Rhythmic Structures and Models

Augusto Sarti

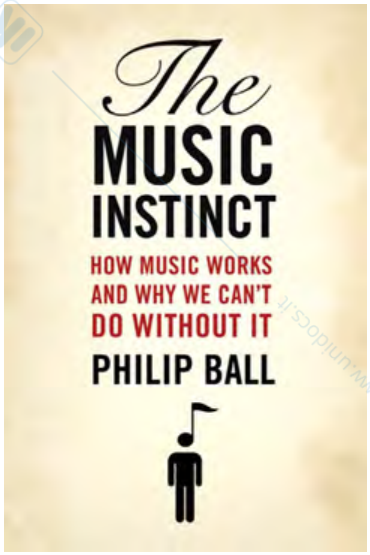
Computer Music Representations and Models

M.Sci. Music and Acoustic Engineering

References and readings

General Music Overview

- Philip Ball “The Music Instinct - How music works and why we can’t do without it”. Oxford Un. Press 2010



Music Notation and Basics

Both units of “Notes and Rhythm” and “Reading Music” are excellent primers for those who don’t have a solid music background

- https://www.khanacademy.org/humanities/music/music-basics2

Khan Academy

POLITECNICO
MILANO 1863

M.S. Music and Acoustic Engineering

Origins

- Several theories
  - human rhythm recalls the regularity with which we walk and the heartbeat (Howard Goodall, composer)
  - beat-based rhythmic processing has ancient evolutionary roots (Aniruddh Patel)
  - the sense of rhythm was developed in the early stages of hominid evolution by the forces of natural selection (Joseph Jordania, ethnomusicologist)
    - to induce battle trance, promote the development of a defense system of early hominids
- Evolved in parallel in all cultures
  - In Sub-Saharan Africa it evolved in complex forms such as multi-layered polyrhythm and simultaneous rhythms in more than one time signature (polymeter)

- cross-rhythms of Sub-Saharan Africa https://youtu.be/ZYhFyF8dvU4
- interlocking kotekan rhythms of the gamelan https://youtu.be/Kfe3DudhY4w

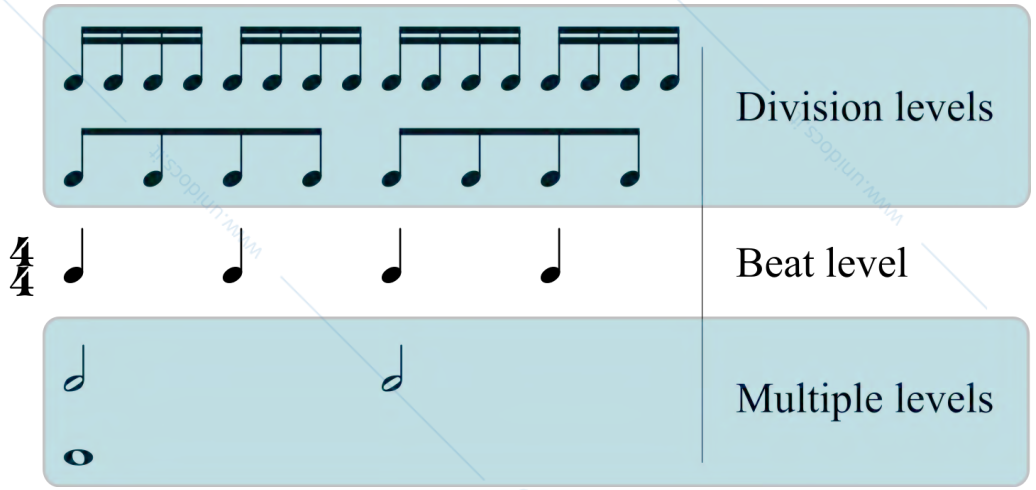
Pulse, beat and measure

Most music, dance and oral poetry establishes and maintains an underlying "metric level", a basic unit of time that may be audible or implied, the pulse or tactus of the mensural level, or beat level, sometimes simply called the beat

- The Beat consists of a (repeating) series of identical yet distinct periodic short-duration stimuli perceived as points in time
- The "beat" pulse is not necessarily the fastest or the slowest component of the rhythm but the one that is perceived as fundamental: it has a tempo to which listeners entrain as they tap their foot or dance to a piece of music
- It is currently most often designated as a crotchet or quarter note in western notation
- Faster levels are division levels, and slower levels are multiple levels
- "Rhythms of recurrence" often arise from the interaction of two levels of motion, the faster providing the pulse and the slower organizing the beats into repetitive groups
- Once a metric hierarchy has been established, we, as listeners, will maintain that organization as long as minimal evidence is present

Pulse, beat and measure

- Regularly repeating events of identical yet distinct periodic short-duration stimuli perceived as points in time occurring at the mensural level
- Pulse is typically what listeners entrain to as they tap their foot or dance along with a piece of music, and is also colloquially termed the 'beat,' or more technically the 'tactus'



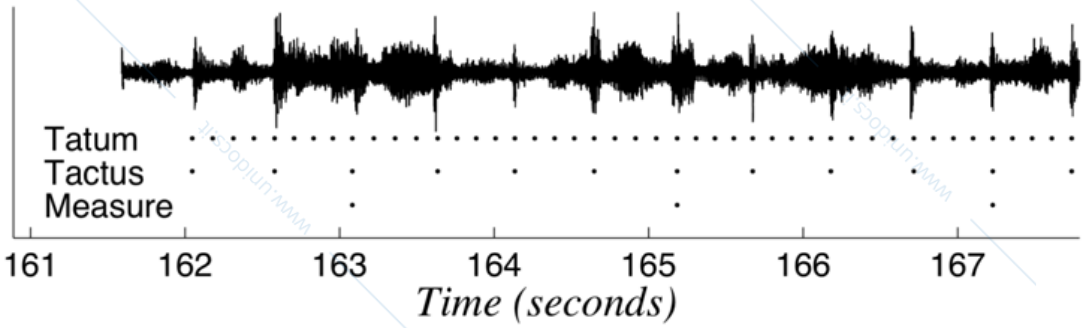
- The **beat** is the basic unit of time, often defined as the rhythm listeners would tap their foot to when listening to a piece of music, or the numbers a musician counts while performing
- In popular use, *beat* can refer to a variety of related concepts including: pulse, tempo, meter, specific rhythms and groove

Pulse, beat and measure

- Rhythm in music is characterized by a repeating sequence of **stressed and unstressed beats** (strong vs. weak; upbeat vs. downbeat) and divided into bars organized by time signature and tempo indications
  - The alternation of the strong and weak beat is fundamental to the languages of poetry, dance and music
  - Syncopation occurs when accenting upbeat and de-emphasizing downbeat as established from the melody or from a preceding rhythm
- Normally, even the most complex of meters may be broken down into a chain of duple and triple pulses either by addition or division

Musical Meter

A hierarchical structure consisting of pulse sensations at different levels (time scales)



- Three metrical levels are particularly important
  - **Tactus** or **beat** (foot-tapping rate)
  - **Tatum** (temporal quantum) – its period corresponds to the shortest durational values in music that are still more than incidentally encountered. All durational values are integer multiples of the tatum period and the onsets of musical events occur approximately at a tatum beats
  - **Measure** - typically related to the harmonic change rate or to the length of a rhythmic pattern
- **Tempo** - rate of the tactus pulse
  - Usually measured in 'beats per minute' (bpm)
  - In order for a meter to make sense, the tempo must be slowly varying

Metric structure

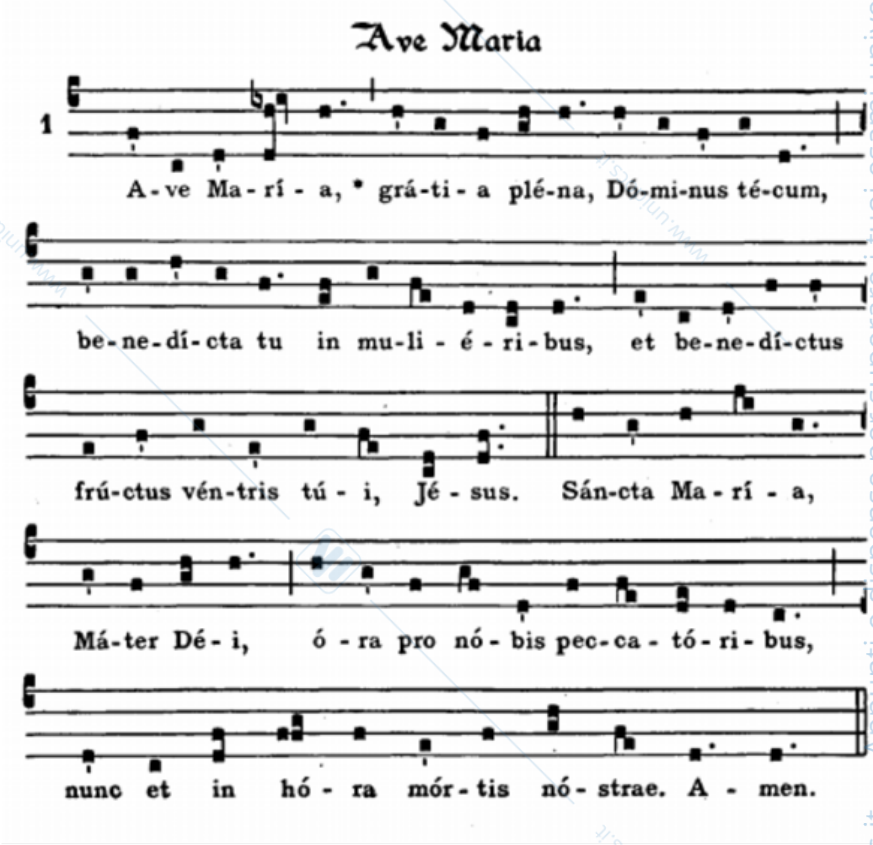
- The study of rhythm, stress, and pitch in speech is called **prosody**
  - it is a topic in linguistics and poetics, where it means the number of lines in a verse, the number of syllables in each line and the arrangement of those syllables as long or short, accented or unaccented
  - Music inherited the term "meter or metre" from the terminology of poetry
- The metric structure of music includes meter, tempo and all other rhythmic aspects that produce temporal regularity against which the foreground details or durational patterns of the music are projected
- Dance music has instantly recognizable patterns of beats built upon a characteristic tempo and measure
  - The Imperial Society of Teachers of Dancing defines the tango, for example, as to be danced in 2/4 time at approximately 66 beats per minute
  - The basic slow step forwards or backwards, lasting for one beat, is called a "slow", so that a full "right–left" step is equal to one 2/4 measure

Metric structure

- Metrical rhythm (divisive rhythm) calculates each time value as a multiple or fraction of the beat, with normal accents that re-occur regularly providing systematical grouping (measures)
- Measured rhythm (additive rhythm) also calculates each time value as a multiple or fraction of a specified time unit but the accents do not recur regularly within the cycle
- Free rhythm is where there is neither
  - Christian chant, which has a basic pulse but a freer rhythm

Mensural notation

- A musical notation system used for European vocal polyphonic music from the later part of the 13th century until about 1600
  - The term "mensural" refers to the ability to describe precisely measured rhythmic durations in terms of numerical proportions between note values
  - Its modern name is inspired by the terminology of medieval theorists, who used terms like *musica mensurata* ("measured music") or *cantus mensurabilis* ("measurable song") to refer to the rhythmically defined polyphonic music of their age, as opposed to *musica plana* or *musica choralis*, i.e., Gregorian plainchant
  - Mensural notation was employed principally for compositions in the tradition of vocal polyphony, whereas plainchant retained its own, older system of neume notation throughout the period

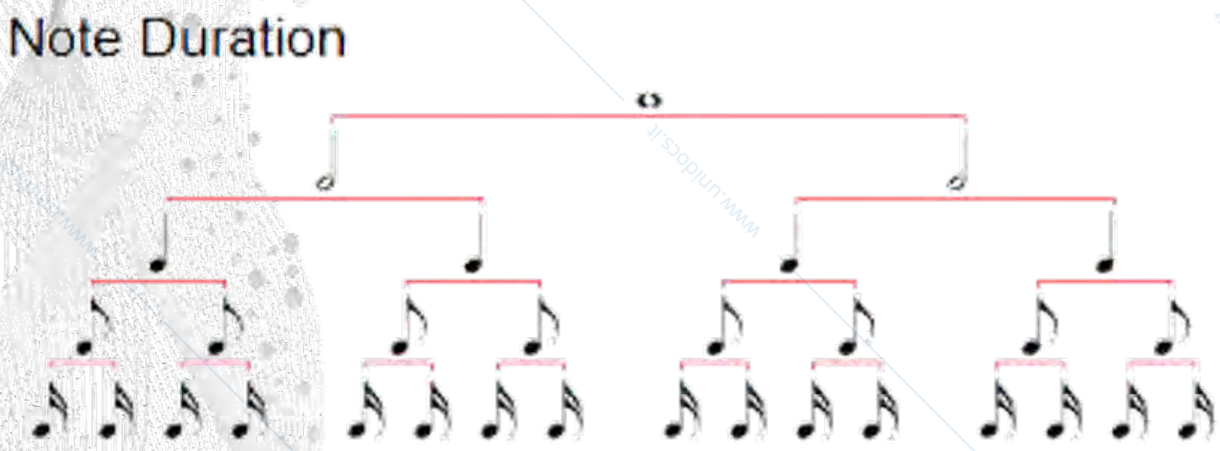


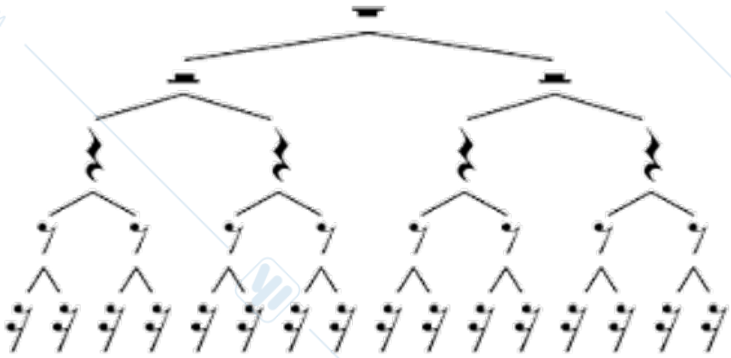
https://www.dropbox.com/s/70ldtjm2t5qsdu7/AVE%20MARIA%20-%20Gregoriano.mp3?dl=0

