INTRODUCTION TO DERIVATIVES MARKETS AND DISCRETE PRICING MODELS

Notes for the course

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Finance and Risk Management

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1 INTRODUCTION TO DERIVATIVE MARKETS

1.1 A Short Synthesis of Financial Markets

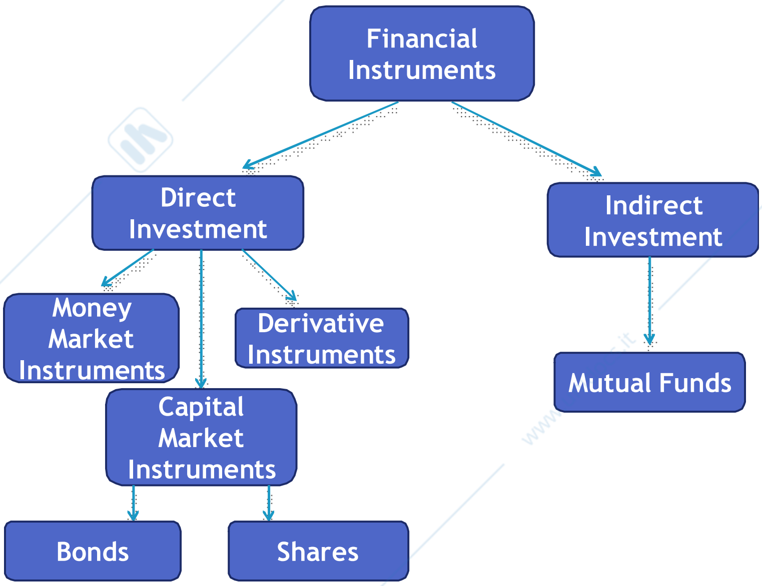
1.1.1 Financial Instruments

A *financial instrument* is a real or virtual document representing a legal agreement involving some sort of monetary value.

In today's financial markets, financial instruments can be classified generally as *equity based*, representing ownership of the asset, or *debt based*, representing a loan made by an investor to the owner of the asset. Foreign exchange instruments comprise a third, unique type of instrument.

Different subcategories of each instrument type exist, such as *preferred share equity* and *common share equity*, for example.

The typology of financial instruments is almost unlimited, and almost everyday, according to the market needs, new instruments are created by market participants. Next scheme is a possible (simplified) classification of financial instruments.



1.1.2 Monetary Market Instruments

Are debt instruments issued by companies, public entities or financial institutions. Their basic feature is a short maturity, usually within one year.

Examples are:

a. Buoni del Tesoro (Italy) and Tresaury Bills (USA)

b. Repurchase Agreement or Repo

c. Deposit Certificates

d. Commercial Paper

e. Interbank Deposits (rates LIBOR, EURIBOR, EONIA)

f. Eurodollars.

In a normal market situation short maturity instruments should give the same rate of return, but some differences can be observed due to the typology of the instrument or the credit rate of the issuer.

1.1.3 Capital Market Instruments

In this class we find instruments which maturity exceeds the year, or instruments without a specified maturity. The most popular capital market instruments are *bonds* (give the right to some cash flow) and *shares* (give the participation to the profits of a company in the future). The risk component is intrinsic and higher for shares, but bonds are also affected by some type of risk (credit risk, market risk, interest rate risk).

Examples of Bonds are the following

a. Certificati del Tesoro Zero Coupon (CTZ)

b. Buoni Poliennali del Tesoro (BTP)

c. Certificati di Credito del Tesoro (CCT)

d. Buoni Poliennali del Tesoro European Inflation linked (BTP€i)

e. Foreign Governments Bonds

f. Municipal bonds

g. Corporate Bonds

h. Structured Bonds (Reverse Floater, Convertible Bonds, cum Warrant Bonds, Index Linked, Equity Linked).

As far as it concerns shares we have:

a. Ordinary Stocks

b. Preferred Stocks

c. Mortgage-Backed Securities (MBS)

1.2 Risk and Return for Financial Instruments

The term return means the benefit arising from an investment in some financial instrument referred to its holding period. Return R is defined as the ratio of the difference of the market price of the instrument (price cashed for selling and price paid to purchase) plus any amount collected in the holding period, over the price paid to buy it:

$$R = \frac{P_s - P_b + D}{P_b} \tag{1}$$

A different definition for return (the one generally used in mathematical models) is the *log-return*: the logarithm of the ratio between sell price and purchase price:

$$\mu = \ln \frac{P_s}{P_b} \tag{2}$$

The ratio of this definition lies in the fact that it reflects the features of the interest intensity (*instantaneous interest rate*) in the compound capitalization.

The risk that is associated to any investment in financial instruments depends on the following factors:

1. The maturity of the asset. (The longer the maturity and higher is the risk).
2. The credit rate of the issuer of the instrument.
3. The nature and the priority of the rights attached to the investment on the capital of the issuer.
4. The liquidity of the instrument and the market where it is traded.

While the return is something that the investor is trying to increase, the risk is an intrinsic component of the investment that one would prefer to avoid or, at least, to reduce.

A necessary relationship (trade-off) connects risk and return such in a way that for any increment in risk a correspondent increment in return is required. Consequently, for any typology of investment, a two values label describes its feature: the value representing the return and the value of its risk benchmark.

For most of the assets traded in the marketplace, the starting point to detect the return parameter is the analysis of the historical series of their prices. The expectation of the return (the average of periodical returns), referred to the time unit of one year, is usually the proxy of the return of the asset.

The standard deviation of periodical returns is in general the most accepted risk parameter.

Next table is an example of estimation of risk and return for several assets, referred to a period when markets were not upset by the last financial crisis (period 2015-2017).

Instrument	Average Return	Standard Deviation
Tresaury Bill	1,8%	1,2%
Tresaury Bond	2,5%	3,3%
Stocks (stars)	13,3%	20,1%
Corporate Bond	3,9%	8,7%
Stocks (small Companies)	17,6	33,6

This empirical analysis confirms the trade-off of risk and return. In the period referred the yearly inflation rate was around 1.2%, and the *Treasury Bills* return should compensate the loss in purchasing power and the available cash: as a matter of fact the TBills return has a spread of 0.6% over the inflation rate. The *Treasury Bonds* have a return that is 0.7% higher than the return of TBills, a premium to be ascribed to the longer maturity of Tbonds (Longer maturity means higher risk). TBonds display also a higher standard deviation with respect to the Tbills to confirm the trade-off of risk and return. This means a higher variability in the prices of Tbonds, a consequence of the fact that the operators perceive the risk associated with longer maturity.

Higher returns in Corporate Bonds ($+1,4\%$) reflect and compensate the higher default risk of private companies with respect to the state government. This increase in risk generates more variability in the corporate bonds and the consequent increase in standard deviation.

A confrontation between the returns of stocks of big companies (stars) and small companies shows a consistent spread (+4.2%) in advantage of the small companies. The spread is to be ascribed to the growth potentiality that normally is higher when the company's dimension is not too big. On the side of risk, the higher standard deviation of returns of small companies (higher variability in the prices of stocks of small companies) suggests that investing in small companies one incurs in more risk as the growth potentiality cannot take place and it can turn into lower financial capacity and higher risk of default.

1.3 Derivative Markets

At a certain point of its development almost every spot market experiences the birth of a parallel *forward market*, where participants can hedge the positions taken in *spot markets*. After a testing period focused on preferences of participants, forward markets were established, with products that, in the early period, were settled at maturity with physical. In a second stage, to promote trading by new market participants and stimulate greater liquidity, the financial contracts were changed, in the majority of the markets, from physical delivery contracts to financial contracts with only cash settlement at maturity.

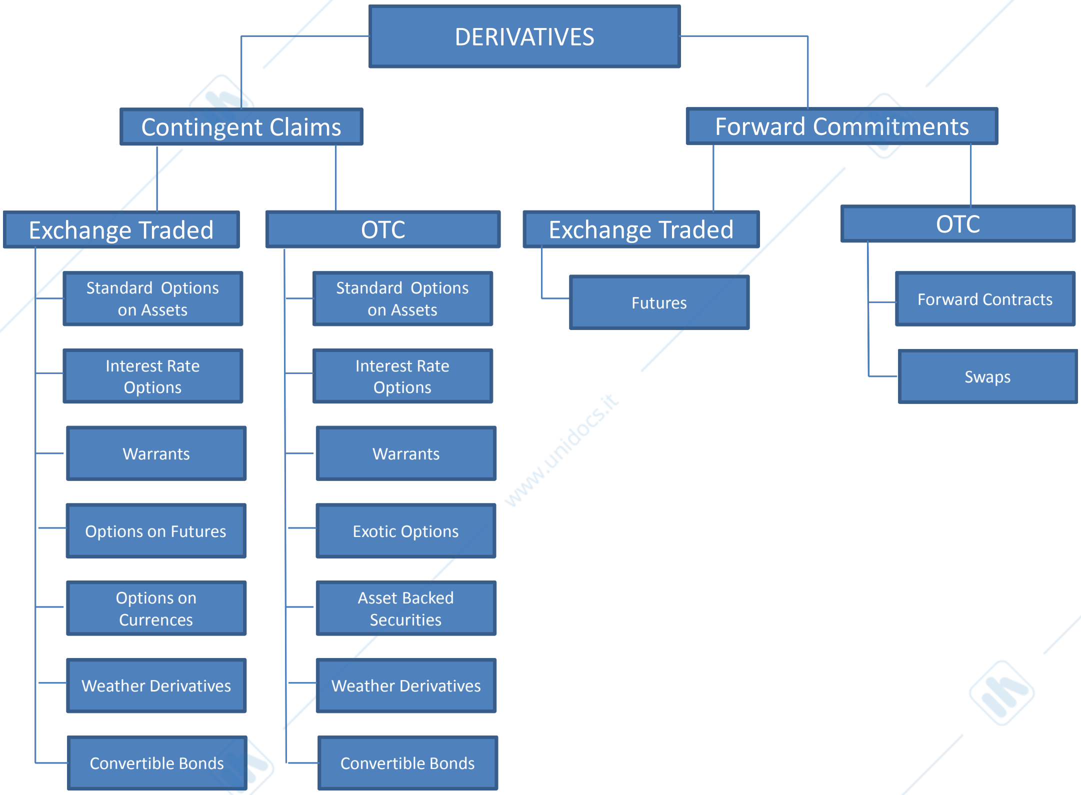
Usually when one speaks about financial markets he thinks of an *exchange*, that is an organized market ruled by specific norms that make all the trades uniform, with a central controlling body and a *clearing house* (see ahead for details of clearing houses). However many financial contracts are executed *over the counter* (OTC) between two parties that agree on particular conditions stated in the contract according to the needs of any of the two, generating a customized contract (tailor made). For OTC traded financial instruments no quotation or other public information are available, but in many cases such markets reach consistent volumes of trades, as is the case of *forwards* or *swaps*. What is important to notice is that when traded in exchanges, financial contracts are entered into without mention to technical conditions that can affect the physical delivery of the underlying. When an exchange organized market becomes "adult", the traditional bilateral trade in *physical-delivery* contracts, are substituted by new pure financial instruments with only *cash settlement*. This is the case of very popular derivatives instruments like *forward contracts*, *futures*, *swaps* and *options* of different nature, today largely traded on specific markets.

