CONTROL AND ACTUATING DEVICES FOR MECHANICAL SYSTEM

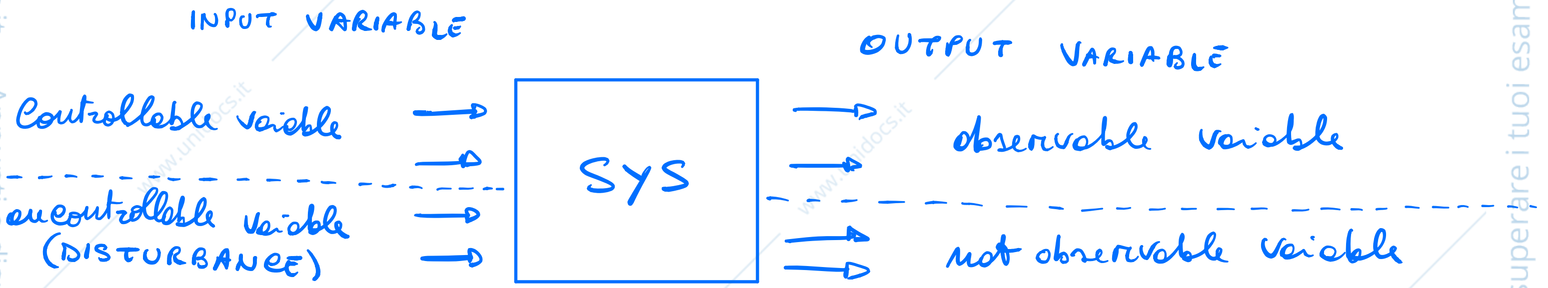
BOOKS:
1) Diana - Cheli (Vol II)
Advanced Dynamics of mechanical system
2) Diana - Resta (Beep)
Material / ref. 2 Control of mech. systems

MAIN TOPICS:
- Introduction (FF e FB)
- Stability force fields, linearization
- Frequency domain (classic control theory)
  - TF (Transfer function, Laplace transform)
  - Nyquist, