, 5, 6) (cont…)

C_{5h}	E	C_5	C_5^2	C_5^3	C_5^4	σ_h	S_5	S_5^7	S_5^3	S_5^9		$\varepsilon = \exp(2\pi i/5)$
A′	1	1	1	1	1	1	1	1	1	1	R_z	x^2+y^2, z^2
E_1'	1	ε	ε^2	ε^{*2}	ε^*	1	ε	ε^2	ε^{*2}	ε^*	(x, y)	
	1	ε^*	ε^{*2}	ε^2	ε	1	ε^*	ε^{*2}	ε^2	ε		
E_2'	1	ε^2	ε^*	ε	ε^{*2}	1	ε^2	ε^*	ε	ε^{*2}	z	$(x^2-y^2, 2xy)$
	1	ε^{*2}	ε	ε^*	ε^2	1	ε^{*2}	ε	ε^*	ε^2		
A″	1	1	1	1	1	−1	−1	−1	−1	−1		
E_1''	1	ε	ε^2	ε^{*2}	ε^*	−1	$-\varepsilon$	$-\varepsilon^2$	$-\varepsilon^{*2}$	$-\varepsilon^*$	(R_x, R_y)	(xz, yz)
	1	ε^*	ε^{*2}	ε^2	ε	−1	$-\varepsilon^*$	$-\varepsilon^{*2}$	$-\varepsilon^2$	$-\varepsilon$		
E_2''	1	ε^2	ε^*	ε	ε^{*2}	−1	$-\varepsilon^2$	$-\varepsilon^*$	$-\varepsilon$	$-\varepsilon^{*2}$		
	1	ε^{*2}	ε	ε^*	ε^2	−1	$-\varepsilon^{*2}$	$-\varepsilon$	$-\varepsilon^*$	$-\varepsilon^2$		

C_{6h} (6/m)	E	C_6	C_3	C_2	C_3^2	C_6^5	i	S_3^5	S_6^5	σ_h	S_6	S_3		$\varepsilon = \exp(2\pi i/6)$
A_g	1	1	1	1	1	1	1	1	1	1	1	1		x^2+y^2, z^2
B_g	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	(R_x, R_y)	(xz, yz)
E_{1g}	1	ε	$-\varepsilon^*$	−1	$-\varepsilon$	ε^*	1	ε	$-\varepsilon^*$	−1	$-\varepsilon$	ε^*		
	1	ε^*	$-\varepsilon$	−1	$-\varepsilon^*$	ε	1	ε^*	$-\varepsilon$	−1	$-\varepsilon^*$	ε		
E_{2g}	1	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$		$(x^2-y^2, 2xy)$
	1	$-\varepsilon$	$-\varepsilon^*$	1	$-\varepsilon$	$-\varepsilon^*$	1	$-\varepsilon$	$-\varepsilon^*$	1	$-\varepsilon$	$-\varepsilon^*$		
A_u	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	z	
B_u	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1		
E_{1u}	1	ε	$-\varepsilon^*$	−1	$-\varepsilon$	ε^*	−1	$-\varepsilon$	ε^*	1	ε	$-\varepsilon^*$	(x, y)	
	1	ε^*	$-\varepsilon$	−1	$-\varepsilon^*$	ε	−1	$-\varepsilon^*$	ε	1	ε^*	$-\varepsilon$		
E_{2u}	1	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$	−1	ε^*	ε	−1	ε^*	ε		
	1	$-\varepsilon$	$-\varepsilon^*$	1	$-\varepsilon$	$-\varepsilon^*$	−1	ε	ε^*	−1	ε	ε^*		

6. The Groups D_{nh} (n = 2, 3, 4, 5, 6)

D_{2h} (mmm)	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$		
A_g	1	1	1	1	1	1	1	1		x^2, y^2, z^2
B_{1g}	1	1	–1	–1	1	1	–1	–1	R_z	xy
B_{2g}	1	–1	1	–1	1	–1	1	–1	R_y	xz
B_{3g}	1	–1	–1	1	1	–1	–1	1	R_x	yz
A_u	1	1	1	1	–1	–1	–1	–1		
B_{1u}	1	1	–1	–1	–1	–1	1	1	z	
B_{2u}	1	–1	1	–1	–1	1	–1	1	y	
B_{3u}	1	–1	–1	1	–1	1	1	–1	x	

D_{3h} $(\bar{6})m2$	E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$		
A_1'	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2'	1	1	–1	1	1	–1	R_z	
E'	2	–1	0	2	–1	0	(x, y)	$(x^2 - y^2, 2xy)$
A_1''	1	1	1	–1	–1	–1		
A_2''	1	1	–1	–1	–1	1	z	
E''	2	–1	0	–2	1	0	(R_x, R_y)	(xy, yz)

D_{4h} ($4/mmm$)	E	$2C_4$	C_2	$2C_2'$	$2C_2''$	i	$2S_4$	σ_h	$2\sigma_v$	$2\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	1	–1	–1	1	1	1	–1	–1	R_z	
B_{1g}	1	–1	1	1	–1	1	–1	1	1	–1		$x^2 - y^2$
B_{2g}	1	–1	1	–1	1	1	–1	1	–1	1		xy
E_g	2	0	–2	0	0	2	0	–2	0	0	(R_x, R_y)	(xz, yz)
A_{1u}	1	1	1	1	1	–1	–1	–1	–1	–1		
A_{2u}	1	1	1	–1	–1	–1	–1	–1	1	1	z	
B_{1u}	1	–1	1	1	–1	–1	1	–1	–1	1		
B_{2u}	1	–1	1	–1	1	–1	1	–1	1	–1		
E_u	2	0	–2	0	0	–2	0	2	0	0	(x, y)	

6. The Groups D_{nh} (n = 2, 3, 4, 5, 6) (cont…)

D_{5h}	E	$2C_5$	$2C_5^2$	$5C_2$	σ_h	$2S_5$	$2S_5^3$	$5\sigma_v$		
A_1'	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2'	1	1	1	–1	1	1	1	–1	R_z	
E_1'	2	2 cos 72°	2 cos 144°	0	2	2 cos 72°	2 cos 144°	0	(x, y)	
E_2'	2	2 cos 144°	2 cos 72°	0	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, 2xy)$
A_1''	1	1	1	1	–1	–1	–1	–1		
A_2''	1	1	1	–1	–1	–1	–1	1	z	
E_1''	2	2 cos 72°	2 cos 144°	0	–2	–2 cos 72°	–2 cos 144°	0	(R_x, R_y)	(xy, yz)
E_2''	2	2 cos 144°	2 cos 72°	0	–2	–2 cos 144°	–2 cos 72°	0		

D_{6h} (6/*mmm*)	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$	i	$2S_3$	$2S_6$	σ_h	$3\sigma_d$	$3\sigma_v$		
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	1	1	–1	–1	1	1	1	1	–1	–1	R_z	
B_{1g}	1	–1	1	–1	1	–1	1	–1	1	–1	1	–1		
B_{2g}	1	–1	1	–1	–1	1	1	–1	1	–1	–1	1		
E_{1g}	2	1	–1	–2	0	0	2	1	–1	–2	0	0	$(R_x - R_y)$	(xz, yz)
E_{2g}	2	–1	–1	2	0	0	2	–1	–1	2	0	0		$(x^2 - y^2, 2xy)$
A_{1u}	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1		
A_{2u}	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	z	
B_{1u}	1	–1	1	–1	1	–1	–1	1	–1	1	–1	1		
B_{2u}	1	–1	1	–1	–1	1	–1	1	–1	1	1	–1		
E_{1u}	2	1	–1	–2	0	0	–2	–1	1	2	0	0	(x, y)	
E_{2u}	2	–1	–1	2	0	0	–2	1	1	–2	0	0		

7. The Groups D_{nd} (n = 2, 3, 4, 5, 6)

$D_{2d} = V_d$ $(\bar{4}2)m$	E	$2S_4$	C_2	$2C_2'$	$2\sigma_d$		
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	–1	–1	R_z	
B_1	1	–1	1	1	–1		$x^2 - y^2$
B_2	1	–1	1	–1	1	z	xy
E	2	0	–2	0	0	(x, y) (R_x, R_y)	(xz, yz)