As well known, derivatives are financial contracts that do not represent ownership rights in an asset but, rather, derive their value from the value of some underlying or other asset. When used prudently, derivatives are effective tools for isolating financial risk and hedging to reduce exposure to risk. Given the increasing, in the last decades, of variability of prices, either for financial instruments or commodities and energy products, hedging is the natural answer to the new reality, and derivatives, being instruments able to transfer risk, play a fundamental role in risk management.

Definition 1 (Derivative Asset) *A derivative is a financial instrument which takes value in dependence of the value of another asset or specified entity. The asset or the entity that gives the value to the derivative is called the underlying.*[1]

Recently the number of derivatives has enormously grown, either in the volume of trades or in the typology, and an exhaustive classification of the different instruments is almost impossible. The next table should give an approximate idea of the different typology of derivative assets.

[1] There are four basic derivatives: forwards, futures, European calls, and European puts. If one understands the four basic derivatives, then one can understand all derivatives. Indeed, it can be shown that one can view the payoffs to any derivative as a limit of a combination of these four basic derivatives.



Every Derivatives Market Exchange has three fundamental principles to cope with:

1. **The Principle of Liquidity**. Liquidity is defined as the number and the volume of contracts of the same type traded. High liquidity is synonymous of market depth. How much a market is deep can be red in the level of supply and demand volume and in the price range pertaining it, as well as in the number of market participants. At high levels of liquidity, direct consequence of the factor mentioned above, it is possible to open or close major positions cost-efficiently at any time with the consequence of reducing price and volume risk. A further indicator of a high liquidity is a close spread between best bid and best ask.

2. **The Principle of Transparency**. Transparency means that buy and sell orders are disclosed to all trading participants in order to ensure the highest circulation of information.

3. **The Principle of Practicability**. Derivatives Markets are designed in a way as to allow all the companies interested to take part in it to operate as easily as possible. This aim is usually achieved by electronic trading and clearing system, as well as through anonymous trading and clearing.

Economic activities always entail risks. As it is well known, to avoid risk is at the core of the risk management activity and, as already mentioned, derivative instruments are fundamental instruments to achieve the objective.

Any Derivatives Market supports the trading participants in managing the following risks:

- *The market price risk* is defined as the threat of losses caused by the volatility of prices. Basically, such market price risks can be controlled with the help of fixed price contracts, forward or futures contracts, or option transactions for future deliveries. The Derivatives Market offers suitable tools for controlling this market price risk.
- *The counterpart risk* is defined as the threat of losses caused by payment or delivery default of a counterpart, such as e.g. as a result of a bankruptcy. Together with the institutions acting as clearing members (clearing banks) assumes the counterpart risk and ensures the fulfillment of obligations arising from all exchange positions.
- *The basis risk* results from a price change in the underlying position and a non-equivalent price alteration of the hedging instrument (e.g. differences in quality, times and/ or places of delivery). The so-called *basis* is the difference between the spot price of the underlying of the derivative used for hedging and the derivative price.
- *The liquidity risk* is defined as the loss which might occur on account of a sudden withdrawal of liquidity, e.g. due to a margin call arising from futures transactions (see in the next section). In almost every Derivatives Market trading and hence price setting, from which the margin calls are derived, is supervised by the trade supervision office. This procedure ensures that margin calls only arise through market price developments. Exchange participants may control this risk by means of carefully liquidity planning.

On principle, derivatives can be traded for three reasons:

Hedging: the sale of futures contracts can be used to hedge against falling prices (short hedge); the purchase of futures contracts can be used to hedge against increasing prices (long hedge). Moreover, options contracts can also be used for hedging.

Arbitration: arbitration uses differences in prices between e.g. futures, which are traded on the exchange, and options and similar contracts, which are traded off the exchange. In this process, the cheaper futures contract is bought and the more expensive futures contract is sold at the same time.

Speculation: a futures contract is e.g. sold in the expectation of falling market prices and with the intention of generating a profit by buying back the contract at a lower price subsequently. Speculators assume risks and provide liquidity for trading participants with contrary market strategies. Speculation is also possible with options contracts.

In any derivative market a special typology of traders is required in order to guarantee a good efficiency of the market itself: the *Market Makers*. Market makers play an important role on the market, contributing to the increase in market liquidity and improving the efficiency of the process of price discovery. The market makers have *quoting obligations* relating to the series, maturities, minimum quantity, maximum spread allowed (determined as the maximum allowable difference between the price of the bid and the price of the sale) and time. Today there are, for example, in the Italian Derivative Market (IDEM Market) more than 20 market makers quoting on an ongoing basis or respond to requests for quotations. The list of operators Market Makers and its activities is available on the web of the Italian Stock Exchange (Banca Aletti & C. S.p.A., Banca Akros S.p.A., Banca IMI S.p.A., Banca Profilo, Banca Simetica S.p.A., Bayerische Hypo-und Vereinsbank AG - Milan Branch, BNP Paribas Italian Branch, Equita SIM S.p.A., Exane S.A., Interactive Brokers (U.K.) LTD, Mediobanca S.p.A., etc.).

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2 FORWARD CONTRACTS

The simplest derivative contract is the *forward contract*. Such contracts where in use since the ancient times and have gone along the economic evolution of mankind. People enter a forward contract with the aim to contrast the uncertainty depending on the ignorance of which price some good or asset will exhibit in the future. While in the spot market goods and payments are exchanged immediately or, at the latest within a short settlement period, when it is agreed to execute the exchange unconditional at some later time, the trade gives rise to a forward contract.

Definition 2 (Forward Contract) *A* forward contract *is an agreement between two parties whereby one of the two takes the obligation to buy and the other the obligation to sell a specified asset for a specified price called* delivery price*, on a specific date in the future, known as the delivery date or* maturity *of the contract.*

The party to the contract that agrees to sell the asset is said to be taking a *short forward position* (he or she goes short), while the other party, obliged to buy the asset at delivery date, is said to have a *long forward position* (he, or she goes long). Two dates are essential in a forward contract: the date of inception of the contract, t_0 and the maturity T at which the contract is settled. At time t_0 the parties rule the content of the contract (maturity, delivery price, delivery place and delivery modalities) and at time T the contract is executed with the delivery of the asset and payment of the price.

The principal reason for entering into a forward contract is to become independent of the unknown future price of a risky asset. There are a variety of examples: a farmer wishing to fix the sale price of his crops in advance (maybe at sowing time), an importer arranging to buy foreign currency at a fixed date in the future, a fund manager who wants to sell assets for a price known in advance. Forward contracts are major examples of OTC contracts, being a direct agreement between the two parties that are free to establish any specific term that fits their needs. It is typically settled by physical delivery of the asset on the agreed date. As an alternative, settlement may sometimes be in cash.

Remark 3 *According to the definition of a forward contract, no money changes hand at the outset of the contract. Therefore it costs nothing to enter into, and this means that the value of a forward contract, at time* t_0 *is zero.*

At delivery time T the party with a long forward position will take advantage from the contract if the delivery price K is less than the spot price S_T of the asset: he can buy the asset for K and immediately sell it at the market price S_T, obtaining an instant profit of $S_T - K$. Meanwhile the part holding the short forward position will suffer a loss of the same amount because he has to sell below the market price. If the market price S_T is lower than the delivery price K, then the situation will be reversed. The payoff at maturity for a long position in a forward contract, being S_T the price of the asset at time T, is $S_T - K$, while $K - S_T$ is the payoff for a short position.

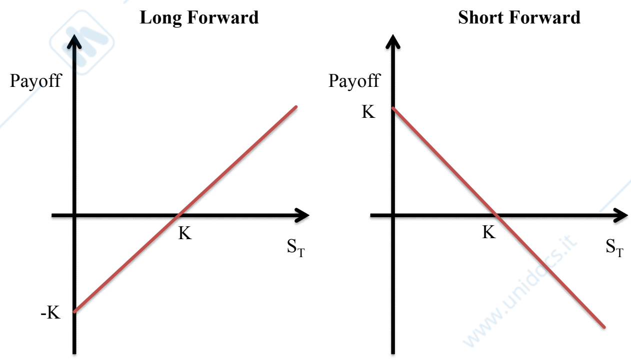
Considering the spot price S_T at delivery time of the asset as a variable and the payoff as a function Ψ of S_T, for a long position we have:

$$\Psi(S_T) = S_T - K \tag{3}$$

and for a short position:

$$\Psi(S_T) = K - S_T \tag{4}$$

with the following graphic representations:



The payoff functions are linear and, as we shall see soon, it is possible to price forward contracts simply using no-arbitrage arguments. The same approach is not allowed for the pricing of options, which payoff is no more linear, and in this case more information is required in order to price.

The delivery price in a forward contract represents, in some way, the perceived value at time t_0 of the value the asset will have at time T. Abstracting from a single specific forward contract, it is possible to define the *forward price* of an asset, referred to time t_0 for the maturity T.

Definition 4 (Forward Price) *Given an asset, at time t_0 its forward price for maturity T, $F_{t_0}(T)$ is the delivery price of an hypothetical forward contract with the same maturity, such that the initial value of the contract is zero.*

It is important not to confuse *delivery price* and *forward price*. The delivery price, once the contract is set up, remains the same until maturity. The forward price today (at time t_0) for maturity T is, in general, different from the forward price tomorrow (at time t_1) for the same maturity T: $F_{t_0}(T) \neq F_{t_1}(T)$.

Considering the forward price $F_{t_0}(T)$ as a function of the maturity T, what is obtained is the so called *forward curve*, a very important piece of information for market operators.

Next step is to investigate the relationship between the spot price S_{t_0} of the asset and the delivery price K. As we shall soon see, such relationships depends on the nature of the asset underlying the forward contract. We have to distinguish between financial instruments and commodities.

A. Financial assets that do not make payments during the life of the forward contract.

B. Financial assets that give rise to payments during the life of the forward contract.

C. Foreign Currencies.

D. Commodities that imply extra costs for storage and/or insurance.

2.1 Forwards on Financial Assets in Absence of Dividends

Let us assume that in a market at time t_0 is traded a financial asset that does not make payments (dividends or coupons for example) during the life of the forward contract. In the market is also possible to borrow or lend money at fixed instantaneous interest rate δ (continuously compounded), and market participants can take either long or short positions in the financial asset. In order to get hold of the financial asset at maturity time T, two different *strategies*[2] can lead to the goal.

- *Portfolio A*: a long position in a forward contract with maturity T and delivery price K. This strategy guarantees, at no cost today, to receive the stock at maturity paying the forward price K.

[2] The term *strategy*, in financial market jargon, means a particular portfolio made up by the assets available in the market.

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- *Portfolio B*: borrow from the bank the amount S_{t_0} (equivalently: sell a bond with face value S_{t_0}), buy the stock (long stock) and keep it in the safe box until time T, when you have to pay out the debt with the bank, interests included, or to reimburse of face value of the bond plus accrued interests. Also with this strategy the balance of cash at initial time is zero (you receive the amount S_{t_0} from the bank, or from the selling of the bond, and this amount is used to buy the stock). And also in this situation, at maturity time T, the stock is owned.

As both strategies do not require any fund at initial time t_0 and both give as a result the possession of the stock at time T, their payoffs at maturity must be equal in order to avoid arbitrage opportunities. The payoff of the forward contract is $S_T - K$, while the payoff of portfolio B is the value S_T of the stock reduced by the amount we have to reimburse to the bank: $S_{t_0}e^{\delta(T-t_0)}$. In conclusion, the no arbitrage condition leads to the relationship:

$$S_T - K = S_T - S_{t_0}e^{\delta(T-t_0)}$$

and therefore:

$$K = S_{t_0}e^{\delta(T-t_0)}. \tag{5}$$

As at time t_0, in both strategies, no cash movement is involved, the delivery price K given in the (5) satisfies the condition that the initial value of the contract is zero.

Now assume that is $K > S_{t_0}e^{\delta(T-t_0)}$. In this case at time t_0:

a. Borrow the amount S_{t_0}.

b. Buy the asset for S_{t_0}.

c. Enter a forward contract in short position at delivery price K for maturity T.

Then at time T:

d. Deliver the asset and cash the amount K to clear the forward commitment,

e. Pay $S_{t_0}e^{\delta(T-t_0)}$ to clear the loan with interest.

This will leave a risk-free profit of $K - S_{t_0}e^{\delta(T-t_0)} > 0$, contrary to the no-arbitrage principle.

Next suppose that is $K < S_{t_0}e^{\delta(T-t_0)}$. In this case we construct the opposite strategy to the one above. At time t_0:

a. Sell short the asset (a share, for example) for S_{t_0}.

b. Invest the proceeds at the risk free rate.

c. enter into a long forward contract with delivery price K.

Then at time T:

d. Cash the risk-free investment with interest, collecting $S_{t_0}e^{\delta(T-t_0)}$.

e. Receive the asset and pay K as required by the contract.

f. Close out the short position in the asset by returning it to the owner.

You will end up with a positive amount: $S_{t_0}e^{\delta(T-t_0)} - K$.

Either in the case $K > S_{t_0}e^{\delta(T-t_0)}$ or in the case $K < S_{t_0}e^{\delta(T-t_0)}$ arbitrage opportunities are present, and hence it must be: $K = S_{t_0}e^{\delta(T-t_0)}$.

Introducing now the forward price $F_{t_0}(T)$ at time t_0, we have:

$$F_{t_0}(T) = S_{t_0}e^{\delta(T-t_0)}. \tag{6}$$

From the last relationship we also have:

$$\frac{\partial F_{t_0}(T)}{\partial T} = \frac{\partial \left[S_0 e^{\delta(T-t_0)}\right]}{\partial T} = S_{t_0}\delta e^{\delta(T-t_0)} > 0$$

reflecting the fact that, keeping fixed the inception date, looking at the forward price of a financial asset that does not pay dividends as a function of maturity T, it is an increasing function of the maturity itself.

Example 5 *Going back to the example* **??** *of the previous section we have seen that the correct delivery price was $K = 120.90$. As a matter of fact, using formula (5) we have: $K = 115e^{0.05} = 120.90$, as expected.*

Example 6 *A pension fund will cash contributions from clients in three months' time and the board decides to invest the amount in stocks of company XYZ that usually pays good dividends. Rumors in the financial markets report that in a month from today the price of XYZ stocks will consistently rise due to a favorable contract that will soon be signed. The board of the pension fund, in order to avoid the risk of a price increase, enters a forward contract in a long position with maturity three months from now. The price today of a stock XYZ is $S_{t_0} = 58.32$ and the continuous interest rate in the market is $\delta = 0.035$. The delivery price in the contract is $K = 59.61$. Is it* *a fair price? And if not how can one set up an arbitrage? The equilibrium delivery price is:*

$$K = 58.32e^{0.035\frac{3}{12}} = 58.83.$$

well less than the price in the contract. To set up an arbitrage one has to sell (short position) the asset that is overpriced, and take a long position in the asset undervalued. In this case one borrows from the bank the amount $S_{t_0} = 58.32$*, buys the stock and enters the forward contract in a short position. At maturity he delivers the stock in his possession, cashes the amount* $K = 59.61$ *from the forward contract and refunds the amount* $S_{t_0}e^{\delta T} = 58.32e^{0.035\frac{3}{12}} = 58.83$ *to the bank. The difference:* $59.61 - 58.83 = 0.78$ *is the arbitrage profit for the contract. Now let us assume that the delivery price in the contract is the correct one* $K = 58.83$*. Suppose that a month away the rumors have been confirmed and that the price of stock is now* $S_{t_1} = 61.12$*. Should someone enter a new forward contract with the same maturity, in the hypothesis that nothing has changed in the interest rate market, the new delivery price has to be:* $K_1 = 61.12e^{0.035\frac{2}{12}} = 61.48$*, higher than* 58.83 *of one month before, reflecting the jump in price exhibited by the stock. An interesting problem arising at this point is the following. The pension fund is long in the original contract with delivery price* $K = 58.83$*. We know that at inception the value of the position (alike the opposite one) was zero. After one month, when the new price of XYZ stock is* 61.12 *and the new delivery price is* 61.48 *what a value has the original contract? In other words, should the pension fund decide to sell to someone else its right to receive at maturity the stock, paying* 58.83*, how much should ask for the negotiation? At first view the answer seems to be the difference:* $61.48 - 58.83 = 2.65$*, but the correct one is* $2.63\ldots$ *(why?).*

Example 7 *Forward contracts can be used also to borrow money. You can do the following: enter today a forward contract in a long position with delivery price* $K = S_{t_0}e^{\delta(T-t_0)}$*, and in the meantime sell short the underlying at current price* S_{t_0}*. At maturity* T *you have to delivery the asset of price* S_T*, but from the forward contract you just receive the asset and have to pay* $K = S_{t_0}e^{\delta(T-t_0)}$*:*

t_0	T
S_{t_0}	$-S_T + S_T - K = -S_{t_0}e^{\delta(T-t_0)}$

The cash flow generated by the strategy is the same as a borrowing today the sum S_{t_0} *and refunding at time* T *the same amount increased by interests.*

2.2 Forward on Assets with Flows of Payments

Some financial assets give the right to the owner to receive one or more payments as dividends or coupons. Let be D_1, D_2, ..., D_k the flow of payments that the underlying the forward contract guarantees at times t_1, t_2, ..., $t_k < T$. Forward contracts on dividend-paying asset are priced in the same way as non-dividend-paying assets. But the flow of cash from the asset during the existence of the contract requires some adjustment in the calculation of the equilibrium delivery price K. Here also we have two strategies A and B that give the same final result as in the previous section. The only difference is that, as far as it concerns strategy B, we do not need to borrow the whole amount S_0, as the cash flow generated by the asset before maturity can be used to pay off part of the loan. At time t_0 the value of the cash flow is

$$D = \sum_{j=1}^{k} D_j e^{-\delta(t_j - t_0)}$$

The strategies are:

- *Portfolio A*: a long position in a forward contract with maturity T and delivery price K. Due to this strategy, at no cost today, at maturity, paying the forward price, K we have possession of the share.
- *Portfolio B*: sell the rights to cash the dividends for the amount D, borrow from the bank the amount $S_{t_0} - D$, buy the stock (long stock) at the spot price S_{t_0} and keep it in the safe box until time T, when the debt with the bank is paid out. With this strategy the balance of cash at initial time is: $D + (S_{t_0} - D) - S_{t_0} = 0$, and the stock is owned at maturity.

Both A and B are zero cost strategies at time t_0 and both give as a result the possession of the stock at time T. Their payoffs at maturity must be equal in order to avoid arbitrage opportunities. The payoff of the forward contract is $S_T - K$, while the payoff of portfolio B is the value $S_T - (S_{t_0} - D)\, e^{\delta(T-t_0)}$. In conclusion, the no arbitrage condition leads to the relationship:

$$K = (S_{t_0} - D)\, e^{\delta(T-t_0)}. \tag{7}$$

In some contests (for instance when dividends are paid with high frequency, as it is the case of indexes that refer to portfolios containing a great number of stocks, and almost every instant some of them pays some dividend) it is useful to think of a continuous flow of payments as it happens when interests are accrued in a continuous capitalization regime. In this case the model assumes that the dividends instantaneously released are immediately reinvested in the asset itself, with the result that at the end of this process the quantity of the stock owned is no more one, but will increase to $e^{\gamma(T-t_0)} > 1$ where γ is the instantaneous rate of release of dividends, called *dividend yield*. Due to this action that increases the quantity owned of the stock, in order to have just one unit at maturity time T, the quantity needed at starting time t_0 is less than one unit, i. e. the quantity $e^{-\gamma(T-t_0)}$ (as a matter of facts, starting with $e^{-\gamma(T-t_0)}$ units, at maturity one has $e^{-\gamma(T-t_0)}e^{\gamma(T-t_0)} = 1$). In conclusion, having continuous release of dividends to be invested in the stock itself, in order to build portfolio B at time t_0, the required loan is reduced to $S_{t_0}e^{-\gamma(T-t_0)}$. And consequently formula (5) has to be modified to:

$$K = S_{t_0}e^{-\gamma(T-t_0)}e^{\delta(T-t_0)} = S_{t_0}e^{(\delta-\gamma)(T-t_0)}. \tag{8}$$

For stocks paying dividend yield at rate γ, the equilibrium delivery price is:

$$K = S_{t_0}e^{(\delta-\gamma)(T-t_0)}. \tag{9}$$

Remark 8 *Taking into consideration the forward curve $F_{t_0}(T) = S_{t_0}e^{(\delta-\gamma)(T-t_0)}$ as a function of the maturity T, the shape of the curve itself depends on the relationship between δ and γ and consequently it is increasing every time $\delta > \gamma$ and decreasing in the opposite situation.*

In term of forward prices, it is evident that the two cases, discrete and continuous flow of dividends, can be reduced to an unique approach, the continuous one, solving with respect to the unknown γ the equation:

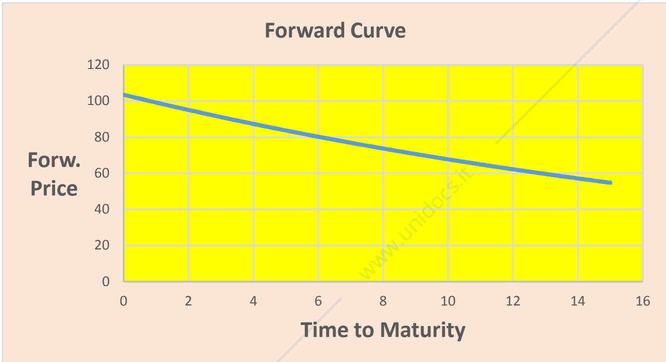
$$S_{t_0} - \sum_{j=1}^{k} D_j e^{-\delta(t_j-t_0)} = S_{t_0}e^{-\gamma(T-t_0)}$$

being:

$$\gamma = \frac{1}{T-t_0}\ln\left[\frac{S_{t_0}}{S_{t_0} - \sum_{j=1}^{k} D_j e^{-\delta(t_j-t_0)}}\right] \tag{10}$$

Example 9 *Assume that a bond which price is today* 103.4 *pays a fixed coupon of* 3.12 *next quarter and in the following two quarters. The instantaneous interest rate is* $\delta = 0.05$. *We have* $D = 3.12\left(e^{-0.05\frac{3}{12}} + e^{-0.05\frac{6}{12}} + e^{-0.05\frac{9}{12}}\right) =$

9.13. A forward contract on the bond with maturity one year has a delivery price $K = (103.4 - 9.13)\, e^{0.05} = 99.103$. *Using the continuous approach we have:* $\gamma = \ln\left[\frac{103.4}{103.4-9.13}\right] = 0.092442$. *And consequently:* $K = 103.4e^{(0.05-0.092442)} = 99.103$. *The graph shows a decreasing forward curve, as the dividend yield is greater than the interest rate.*



2.3 Forwards on Foreign Currency

When the underlying is a foreign currency we have to take into account the fact that a person holding a foreign currency can receive interest at the current risk-free rate of that foreign country.

Going again to the portfolio B. made up of the long position in the underlying (the foreign currency) financed by a loan, the holder of the currency can invest it in a bond issued by the foreign country, or start a bank deposit in the foreign country. Assumed that δ_f is the continuously compounded interest rate, one unit of foreign currency invested will grow at the rate δ_f. This situation is the analogous of the one already analyzed in the case of dividend yield: in order to set up the portfolio B a quantity $e^{-\delta_f(T-t_0)}$ of foreign currency will suffice, and consequently the amount to borrow is $S_{t_0}e^{-\delta_f(T-t_0)}$. The equilibrium delivery price K is then:

$$K = S_{t_0}e^{(\delta-\delta_f)(T-t_0)}. \tag{11}$$

Example 10 *The financial director of an import-export company of the UK knows that the company will receive a payment in six months' time of* 1.5 *billion yen. Being concerned about fluctuations in the currency markets he would like to know today the amount he will get, in sterling, once he will*

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sell the yen on the day they are received. A short position in a forward contract on Japanese currency is the appropriate instrument. The pound/yen exchange rate is today 152.3667 *and the company can borrow pounds at the domestic continuously compounded rate* $\delta = 0.045$*. The Japanese interest rate is* $\delta_f = 0.031$*. On the basis of formula (8), being in this case* $\gamma = \delta_f$*, the interest rate one get investing in Japan, the four-month forward price of the yen is:*

$$\frac{1}{152.3667}e^{(0.045-0.031)\frac{6}{12}} = 0.006\,6092$$

Here $S_{t_0} = \frac{1}{152.3667}$ *is the price of one yen expressed in pounds units. It is possible to explain the last result in term of a combination of two strategies. The first consists in buying today* $e^{-\delta_f T} = e^{-0.031\frac{6}{12}} = 0.984\,62$ *units of yen (a quantity that invested at the Japanese rate* δ_f *will give exactly one yen after six months). To do that an amount of* $0.98462\frac{1}{152.3667} = 0.006462\,2$ *pound is required. A debt with the bank for this quantity is started. The second strategy consists in entering short in a forward contract for delivery of one yen after six months at the delivery price* K *(in pounds). At maturity, six months later, one has to pay off the loan grown to* $0.0064622e^{0.045\frac{6}{12}} = 0.006609\,2$ *in pounds. The Japanese investment gives just one yen that is delivered to the counterpart of the forward contract. As, at initial time, the value* V_0 *of the combined strategies is zero (the forward contract does not require any cash and the buying of the* 0.006422 *unit of yen is financed by the bank), the final value*

$$V_T = (K - S_T) + (S_T - 0.0066092) = K - 0.0066092$$

must be zero in order to avoid arbitrage opportunities[3] *(here* S_T *is the value of the yen at maturity,* $(K - S_T)$ *is the payoff of the forward contract,* $(S_T - 0.0066092)$ *the payoff of the first strategy). In conclusion we have* $K = 0.0066092$*.*

2.4 Forward on Commodities

Sometimes the underlying the forward contract gives rise to expenses during the life of the contract. This may happen when the underlying is a commodity that has to be stored or insured (think of gold that has to be stored in safe boxes, or of crude oil stored in some container). Such expenses play a role similar to the cash flow generated by the asset, with the difference that the signs of the elements of the flow are here negative. Hence setting

[3] As the cost of the combined two strategies is zero, zero must be also their final payoff.

up the B strategy the amount to be borrowed is the spot price of the asset plus the discounted value of future expenses. Indicating with E the present value of the expenses, the delivery price K is given by the relationship:

$$K = (S_{t_0} + E)\, e^{\delta(T-t_0)}. \tag{12}$$

Here also, as in the previous section, it is possible to reduce relationship (12) to a continuous form introducing an instantaneous rate of expense β, having:

$$K = S_{t_0} e^{(\delta+\beta)(T-t_0)}. \tag{13}$$

Here $\beta = \frac{1}{T-t_0} \ln \frac{S_{t_0}+E}{S_{t_0}}$.

The forward price $F_{t_0}(T)$ is now:

$$F_{t_0}(T) = S_{t_0} e^{(\delta+\beta)(T-t_0)}. \tag{14}$$

In the present contest, as shown by formula (14), the forward curve turns out to be an increasing one.