Modern notation

For a primer on **modern rhythmic notation** please refer to Khan Academy:

https://www.khanacademy.org/humanities/music/music-basics2





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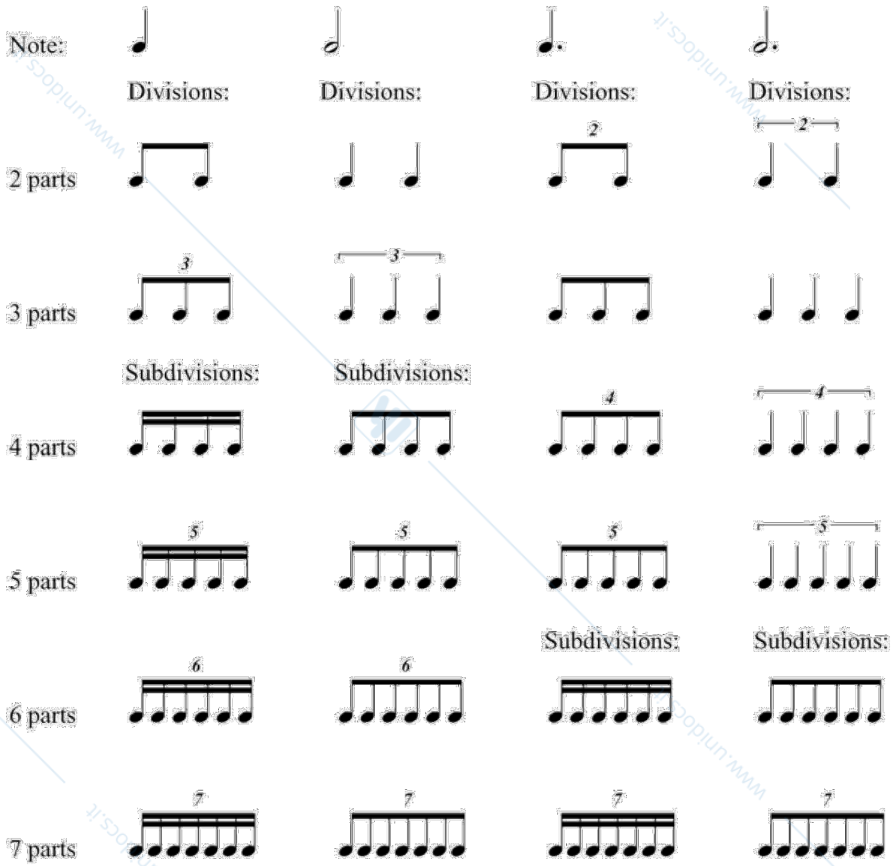
Name	Note	Rest	Equivalents
Breve (Double Whole Note)	𝅜 or 𝅜		Two Whole Notes
Whole Note	𝅝	𝄻	Two Half Notes
Half Note	𝅗𝅥	𝄼	Two Quarter Notes
Quarter Note	♩	𝄽	Two Eighth Notes
Eighth Note	♪	𝄾	Two Sixteenth Notes
Sixteenth Note	𝅘𝅥𝅯	𝄿	Two Thirty-second Notes
Thirty-second Note	𝅘𝅥𝅰	𝅀	Two Sixty-fourth Notes
Sixty-fourth Note	𝅘𝅥𝅱	𝅁	Two One Hundred Twenty-eighth Notes



Rhythm and meter in poetry

Rhythm and meter in poetry resembles that in music as it relies on the alternation of stressed and unstressed syllables

- A fundamental aspect in writing songs consists of reconciliating rhythmic structures of lyrics and music
- Often the rhythmic structure of lyrics suggests the rhythmic structure of a song

Rhythm and meter in poetry

Foot: basic unit of measurement of accentual-syllabic meter

- It usually contains one stressed syllable and at least one unstressed syllable
- The standard types of **feet** in **poetry** are the iamb, trochee, dactyl, anapest, spondee, and pyrrhic (two unstressed syllables)

Five basic rhythm of varying stressed (/) and unstressed (x) syllables

- Meters with two-syllable feet are
  - IAMBIC (x /): That **time** of **year** thou **mayst** in **me** be**hold** (§)
  - TROCHAIC (/ x): **Tell** me **not** in **mourn**ful **num**bers (°)
  - SPONDAIC (/ /): **Break, break**, break/ On thy **cold gray** stones, **O Sea**! (*)
- Meters with three-syllable feet are
  - ANAPESTIC (x x /): And the **sound** of a **voice** that is **still** (*)
  - DACTYLIC (/ x x): **This** is the **for**est pri**me**val, the **mur"**muring **pines** and the **hem**lock (°)

(§) Sonnet 73: “That time of year thou mayst in me behold”, by W. Shakespeare

(*) “Break, break, break” by Alfred, Lord Tennyson (1842)

(°) “Evangeline: A Tale of Acadie” by Henry Wadsworth Longfellow - 1807-1882

Rhythm and meter in poetry

Each line of a poem contains a certain number of feet of iambs, trochees, spondees, dactyls or anapests

- A line of one foot is a monometer (1)
- A line of 2 feet is a dimeter (2)
- A line of 3 feet is a trimeter (3)
- A line of 4 feet is a tetrameter (4)
- A line of 5 feet is a pentameter (5)
- A line of 6 feet is a hexameter (6)
- A line of 7 feet is a heptameter (7)
- A line of 8 feet is a octameter (8)

Rhythm and meter in poetry: examples

Iambic pentameter (5 iambs, 10 syllables)

- That **time** | of **year** | thou **mayst** | in **me** | be**hold**

Trochaic tetrameter (4 trochees, 8 syllables)

- **Tell** me | **not** in | **mourn**ful | **num**bers

Anapestic trimeter (3 anapests, 9 syllables)

- And the **sound** | of a **voice** | that is **still**

Dactylic hexameter (6 dactyls, 18 syllables)

- **This** is the | **for**est pri | **me**val, the | **mur**muring | **pine** and the | **hem**locks

(it actually has 17 syllables because a trochee replaces the last dactyl)

Rhythm and meter in poetry: examples

Dactylic pentameter (5 dactyls, 15 syllables)

Puncha, puncha, la rosa huele
Que l'amor muncho dueles

Puncha, pun | **cha**,- la | **ro** - sa | **hue** - -| **le**
Que l'a | **mor** mun | **cho** - - | **due** - - | **les**

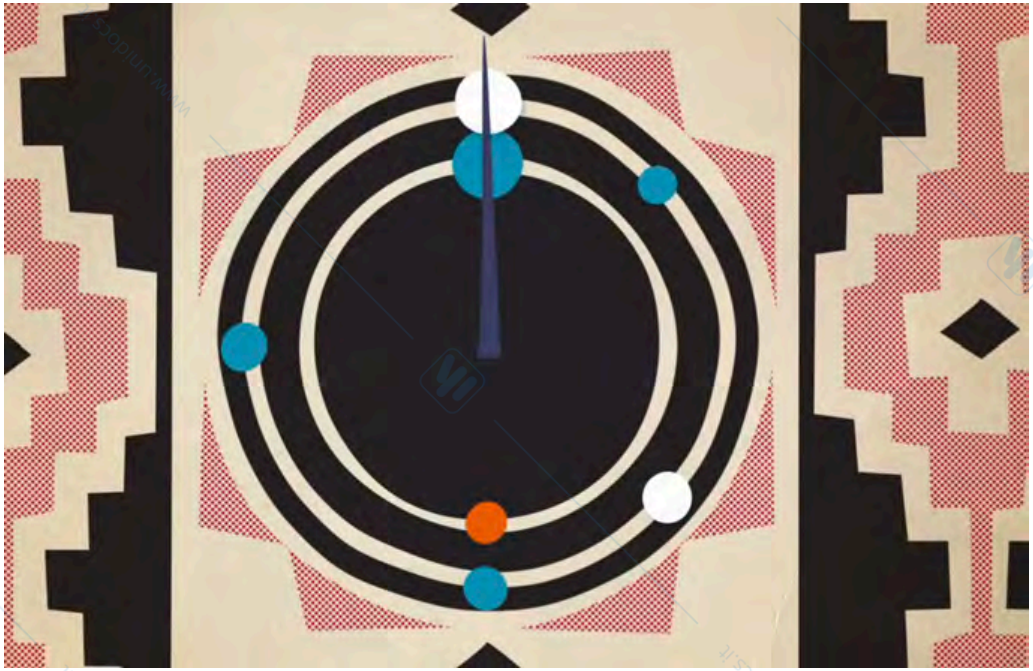
5 dactyls, 9 syllables (several trochees replace dactyls)

Avishai Cohen and his trio with l'Orchestre National d'Île-de-France (2016)

https://youtu.be/Bk5Z5cCTHJI Puncha Puncha (9’09’’)

Rhythmic wheel

- https://youtu.be/2UphAzryVpY



A different way to visualize rhythm - John Varney https://youtu.be/2UphAzryVpY

Examples of common rhythms

See video from NYU – Music Experience Design Lab

- https://www.youtube.com/watch?v=tm2BgO1VaRY

Examples

cut time	1	+	2	+	3	+	4	+	1	+	2	+	3	+	4	+
common time	1	e	+	a	2	e	+	a	3	e	+	a	4	e	+	a
kick																
snare																
closed hh																

https://youtu.be/nrHRYliYNQk

Examples

Classic Breakbeats - 8 steps

Billie Jean

cut time	1	+	2	+	3	+	4	+	1	+	2	+	3	+	4	+
common time	1	e	+	a	2	e	+	a	3	e	+	a	4	e	+	a
kick	■								■							
snare					■								■			
closed hh	■		■		■		■		■		■		■		■	

Cold Sweat (opening bar)

cut time	1	+	2	+	3	+	4	+	1	+	2	+	3	+	4	+
common time	1	e	+	a	2	e	+	a	3	e	+	a	4	e	+	a
kick	■		■						■		■					
snare					■			■					■			■
ride	■		■		■		■		■		■		■		■	

1) https://www.dropbox.com/s/o62k2r543szytzw/Michael%20Jackson%20-%20Billie%20Jean.mp3?dl=0
2) https://www.dropbox.com/s/ns088pt4l5592yf/James%20Brown%20Cold%20Sweat.mp3?dl=0

Examples

Classic Breakbeats - 16 steps

Impeach The President

cut time	1	+	2	+	3	+	4	+	1	+	2	+	3	+	4	+
common time	1	e	+	a	2	e	+	a	3	e	+	a	4	e	+	a
kick	■							■	■						■	
snare					■								■			
closed hh	■		■		■		■	■	■				■		■	
open hh											■					

The Honey Drippers
Impeach The President

When The Levee Breaks

cut time	1	+	2	+	3	+	4	+	1	+	2	+	3	+	4	+
common time	1	e	+	a	2	e	+	a	3	e	+	a	4	e	+	a
kick	■	■						■			■	■				
snare					■								■			
closed hh	■		■		■		■		■		■		■		■	

Led Zeppelin
When the Levee Breaks

1) https://www.dropbox.com/s/q9jelgtsxdaqzdi/The%20Honey%20Drippers%20-%20Impeach%20The%20President.mp3?dl=0
2) https://www.dropbox.com/s/vw60jr6k8rtnivy/When%20The%20Levee%20Breaks%20-%20Dark%20Remaster.mp3?dl=0

Examples

Rumba																
cut time	1	+	2	+	3	+	4	+	1	+	2	+	3	+	4	+
common time	1	e	+	a	2	e	+	a	3	e	+	a	4	e	+	a
kick	■			■	■			■	■			■	■			■
rim	■			■				■			■		■			
ride	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■

https://www.dropbox.com/s/ayzhd9vkdhoukc7/Rumba.mp3?dl=0

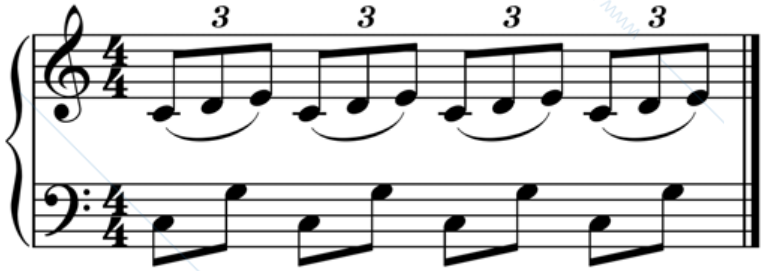
Examples

Bossa Nova																
cut time	1	+	2	+	3	+	4	+	1	+	2	+	3	+	4	+
common time	1	e	+	a	2	e	+	a	3	e	+	a	4	e	+	a
kick	■			■	■			■	■			■	■			■
rim	■			■			■				■			■		
ride	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■

https://youtu.be/mZ_mEmaJu98

Polyrhythms

- A polyrhythm is the simultaneous use of two or more conflicting rhythms, that are not readily perceived as deriving from one another, or as simple manifestations of the same meter
  - Polyrhythms can be distinguished from irrational rhythms (tuplets), which can occur within the context of a single part
  - Polyrhythms require at least two rhythms to be played concurrently, one of which is typically an irrational rhythm

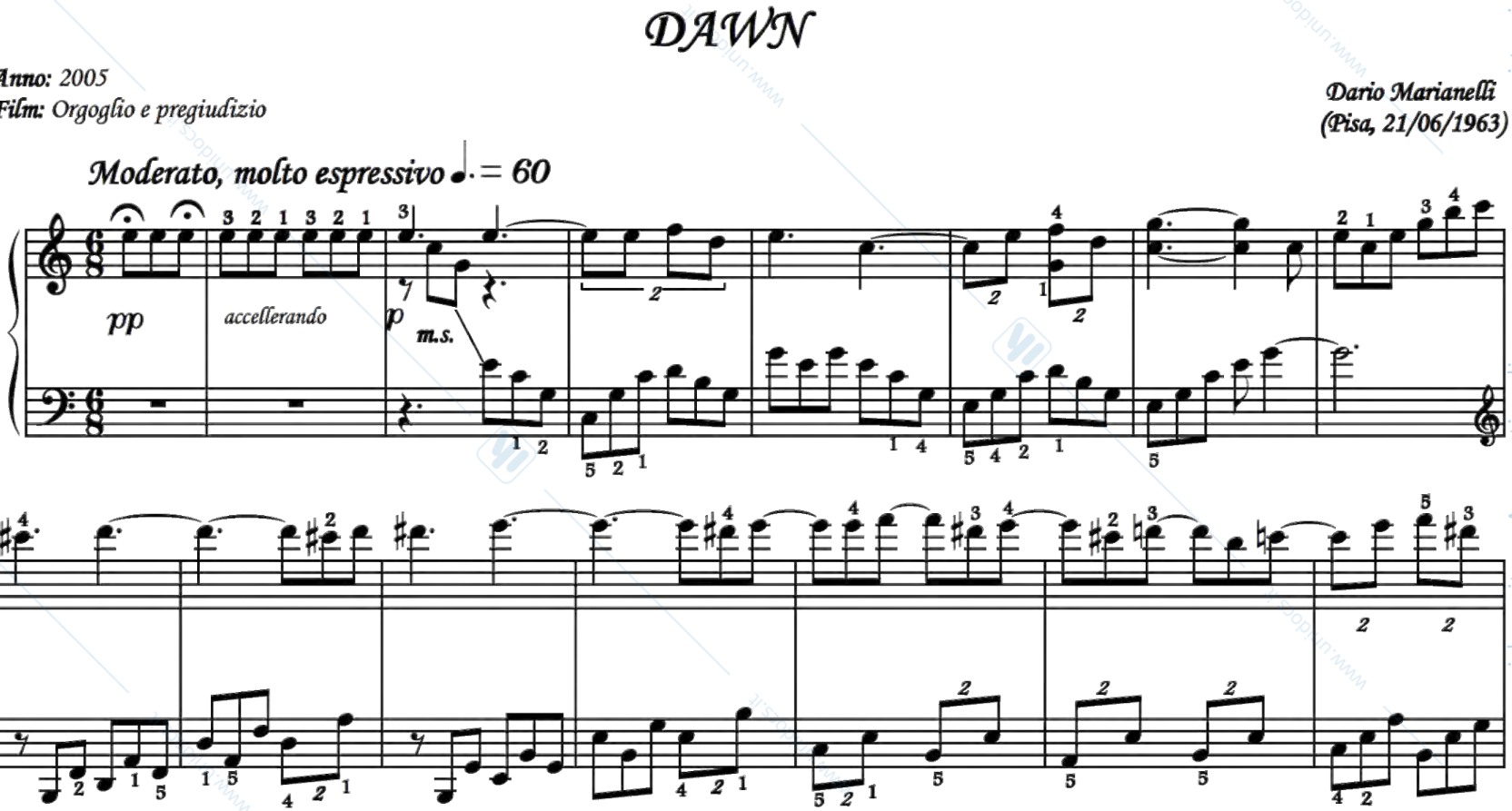


Polyrhythms

- **Hemiola:** based on a two-over-three (2:3) structure

https://youtu.be/DHYF1UchYgs

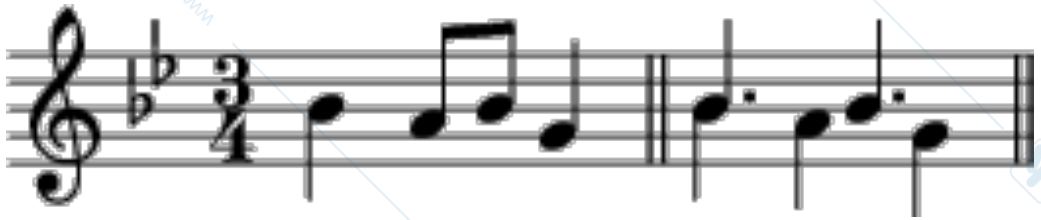
Written using tuplets



Polyrhythms

Composite Hemiola

- The four-note ostinato pattern of «carol of the bells» is the composite of the two-against-three hemiola



https://youtu.be/yfPCxCua0xY

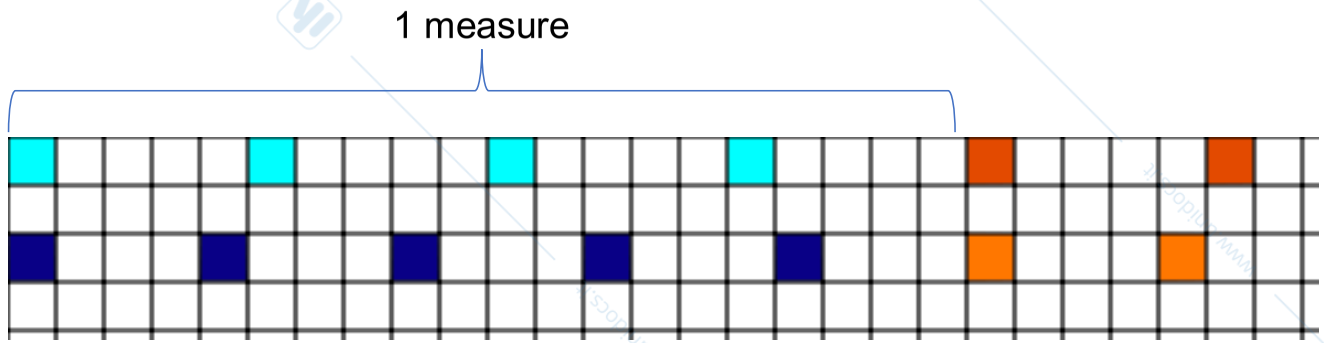
Polyrhythms

Cross-rhythm – refers to systemic polyrhythm

- a rhythm in which the regular pattern of accents of the prevailing meter is contradicted by a conflicting pattern and not merely a momentary displacement that leaves the prevailing meter fundamentally unchallenged
- the physical basis of cross-rhythms can be described in terms of interference of different periodicities

Examples

- 3:2 (Hemiola)
- 4:3
- 5:2
- 5:3
- 5:4



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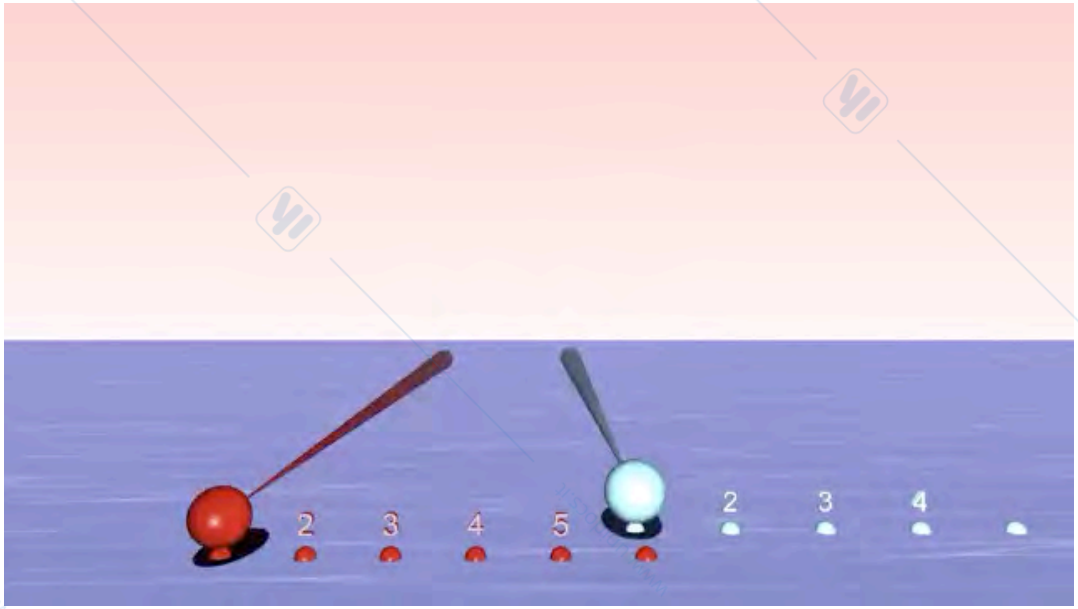
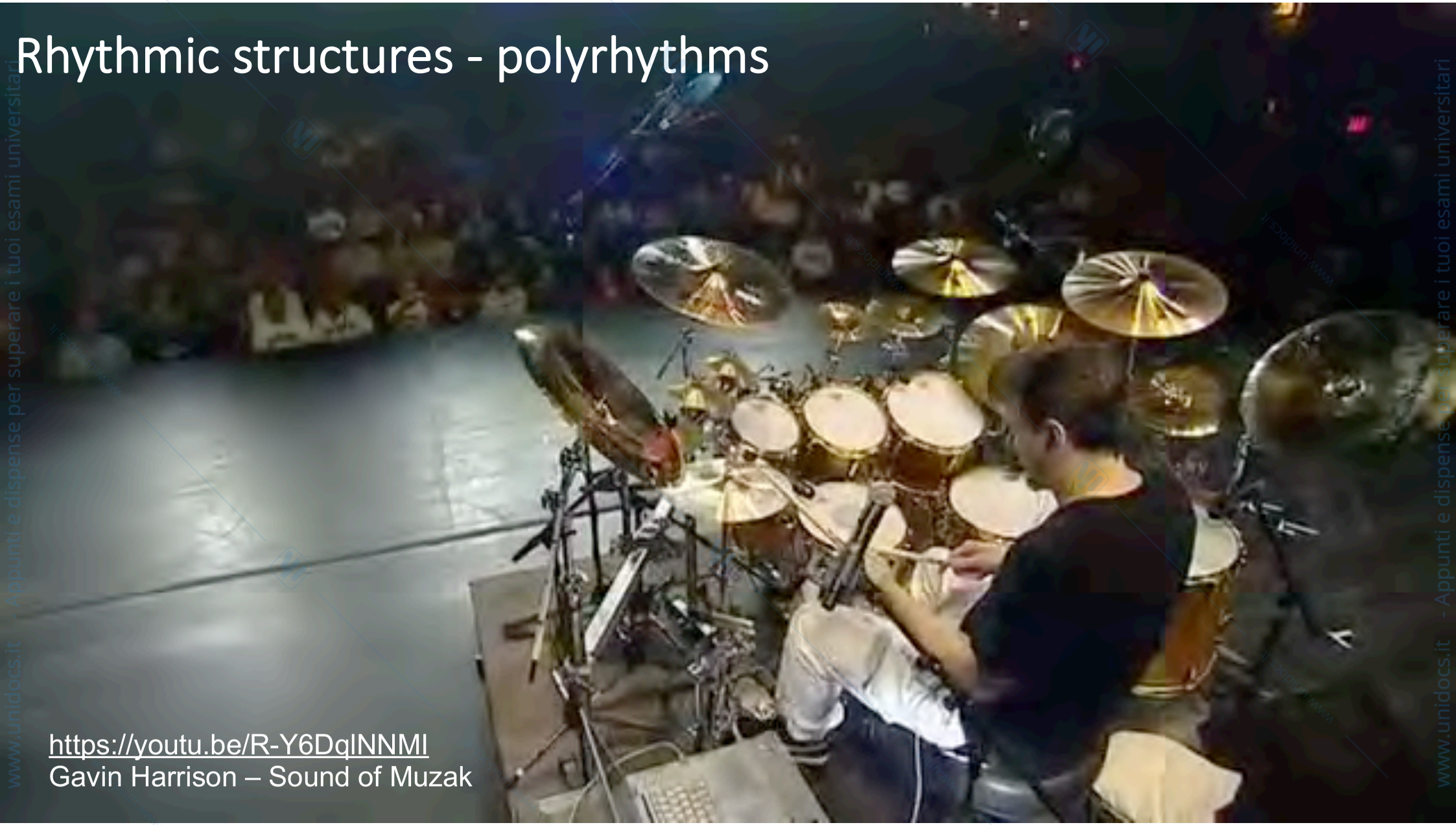


https://youtu.be/7cJVpF6cN7E

Rhythmic structures - polyrhythms



https://youtu.be/R-Y6DqINNMI
Gavin Harrison – Sound of Muzak

Rhythmic structures - polyrhythms

```
patternDescription = {midi.Drums.SplashCymbal:      "S[16] 6--------------------------- ",
                      midi.Drums.ClosedHiHat:       "S[16] --5-9-5-9-5-9-5-9-5-9-5----- ",
                      midi.Drums.OpenHiHat:         "S[16] ------------------------9--- ",
                      midi.Drums.SnareDrum2:        "S[16] -----9------9------9------9- ",
                      midi.Drums.Kick:              "S[16] 9--9---9--9---9--9---9--9--- "
                      }
```

https://www.dropbox.com/s/y16tmp21o75xhyz/sound%20of%20muzak%20drum%20only.mp3?dl=0

https://youtu.be/R-Y6DqlNNMI
Gavin Harrison – Sound of Muzak

Rhythmic structures - polyrhythms

```
patternDescription = {midi.Drums.SplashCymbal:     "S[16] 6---|----|----|----|----|----|----",
                      midi.Drums.ClosedHiHat:      "S[16] --5-|9-5-|9-5-|9-5-|9-5-|9-5-|----",
                      midi.Drums.OpenHiHat:        "S[16] ----|----|----|----|----|----|9---",
                      midi.Drums.SnareDrum2:       "S[16] -----9-|-----9-|-----9-|-----9-",
                      midi.Drums.Kick:             "S[16] 9--9---|9--9---|9--9---|9--9---"
                      }
```

https://www.dropbox.com/s/y16tmp21o75xhyz/sound%20of%20muzak%20drum%20only.mp3?dl=0

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
Kick	x			x				x			x				x			x				x			x				set
Snare						x							x							x							x		coset
Cymbs	X		x		X		x		X		x		X		x		X		x		X		x		X		(x)		set

https://youtu.be/R-Y6DqlNNMI
Gavin Harrison – Sound of Muzak

Rhythmic structures - polyrhythms

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Kick	X	x						X							set
Snare					x							x			coset
Cymbs	x		x		x		x		x		x		x		set

https://youtu.be/7UlYwBdo_e8
Porcupine Tree – Dark Matter

Algebraic structures and rhythmic structures

- **Group:** set of elements equipped with an operation (+) that satisfies four conditions: closure, associativity, identity, invertibility
- If G is a group, and H is a subgroup of G, and g is an element of G, then
  - g+H = {g+h : h∈H } is the left coset of H in G with respect to g
  - H+g = {h+g : h∈H } is the right coset of H in G with respect to g
  - Groups are «normal» if and only if right cosets and left cosets of H coincide
  - Although derived from a subgroup, cosets are not usually subgroups of G, only subsets
- In our case:
  - + represents temporal transalation
  - sets in the previous diagrams form a group (rhythmic substructures in downbeat)
  - cosets are shifted versions of subgroups and usually contribute to generate syncopation
  - cosets are generated by «turning» the corresponding rhythmic wheel

Notation for complex rhythms

- Brian Ferneyhough's Notation (BFN)
  - m/n means: m beats, each an n-th of a whole note
  - m/n : i/j means that
    - The first rhythm fits a measure of m beats (each lasting 1/n of a whole note)
    - The second rhythm fits a measure of i beats (each lasting 1/j of a whole note)

Rhythm 1 (R_1) (R_2) Rhythm 2

4/4 : 7/8

- 4 beats of R_1 last like 8 beats of R_2 (a beat of R_2 is twice as fast as a beat of R_1)
- R_1 repeats every 4 of its own beats, R_2 repeats every 7 of its own beats

Notice that the above example does NOT correspond to the previous case of «sound of muzak»!

A Polyrhythm K against N typically has a BFN of K/K : N/N

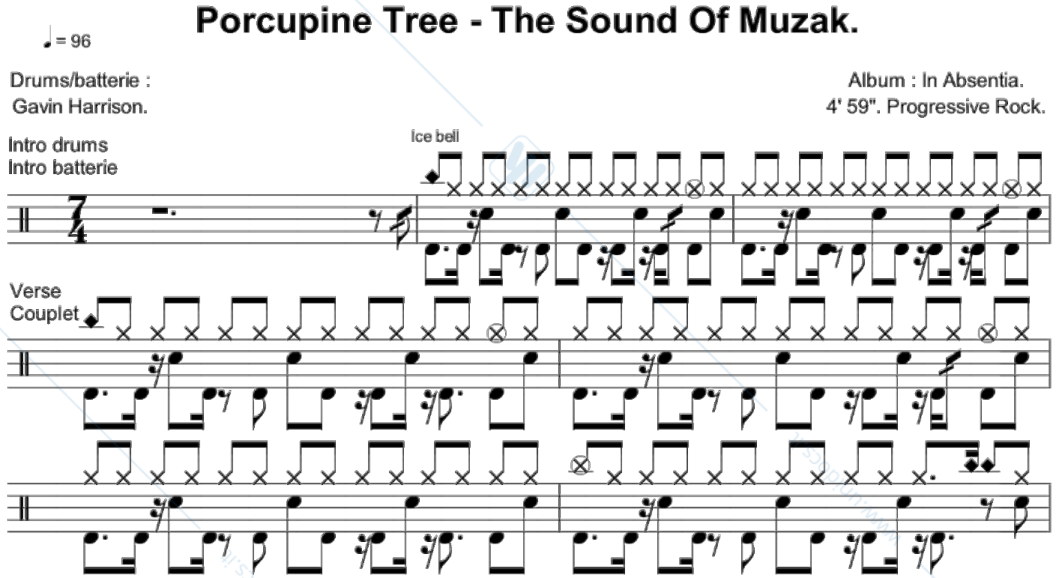
Notice that in order to somehow match musical notations, the denominator should be a power of 2

Notation for complex rhythms

- Notice that 4/4 : 7/8 is not a systemic polyrhythm of 7 against 4 (e.g. «sound of muzak»)

> In order to represent a systemic polyrhythm of K against N we would need a BFN double signature of K/K : N/N (if it weren't for that power-of-2 constraint...)

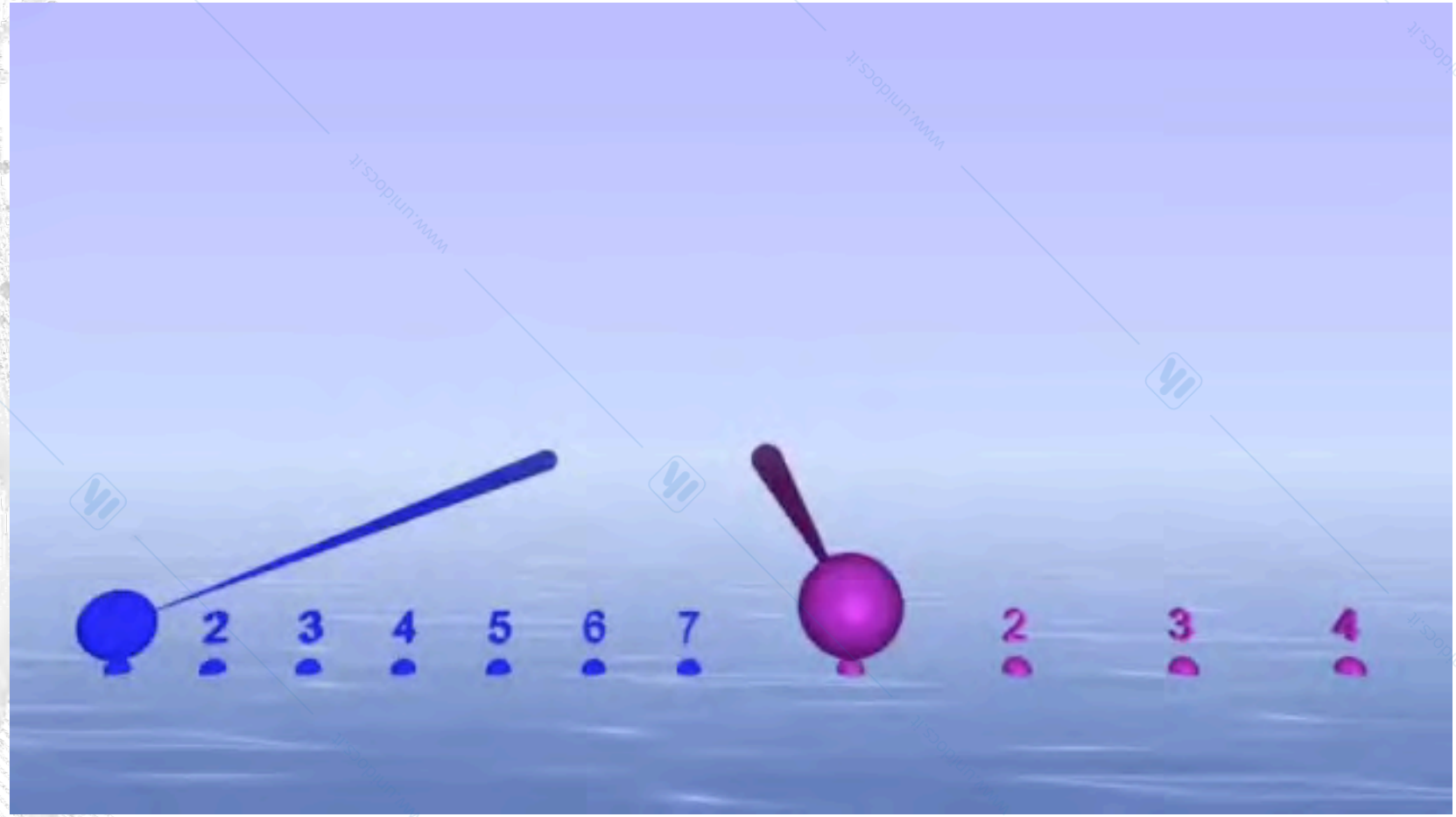
- Music notation does not easily accommodate notes whose duration is not a negative power of 2: i.e. $1/2^k$ (1/2, 1/4, 1/8, 1/16, 1/32, 1/64), therefore a polyrhythm can only be written using groups of n-tuples, which makes the writing overly cumbersome



Rhythmic structures

Polymeter => 4/4:7/8

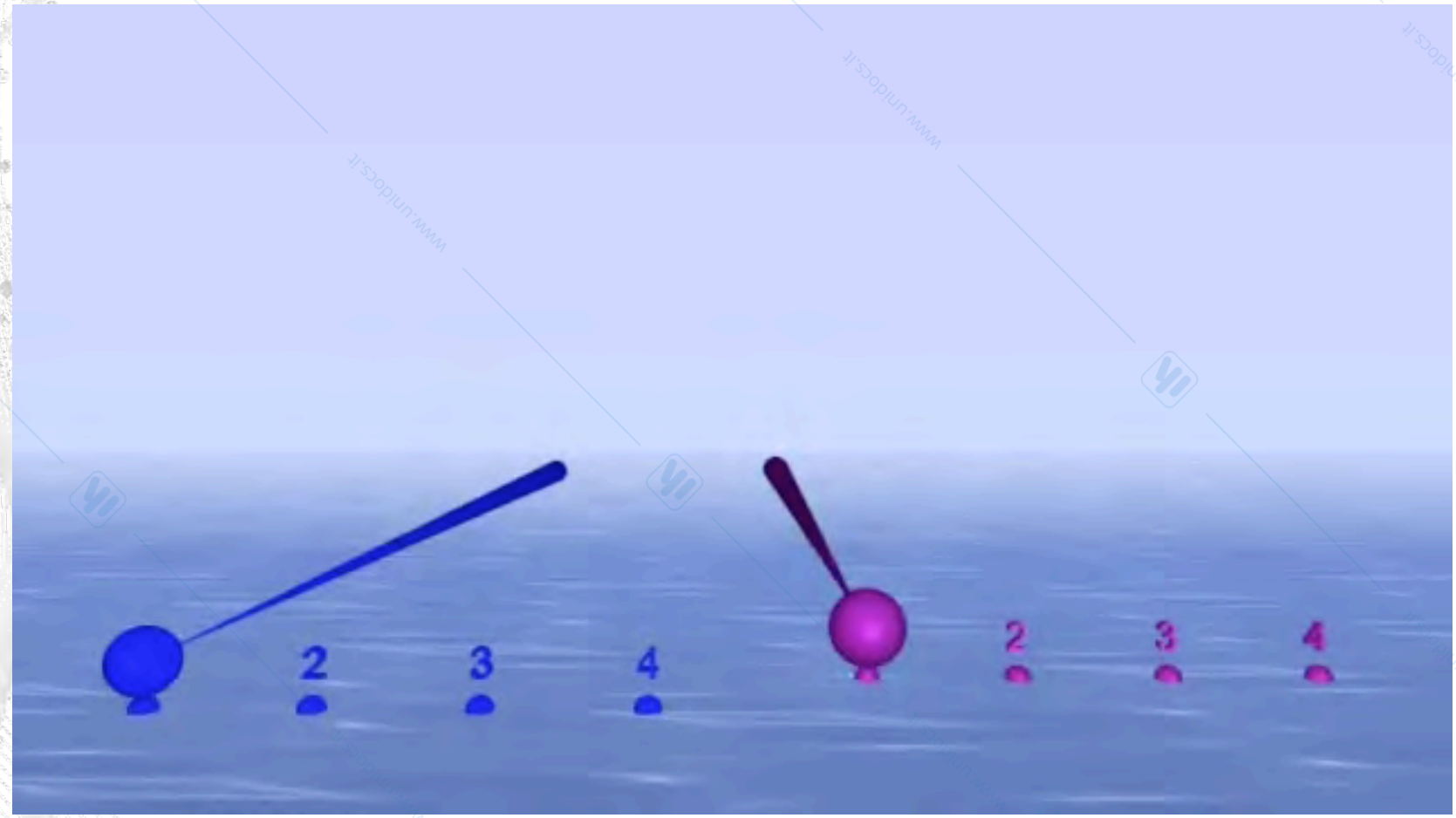
https://youtu.be/O8c79ShxJgM



Rhythmic structures

Polymeter => 4/4:4/3

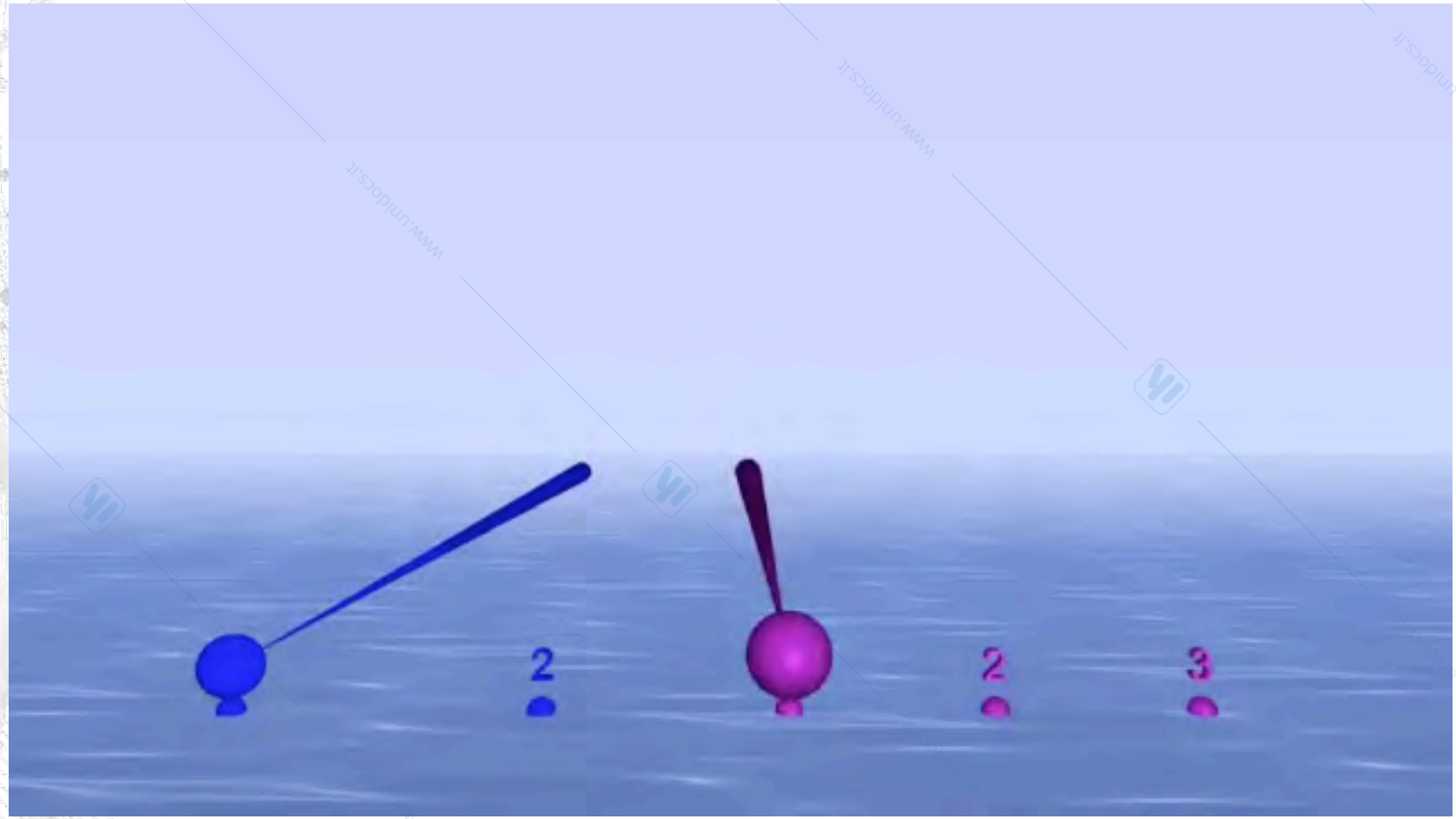
https://youtu.be/qWU2xi4mDMU



Rhythmic structures

Polymeter => 3/4:2/6

https://youtu.be/MUSdQ8M88cg



Rhythmic structures

- Earlier examples of Ferneyhough's notation
  - Dark matter:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Kick	X	x						X							set
Snare					x							x			coset
Cymbs	x		x		x		x		x		x		x		set

Signature => 7/8 Cross-Rhythm «7 against 2» BFN notation => 7/7:2/2

Notice the forward shift of the snare's pattern (5/14) -> coset

  - Sound of Muzak:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
Kick	x			x				x			x				x			x				x			x				set
Snare						x							x							x							x		coset
Cymbs	X		x		X		x		X		x		X		x		X		x		X		x		X		(x)		set

Signature => 7/8 Cross-Rhythm «7 against 4» BFN notation => 7/7:4/4

Notice the forward shift of the snare's pattern (5/28) -> coset

Duality btw Polymetrics and Polyrhythms

In order to find a **single signature** that accommodates both sub-signatures of a polymetric structure we proceed as follows:

$$\frac{N_1}{D_1} : \frac{N_2}{D_2} \rightarrow \frac{N}{D}$$

Where

$D=LCM(D_1,D_2)$ (LCM=Lowest Common Multiple)
$N=LCM(N_1K_1,N_2K_2)$, where $K_1=D/D_1$ and $K_2=D/D_2$

In fact, we have

$$\frac{N_1}{D_1} : \frac{N_2}{D_2} \rightarrow \frac{N_1}{\frac{D}{K_1}} : \frac{N_2}{\frac{D}{K_2}} \rightarrow \frac{N_1K_1}{D} : \frac{N_2K_2}{D} \rightarrow \frac{N}{D}$$

Notice, however, that often $D > D_1$ and $D > D_2$, therefore the resulting signature might no longer be indicative of the beat (dominant pulse)

Examples

- Porcupine Tree: Sound of Muzak
  https://youtu.be/R-Y6DqlNNMI

$$\frac{N_1}{D_1}:\frac{N_2}{D_2} \rightarrow \frac{N_1}{\frac{D}{K_1}}:\frac{N_2}{\frac{D}{K_2}} \rightarrow \frac{N_1K_1}{D}:\frac{N_2K_2}{D} \rightarrow \frac{N}{D}$$

$$\frac{4}{4}:\frac{7}{7} \rightarrow \frac{4}{\frac{28}{7}}:\frac{7}{\frac{28}{4}} \rightarrow \frac{4\cdot 7}{28}:\frac{7\cdot 4}{28} \rightarrow \frac{28}{28}:\frac{28}{28} \rightarrow \frac{28}{28}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
Kick	x			x				x			x				x			x				x			x				set
Snare						x							x							x							x		coset
Cymbs	X		x		X		x		X		x		X		x		X		x		X		x		X		(x)		set

Examples

- Porcupine Tree: Dark Matter
  https://youtu.be/7UlYwBdo_e8

$$\frac{N_1}{D_1}:\frac{N_2}{D_2} \rightarrow \frac{N_1}{\frac{D}{K_1}}:\frac{N_2}{\frac{D}{K_2}} \rightarrow \frac{N_1K_1}{D}:\frac{N_2K_2}{D} \rightarrow \frac{N}{D}$$

$$\frac{7}{7}:\frac{2}{2} \rightarrow \frac{7}{\frac{14}{2}}:\frac{2}{\frac{14}{7}} \rightarrow \frac{7\cdot 2}{14}:\frac{2\cdot 7}{14} \rightarrow \frac{14}{14}:\frac{14}{14} \rightarrow \frac{14}{14}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Kick	X	x						X							set
Snare					x							x			coset
Cymbs	x		x		x		x		x		x		x		set

Examples

«Etude No. 1»,
by Tigran Hamasyan

Original recording:
https://youtu.be/xmScqhDSJrI

$$\frac{N_1}{D_1} : \frac{N_2}{D_2} \rightarrow \frac{N_1}{\frac{D}{K_1}} : \frac{N_2}{\frac{D}{K_2}} \rightarrow \frac{N_1 K_1}{D} : \frac{N_2 K_2}{D} \rightarrow \frac{N}{D}$$

$$\frac{5}{8} : \frac{7}{8} \rightarrow \frac{5}{\frac{8}{1}} : \frac{7}{\frac{8}{1}} \rightarrow \frac{5 \cdot 1}{8} : \frac{7 \cdot 1}{8} \rightarrow \frac{35}{8}$$

Example: Etude No. 1 (Tigran Hamasyan)

Polymetric structure => 5/8 : 7/8 - Melodic Periodicity (both melodies): 140 beats (20=5*4 measures of 7/8, or 28=7*4 mesures of 5/8)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Mel 1 (7/8)	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C
Mel 2 (5/8)	E			C		D			C		C			B		C			C		E			C		D			C		C			B	
Mel 1 (7/8)	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C
Mel 2 (5/8)	C			C		E			C		D			C		C			B		C			C		E			C		D			C	
Mel 1 (7/8)	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C
Mel 2 (5/8)	C			B		C			C		E			C		D			C		C			B		C			C		E			C	
Mel 1 (7/8)	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C	E		G	F	E		C
Mel 2 (5/8)	D			C		C			B		C			C		E			C		D			C		C			B		C			C	

Rhythmic structures

Metric modulation

- A metric modulation is a modulation from one metric unit or meter to another
- Sometimes used to stress a subdivision of an asymmetrical rhythms
  - 2+2+3 time, where each bar has two 2-beat units and a 3-beat unit with a stress at the beginning of each unit
- Sometimes necessary when complexity is excessive

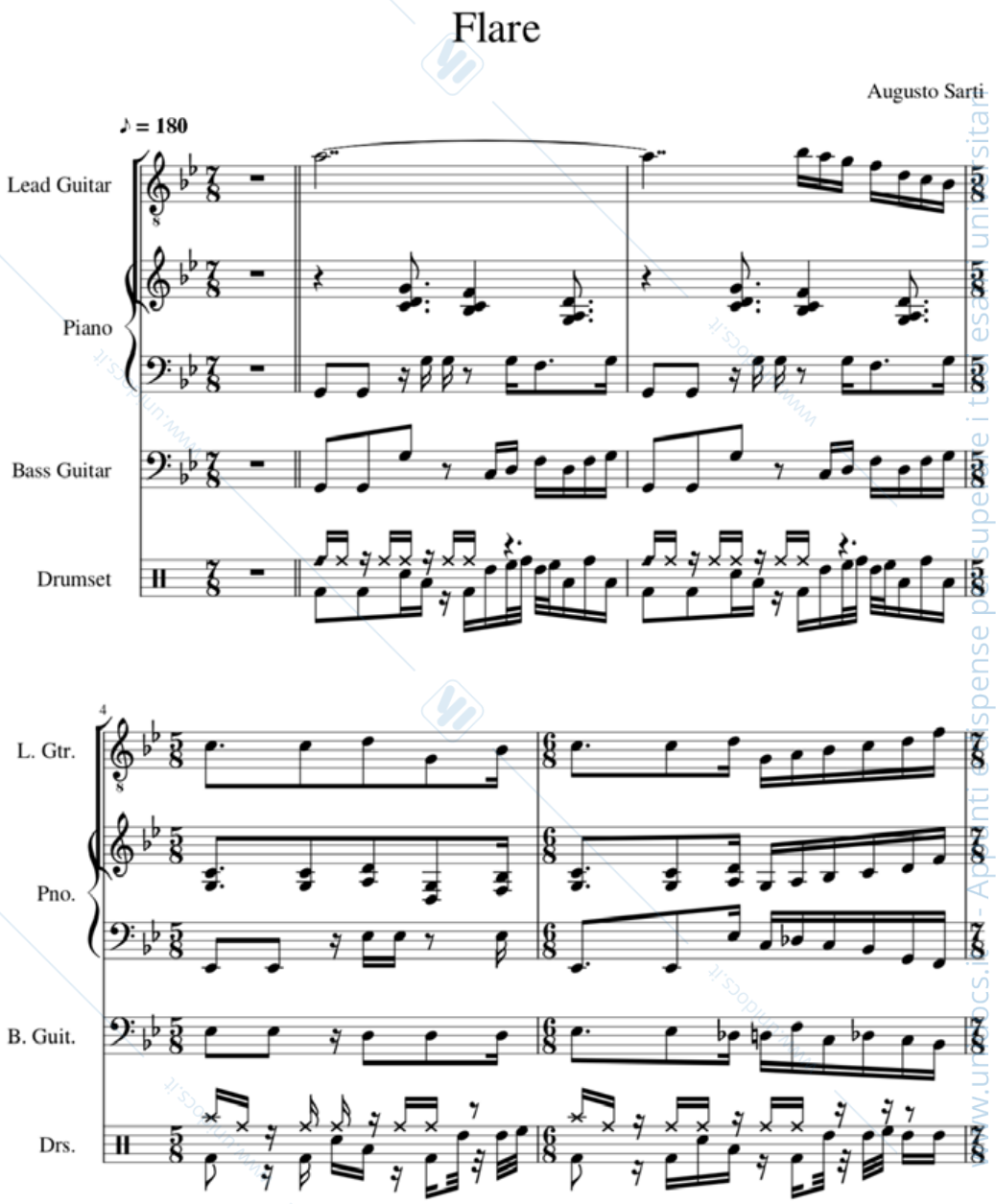
https://youtu.be/BFa__HYxNxU
Gavin Harrison – 19 days

Rhythmic structures – metric modulation

Examples

- «Flare», Interplay
  https://youtu.be/HqWSkOoKrtI?t=112

$$\frac{7}{8} \rightarrow \frac{7}{8} \rightarrow \frac{5}{8} \rightarrow \frac{6}{8}$$
$$\frac{7}{8} \rightarrow \frac{7}{8} \rightarrow \frac{5}{8} \rightarrow \frac{6}{8}$$
$$\frac{6}{8} \rightarrow \frac{6}{8} \rightarrow \frac{5}{8} \rightarrow \frac{5}{8}$$
$$\frac{6}{8} \rightarrow \frac{6}{8} \rightarrow \frac{5}{8} \rightarrow \frac{5}{8}$$



Other examples

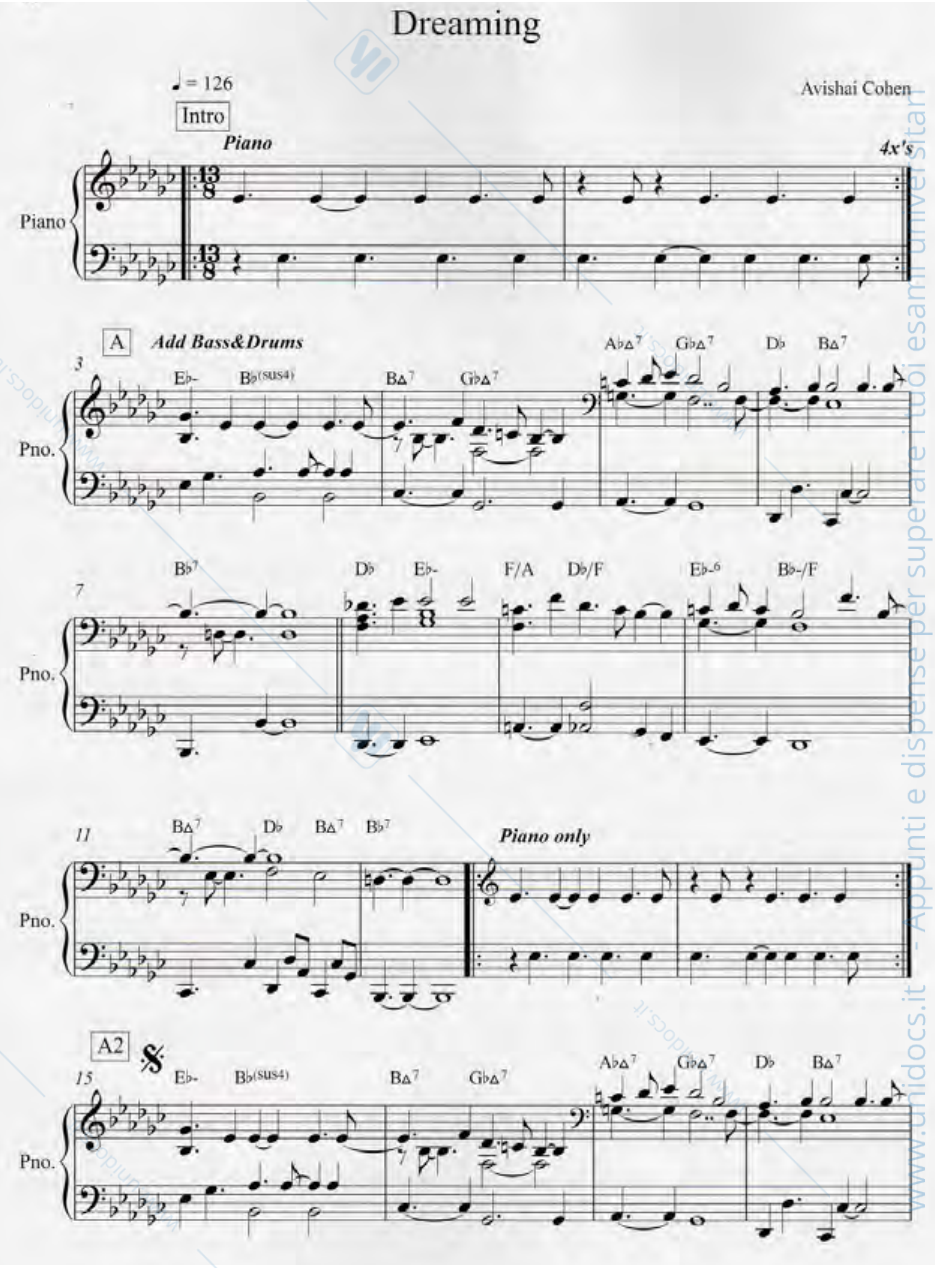
- The Beatles: «Martha my dear» - https://youtu.be/RXawa90YU2s
  - multiple metric modulations
- Asgeir: «King and Cross» - https://youtu.be/D-dHIkF_lO4
  - polymetric structure $\frac{3}{4} : \frac{4}{4}$
- Avishai Cohen – «Simonero» - https://youtu.be/6uwBIywSrgw
  - Regular 4/4 metric signature with irregular subdivisions
- Steven Wilson – «Three years older» - https://www.youtube.com/watch?v=dTyHsIQqDa8 (skip first 2')
  - Regular 4/4 metric signature with irregular subdivision
- Billy Cobham – «Spanish Moss» - https://youtu.be/PR9YH64cqn0
  - Odd rhythmic signature of 17/16
- Slobodan Trkulja & Metropol Orchestra – «Pitagorina pesma» https://youtu.be/5h8PoUyL-hk
  - Odd rhythmic signature of 25/16
- Dhafer Youssef – «Odd Elegy» - https://youtu.be/TYWgYRaHwbU
  - Odd rhythmic signature of 39/16

A closer look

https://youtu.be/ano9AhcpMOE

«Dreaming», by Avishai Cohen
Performed and explained by
Kevin Sarti

Original version (live):
https://youtu.be/JVijn6v_cLM



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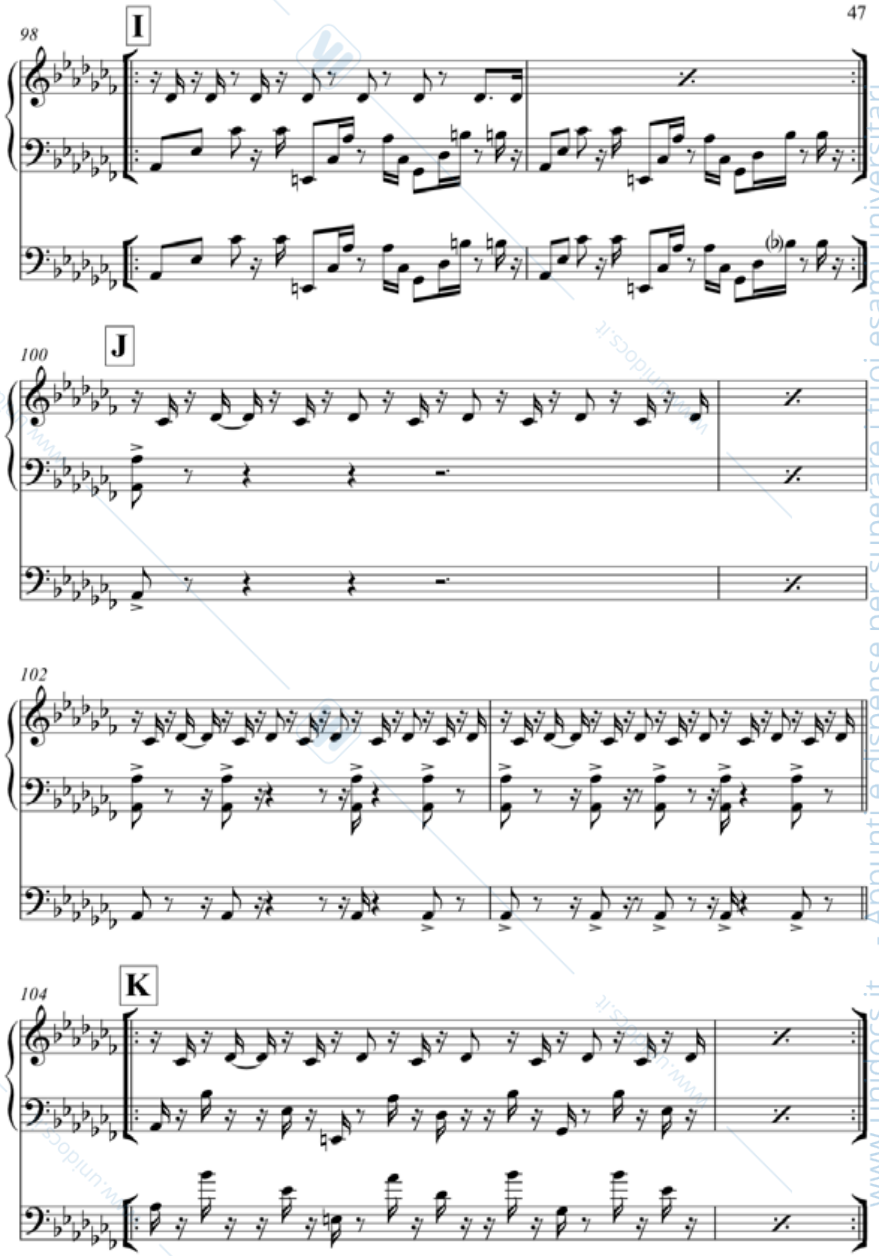
A closer look

https://youtu.be/YMKnsnOD9ro

«Structure in Emotion»,
by Avishai Cohen
Performed and explained by
Kevin Sarti

Original version:
https://youtu.be/laKnB_Eeyb0



POLITECNICO
MILANO 1863

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A closer look

https://youtu.be/zljW1UJ1K0o

«The ever-evolving etude»,
by Avishai Cohen
Performed and explained by
Kevin Sarti

Original version at:
https://youtu.be/pq7-oVKpn6M

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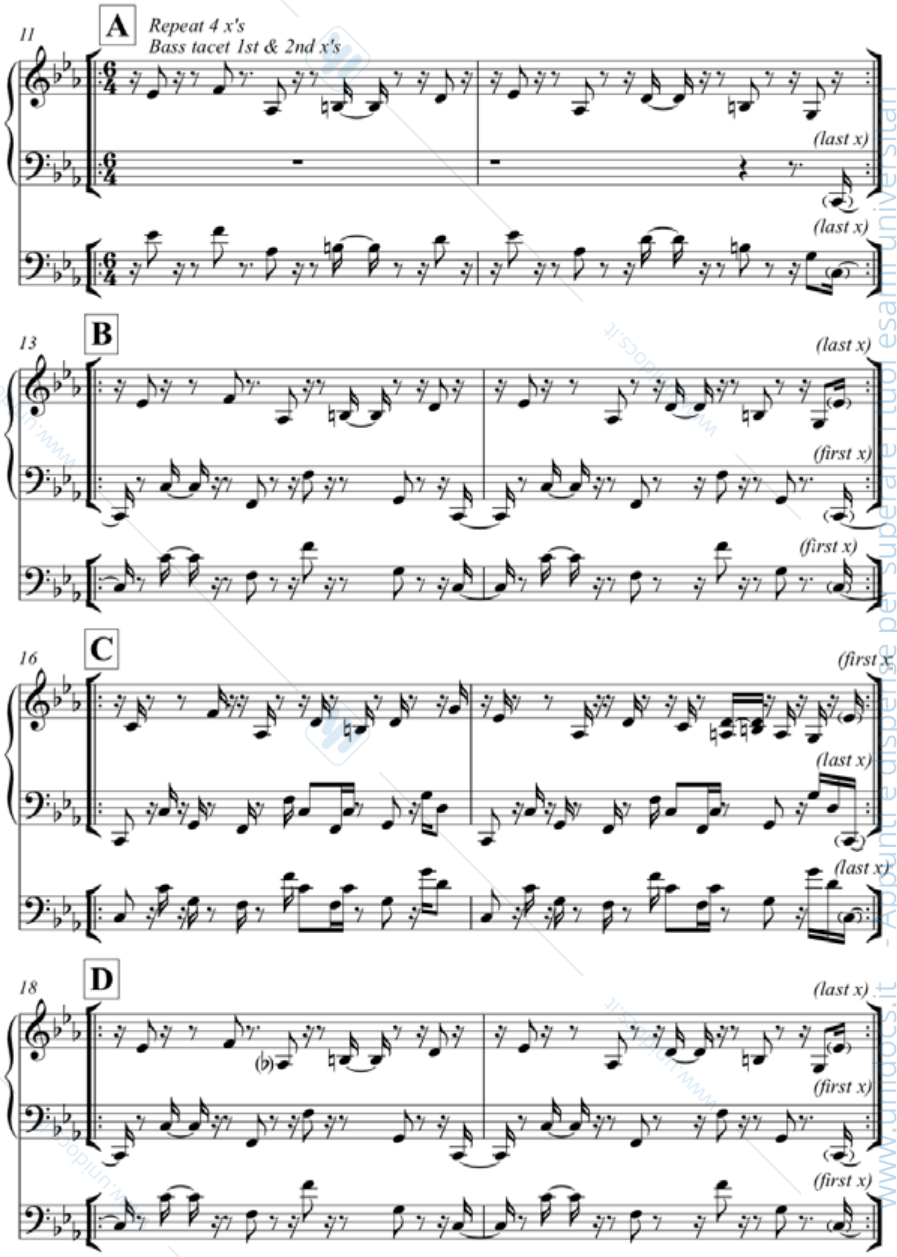

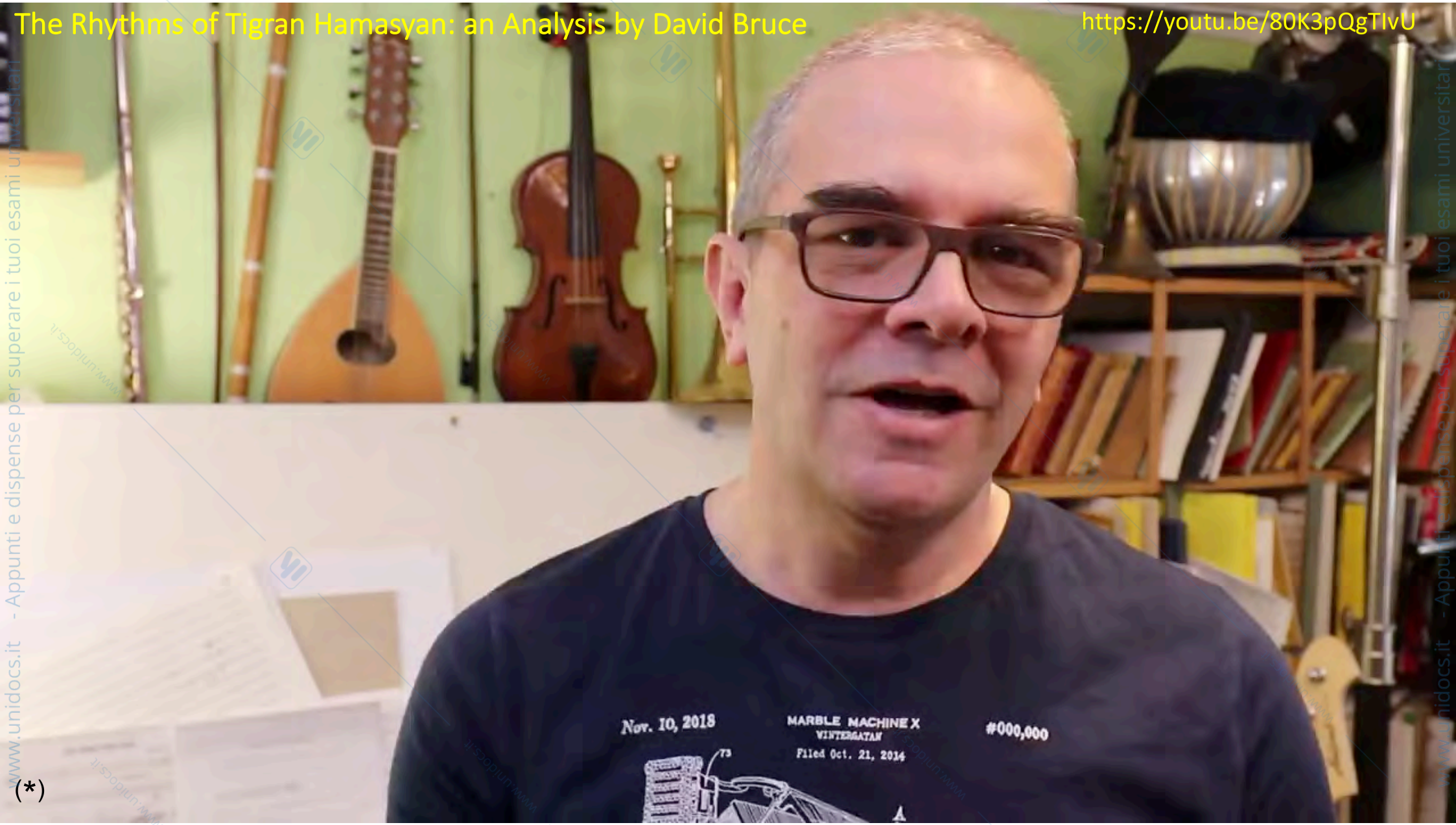

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Bringing perception into the picture

- Our perception of rhythms operates in a multi-resolution fashion, like anything else...
  - A cursory listening session might reveal a coarse rhythmic grand structure made of a succession of chunks or subdivisions
  - A more attentive listening session will allow you to discover how the chunks are built
- This can be accommodated using a multi-resolution representation

Example

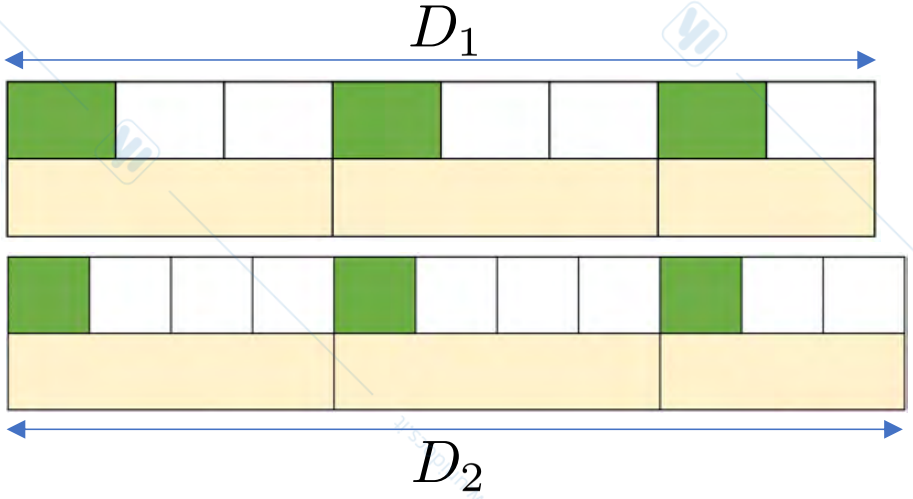
Low-resolution: Long-Long-Short

Higher-resolution:
Long-Long-Short subdivision of a slower 8/8

Replace the triplets with quadruples
Replace the twoplet with a triplet

Higher-resolution:
Long-Long-Short subdivision of a faster 11/8

Low-resolution: Long-Long-Short



$$\frac{\Delta D}{D_1} 100 \approx 3\%$$

Follow the clinic on rhythmic complexity with Yogev Gabay at PoliMI: Dec 9, 2021!

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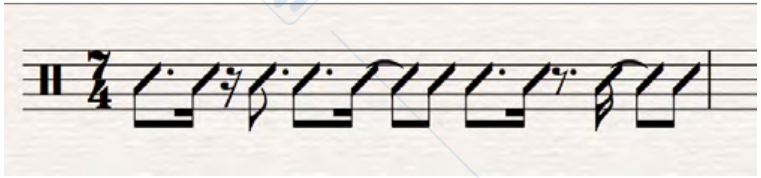
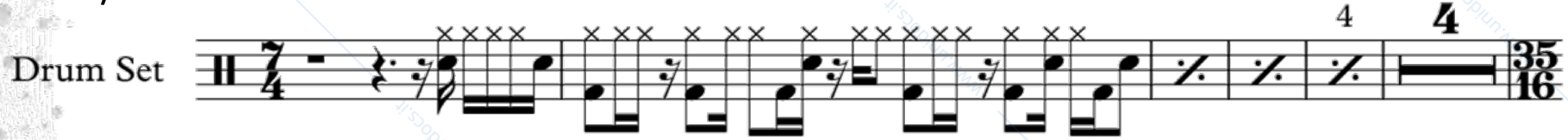
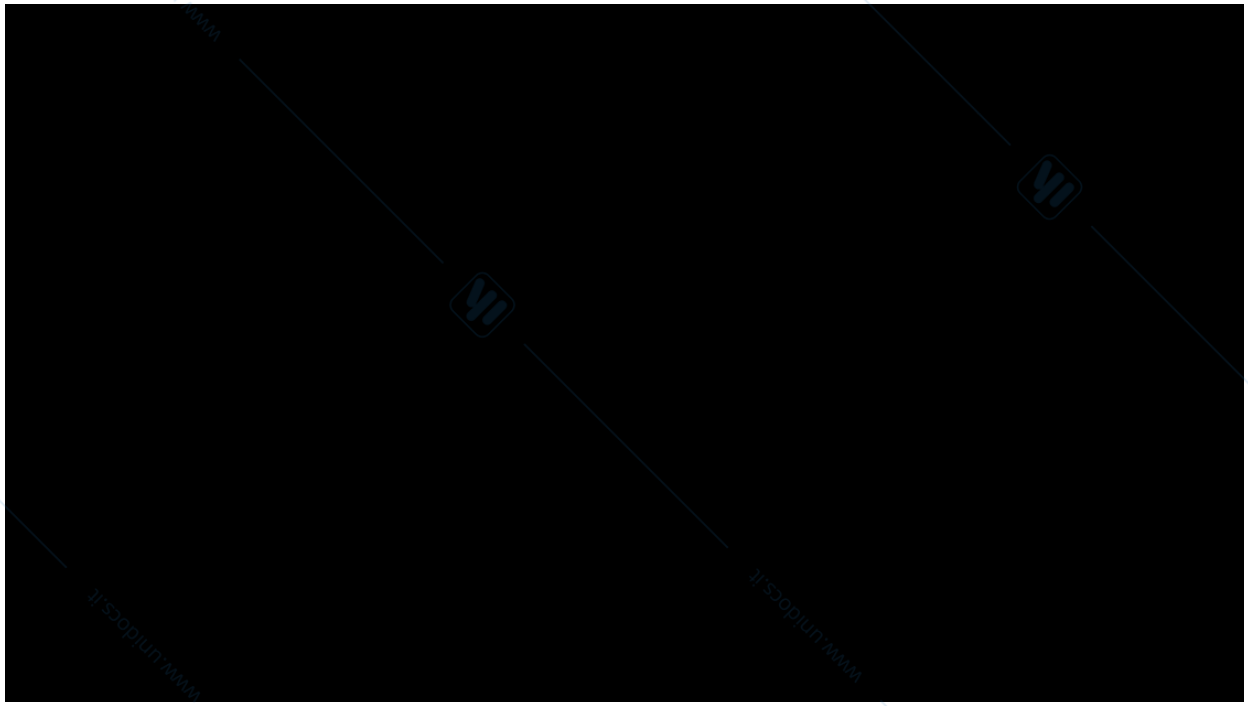
Charting the solo (Yogev's notes)



- The first section of this solo is in 7/4
- when he beatboxes over this, he's basically playing around groupings of 5, in 16 notes over the 7/4





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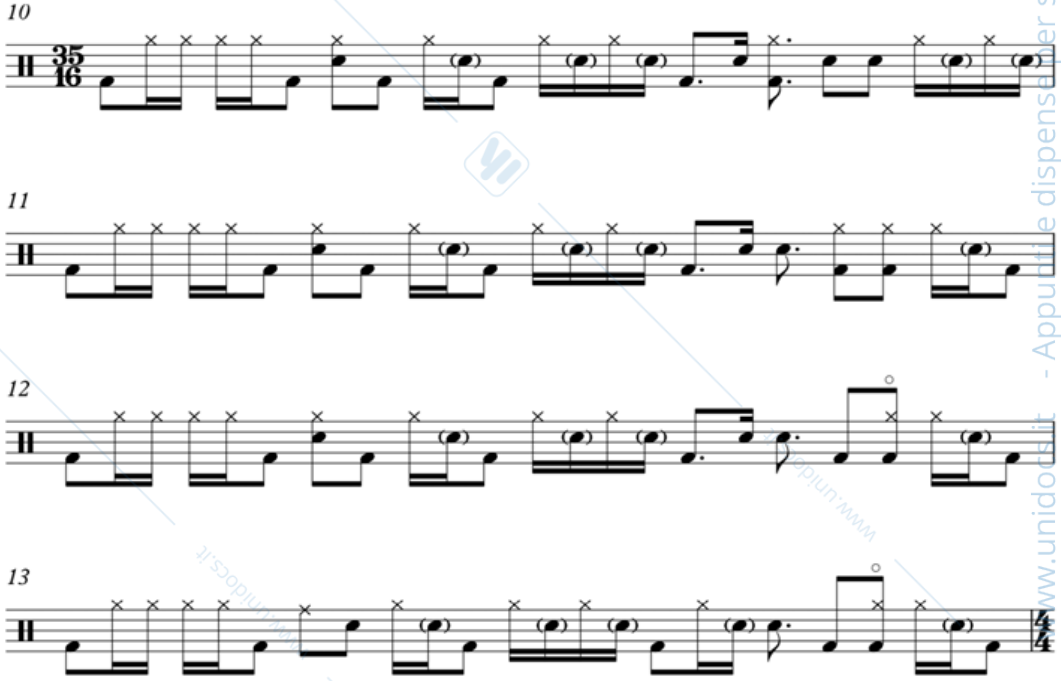
Charting the solo (Yogev's notes)

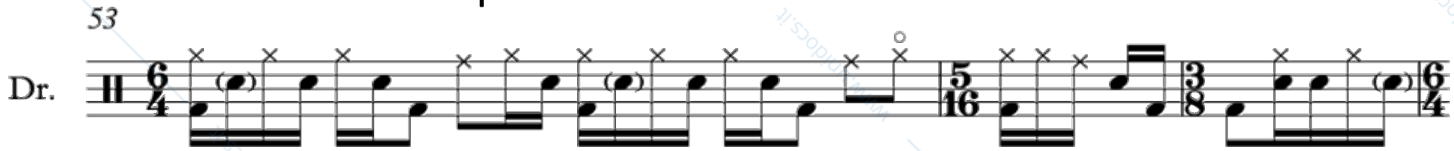
- The next section is where the fun begins: Tigran now shifts from 16 notes to quintuplets. So now, instead of having 7x4=28 "sub beats" or 16 notes in a bar, we have 7x5=35 "sub beats" in a bar.
- He chose to not play "in 7", meaning that his main pulse isn't the big 7 any more. If it was, we would hear a very clear quintuplet feel. Instead, he chose to divide these 35 notes or beats in a different way. Technically we're still in 7/4, but writing all this in quintuplets would be too complicated, so for aesthetic purposes and reading ease, I wrote the rest of the solo as 16 notes in a "different tempo".