D_{3d} $(\bar{3})m$	E	$2C_3$	$3C_2$	i	$2S_6$	$3\sigma_d$		
A_{1g}	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	–1	1	1	–1	R_z	
E_g	2	–1	0	2	–1	0	(R_x, R_y)	$(x^2 - y^2, 2xy)$ (xz, yz)
A_{1u}	1	1	1	–1	–1	–1		
A_{2u}	1	1	–1	–1	–1	1	z	
E_u	2	–1	0	–2	1	0	(x, y)	

D_{4d}	E	$2S_8$	$2C_4$	$2S_8^3$	C_2	$4C_2'$	$4\sigma_d$		
A_1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	1	–1	–1	R_z	
B_1	1	–1	1	–1	1	1	–1		
B_2	1	–1	1	–1	1	–1	1	z	
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	–2	0	0	(x, y)	
E_2	2	0	–2	0	2	0	0		$(x^2 - y^2, 2xy)$
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	–2	0	0	(R_x, R_y)	(xz, yz)

7. The Groups D_{nd} (n = 2, 3, 4, 5, 6) (cont..)

D_{5d}	E	$2C_5$	$2C_5^2$	$5C_2$	i	$2S_{10}^3$	$2S_{10}$	$5\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	1	–1	1	1	1	–1	R_z	
E_{1g}	2	2 cos 72°	2 cos 144°	0	2	2 cos 72°	2 cos 144°	0	(R_x, R_y)	(xy, yz)
E_{2g}	2	2 cos 144°	2 cos 72°	0	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, 2xy)$
A_{1u}	1	1	1	1	–1	–1	–1	–1		
A_{2u}	1	1	1	–1	–1	–1	–1	1	z	
E_{1u}	2	2 cos 72°	2 cos 144°	0	–2	–2 cos 72°	–2 cos 144°	0	(x, y)	
E_{2u}	2	2 cos 144°	2 cos 72°	0	–2	–2 cos 144°	–2 cos 72°	0		

D_{6d}	E	$2S_{12}$	$2C_6$	$2S_4$	$2C_3$	$2S_{12}^5$	C_2	$6C_2'$	$6\sigma_d$		
A_1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	1	1	1	–1	–1	R_z	
B_1	1	–1	1	–1	1	–1	1	1	–1		
B_2	1	–1	1	–1	1	–1	1	–1	1	z	
E_1	2	$\sqrt{3}$	1	0	–1	$-\sqrt{3}$	–2	0	0	(x, y)	
E_2	2	1	–1	–2	–1	1	2	0	0		$(x^2 - y^2, 2xy)$
E_3	2	0	–2	0	2	0	–2	0	0		
E_4	2	–1	–1	2	–1	–1	2	0	0		
E_5	2	$-\sqrt{3}$	1	0	–1	$\sqrt{3}$	–2	0	0	(R_x, R_y)	(xy, yz)

8. The Groups S_n (n = 4, 6, 8)

S_4 ($\bar{4}$)	E	S_4	C_2	S_4^3		
A	1	1	1	1	R_z	x^2+y^2, z^2
B	1	−1	1	−1	z	$(x^2-y^2, 2xy)$
E	{1	i	−1	−i}	(x, y) (R_x, R_y)	(xz, yz)
	{1	−i	−1	i}		

S_6 ($\bar{3}$)	E	C_3	C_3^2	i	S_6^5	S_6		$\varepsilon = \exp(2\pi i/3)$
A_g	1	1	1	1	1	1	R_z	x^2+y^2, z^2
E_g	{1	ε	ε^*	1	ε	ε^*}	(R_x, R_y)	$(x^2-y^2, 2xy)$ (xy, yz)
	{1	ε^*	ε	1	ε^*	ε}		
A_u	1	1	1	−1	−1	−1	z	
E_u	{1	ε	ε^*	1	ε	ε^*}	(x, y)	
	{1	ε^*	ε	1	ε^*	ε}		

S_8	E	S_8	C_4	S_8^3	C_2	S_8^5	C_4^3	S_8^7		$\varepsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	R_z	x^2+y^2, z^2
B	1	−1	1	−1	1	−1	1	−1	z	
E_1	{1	ε	i	$-\varepsilon^*$	−1	$-\varepsilon$	−i	ε^*}	(x, y) (R_x, R_y)	
	{1	ε^*	−i	$-\varepsilon$	−1	$-\varepsilon^*$	i	ε}		
E_2	{1	i	−1	−i	1	i	−1	−i}		$(x^2-y^2, 2xy)$
	{1	$-i$	−1	i	1	−i	−1	i}		
E_3	{1	$-\varepsilon^*$	−i	ε	−1	ε^*	i	$-\varepsilon$}		(xy, yz)
	{1	$-\varepsilon$	i	ε^*	−1	ε	−i	$-\varepsilon^*$}		

9. The Cubic Groups

T (23)	E	$4C_3$	$4C_3^2$	$3C_2$		$\varepsilon = \exp(2\pi i/3)$
A	1	1	1	1		$x^2+y^2+z^2$
E	$\begin{cases}1\\1\end{cases}$	ε ε^*	ε^* ε	$\begin{rcases}1\\1\end{rcases}$		$(\sqrt{3}(x^2-y^2), 2z^2-x^2-y^2)$
T	3	0	0	−1	(x, y, z) (R_x, R_y, R_z)	(xy, xz, yz)

T_d ($\bar{4}3m$)	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$		
A_1	1	1	1	1	1		$x^2+y^2+z^2$
A_2	1	1	1	−1	−1		
E	2	−1	2	0	0		$(2z^2-x^2-y^2, \sqrt{3}(x^2-y^2))$
T_1	3	0	−1	1	−1	(R_x, R_y, R_z)	
T_2	3	0	−1	−1	1	(x, y, z)	(xy, xz, yz)

T_h ($m3$)	E	$4C_3$	$4C_3^2$	$3C_2$	i	$4S_6$	$4S_6^2$	$3\sigma_d$		$\varepsilon = \exp(2\pi i/3)$
A_g	1	1	1	1	1	1	1	1		$x^2+y^2+z^2$
E_g	$\begin{cases}1\\1\end{cases}$	ε ε^*	ε^* ε	1 1	1 1	ε ε^*	ε^* ε	$\begin{rcases}1\\1\end{rcases}$		$(2z^2-x^2-y^2, \sqrt{3}(x^2-y^2))$
T_g	3	0	0	−1	3	0	0	−1	(R_x, R_y, R_z)	(xy, yz, xz)
A_u	1	1	1	1	−1	−1	−1	−1		
E_u	$\begin{cases}1\\1\end{cases}$	ε ε^*	ε^* ε	1 1	−1 −1	$-\varepsilon$ $-\varepsilon^*$	$-\varepsilon^*$ $-\varepsilon$	$\begin{rcases}-1\\-1\end{rcases}$		
T_u	3	0	0	−1	−3	0	0	1	(x, y, z)	

O (432)	E	$8C_3$	$3C_2$	$6C_4$	$6C_2'$		
A_1	1	1	1	1	1		$x^2+y^2+z^2$
A_2	1	1	1	−1	−1		
E	2	−1	2	0	0		$(2z^2-x^2-y^2, \sqrt{3}(x^2-y^2))$
T_1	3	0	−1	1	−1	(x, y, z) (R_x, R_y, R_z)	
T_2	3	0	−1	−1	1		(xy, xz, yz)

9. The Cubic Groups (cont…)

O_h ($m3m$)	E	$8C_3$	$6C_2$	$6C_4$	$3C_2$ ($= C_4^2$)	i	$6S_4$	$8S_6$	$3\sigma_h$	$6\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
A_{2g}	1	1	–1	–1	1	1	–1	1	1	–1		
E_g	2	–1	0	0	2	2	0	–1	2	0		$(2z^2 - x^2 - y^2, \sqrt{3}\,(x^2 - y^2))$
T_{1g}	3	0	–1	1	–1	3	1	0	–1	–1	(R_x, R_y, R_z)	
T_{2g}	3	0	1	–1	–1	3	–1	0	–1	1		(xy, xz, yz)
A_{1u}	1	1	1	1	1	–1	–1	–1	–1	–1		
A_{2u}	1	1	–1	–1	1	–1	1	–1	–1	1		
E_u	2	–1	0	0	2	–2	0	1	–2	0		
T_{1u}	3	0	–1	1	–1	–3	–1	0	1	1	(x, y, z)	
T_{2u}	3	0	1	–1	–1	–3	1	0	1	–1		