For underlying that demand for storage costs or insurance costs, the forward price can be thought as made up by three different components: the spot price of the underlying, the cost of funds (the interest component) and the storage cost. The last two are also called the *cost of carry*, so we can say that the forward price is the sum of the spot price plus the cost of carry. In commodity and energy markets the basic equilibrium relationship (13):

$$F_{t_0}(T) = S_{t_0} e^{(\delta+\beta)(T-t_0)}$$

often does not hold.

2.4.1 The Convenience Yield

What frequently is observed in the real commodity markets is the persistence of the inequality:

$$F_{t_0}(T) < S_{t_0} e^{(\delta+\beta)(T-t_0)} \tag{15}$$

that is, the forward price is less than its equilibrium price. The reason for which a situation that, in spite of being temporary, seems to be permanent lies on the fact that the non-arbitrage argument that leads to relationship (13) implies the possibility, for market participants, to set up strategies that involve purchases or sells either for the underlying or the forward contract. As a matter of facts, the condition that allows to reach equilibrium selling the overpriced asset and buying the underpriced one, is that both the assets are financial in their nature, the so called *tradable*. In other terms, every time one of the assets is not a financial instrument (as is the case in which the underlying is a commodity) the no-arbitrage mechanism does not work. This is due to the fact that, in general, the commodity in the contract has an intrinsic utility because is a raw material to be used in some production process, or can be directly consumed providing direct utility to the subject. Consequently, although it could be convenient to sell it in the market, an economic operator may prefer to keep it as an inventory. Observing in the market a relationship like the (15), not every operator will decide to sell the commodity and to enter long the forward contract, letting the inequality persist.

The only way to force the relationship (15) to become an equality is to introduce a new parameter $y > 0$ to obtain:

$$F_{t_0}(T) = S_{t_0} e^{(\delta+\beta-y)(T-t_0)}. \tag{16}$$

Due to the similarity of the right member of (16) and the right member of (9) it is natural to think of parameter y as something similar to the dividend yield γ, in this case a yield accrued by the commodity as a consequence of its possession. Parameter y is named *convenience yield*, according the following definition.

Definition 11 (Convenience Yield) *The convenience yield is the* premium *associated with holding an underlying commodity or physical good, rather than the contract or derivative product.*

It represents sort of immaterial income generated by the possession of the asset, due to the fact that having stored the commodity one can immediately use it, avoiding market price risk. Obviously, differently from other parameters in formula (16), the convenience yield has to be estimated as no direct piece of information about its value is available in the market.

At this point a general formulation for the *spot-forward parity* relationship is possible. Let $k = y - \beta$. We have:

$$F_{t_0}(T) = S_{t_0} e^{(\delta-k)(T-t_0)}. \tag{17}$$

Remark 12 *Assume that for a certain asset is known the spot price S_0 together with δ, β and y. Then, from relationship (17) the whole forward curve $F_{t_0}(t)$ can easily obtained for any future maturity t. The* forward curve *is a fundamental instrument for operators, specially in commodity markets. The shape of the curve depends on the sign of $\delta - k$, and it can result* *decreasing, or increasing. In the first occurrence (downward shaped curve) the market is said to be in* normal backwardation*, in the second (upward shaped curve) in* contango*. The difference $\delta - k$ is negative, and hence the market is in a normal backwardation situation, only if $\delta + \beta < y$, indicating that interest rate, as well storage costs, are low enough, with respect to the benefits arising from the physical holding off the commodity. Conversely, in a contango market, the premium for the holding of the commodity is low, compared with relative high costs of carry.*

2.5 Market Value of Forward Contracts

Due to equilibrium conditions, at inception, the value of any forward contract must be zero because none of the contracting parts has to pay or receive any amount of money. Once the parts have entered the contract and time elapses, the value changes and will assume positive or negative value depending on the evolution of the spot price of he underlying asset. For the long side of the contract any increase in the spot price will add a positive value to the position, while a negative value will affect the position when spot price decreases.

Assume that at time t_0 a forward contract with maturity T and delivery price $K = S_{t_0} e^{\delta(T-t_0)}$ has been created. $S_{t_0}$is the current price of the underlaying, here an asset that does not make any payment during the life of the forward contract. Let us also assume that at an intermediate time t: $(t_0 < t < T)$ one of the two counterparts (for example the one in the long position) decides to transfer to a third party his position in the contract. Due to the fact that the price of the underlying, from t_0 to t, can undergo some change, when a new contract is started at time t with the same maturity T, the equilibrium delivery price is $K_t = S_t e^{\delta(T-t)}$. Consequently the difference $K_t - K$ must be the value of the old contract at maturity time T.

Let be V_t the value of the old contract at time t when it is transferred, it must be:

$$V_t = e^{-\delta(T-t)} \left(K_t - K\right). \tag{18}$$

If is $K_t - K > 0$, the delivery price of the old contract is higher than the delivery price of the equivalent new contract having birth at time t. Consequently, in order to maintain the equilibrium, taking at t a short position in the old contract requires a payment equal to V_t, while if a long position is taken, the same amount is cashed.

Being $K_t - K < 0$, the delivery price of the old contract is lower than the delivery price of the equivalent new contract having birth at time t, and the new party entering a long position in the old contract will cash V_t, while for the short position the same amount has to be paid.

The next table summarized all the possibilities.

Enter the old contract	$K_t > K$	$K_t < K$
Long Position	Pay V_t	Cash V_t
Short Position	Cash V_t	Pay V_t

Relationship (18) can be easily proved using no arbitrage arguments.

Proof. Assume now that $V_t < e^{-\delta\,(T-t)}\,(K_t - K)$.

If so, then at time t:

A. Case $K_t > K$, $V_t > 0$:

a. Borrow the amount V_t.

b. Use the money to enter into an existing long forward contract started at t_0, with delivery price K and delivery time T .

c. Initiate a new short forward position with delivery price K_t, at no cost.

Next, at time T:

d. close out the forward contracts collecting (or paying, if negative) the amounts $S_T - K$ for the long position, and $K_t - S_T$ for the short position.

e. Pay back the loan with interest for the amount $V_t\,e^{\delta(T-t)}$.

At time t the balance is zero (the borrowed money V_t is used to enter the existing contract, and the forward has zero cost), while at time T the balance is

$$S_T - K - S_T + K_t - V_t\,e^{\delta(T-t)} = K_t - K - V_t\,e^{\delta(T-t)}.$$

This is an arbitrage because

$$K_t - K - V_t\,e^{\delta(T-t)} = \left[e^{-\delta(T-t)}\,(K_t - K) - V_t\right]e^{\delta(T-t)} > 0$$

as, by assumption, is $e^{-\delta(T-t)}\,(K_t - K) - V_t > 0$. The global strategy has no cost and a positive final value.

B. Case $K_t < K$, $V_t < 0$

a. Enter into an existing long forward contract started at t_0, with forward price K and delivery time T.

b. Cash the amount V_t and put it in a bank deposit.

c. Initiate a new short forward position with forward price K_t, at no cost.

Next, at time T:

d. close out the forward contracts collecting (or paying, if negative) the amounts $S_T - K$ for the long position, and $K_t - S_T$ for the short position.

e. Cash the bank deposit with interest for the amount $V_t\, e^{\delta(T-t)}$

At time t the balance is zero, while at time T, remembering that V_t is a negative number, the balance is:

$$S_T - K - S_T + K_t - V_t\, e^{\delta(T-t)} = K_t - K - V_t\, e^{\delta(T-t)}.$$

We have here an arbitrage being:

$$K_t - K - V_t\, e^{\delta(T-t)} = e^{\delta(T-t)} \left[e^{-\delta(T-t)} \left(K_t - K\right) - V_t \right].$$

Here again we have an arbitrage because, being $V_t < e^{-\delta\,(T-t)}\,(K_t - K)$:

$$e^{\delta(T-t)} \left[e^{-\delta(T-t)} \left(K_t - K\right) - V_t \right] > 0.$$

■

Exercise 13 *Prove that if* $V_t > e^{-\delta\,(T-t)}\,(K_t - K)$ *then an arbitrages are possible.*

Going back to relationship (18) and using the forward prices: $F_{t_0}\,(T) = K$ and $F_t\,(T) = K_t$, we have:

$$V_t = e^{-\delta(T-t)} \left[F_t\,(T) - F_{t_0}\,(T) \right]. \tag{19}$$

Substituting into to $F_{t_0}\,(T)$ and $F_t\,(T)$ their expressions in terms of the spot prices S_{t_0} and S_t (both known at time t) :

$$\begin{aligned} F_{t_0}\,(T) &= S_{t_0}\, e^{\delta\,(T-t_0)} \\ F_t\,(T) &= S_t\, e^{\delta\,(T-t)} \end{aligned}$$

the relationship (19) assumes now the form

$$\begin{aligned} V_t &= e^{-\delta\,(T-t)} \left[S_t\, e^{\delta\,(T-t)} - S_{t_0}\, e^{\delta\,(T-t_0)} \right] = \\ &= S_t - S_0\, e^{\delta\,(t-t_0)} \end{aligned} \tag{20}$$

The meaning of the last formula is clear: the value of a forward contract during its life remains the same as it was at inception ($V_0 = 0$) only if the stock price grows at the same rate as a risk-free investment. Growth above the risk-free rate δ results in a gain for the holder of a long forward position. In this case, selling the long position in the contract one receives a cash amount. In the opposite case, in order to transfer the existing contract, some money has to be paid.