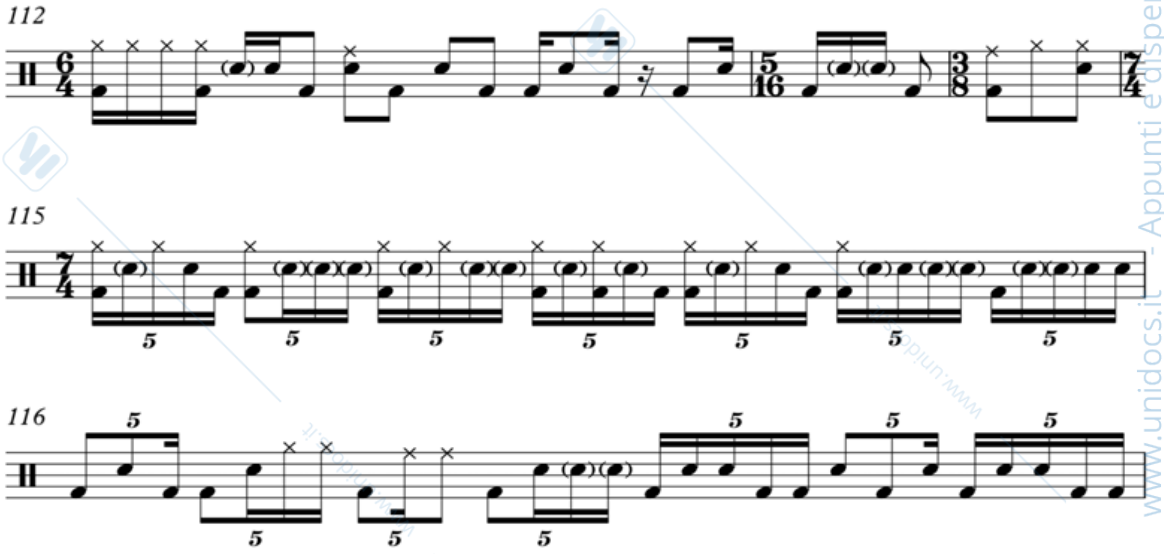
Charting the solo (Yogev's notes)

(*)

- Mainly, the bars are divided as: 6/4 (24 sixteen notes), 5/16 (a total of 5) and 3/8 (a total of 6 sixteen notes) so 24+5+6=35. Sometimes he switches between the 5/16 and the 3/8 but he stays on this clave for the most part



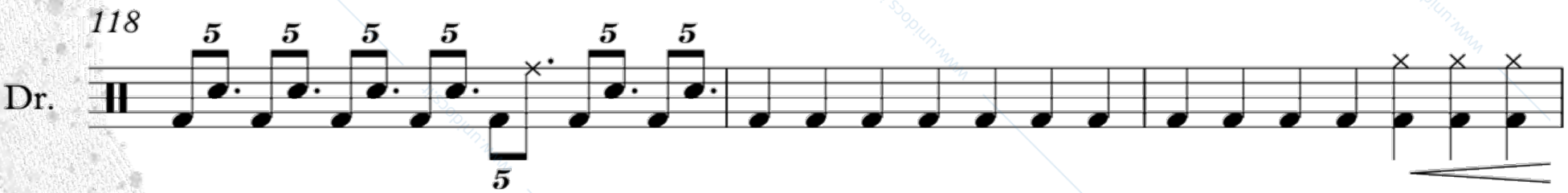
- At the end, bar 115, he shifts the feel back to the big 7 and the quintuplet feel is back!



Charting the solo (Yogev's notes)

(*)

- Then at bar 118 he clearly sings the quarter notes of the 7, with no subdivision what so ever, which gives him a "clean" start to go back to any other subdivision, in our case 16 notes like he started. And he does. Yes, they're kinda swang, but still 16 notes



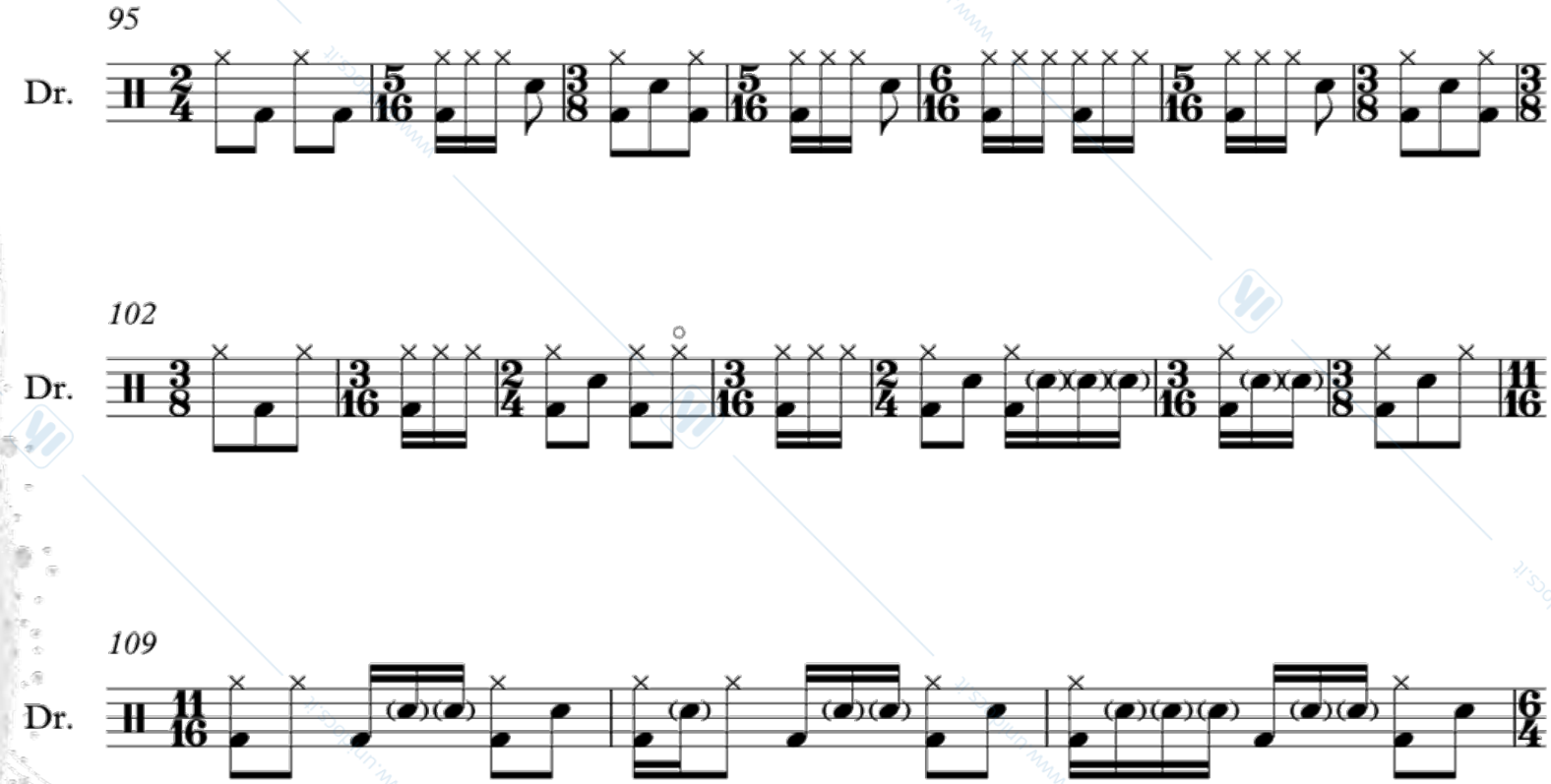
- Now. You may have noticed that bars 95 though 112 are a bit different and don't add up to 35. I didn't find an explanation other than the one I wrote. I like to think that because it's live, he might added a beat here and there, but yet again it's Tigran. So I mainly assume he did some other rhythmic trick I haven't figured out yet...

Download full chart at this link:
https://www.dropbox.com/s/7xii0faee4532rz/Tigran%20Beatbox%20%232.pdf?dl=0

Charting the solo (Yogev's notes)

(*)

- You may have noticed that bars 95 though 112 are a bit different and don't add up to 35. I didn't find an explanation other than the one I wrote. I like to think that because it's live, he might added a beat here and there, but yet again it's Tigran... So I mainly assume he did some other rhythmic trick I haven't figured out yet...



Time in Music

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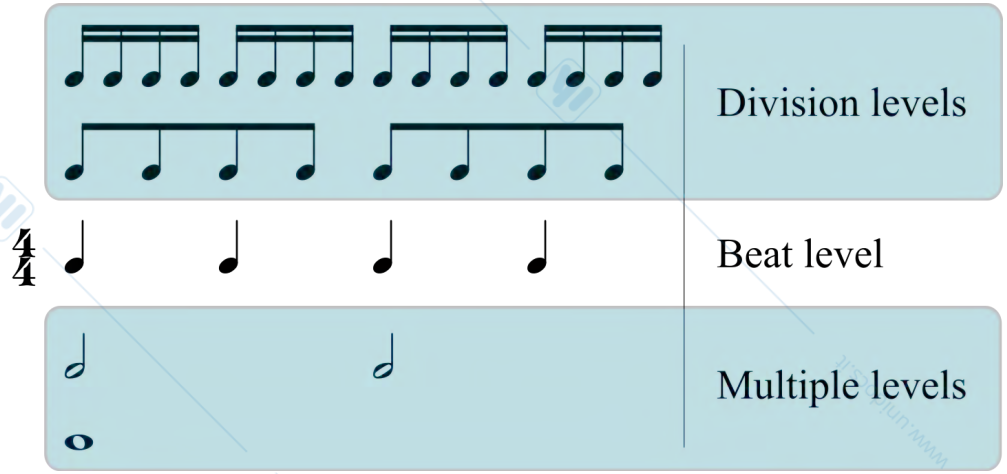
Properties of time in music

- **Duality**
  - Distinction between *meter* and *rhythm*
    - (Wikipedia) In music, **meter** refers to the regularly recurring patterns and accents such as bars and beats. Unlike rhythm, metric onsets are not necessarily sounded, but are nevertheless expected by the listener
    - (Wikipedia) **Rhythm** generally means a *movement marked by the regulated succession of strong and weak elements, or of opposite or different conditions*
- **Hierarchy**
  - Both meter and rhythm can be viewed at increasingly broader time spans
- **Motion**
  - *Tempo* as example of motion in musics



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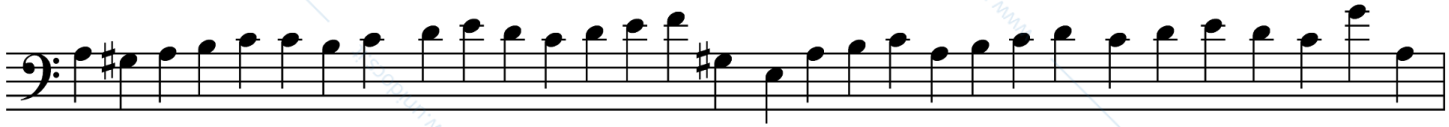
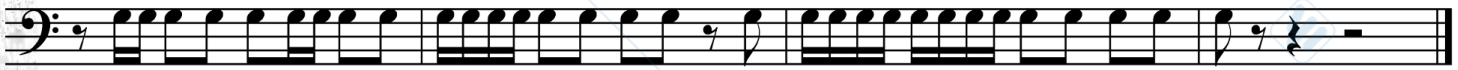
Pitch and Rhythm Interactions

Rhythm is defined by any perceivable changes (tonal, harmonic, melodic) not just by onsets

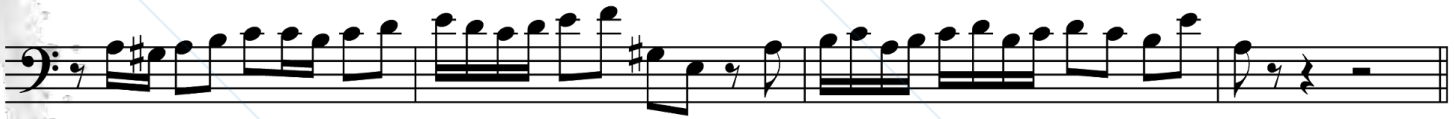
Pitch contour (derivative):



Temporal changes:



Melody:



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Pitch and Rhythm Interactions

There have been various attempts to modeling pitch-duration interactions

An example is by Palmer & Krumhansl (1987)

Additive Model

- Melody = α Pitch + β Temporal

Interactive Model

- Melody = α Pitch + β Temporal + ρ PT

where PT some measurement of pitch and temporal interaction (e.g., Pitch x Temporal; Pitch / Temporal)