10. The Groups *I*, *I*

I	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$		$\eta^{\pm} = \frac{1}{2}\left(1 \pm 5^{\frac{1}{2}}\right)$
A	1	1	1	1	1		$x^2+y^2+z^2$
T_1	3	η^+	η^-	0	–1	(x, y, z) (R_x, R_y, R_z)	
T_2	3	η^-	η^+	0	–1		
G	4	–1	–1	1	0		
H	5	0	0	–1	1		$(2z^2 - x^2 - y^2,$ $\sqrt{3}\,(x^2 - y^2)$ $xy, yz, zx)$

I_h	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$	i	$12S_{10}$	$12S_{10}^3$	$20S_6$	15σ		$\eta^{\pm} = \frac{1}{2}\left(1 \pm 5^{\frac{1}{2}}\right)$
A_g	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
T_{1g}	3	η^+	η^-	0	–1	3	η^-	η^+	–1	–1	(R_x,R_y,R_z)	
T_{2g}	3	η^-	η^+	0	–1	3	η^+	η^-	0	–1		
G_g	4	–1	–1	1	0	4	–1	–1	1	0		
H_g	5	0	0	–1	1	5	0	0	–1	1		$(2z^2 - x^2 - y^2,$ $\sqrt{3}\,(x^2 - y^2))$ (xy, yz, zx)
A_u	1	1	1	1	1	–1	–1	–1	–1	–1		
T_{1u}	3	η^+	η^-	0	–1	–3	η^-	η^+	0	1	(x, y, z)	
T_{2u}	3	η^-	η^+	0	–1	–3	η^+	η^-	0	1		
G_u	4	–1	–1	1	0	–4	1	1	–1	0		
H_u	5	0	0	–1	1	–5	0	0	1	–1		

11. The Groups $C_{\infty v}$ and $D_{\infty h}$

$C_{\infty v}$	E	C_2	$2C_{\infty}^{\phi}$	…	$\infty\sigma_v$		
$A_1 \equiv \Sigma^+$	1	1	1	…	1	z	x^2+y^2, z^2
$A_2 \equiv \Sigma^-$	1	1	1	…	−1	R_z	
$E_1 \equiv \Pi$	2	−2	$2\cos\phi$	…	0	(x,y) (R_x,R_y)	(xz, yz)
$E_2 \equiv \Delta$	2	2	$2\cos 2\phi$	…	0		$(x^2-y^2, 2xy)$
$E_3 \equiv \Phi$	2	−2	$2\cos 3\phi$	…	0		
…	…	…	…	…	…		
…	…	…	…	…	…		

$D_{\infty h}$	E	$2C_{\infty}^{\phi}$	…	$\infty\sigma_v$	i	$2S_{\infty}^{\phi}$	…	∞C_2		
Σ_g^+	1	1	…	1	1	1	…	1		x^2+y^2, z^2
Σ_g^-	1	1	…	−1	1	1	…	−1	R_z	
Π_g	2	$2\cos\phi$	…	0	2	$-2\cos\phi$	…	0	(R_x, R_y)	(xz, yz)
Δ_g	2	$2\cos 2\phi$	…	0	2	$2\cos 2\phi$	…	0		$(x^2-y^2, 2xy)$
…	…	…	…	…	…	…	…	…		
Σ_u^+	1	1	…	1	−1	−1	…	−1	z	
Σ_u^-	1	1	…	−1	−1	−1	…	1		
Π_u	2	$2\cos\phi$	…	0	−2	$2\cos\phi$	…	0	(x,y)	
Δ_u	2	$2\cos 2\phi$	…	0	−2	$-2\cos 2\phi$	…	0		
…	…	…	…	…	…	…	…	…		

12. The Full Rotation Group (SU_2 and R_3)

$$\chi^{(j)}(\phi) = \begin{cases} \dfrac{\sin\left(j+\frac{1}{2}\right)\phi}{\sin\frac{1}{2}\phi} & \phi \neq 0 \\ 2j+1 & \phi = 0 \end{cases}$$

Notation : Representation labelled $\Gamma^{(j)}$ with j = 0,1/2, 1, 3/2,…∞, for R_3 j is confined to integral values (and written l or L) and the labels $S \equiv \Gamma^{(0)}$, $P \equiv \Gamma^{(1)}$, $D \equiv \Gamma^{(2)}$, etc. are used.

Direct Products

1. General rules

(*a*) For point groups in the lists below that have representations *A, B, E, T* without subscripts, read $A_1 = A_2 = A$, etc.

(*b*)

	g	u
g	g	u
u		g

	′	″
′	′	″
″		′

(*c*) Square brackets [ ] are used to indicate the representation spanned by the antisymmetrized product of a degenerate representation with itself.

Examples

For D_{3h} $E' \times E''\ \ A_1'' + A_2'' + E$

For D_{6h} $E_{1g} \times E_{2g} = 2B_g + E_{1g}$.

2. For C_2, C_3, C_6, D_3, D_6, C_{2v}, C_{3v}, C_{6v}, C_{2h}, C_{3h}, C_{6h}, D_{3h}, D_{6h}, D_{3d}, S_6

	A_1	A_2	B_1	B_2	E_1	E_2
A_1	A_1	A_2	B_1	B_2	E_1	E_2
A_2		A_1	B_2	B_1	E_1	E_2
B_1			A_1	A_2	E_2	E_1
B_2				A_1	E_2	E_1
E_1					$A_1 + [A_2] + E_2$	$B_1 + B_2 + E_1$
E_2						$A_1 + [A_2] + E_2$

3. For D_2, D_{2h}

	A	B_1	B_2	B_3
A	A	B_1	B_2	B_3
B_1		A	B_3	B_2
B_2			A	B_1
B_3				A

4. For C_4, D_4, C_{4v}, C_{4h}, D_{4h}, D_{2d}, S_4

	A_1	A_2	B_1	B_2	E
A_1	A_1	A_2	B_1	B_2	E
A_2		A_1	B_2	B_1	E
B_1			A_1	A_2	E
B_2				A_1	E
E					$A_1 + [A_2] + B_1 + B_2$

5. For C_5, D_5, C_{5v}, C_{5h}, D_{5h}, D_{5d}

	A_1	A_2	E_1	E_2
A_1	A_1	A_2	E_1	E_2
A_2		A_1	E_1	E_2
E_1			$A_1 + [A_2] + E_2$	$E_1 + E_2$
E_2				$A_1 + [A_2] + E_1$

6. For D_{4d}, S_8

	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_2		A_1	B_2	B_1	E_1	E_2	E_3
B_1			A_1	A_2	E_3	E_2	E_1
B_2				A_1	E_3	E_2	E_1
E_1					$A_1 + [A_2] + E_2$	$E_1 + E_2$	$B_1 + B_2 + E_2$
E_2						$A_1 + [A_2] + B_1 + B_2$	$E_1 + E_3$
E_3							$A_1 + [A_2] + E_2$

7. For T, O, T_h, O_h, T_d

	A_1	A_2	E	T_1	T_2
A_1	A_1	A_2	E	T_1	T_2
A_2		A_1	E	T_2	T_1
E			$A_1 + [A_2] + E$	$T_1 + T_2$	$T_1 + T_2$
T_1				$A_1 + E + [T_1] + T_2$	$A_2 + E + T_1 + T_2$
T_2					$A_1 + E + [T_1] + T_2$