3 FUTURES CONTRACTS

Even though forward contracts were stipulated since Ancient Greeks times, the first contract negotiated in an exchange was a corn forward written in 1851 at the Chicago Board of Trade (CBOT), an exchange born in 1848. It soon became clear that, depending on fluctuation in the market prices, an existing forward contract could become a valuable asset. Buying or selling preexistent contract could generate profits, once the contracts themselves were sufficiently standardized to be suitable for many operators, so to make easy their transfer from operator to operator. In order to gain high liquidity and to give substantial guarantees for the correct execution through an exchange management of the contracts, the original forward contract has been transformed, giving origin to a new and more flexible instrument: the *futures contract*. On the side of the rights arising from the contract, nothing differentiates a forward and a futures contract: in both cases one of the two parties assumes the obligation to delivery some specified asset at a specified maturity, and the other party assumes the obligation to pay the amount fixed in the contract at its inception. A futures contract is in essence a forward contract, but with some technical differences. Those differences are to be sought on the side of the regulation modalities.

Since futures contracts trade on an exchange, all futures contracts are standardized in terms of their size, grade, and time and place of delivery. Other features of futures contracts that are standardized include their trading hours, minimum price fluctuations and, for contracts that have them, maximum daily price limits. All contract terms, except price, are defined by the exchange on which they trade. This standardization can have an impact on hedging, as delivery dates and terms are not flexible. Entering the details of standardization, the elements of the contract that are ruled by the exchange are the following.

A. The precise nature of the asset to be traded, that is, its intrinsic quality if it is a commodity, the maturity and the issuer if is a bond, etc.

B. The size of the contract. For many agricultural commodities, for example, the size is 5000 bushels in almost all the exchanges in the world, and no other quantity is allowed. For stocks the size is usually 100 units of stock.

C. Delivery arrangements: where delivery will take place, which of the parts has to assume some complementary obligations connected with delivery.

D. Maturity of the contract. The parts are not free to choose the maturity. It has to be one of prefixed months (September, December, March, May and July, for many commodities).

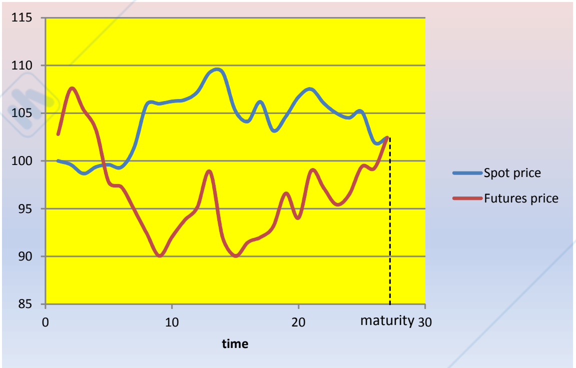
For example at the *Chicago Mercantile Exchange* (CME), the futures contract for corn undergoes the following standard conditions:

Corn Futures		
Contract Size	5,000 bushels (~ 127 Metric Tons)	
Deliverable Grade	#2 Yellow at contract Price, #1 Yellow at a 1.5 cent/bushel premium #3 Yellow at a 1.5 cent/bushel discount	
Pricing Unit	Cents per bushel	
Tick Size (minimum fluctuation)	1/4 of one cent per bushel ($12.50 per contract)	
Contract Months/Symbols	March (H), May (K), July (N), September (U) & December (Z)	
Trading Hours	CME Globex (Electronic Platform)	Sunday – Friday, 7:00 p.m. – 7:45 a.m. CT and Monday – Friday, 8:30 a.m. – 1:15 p.m. CT
	Open Outcry (Trading Floor)	Monday – Friday, 8:30 a.m. – 1:15 p.m. CT
Daily Price Limit	$0.40 per bushel expandable to $0.60 when the market closes at limit bid or limit offer. There shall be no price limits on the current month contract on or after the second business day preceding the first day of the delivery month.	

An important feature of futures markets is the convergence, at maturity, of the futures price to the spot price. Or in a formula:

$$F_T(T) = S_T. \tag{21}$$

Here $F_T(T)$ is the futures price just at (or very short time before) maturity of the contract. As a matter of facts, reasoning in terms of equilibrium of the market, one can easily convince himself that if in the proximity of the maturity of the contract the spot price of the asset is lower than the futures price, a trader can set up an arbitrage by buying the asset and simultaneously taking a short position in the futures contract. Few minutes later, at maturity, he delivers the asset and cashes the higher futures price. Conversely, if the futures price is lower than the spot price the strategy is reversed: sell short the asset and take a long position in the futures market. Next graph illustrates two possible hypothetical price evolutions, one for the spot price and the other for the futures price and the behavior toward maturity of the contract.



3.1 Futures Exchanges and Clearing Houses

Each futures exchange has an associated organization that takes care of financial settlement, and helps to ensure that markets operate efficiently. This organization, which is called a *clearing house*, can be set up either as a separate corporation or as a department of the exchange. A clearing house guarantees the financial obligations of every contract that it clears. Only clearinghouse members can submit trades to the clearinghouse, and while every member of a clearinghouse must also be a member of the related exchange, not all exchange members are members of the clearinghouse. Clearinghouse membership involves financial requirements and responsibilities over and above those of exchange membership, including the maintenance of a guaranty deposit at the clearinghouse. This deposit serves as a reserve fund that can be used, if necessary, to meet the financial obligations of a defaulting clearing member. It does this by acting as the buyer for every seller, and the seller for every buyer (*principle of substitution*). A participant who has bought or sold a futures contract has an obligation not to the party on the other side of the transaction, but to the clearing house, just as the clearing house has an obligation to the participant. The existence of the clearing house means that market participants need not be concerned about the honesty or reliability of other trading parties. The integrity of the clearing house is the only issue. As clearing houses have a good record in honouring their obligations, the counterpart risk in futures trading is considered to be negligible. This is one of the principal advantages of futures trading as opposed to OTC trading.

Clearing houses are able to guarantee the financial integrity of futures contracts through a layered system of financial protection. *Margin deposits* (see next paragraph for details) provide the first layer of protection. Parties to a futures trade must deposit an initial or original margin when the contract is first entered. A primary activity of a clearing house is to match trades submitted by clearing member firms. Throughout each trading day, clearing members report the details of executed trades, whether they are on behalf of their clients or are on their own accounts, to the clearing house. Once the clearing house verifies the accuracy of all reported transactions, ensures that there is a buy for every sell, and receives original margin from clearing members, it takes over the financial obligations inherent in the futures contract.

The clearing house does not need to know the actual identities of the parties to the transactions. It only needs to know the net positions of the clearing members. The clients are financially responsible to the member firms, while the member firms are financially responsible to the clearing houses. Once a transaction is consummated and confirmed, the clearing house substitutes itself as the buyer for the seller and as the seller for the buyer. This substitution enables the individual trader to liquidate a position without having to wait until the other party to the original contract decides to liquidate. The trader has in effect bought the contract from or sold it to the clearing house.

It is also the job of the clearing house to ensure that all deliveries are carried out smoothly. It is important to keep in mind that the principle of substitution does not apply to deliveries. The clearing house merely matches up the buyers and sellers who then can make arrangements for delivery either outside the clearing house or within the clearing house (in which case the clearing house merely acts as custodian). Once the long party accepts the *delivery notice*, the clearing house's obligation is honoured. The clearing house does not take on the obligations of delivery if one side does not satisfy the conditions of delivery. The clearing members must settle any disputes between themselves in accordance with regulatory by-laws. Neither member has any recourse to the clearing house.

Futures exchanges provide a forum for market participants to buy and sell futures contracts. Regardless of the type of futures exchange (*open-cry* or *electronic*), the price buyers and sellers agree upon is arrived at through an *auction process*. The term *open outcry* auction process is used to describe trading on a physical exchange. In this type of auction system, bids and offers are communicated between floor traders in a trading ring or pit through both verbal and hand communications. Once a trade is concluded, market reporters, who operate from strategic locations around the floor of the exchange, record and input the information into a communications system. Once inputted, the price information can be disseminated almost instantaneously around the world. On an *electronic exchange*, a specific futures contract's best bid and offer prices are displayed on computer terminals located in member firms' offices. The terminals also allow member firms' traders to enter orders for any contract trading on the system. As orders are entered, the exchanges' trading systems will sort, display and, when the rules of auction trading say so, match them (i.e. create a trade). Only *registered members* of an exchange have privileges to trade on that exchange. On a physical exchange, there are two types of floor traders. Those who primarily trade on their own account are referred to as *locals*, while those who fill orders from customers are referred to as *floor brokers*. Locals either own an exchange membership, known as a *seat*, or lease one from an owner.

3.2 Orders in Futures Markets, Typologies of Traders, Volumes and Open Interest

As far as it concerns the modalities to place orders in the futures markets, many different possibilities are at hand, depending on the specific exchange. Consequently, when placing an order it is essential to understand the correct terminology in use, the jargon of the market, so to be sure that order itself is executed properly. A list of some of the most common order types, valid either for buying and selling contracts is the following.

- **Market order**. This order is used to buy or sell immediately at the *market price*. There is no guarantee what that price will be, so the customer relies on the broker and trader for timely and effective execution.

- **Good until cancelled** (GTC). This is an order to execute a trade that stays live until the customer cancels the trade.

- **Market on open** ($\mathcal{MOO}$). This is a 'market' order that will be executed when the market opens, at a price within the opening range of prices. The *opening* is the period at the beginning of the trading session officially designated by the exchange during which all transactions are considered made *at the opening*. Opening price ranges can be quite wide, so this type of trade is to be used with discretion.

- **Market on close** (MOC). This is a 'market' order that will be executed when the market closes. The price of the trade will be within the closing range of the day, which may be quite large and vary substantially from the *settlement price* (see next section).