Pitch and Rhythm Interactions

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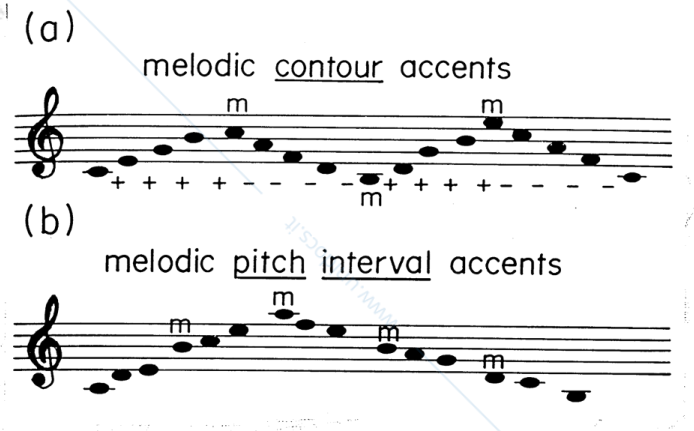
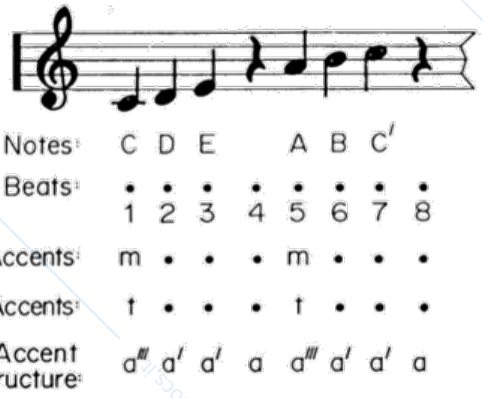
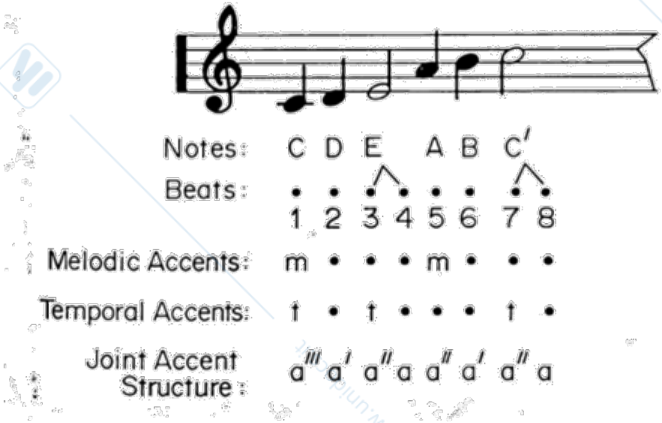
Joint accent structure (Mari Jones, Marilyn Boltz)

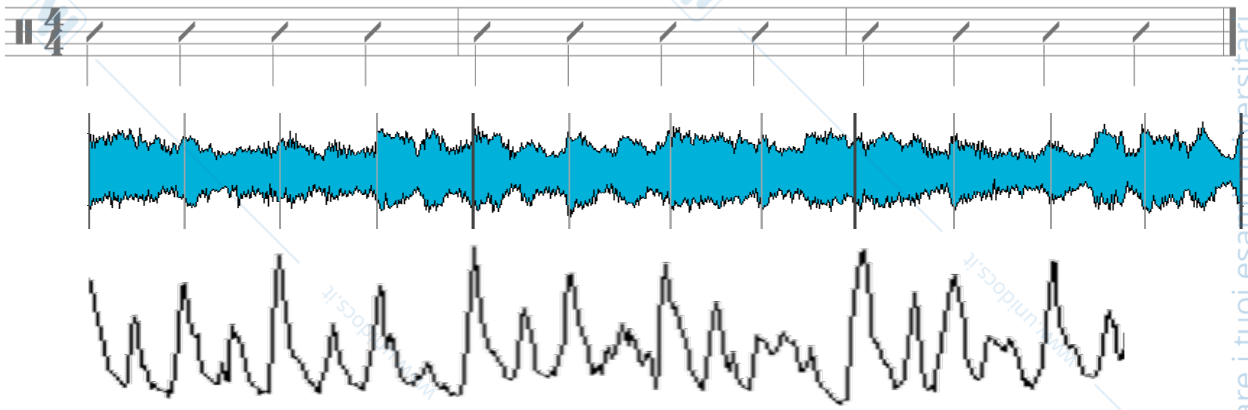


Synth solo in «follow you follow me» (Genesis)
Melodic contour accents create a rhythmic ternary counter-pattern





Analysis tools



Onset detection

- Methods
  - In the time domain
  - In the frequency domain
  - In the phase domain
- They look for
  - Increases in spectral energy
  - Changes in spectral energy distribution (spectral flux) or phase
  - Changes in detected pitch - e.g. using a polyphonic pitch detection algorithm
  - Spectral patterns recognisable by machine learning techniques such as neural networks.
- Simpler techniques (e.g. detecting amplitude changes in the time-domain) typically generate many false positives or false negatives
- Better methods tend to judge onsets similarly to how a human would: so psychoacoustically-motivated strategies may be employed. Sometimes the onset detector can be restricted to a particular domain (depending on intended application), for example being targeted at detecting percussive onsets
- Onset detection is an active research area.
  MIREX annual competition features an Audio Onset Detection contest

Analysis tools

Relevant features for rhythmic analysis and musical content analysis in general

- Perceptual Spectral Flux (PSF)
  - estimate all partial amplitudes
  - sum together all of their time derivatives
  - multiply them with a frequency-dependent weight in order to have perceptually normalized amplitudes
- Rhythmogram
  - rhythm feature obtained by computing a windowed autocorrelation function on the PSF in which the regularity of the note onsets intervals are found in overlapping segments of the music
- Timbregram
  - Based on an acoustic preprocessor used in speech recognition called Perceptual Linear Predictive (PLP) analysis
  - Noise and intermittent events are removed through time smoothing using a Gaussian weight
- Chromagram
  - Feature that maps the partials into twelve bands, corresponding to the twelve notes of one octave

(*)

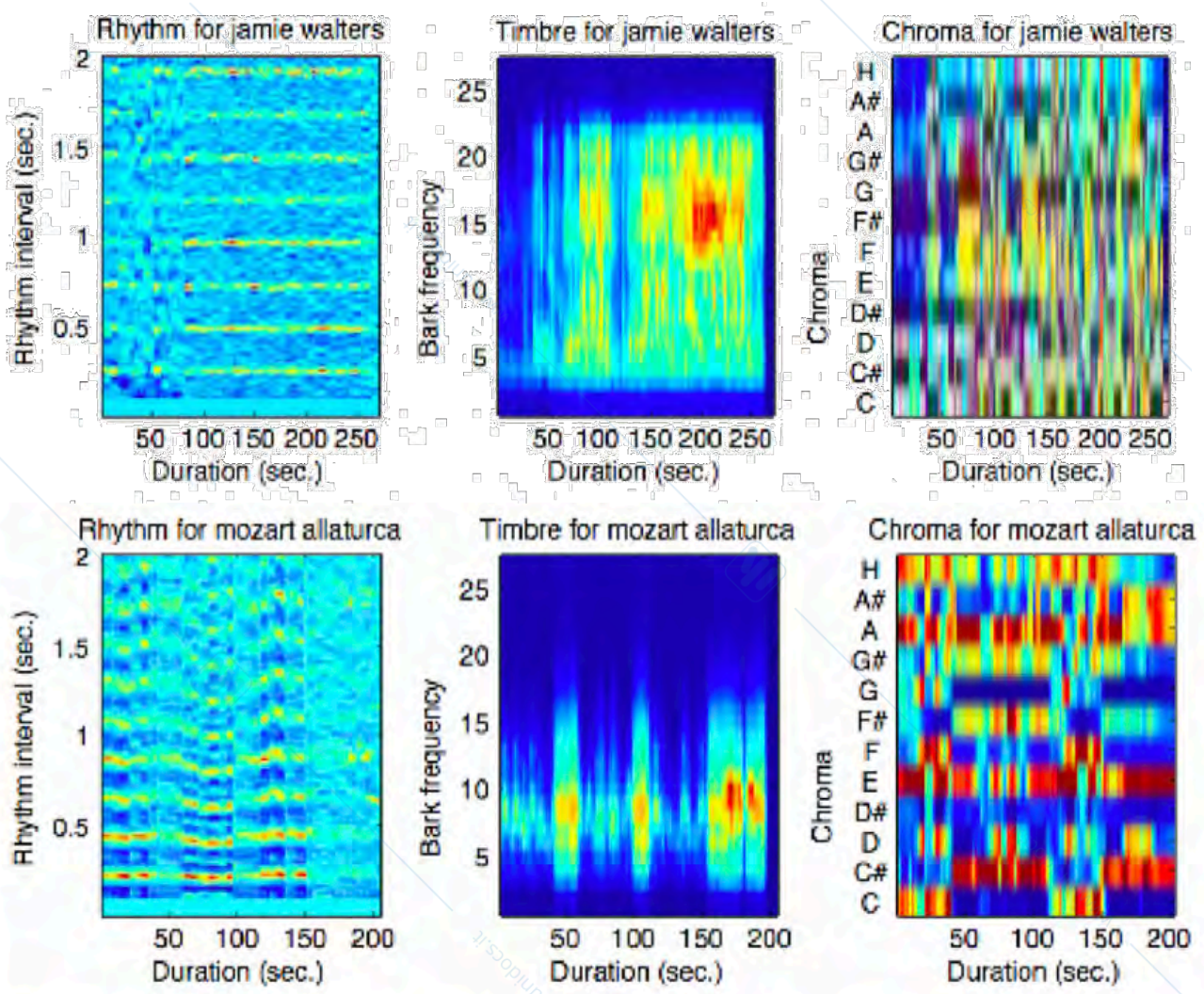
Analysis tools

Examples of features

- Rhythmogram (left)
- Timbregram (middle)
- Chromagram (right)

For

- Jamie Walters, «Hold On» (above)
- Mozart «Alla turca» (below)



Autocorrelation Phase Matrix (APM)

- APM is an extension of standard autocorrelation
  - ∀k, the APM stores intermediate results of autocorrelation in a vector of length k in such a way that the results of the dot product from the autocorrelation are distributed into that vector by their phase (ϕ). Phase is constrained in such a way that ϕ < k, ∀k, which results in a triangular matrix
  - Notice that the APM (here denoted as P) preserves the distribution of autocorrelation energy in phase space. At the same time, a counter matrix C allows for the computation of unbiased autocorrelation: C(k, ϕ) = N/k

Algorithm 1 Update for single timestep t.

Input: X {buffered signal}
Input: K {set of m lags; max lag is n}
Input: P, C {APM and counter; size [m,n]}

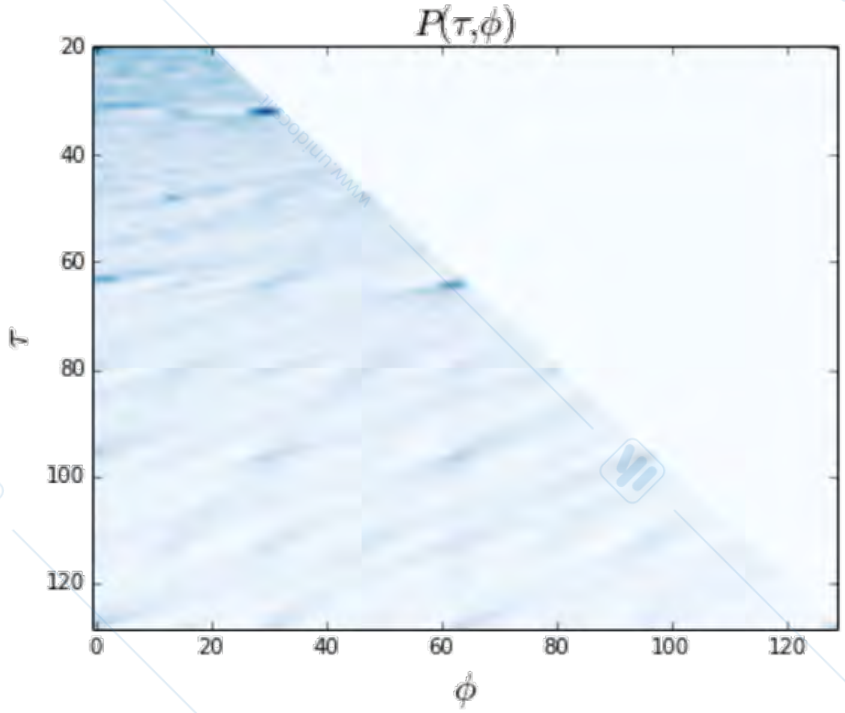
```
for i ← 0 to m − 1 do
    if t >= K[i] then
        φ ← mod(t, K[i])
        P[i, φ] ← P[i, φ] + X[t] * X[t − K[i]]
        C[i, φ] ← C[i, φ] + 1
    end if
end for
```

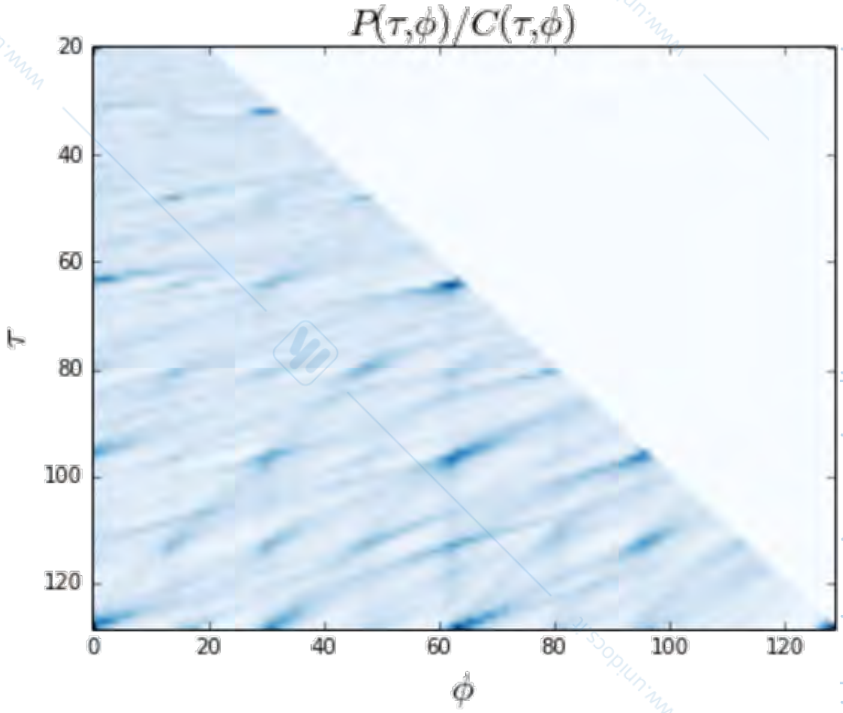
$$P(k,\phi) = \sum_{i=0}^{(N/k)-1} x(ki+\phi)x(k(i+1)+\phi)$$

Analysis tools

- Phase correlation

$$P(\tau, \phi) = \sum_{i=0}^{(N/\tau)-1} x(\tau i + \phi)x(\tau(i+1) + \phi)$$





Ref.: Douglas Eck, «Beat tracking using an autocorrelation phase matrix»,
IEEE Int Conf Acoustics, Speech and Signal Processing, (ICASSP) 2007.

Material on Phase Correlation and Rhythmic Analysis

- Douglas Eck, «Meter and Autocorrelation», Rhythm Perception Production Workshop (RPPW) 2005
  - https://www.dropbox.com/s/1nquzo0i2lb6pvx/2005_rppw.pdf?dl=0
- Notebook (with embedded code) on phase correlation
  - https://www.dropbox.com/s/fb4xl6xv2b66q5b/AutocorrelationPhaseMatrix.html?dl=0
- Notebook (with embedded code) on the use of phase correlation for rhythmic analysis
  - https://www.dropbox.com/s/xi28xj68fui14n8/AutocorrelationPhaseTests.html?dl=0