8. ***For*** D_{6d}

	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4	E_5
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4	E_5
A_2		A_1	B_2	B_1	E_1	E_2	E_3	E_4	E_5
B_1			A_1	A_2	E_5	E_4	E_3	E_2	E_1
B_2				A_1	E_5	E_4	E_3	E_2	E_1
E_1					$A_1 + [A_2] + E_2$	$E_1 + E_3$	$E_2 + E_4$	$E_3 + E_5$	$B_1 + B_2 + E_4$
E_2						$A_1 + [A_2] + E_4$	$E_1 + E_5$	$B_1 + B_2 + E_2$	$E_3 + E_5$
E_3							$A_1 + [A_2] + B_1 + B_2$	$E_1 + E_5$	$E_2 + E_4$
E_4								$A_1 + [A_2] + E_4$	$E_1 + E_3$
E_5									$A_1 + [A_2] + E_2$

9. For I, I_h

	A	T_1	T_2	G	H
A	A	T_1	T_2	G	H
T_1		$A + [T_1] + H$	$G + H$	$T_2 + G + H$	$T_1 + T_2 + G + H$
T_2			$A + [T_2] + H$	$T_1 + G + H$	$T_1 + T_2 + G + H$
G				$A + [T_1 + T_2] + G + H$	$T_1 + T_2 + G + 2H$
H					$A_1 + [T_1 + T_2 + G] + G + 2H$

10. ***For*** $C_{\infty v}, D_{\infty h}$

	Σ^+	Σ^-	Π	Δ
Σ^+	Σ^+	Σ^-	Π	Δ
Σ^-		Σ^+	Π	Δ
Π			$\Sigma^+ + [\Sigma^-] + \Delta$	$\Pi + \Phi$
Δ :				$\Sigma^+ + [\Sigma^-] + \Gamma$

Notation

Σ	Π	Δ	Φ	Γ	…
$\Lambda = 0$	1	2	3	4	…

$\Lambda_1 \times \Lambda_2 = |\Lambda_1 - \Lambda_2| + (\Lambda_1 + \Lambda_2)$

$\Lambda \times \Lambda = \Sigma^+ + [\Sigma^-] + (2\Lambda)$.

11. The Full Rotation Group (*SU*$_2$ and *R*$_3$)

$$\Gamma^{(j)} \times \Gamma^{(j')} = \Gamma^{(j+j')} + \Gamma^{(j+j'-1)} + \ldots + \Gamma^{(|j-j'|)}$$
$$\Gamma^{(j)} \times \Gamma^{(j)} = \Gamma^{(2j)} + \Gamma^{(2j-2)} + \ldots + \Gamma^{(0)} + [\Gamma^{(2j-1)} + \ldots + \Gamma^{(1)}]$$

Extended rotation groups (double groups):

Character tables and direct product tables

D_2^*	E	R	$2C_2(z)$	$2C_2(y)$	$2C_2(x)$
$E_{1/2}$	2	–2	0	0	0

D_3^*	E	R	$2C_3$	$2C_3R$	$3C_2$	$3C_2R$
$E_{1/2}$	2	–2	1	–1	0	0
$E_{3/2}$	{1	–1	–1	1	i	–i}
	{1	–1	–1	1	–i	i}

D_4	E	R	$2C_4$	$2C_4R$	$2C_2$	$4C_2'$	$4C_2''$
$E_{1/2}$	2	–2	$\sqrt{2}$	$-\sqrt{2}$	0	0	0
$E_{3/2}$	2	–2	$-\sqrt{2}$	$\sqrt{2}$	0	0	0

D_6^*	E	R	$2C_6$	$2C_6R$	$2C_3$	$2C_3R$	$2C_2$	$6C_2'$	$6C_2''$
$E_{1/2}$	2	–2	$\sqrt{3}$	$-\sqrt{3}$	1	–1	0	0	0
$E_{3/2}$	2	–2	$-\sqrt{3}$	$\sqrt{3}$	–1	1	0	0	0
$E_{5/2}$	2	–2	0	0	–2	2	0	0	0

T_d^*	E	R	$8C_3$	$8C_3R$	$6C_2$	$6S_4$	$6S_4R$	$12\sigma_d$
O^*	E	R	$8C_3$	$8C_3R$	$6C_2$	$6C_4$	$6S_4R$	$12C_2'$
$E_{1/2}$	2	–2	1	–1	0	$\sqrt{2}$	$-\sqrt{2}$	0
$E_{5/2}$	2	–2	1	–1	0	$-\sqrt{2}$	$\sqrt{2}$	0
$G_{3/2}$	4	–4	–1	1	0	0	0	0

$E_{1/2} \times E_{1/2} = [A] + B_1 + B_2 + B_3$

	$E_{1/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + A_2 + E$	$2E$
$E_{3/2}$		$[A_1] + A_1 + 2A_2$

	$E_{1/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + A_2 + E$	$B_1 + B_2 + E$
$E_{3/2}$		$[A_1] + A_2 + E$

	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
$E_{1/2}$	$[A_1] + A_2 + E_1$	$B_1 + B_2 + E_2$	$E_1 + E_2$
$E_{3/2}$		$[A_1] + A_2 + E_1$	$E_1 + E_2$
$E_{5/2}$			$[A_1] + A_2 + B_1 + B_2$

	$E_{1/2}$	$E_{5/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + T_1$	$A_2 + T_2$	$E + T_1 + T_2$
$E_{5/2}$		$[A_1] + T_1$	$E + T_1 + T_2$
$G_{3/2}$			$[A_1 + E + T_2] + A_2 + 2T_1 + T_2]$

Direct products of ordinary and extended representations for T_d^* and O*

	A_1	A_2	E	T_1	T_2
$E_{1/2}$	$E_{1/2}$	$E_{5/2}$	$G_{3/2}$	$E_{1/2} + G_{3/2}$	$E_{5/2} + G_{3/2}$
$E_{5/2}$	$E_{5/2}$	$E_{1/2}$	$G_{3/2}$	$E_{5/2} + G_{3/2}$	$E_{1/2} + G_{3/2}$
$G_{3/2}$	$G_{3/2}$	$G_{3/2}$	$E_{1/2} + E_{5/2} + G_{3/2}$	$E_{1/2} + E_{5/2} + 2G_{3/2}$	$E_{1/2} + E_{5/2} + 2G_{3/2}$

Descent in symmetry and subgroups

The following tables show the correlation between the irreducible representations of a group and those of some of its subgroups. In a number of cases more than one correlation exists between groups. In C_s the σ of the heading indicates which of the planes in the parent group becomes the sole plane of C_s; in C_{2v} it becomes must be set by a convention); where there are various possibilities for the correlation of C_2 axes and σ planes in D_{4h} and D_{6h} with their subgroups, the column is headed by the symmetry operation of the parent group that is preserved in the descent.

C_{2v}	C_2	C_s $\sigma(zx)$	C_s $\sigma(yz)$
A_1	A	A'	A'
A_2	A	A''	A''
B_1	B	A'	A'
B_2	B	A''	A''

C_{3v}	C_3	C_s
A_1	A	A'
A_2	A	A''
E	E	$A' + A''$

C_{4v}	C_{2v} σ_v	C_{2v} σ_d
A_1	A_1	A_1
A_2	A_2	A_2
B_1	A_1	A_2
B_2	A_2	A_1
E	$B_1 + B_2$	$B_1 + B_2$

[Other subgroups: C_4, C_2, C_6]

D_{3h}	C_{3h}	C_{3v}	C_{2v} $\sigma_h \rightarrow \sigma_v$	C_s σ_h	C_s σ_v
A_1'	A'	A_1	A_1	A'	A'
A_2'	A'	A_2	B_2	A'	A''
E'	E'	E	$A_1 + B_2$	$2A'$	$A' + A''$
A_1''	A''	A_2	A_2	A''	A''
A_2''	A''	A_1	B_1	A''	A'
E''	E''	E	$A_2 + B_1$	$2A''$	$A' + A''$

[Other subgroups: D_3, C_3, C_2]