- **Limit order.** This order is placed when a market participant is looking to buy or sell at a specific price or better. This kind of order is placed by an operator that intends to enter the futures contract at a price no higher than the limit if long, or at a price no lower than the limit if short. When using this type of order, the operator knows that the order may still not be filled if market prices stay out of the limit.

- **Stop order.** This is an order to buy or sell when the market reaches a certain price. Once that price has been reached, the order becomes an effective market order. A buy stop is placed above the market and a sell stop is placed below the market. Stop orders are commonly used to protect profits or to attempt to limit losses.

- **Fill or kill** (*FOK*). This order is a limit order that is sent to be executed immediately and if the order is unable to be filled right away, it is cancelled.

- **Spread order**. A simple spread order involves two positions, one bought and one sold. The trades generally involve the same market with different months (calendar spread) or closely related markets, such as interest rates of different maturities.

A specific terminology is used in placing orders on the futures markets, as well on options markets. *Buy to open* or *sell to open* orders refers to the opening of a new position in the futures (or options) market, a long position in the first case, a short position in the second. *Buy to close* or *sell to close* orders have the aim to kill an existing position, closing a short position in the first case, a long position in the second.

The existence of orders to close highlights the fact that the process of leaving the market, once one has entered a futures contract, is an easy task: all what is to do is to enter a new contract in the opposite position respect to the original, on the same asset and with the same delivery date. This process, called *closing out a position*, is very important, as almost all the futures contracts entered do not proceed to maturity. This happens mostly for contracts that have commodity as underlying, because operators probability prefer not to receive huge quantity of goods, or living cattle... In many exchanges the physical delivery of the underlying is not a possible option for the seller and at maturity, for contracts still alive, only financial settlement is allowed.

Like forward contracts, futures contracts are used to lock in a stock price, an interest rate, an exchange rate or a commodity price. But, as already seen above, futures contracts are organized in such a way that the counterpart risk of default is always completely eliminated because the *clearing house* steps in between a buyer and a seller, each time a deal is concluded. Consequently every trader in the futures markets has obligations only to the clearing house, and has strong expectations that the clearing house will maintain its side of the bargain as well. In the futures market, as in any other market ruled by an exchange, all the contracts are *anonymous* in the sense that the buyer does not know the seller and the seller does not know the counterpart. And neither do they need to know each other. Organization, administration and the running of the contracts are all handled by the exchange, and for all those activities transaction cost are to be paid by the two parties. The credibility of the system is maintained through the requirements of *margin* and *daily settlements*, as it will clarified soon.

Futures contracts are used for *speculation*, *hedging* (long or short), *arbitrage* or *investment*. Each market participant in the futures market has his own specific function. We can group the market participants in the following classes

- *Hedgers.* The primary function of a futures market is to allow participants who wish to reduce or eliminate risk to do so by shifting the risk to those who want to assume it in return for the possibility of earning a profit. A market participant may need to either reduce the risk of holding a particular asset for future sale or reduce the risk involved in anticipating the purchase of a particular asset. Hedgers is the name given to market participants that use futures contracts with the aim to reduce price or quantity risk.

  A *short hedge* is executed by someone who owns an asset in the spot market that will be sold at some point in the future. In order to protect against a decline in price between the present and the time when the asset will be ready for sale, the hedger can take a short position in a futures contract on the same underlying asset which matures approximately at the time of the anticipated sale. By taking this action in the futures market, the hedger will be able to receive an amount equal to the price agreed in the contract, despite the fact that the spot price of the asset at the time of the sale might be considerably different.

  A *long hedge* is executed by someone who has to buy some underlying asset at some point in the future. In order to protect against rising prices between the present and the time when the asset is needed, the hedger can take a long position in a futures contract on the underlying asset which matures approximately at the time of the anticipated purchase of the asset. By taking this action in the futures market the hedger has fixed the purchase price.

Example 14 *To give a better idea of how hedging works think of an operator who has purchased some commodity (soybeans, as an example), but not yet sold. In market terminology the operator has a* long cash market position*. It is May and the operator plans to sell his stock of soybeans in October. The current cash market price for soybeans for delivery in October is* \$12.00 *per bushel. If the price goes up between now to October when he plans to sell, he will gain. On the other hand if the price goes down during that time, he will have a loss. To protect himself against a possible price decline during the coming months, he can hedge by selling corresponding number of bushel in the futures market now, and buying them back later when it is time to sell the soybeans in the spot market. If the price declines in the time interval, any loss incurred in the spot market will be offset by a gain from the hedge in the futures market. This particular type of hedge is called* short hedge *because of the initial short futures position.*

Due to the standardization of futures contracts, several problems show up in connection with the possibility of hedging positions in the spot markets. This is the case, for example, when in the market are not traded futures which underlying is just the one to be hedged. With such problems we shall cope in a next section when a general theory of hedging will be presented.

- *Speculators.* Speculators are those market participants who, having some expectations concerning the prices evolution, in the pursuit of profit, are willing to assume the risk that hedgers are seeking to shift. There are several different types of speculator who operate in the futures market. They are distinguished from each other by a number of factors, including the length of time they plan to hold a particular futures position. We can group them in *locals*, *day traders*, *position traders* and *spreaders*.

Locals are also referred to as *scalpers*. This type of speculator operates having the shortest time horizon of all. The local attempts to profit from small price changes that take place in very short periods of time. The time horizon for a local can often be measured in minutes, rather than hours or days. Since the local is only looking to profit from very small price changes, the amount that is typically at risk on any given trade is small.

Day traders, as the name suggests, are speculators whose time horizon is a single day. Positions taken during a trading day are liquidated by the end of that day. Positions are not carried overnight. They are looking to profit from larger price moves than locals, and as a result they are willing to risk more. However, as is evidenced by their desire not to hold any positions overnight, they are not willing to tolerate a lot of risk.

Position traders have a time horizon that can be measured in terms of weeks or even months. Position traders attempt to profit from expectations on longer-term price trends. Timing is not as important for a position trader as it is for a local or day trader. The position trader is typically well financed and is willing and able to withstand adverse short-term price changes to a larger extent than locals or day traders, in order to maintain a position consistent with their long-term view of the market.

Spreaders are speculators operating through the purchase of one futures contract against the sale of another which is related in some fashion. Spread traders attempt to identify market situations where the price relationship between two related assets has deviated from its theoretical norm. When such a situation is identified, the trader will take a spread position designed to profit from a move back towards a level or a range that is more in line with historical performance.

- *Investors*. They use futures contracts as diversification device that can join other financial instruments already present in their portfolios, such as stocks or bonds.

Futures market also carry out a very useful function in spreading out price information. The recording and the immediate publication of every detail of the concluded contracts gives rise to an informational cascade that market participants can use with profit in their strategies. Prices are the first and most important piece of information conveyed by futures market. Remembering that futures prices are, in their intrinsic nature, forward prices, the knowledge of a complete set of prices spread along all the maturities gives to the market operators a sort of snapshot of the market sentiment concerning the future evolution of prices. But also *volume* and *open interest* provide a relevant piece of information, leading indication of a possible impending change of trend in the price movements. Volume represents the total amount of trading activity or contracts that have changed hands in a given market for a single trading day. The greater the amount of trading during a market session the higher will be the trading volume. According to the traders opinion, the volume represents a measure of intensity or pressure behind a price trend. In the operators experience, in general, volume precedes price and hence the greater the volume the more one can expect the existing trend (growing, or decreasing, or sideways prices trend) to continue rather than reverse.

Open interest is another recorded data that one can reed (besides prices and volume) in any exchange quotation list. It also provides useful information that should be considered when entering a futures position. First, let's look at exactly what open interest represents. Unlike stock trading, in which there is a fixed number of shares to be traded, futures (and options) trading can involve the creation of a new contract when a trade is placed. Open interest is nothing else than the total number of futures contracts that are currently open, in other words, contracts that have been traded but not yet liquidated by either an offsetting trade or an exercise. What must be clear in interpreting the open interest number, is that when a futures contract is traded with one party opening and one party closing, the open interest remains unchanged. If both parties in the transaction are closing existing positions then the open interest decreases of one unit. If both parties are opening positions then the open interest goes up accordingly. Where volume measures the pressure or intensity behind a price trend, open interest measures the flow of money into the futures market.

By monitoring the changes in the open interest figures at the end of each trading day, some conclusions about the day's activity can be drawn. Increasing open interest means that new money is flowing into the marketplace. The result will be that the present trend will continue. Declining open interest means that the market is liquidating positions started in the past days, and implies that the prevailing price trend is coming to an end. A knowledge of open interest can prove useful toward the end of major market moves. Practitioners's experiences has led to recognize some relationship between the prevailing price trend, volume, and open interest, as summarized by the following table.

Prices Trend	Volume	Open Interest	Interpretation
Rising	Rising	Rising	Trend Confirmed - Market Strengthening
Rising	Falling	Falling	Trend Change- Market Weakening
Falling	Rising	Rising	Trend Confirmed- Market Strengthening
Falling	Falling	Falling	Trend Change- Market Weakening

3.3 The Types of Futures Contracts

There are many categories of futures contracts. There are futures contracts that trade based on the underlying markets of commodities, equity indices, foreign exchange, interest rate, metals,

real estate and even weather.

- **Commodity Futures** Commodity futures represent some of the most important futures contracts for hedging. The reason for this is that they help farmers, manufacturers, and other businesses reduce unwanted risks. There are various types of commodity futures including agricultural futures and energy futures. Some of the many agricultural commodities include corn, soybeans, wheat, hog, cattle, milk, rice, lumber, butter, cotton, cheese, sugar, and coffee.

The energy commodities include natural gas, crude oil, ethanol, coal, and propane. The trading of commodities is a detailed and complicated task.