D_{4h}	D_{2d} C_2' ($\rightarrow C_2'$)	D_{2d} C_2'' ($\rightarrow C_2'$)	D_{2h} C_2'	D_{2h} C_2''	D_2 C_2'	D_2 C_2''	C_{4h}	C_{4v}	C_{2v} C_2, σ_v	C_{2v} C_2, σ_d
A_{1g}	A_1	A_1	A_g	A_g	A	A	A_g	A_1	A_1	A_1
A_{2g}	A_2	A_2	B_{1g}	B_{1g}	B_1	B_1	A_g	A_2	A_2	A_2
B_{1g}	B_1	B_2	A_g	B_{1g}	A	B_1	B_g	B_1	A_1	A_2
B_{2g}	B_2	B_1	B_{1g}	A_g	B_1	A	B_g	B_2	A_2	A_1
E_g	E	E	$B_{2g} + B_{3g}$	$B_{2g} + B_{3g}$	$B_2 + B_3$	$B_2 + B_3$	E_g	E	$B_1 + B_2$	$B_1 + B_2$
A_{1u}	B_1	B_1	A_u	A_u	A	A	A_u	A_2	A_2	A_2
A_{2u}	B_2	B_2	B_{1u}	B_{1u}	B_1	B_1	A_u	A_1	A_1	A_1
B_{1u}	A_1	A_2	A_u	B_{1u}	A	B_1	B_u	B_2	A_2	A_1
B_{2u}	A_2	A_1	B_{1u}	A_u	B_1	A	B_u	B_1	A_1	A_2
E_u	E	E	$B_{2u} + B_{3u}$	$B_{2u} + B_{3u}$	$B_2 + B_3$	$B_2 + B_3$	E_u	E	$B_1 + B_2$	$B_1 + B_2$

Other subgroups: D_4, C_4, S_4, $3C_{2h}$, $3C_s$, $3C_2$, C_i, $(2C_{2v})$

D_6	$D_{3d}C_2''$	$D_{3d}C_2'$	D_{2h} $\sigma_h \rightarrow \sigma(xy)$ $\sigma_v \rightarrow \sigma(yz)$	C_{6v}	C_{3v} σ_v	C_{2v} C_2'	C_{2v} C_2''	C_{2h} C_2	C_{2h} C_2'	C_{2h} C_2''
A_{1g}	A_{1g}	A_{1g}	A_g	A_1	A_1	A_1	A_1	A_g	A_g	A_g
A_{2g}	A_{2g}	A_{2g}	B_{1g}	A_2	A_2	B_1	B_1	A_g	B_g	B_g
B_{1g}	A_{2g}	A_{1g}	B_{2g}	B_2	A_2	A_2	B_2	B_g	A_g	B_g
B_{2g}	A_{1g}	A_{2g}	B_{3g}	B_1	A_1	B_2	A_2	B_g	B_g	A_g
E_{1g}	E_g	E_g	$B_{2g} + B_{3g}$	E_1	E	$A_2 + B_2$	$A_2 + B_2$	$2B_g$	$A_g + B_g$	$A_g + B_g$
E_{2g}	E_g	E_g	$A_g + B_{1g}$	E_2	E	$A_1 + B_1$	$A_1 + B_1$	$2A_g$	$A_g + B_g$	$A_g + B_g$
A_{1u}	A_{1u}	A_{1g}	A_u	A_2	A_2	A_2	A_2	A_u	A_u	A_u
A_{2u}	A_{2u}	A_{2g}	B_{1u}	A_1	A_1	B_2	B_2	A_u	B_u	B_u
B_{1u}	A_{2u}	A_{1u}	B_{2u}	B_1	A_1	B_1	B_1	B_u	A_u	B_u
B_{2u}	A_{1u}	A_{2u}	B_{3u}	B_2	A_2	A_1	A_1	B_u	B_u	A_u
E_{1u}	E_u	E_u	$B_{2u} + B_{3u}$	E_1	E	$A_1 + B_1$	$A_1 + B_1$	$2B_u$	$A_u + B_u$	$A_u + B_u$
E_{2u}	E_u	E_u	$A_u + B_{1u}$	E_2	E	$A_2 + B_2$	$A_2 + B_2$	$2A_u$	$A_u + B_u$	$A_u + B_u$

Other subgroups: D_6, $2D_{3h}$, C_{6h}, C_6, C_{3h}, $2D_3$, S_6, D_2, C_3, $3C_2$, $3C_g$, C_i

T_d	T	D_{2d}	C_{3v}	C_{2v}
A_1	A	A_1	A_1	A_1
A_2	A	B_1	A_2	A_2
E	E	$A_1 + B_1$	E	$A_1 + A_2$
T_1	T	$A_2 + E$	$A_2 + E$	$A_2 + B_1 + B_2$
T_2	T	$B_2 + E$	$A_1 + E$	$A_1 + B_2 + B_1$

Other subgroups: S_4, D_2, C_3, C_2, C_s.

O_h	O	T_d	T_h	D_{4h}	D_{3d}
A_{1g}	A_1	A_1	A_g	A_{1g}	A_{1g}
A_{2g}	A_2	A_2	A_g	B_{1g}	A_{2g}
E_g	E	E	E_g	$A_{1g} + B_{1g}$	E_g
T_{1g}	T_1	T_1	T_g	$A_{2g} + E_g$	$A_{2g} + E_g$
T_{2g}	T_2	T_2	T_g	$B_{2g} + E_g$	$A_{1g} + E_g$
A_{1u}	A_1	A_2	A_u	A_{1u}	A_{1u}
A_{2u}	A_2	A_1	A_u	B_{1u}	B_{1u}
E_u	E	E	E_u	$A_{1u} + B_{1u}$	E_u
T_{1u}	T_1	T_2	T_u	$A_{2u} + E_u$	$A_{2u} + E_u$
T_{2u}	T_2	T_1	T_u	$B_{2u} + E_u$	$A_{1u} + E_u$

Other subgroups: T, D_4, D_{2d}, C_{4h}, C_{4v}, $2D_{2h}$, D_3, C_{3v}, S_6, C_4, S_4, $3C_{2v}$, $2D_2$, $2C_{2h}$, C_3, $2C_2$, S_2, C_s

R_3	O	D_4	D_3
S	A_1	A_1	A_1
P	T_1	$A_2 + E$	$A_2 + E$
D	$E + T_2$	$A_1 + B_1 + B_2 + E$	$A_1 + 2E$
F	$A_2 + T_1 + T_2$	$A_2 + B_1 + B_2 + 2E$	$A_1 + 2A_2 + 2E$
G	$A_1 + E + T_1 + T_2$	$2A_1 + A_2 + B_1 + B_2 + 2E$	$2A_1 + A_2 + 3E$
H	$E + 2T_1 + T_2$	$A_1 + 2A_2 + B_1 + B_2 + 3E$	$A_1 + 2A_2 + 4E$

Notes and Illustrations

General Formulae

(a) Notation

h the *order* (the number of elements) of the group.

$\Gamma^{(i)}$ labels the *irreducible representation.*

$X^{(i)}(R)$ the *character* of the operation R in $\Gamma^{(i)}$.

$D^{(i)}_{\mu\nu}(R)$ the μν element of the *representative matrix* of the operation R in the irreducible representation $\Gamma^{(i)}$.

l_i the *dimension* of $\Gamma^{(i)}$ (the number of rows or columns in the matrices $\mathbf{D}^{(i)}$)

(b) Formulae

(i) *Number of irreducible representations of a group = number of classes.*

(ii) $\sum_i l_i^2 = h$

(iii) $\chi^{(i)}(R) = \sum_\mu D^{(i)}_{\mu\mu}(R)$

(iv) Orthogonality of representations:

$$\sum D^{(i)}_{\mu\nu}(R)^* D^{(i')}_{\mu'\nu'}(R) = (h/l_i)\delta_{ii'}\delta_{\mu\mu'}\delta_{\nu\nu'}$$

(δ_{ij}=1 if $i = j$ and δ_{ij} = 0 if $i \neq j$)

(v) Orthogonality of characters:

$$\sum_R \chi^{(i)}(R)^* \chi^{(i)}(R) = h\delta_{ii'}$$

(vi) Decomposition of a direct product, reduction of a representation: If

$$\Gamma = \sum_i a_i \Gamma^{(i)}$$

and the character of the operation R in the reducible representation is $\chi(R)$, then the coefficients a_t are given by

$$a_i = (l/h)\sum_R \chi^{(i)}(R)^* \chi(R).$$

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(vii) Projection operators:

The projection operator

$$P^{(i)} = (l_i / h)\sum_R \chi^{(i)}(R)^* R$$

when applied to a function f, generates a sum of functions that constitute a component of a basis for the representation $\Gamma^{(i)}$; in order to generate the complete basis $P^{(i)}$ must be applied to l_i distinct functions f. The resulting functions may be made mutually orthogonal. When $l_i = 1$ the function generated is a basis for $\Gamma^{(i)}$ without ambiguity:

$$P^{(i)} f = f^{(i)}$$

(viii) Selection rules:

If $f^{(i)}$ is a member of the basis set for the irreducible representation $\Gamma^{(i)}$, $f^{(k)}$ a member of that for $\Gamma^{(k)}$, and $\hat{\Omega}^{(j)}$ an operator that is a basis for $\Gamma^{(j)}$, then the integral

$$\int d\tau f^{(i)*} \hat{\Omega}^{(j)} f^{(k)}$$

is zero unless $\Gamma^{(i)}$ occurs in the decomposition of the direct product $\Gamma^{(j)} \times \Gamma^{(k)}$

(ix) The *symmetrized* direct product is written $\Gamma^{(i)} \overset{s}{\times} \Gamma^{(i)}$, and its characters are given by

$$\chi^{(i)}(R) \overset{s}{\times} \chi^{(i)}(R) = \tfrac{1}{2}\chi^{(i)}(R)^2 + \tfrac{1}{2}\chi^{(i)}(R^2)$$

The *antisymmetrized* direct product is written $\Gamma^{(i)} \overset{a}{\times} \Gamma^{(i)}$ and its characters are given by

$$\chi^{(i)}(R) \overset{a}{\times} \chi^{(i)}(R) = \tfrac{1}{2}\chi^{(i)}(R)^2 + \tfrac{1}{2}\chi^{(i)}(R^2)$$

Worked examples

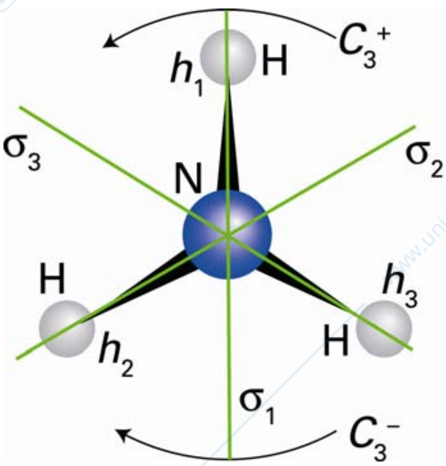
1. *To show that the representation* Γ *based on the hydrogen* 1*s-orbitals in* NH_3 (C_{3v}) *contains* A_1 *and* E, *and to generate appropriate symmetry adapted combinations.*

A table in which symmetry elements in the same class are distinguished will be employed:

C_{3v}	E	C_3^+	C_3^-	σ_1	σ_2	σ_3
A_1	1	1	1	1	1	1
A_2	1	1	1	–1	–1	–1
E	2	–1	–1	0	0	0
$D(R)$	$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	$\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$	$\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$	$\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$	$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$
$x(R)$	3	0	0	1	1	1
Rh_1	h_1	h_2	h_3	h_1	h_3	h_2
Rh_2	h_2	h_3	h_1	h_3	h_2	h_1

The *representative matrices* are derived from the effect of the operation R on the basis *(h_1, h_2, h_3);* see the figure below. For example

$$C_3^+(h_1,h_2,h_3)=(h_2,h_3,h_1)=(h_1,h_2,h_3)\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$$



According to the general formula (b)(iii) the *character* $\chi(R)$ is the sum of the diagonal elements of **D**(R). For example, $\chi(\sigma_2) = 0 + 1 + 0 = 1$. The *decomposition* of Γ follows from the formula (b)(vi):

$$\Gamma = a_1A_1 + a_2A_2 + a_EE$$

where

$$a_1 = \tfrac{1}{6}\{1\times3+2\times1\times0+3\times1\times1\}=1$$
$$a_2 = \tfrac{1}{6}\{1\times3+2\times1\times0+3\times1\times(-1)\}=0$$
$$a_E = \tfrac{1}{6}\{2\times3+2\times(-1)\times0+3\times0\times1\}=1$$

Therefore

$$\Gamma = A_1 + E$$

Symmetry adapted combinations are generated by the projection operator in (b)(vii). Using the last two rows of the table,

$$\phi(A_1) = \wp^{(A_1)}h_1 = \tfrac{1}{6}(1\times h_1 + 1\times h_2 + 1\times h_3 + 1\times h_1 + 1\times h_3 + 1\times h_2) = \tfrac{1}{3}(h_1 + h_2 + h_3)$$

$$\begin{cases} \phi(E) = \wp^{(E)}h_1 = \tfrac{2}{6}(2\times h_1 - 1\times h_2 - 1\times h_3 + 0\times h_1 + 0\times h_3 + 0\times h_2) = \tfrac{1}{3}(2h_1 - h_2 - h_3) \\ \phi'(E) = \wp^{(E)}h_2 = \tfrac{2}{6}(2\times h_2 - 1\times h_3 - 1\times h_1 + 0\times h_3 + 0\times h_2 + 0\times h_1) = \tfrac{1}{3}(-h_1 + 2h_2 - h_3) \end{cases}$$

$\phi(E)$ and $\phi'(E)$ provide a valid basis for the E representation, but the orthogonal combinations

$$\phi_a(E) = (1/6)^{\frac{1}{2}}(2h_1 - h_2 - h_3) = (3/2)^{\frac{1}{2}}\phi(E)$$
$$\phi_b(E) = (1/2)^{\frac{1}{2}}(h_2 - h_3) = (1/2)^{\frac{1}{2}}\{\phi(E) + 2\phi'(E)\}$$

would be a more useful basis in most applications.

2. *To determine the symmetries of the states arising from the electronic configurations e^2 and $e^1t_2^1$ for a tetrahedral complex (T_d), and to determine the group theoretical selection rules for electric dipole transitions between them.*

The spatial symmetries of the required states are given by the direct products in Table 7.

$$E \times E = A_1 + [A_2] + E \qquad E \times T_2 = T_1 + T_2$$

Combination of the electron spins yields both singlet and triplet states, but for the e^2 configuration some possibilities are excluded. Since the total (spin and orbital) state must be antisymmetric under electron interchange, the antisymmetrized spatial combination $[A_2]$ must be a triplet, and the symmetrized combinations A_1 and E are singlets. For the $e^1t_2^1$ configuration there are no exclusions. The required terms are therefore

$$e^2 \rightarrow {}^1A_1 + {}^3A_2 + {}^1E$$

$$e^1t_2^1 \rightarrow {}^1T_1 + {}^1T_2 + {}^3T_1 + {}^3T_2$$

The selection rules are obtained from formula (b)(viii). For electric dipole transitions the operator $\Omega^{(j)}$ has the symmetry of a vector (x, y, z), which from the character table for T_d transforms as T_2. From the table of direct products, Table 7,

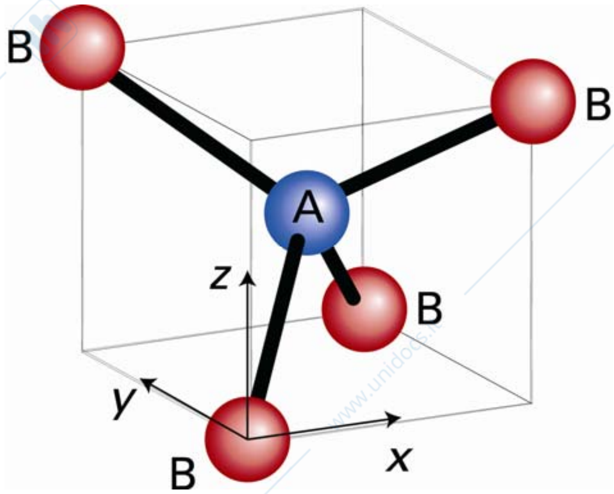
$$A_1 \times T_2 = T_2 \qquad A_2 \times T_1 = T_2 \qquad E \times T_2 = E \times T_1 = T_1 + T_2$$

Assuming the spin selection rule $\Delta S = 0$, the allowed transitions are

$$e^2\ {}^1A_1 \leftrightarrow e^1t_2^1\ {}^1T_2 \qquad e^2\ {}^3A_2 \leftrightarrow e^1t_2^1\ {}^3T_1 \qquad e^2\ {}^1E \leftrightarrow e^1t_2^1\ {}^1T_1, {}^1T_2$$

3. *To determine the symmetries of the vibrations of a tetrahedral molecule* AB_4*, and to predict the appearance of its infrared and Raman spectra.*

The molecule is depicted in the figure below and the character table for the point group T_d is given on page 15.