- **Equity Index Futures** Equity futures are used for all sorts of reasons for investors. They are used to create leveraged portfolios, to gain cheap exposure to equities, and for hedging purposes. The hedging can take a variety of forms. For example, if an investor has a long exposure to equities, but worries that the stock market will decline, he can short or sell futures contracts to reduce his exposure. There are equity futures on a host of major stock market indices both in the United States and around the world. These include futures on the S&P 500 index, the Nasdaq Index, the Dow Jones Industrial Average index, the Russell 2000 index, and various sectors of the U.S. economy.
- **Foreign Exchange Futures** One of the most actively traded financial instruments in the world is foreign exchange. Foreign exchange is central to international business. Companies tap into the foreign

exchange markets to hedge all kinds of business risk associated with cross-border transactions. Businesses aren't the only ones that use currency futures to hedge. Global money managers use futures frequently to hedge their investments in other currencies so as to protect their investments. Currency trading is more than $4 trillion per day. There exist futures on many currencies around the world, including on the Yen-U.S. Dollar (referred to as the yen-dollar), the Pound-U.S. Dollar (referred to by traders as the Cable), the Euro-U.S. Dollar, the Australian dollar-US Dollar (referred to as the aussie-dollar), etc.

- **Interest Rates Futures** There are a whole host of futures that are based on the movement of interest rates in the economy. They are based on all types of underlying interest rates, including interest rates between different currencies in the interbank market, government interest rates.
- **Metal Futures** The world of metal futures are divided into three categories. They are base metals, industrial metals, and precious metals. Some of the many base metal futures include aluminum, copper, iron ore, lead, nickel, steel, tin, and zinc. Some of the industrial metal futures include lumber, polyethylene, rubber, and glass. The precious metals include gold, palladium, and silver.
- **Real Estate Futures** The real estate market is important to almost everyone. It was the principal cause behind the financial crisis of 2008. Due to its importance in the economy, futures contracts were created that are based on an index of home prices in major cities in the United States and other parts of the world.
- **Weather Futures** Although it seems strange, there are also futures contracts that trade based on the weather in various cities of the world. That is, a hedger or a trader can buy or sell a contract based on future weather conditions. Weather derivatives allow businesses to smooth out fluctuations in the demand for their products as weather conditions change.

For more pieces of information regarding Forward Contracts and Futures Contracts read the corresponding chapters in J. Hull: *Options, Futures and Other Derivatives.*

Details on futures contracts (quotations, volumes, open interests) can be found in the following website:

https://www.borsaitaliana.it/borsa/derivati/idem-stock-futures/lista.html?underlyingId=ENEL&

https://www.cmegroup.com/trading/why-futures/welcome-to-nymex-wti-light-sweet-crude-oil-futures.html

https://www.cmegroup.com/trading/weather/temperature/us-monthly-weather-cooling_contract_specifications.html

https://derivatives.euronext.com/en/products/stock-futures/AD6-DAMS/contract-specification

https://derivatives.euronext.com/en/products/commodities-futures/EMA-DPAR

3.4 The Margin Mechanism

To guarantee the parties intervening in the contract that the counterpart will fulfill his obligation, a complex mechanism based on a margin has been introduced in all the futures exchanges.

On the day a futures contract is entered, both parties have to open a margin. The margin is a deposit in the form of cash, government securities, stock in the clearing corporation or letters of credit issued by an approved bank. The main purpose of the margin is to provide a safeguard to ensure that traders will honor their obligations. It is usually set to the maximum loss a trader can experience in a normal trading day. Usually the amount of the margin is calculated as a percentage of the value of the contract (futures price multiplied by the quantity in the contract multiplied by the number of contracts). The percentage level depends on the exchange, the typology of contract and the objective of the operator. The amount to put in the exchange account is called *initial margin*. Aim of the margin, also known as *Performance Bonds*, is to ensure that a trader can cover potential losses with his or her trading positions. The account may be rewarded by interest or not, depending on the exchange rules.

Example 15 *For futures contract on soybean oil traded at CME, the price August 7th 2015 for maturity September 2015 was* 42.78 *US dollar cents for pound. The size of the contract is* 60000 *pounds and the value of a single contract is US dollars* $\frac{42.78(60000)}{100} = 25668$*. Assume that for a non-speculative trader (an* hedger*) the percentage required is* 4.8%*, then the initial margin for both traders (either the one who takes a long position, or the one taking a short position) is* $25668\,(0.048) = 1232.10$ *US dollars. This amount is the only one a trader has to invest in his or her contract at inception. As* *the value of the contract is much higher with respect to the required initial money sum, the futures contract incorporates a* leverage *opportunity.*

During the course of a trading day, the futures price for a specified asset and maturity will change according to the law of offer and demand. Let us assume that at the end of the day the price has risen. Anyone holding a contract to buy (an operator with a long position in the futures contract) will make a gain, as in his or her contract a lower price is required for the purchase at maturity of the underlying asset. Vice versa any trader with a short position (obligation to sell at maturity) will make a loss. If the price falls, the results are reversed. At the end of each trading day the exchange will calculate the gains and the losses associated to every contracts on the basis of a final official price called the *settlement price*. The way the settlement price is calculated vary according the different rules of the exchanges: usually it is some average of the prices of the last contracts stipulated in the day. The exchange closes all the existing contracts, and in the margin accounts of the operators holding a futures contract that has made a loss, a sum of money equal to the loss is deducted, while a sum equal to the day's gain is credited in the margin accounts of operators that have got a gain. At this point new contracts are opened, any having as futures price the *settlement price* of the day and the same original maturity. This process is called *marking to market*. Due to the marking to market mechanism, changes in the value of futures contracts are settled daily from available resources (the margin account), and hence the risk of default by either party is consistently reduced. In order to make more effective the reduction of default risk a minimum level of the margin account is established for every typology of contract, called *maintenance margin*. Maintenance margin is something less than initial margin (often $70\% - 75\%$ of the initial value of the contract). If funds in the margin account fall below the maintenance margin, the holder receives a *margin call* from the exchange: the operator has to top up at the initial margin level his or her account in a very short time. In some exchanges the margin call requires that the new margin level is set at a percentage (the same adopted for the initial margin) of the current value of the contract, instead of the initial value of the contract. The additional funds to be deposited in the margin account are called *variation margin*. Should the holder not provide the variation margin in time, the exchange does not open the new contract and it is possible that the balance in the margin account is retained by the exchange itself. A theoretical example of a trading account's balance through time is shown below.

Example 16 *Suppose a trader established a position to buy (go long) a*

September E-mini S&P 500 *futures contract on June 13, when the contract was trading at* 1050.00 *points. CME, at the time, required an initial margin of* \$ 4000 *to trade that contract, with a maintenance margin of* \$ 3200*. If the price variations of the contract bring the account balance under* \$3200*, the trader will have to deposit additional funds to bring the account back up to* \$ 4000*. The total value of the contract is determined by multiplying* \$ 50 *times the S&P 500 Stock Index futures index. On June 13, therefore, the contract was valued at* $1050\,(\$50) = \$52\,500$*. The account balance will vary at the end of each day based on the closing value of the index multiplied by* \$ 50*. For example, if the index goes down by* 10 *points the next day, the account goes down by* $10\,(\$50)$ *or* \$ 500*. If it goes up by six points, the account goes up in value by* $6\,(\$50)$ *or* \$ 300*. In the next three days the settlement level of the index is* 1052, 1035, 1028*. The next table illustrates the evolution of the margin account under the hypothesis that no interest is paid is recognized to the holder:*

DAY	*INDEX*	*MARK TO MARKET*	*MARGIN BALANCE*	*MARGIN CALL*
13/6	1050	0	4000	0
14/6	1052	$+2\,(50) = +100$	$4000 + 100 = 4100$	0
15/6	1035	$-17\,(50) = -850$	$4100 - 850 = 3250$	0
16/6	1027	$-8\,(50) = -400$	$3250 - 400 + 1150 = 4000$	1150
17/6	1017	$-10\,(50) = -500$	$4000 - 500 = 3500$	0

Should have the exchange established that the initial margin would be 10% *of the initial value of the contract, with maintenance margin at* 80% *and top up at initial margin, the result should have been:*

DAY	*INDEX*	*MARK TO MARKET*	*MARGIN BALANCE*	*MARGIN CALL*
13/6	1050	0	$1050\,(50)\,(0.10) = 5250$	0
14/6	1052	$+2\,(50) = +100$	$5250 + 100 = 5350$	0
15/6	1035	$-17\,(50) = -850$	$5350 - 850 = 4500$	0
16/6	1027	$-8\,(50) = -400$	$4500 - 400 = 4100$	0
17/6	1017	$-10\,(50) = -500$	$4100 - 500 + 1650 = 5250$	1650

In this case the maintenance margin is $5250\,(0.7) = 3675$.

What above presented refers to operators that are members of the futures exchange. If a private operator wants to enter a futures contract he, or she, has to contact a broker who will start a contract on behalf of the private operator. The broker will apply margins that can be consistently higher than what the exchange requires.

Example 17 *Going back to the previous example, suppose that the trader closes out the position on June 17th. He, only has to enter that day a futures contract, with the same underlying and maturity, in short position. Long position (the one at inception of the contract) and short position (the new contract) offset each other. In this case the final payoff of the trade is a loss of* $(1017 - 1050)(50) = -1650$*. This result can be computed on the basis of the margin account. In the first case (fixed margin) we have an initial outlay of* 4000 *another outlay of* 1150 *for the margin call in front of a balance of* 3500 *to cash from the account:* $(3500 - 4000 - 1150) = -1650$*. In the second case the final payoff is the same: a loss of* 1650*, and from the margin account:* $5250 - 5250 - 1650 = -1650$*.*