The representations spanned by the vibrational coordinates are based on the 5 × 3 cartesian displacements less the representations T_1 and T_2, which are accounted for by the rotations (R_x, R_y, R_z) and the translations (x, y, z). The stretching vibrations are the subset based on the 4 bonds of the molecule. For a particular symmetry operation, only atoms (or bonds) that remain invariant can contribute to the character of the cartesian displacement representation, $\Gamma^{(all)}$ (or the stretching representation, $\Gamma^{(stretch)}$).

C_3: Two atoms invariant, x, y, z, interchanged $\chi^{(all)}(C_3) = 0$
One bond invariant $\chi^{(stretch)}(C_3) = 1$

$C_2(z)$: Central atom invariant; $x, y,$ sign reversed, z invariant $\chi^{(all)}(C_3) = 0$
No bonds invariant $\chi^{(stretch)}(C_2) = 0$

$S_4(z)$: Central atom invariant; $x, y,$ interchanged, z sign reversed $x^{(all)}(S_4) = -1$
No bonds invariant $\chi^{(stretch)}(S_4) = 0$

$\sigma_d(z)$: Three atoms invariant; $x, y,$ interchanged, z invariant $x^{(all)}(\sigma_d) = 3$
Two bonds invariant $\chi^{(stretch)}(\sigma_d) = 2$

The characters of the representations $\Gamma^{(all)}$ and $\Gamma^{(stretch)}$ are therefore

	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	
$\Gamma^{(all)}$	15	0	–1	–1	3	$= A_1 + E + T_1 + 3T_2$
$\Gamma^{(stretch)}$	4	1	0	0	2	$= A_1 + T_2$

$\Gamma^{(all)}$ and $\Gamma^{(stretch)}$ have been decomposed with the help of formula (b)(vi) (compare Example 1). Allowing for the rotations and translations contained in $\Gamma^{(all)}$ there are therefore four fundamental vibrations, conventionally labelled $\nu_1(A_1)$, $\nu_2(E)$, $\nu_3(T_2)$, and $\nu_4(T_2)$. ν_1 and ν_2 are stretching and bending vibrations respectively, ν_3 and ν_4 involve both stretching and bending motions.

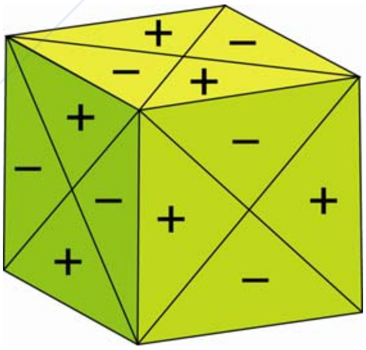
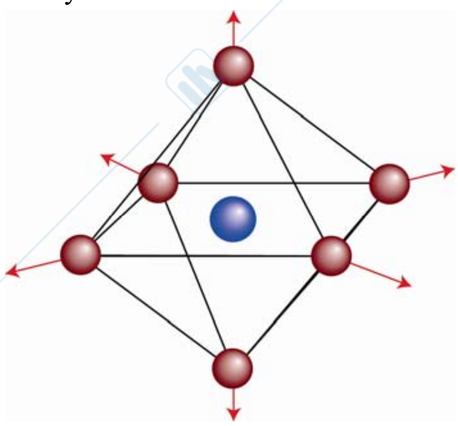
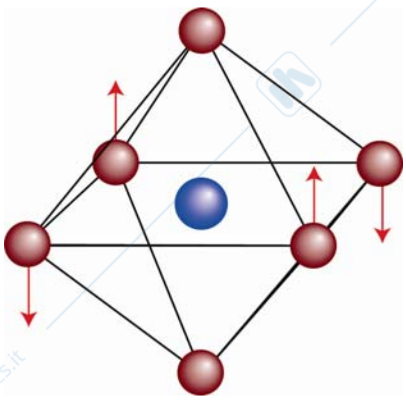
The selection rule (b)(viii) gives the spectroscopic properties of the vibrations. Infrared activity is induced by the dipole moment (a vector with symmetry T_2, according to the character table for T_d) as the operator $\hat{\Omega}^{(j)}$ In the case of the Raman effect, $\hat{\Omega}^{(j)}$ is the component of the polarizability tensor ($A_1 + E + T_2$). $f^{(i)}$ is the ground vibrational state (A_1), and $f^{(k)}$ is the excited state (with the same symmetry as the vibration in the case of the fundamental; as the direct product of the appropriate representations in the case of an overtone or a combination band). $\nu_1(A_1)$and $\nu_2(E)$ are therefore Raman active and $\nu_3(T_2)$ and $\nu_4(T_2)$ are infrared and Raman active. The following overtone and combination bands are allowed in the infrared spectrum:

$\nu_1 + \nu_3$, $\nu_1 + \nu_4$, $\nu_2 + \nu_3$, $\nu_2 + \nu_4$, $2\nu_3$, $\nu_3 + \nu_4$, $2\nu_4$

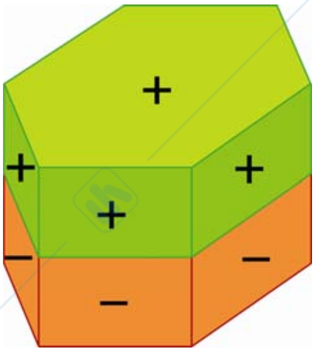
Examples of bases for some representations

The customary bases—polar vector (e.g. translation *x*), axial vector (e.g. rotation R_x), and tensor (e.g. *xy*)—are given in the character tables.

It may be of some assistance to consider other types of bases and a few examples are given here.

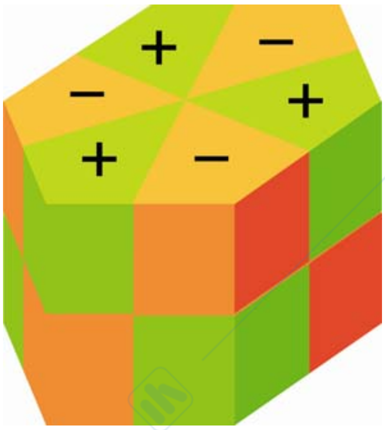
	Base	Irreducible Representation
1		A_2 in T_d
2	$x(1)y(2) - x(2)y(1)$	A_2 in C_{4v}
3	The normal vibration of an octahedral molecule represented by 	A_{1g} in O_h
4	The three equivalent normal vibrations of an octahedral molecule, one of which is represented by 	T_{2u} in O_h

5 The π-orbital of the benzene molecule represented by



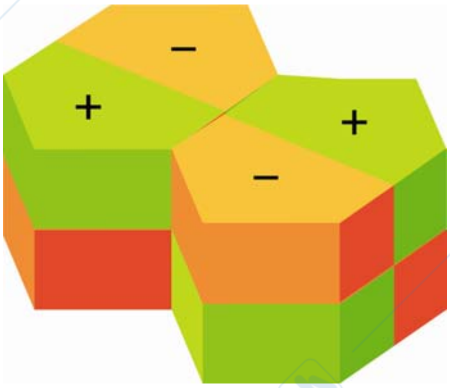
A_{2u} in D_{6h}

6 The π-orbital of the benzene molecule represented by



B_{2g} in D_{6h}

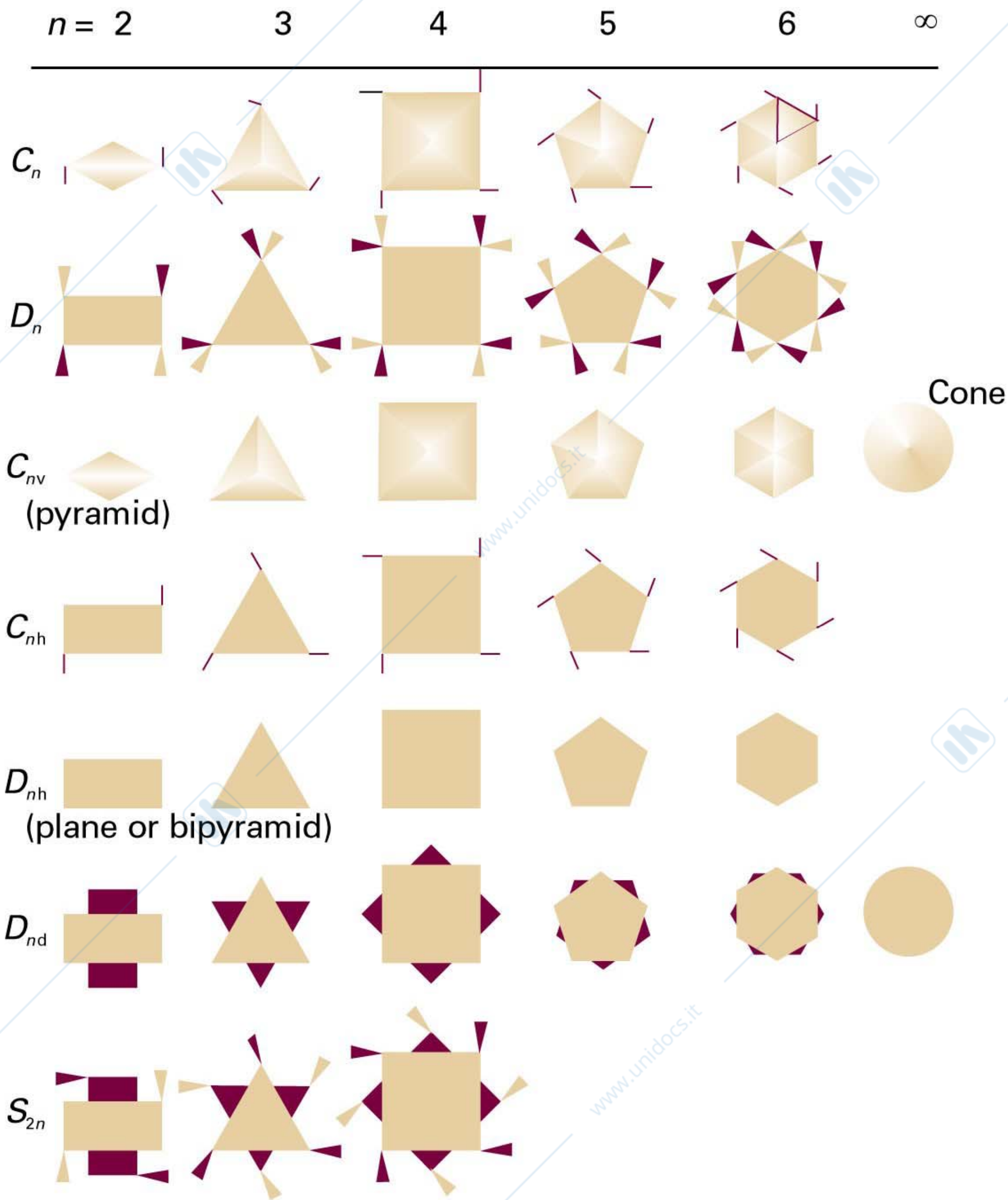
7 The π-orbital of the naphthalene molecule represented by



A_u in D_{2h}

Illustrative Examples of Point Groups

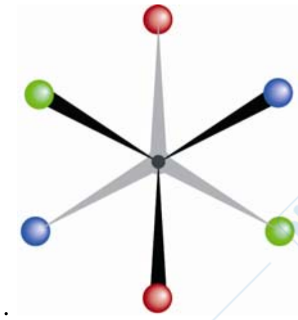
I *Shapes*



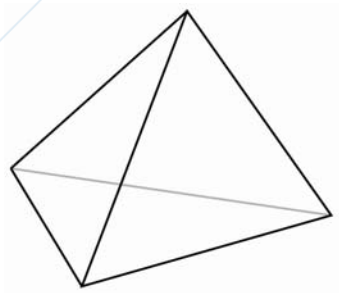
The character tables for (a), C_n, are on page 4; for (b), D_n, on page 6; for (c), C_{nv}, on page 7; for (d), C_{nh}, on page 8; for (e), D_{nh}, on page 10; for (f), D_{nd}, on page 12; and for (g), S_{2n}, on page 14.

C_s

C_i

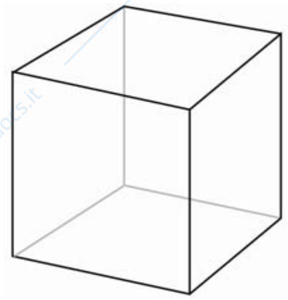


T_d



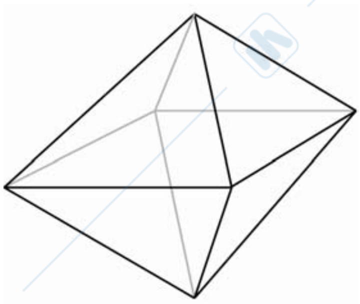
tetrahedron

O_h



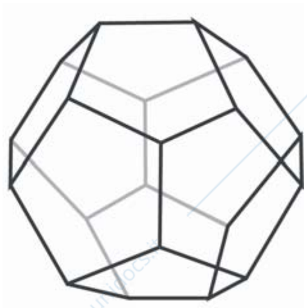
cube

O_h



octahedron

I_h

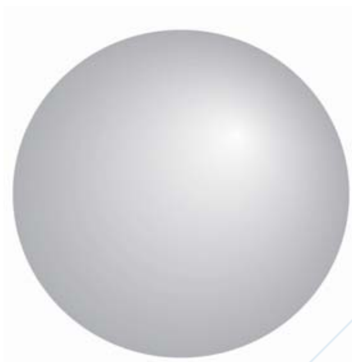


dodecahedron

I_h

icosahedron

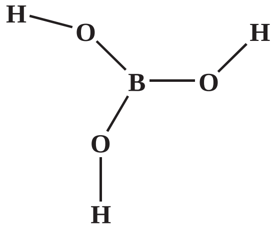
R_3



sphere

The character table for C_s is on page 3, for C_i on page 3, for T_d on page 15, for O_h on page 16, for I_h on page 17, and for R_3 on page 19.

II *Molecules*

Point group	Example	Page number for character table
C_1	CHFClBr	3
C_s	BFClBr (planar), quinoline	3
C_i	meso-tartaric acid	3
C_2	H_2O_2, S_2Cl_2 (skew)	4
C_{2v}	H_2O, HCHO, C_6H_5Cl	7
C_{3v}	NH_3 (pyramidal), $POCl_3$	7
C_{4v}	SF_5Cl, $XeOF_4$	7
C_{2h}	*trans*-dichloroethylene	8
C_{3h}	 (in planar configuration)	8
D_{2h}	*trans*-PtX_2Y_2, C_2H_4	10
D_{3h}	BF_3 (planar), PCl_5 (trigonal bipyramid), 1:3: 5–trichlorobenzene	10
D_{4h}	$AuCl_4^-$ (square plane)	10
D_{5h}	ruthenocene (pentagonal prism), IF_7 (pentagonal bipyramid)	11
D_{6h}	benzene	11
D_{2d}	$CH_2{=}C{=}CH_2$	12
D_{4d}	S_8 (puckered ring)	12
D_{5d}	ferrocene (pentagonal antiprism)	13
S_4	tetraphenylmethane	14
T_d	CCl_4	15
O_h	SF_6, FeF_6^{3-}	16
I_h	$B_{12}H_{12}^{2-}$	17
$C_{\infty v}$	HCN, COS	18
$D_{\infty h}$	CO_2, C_2H_2	18
R_3	any atom (sphere)	19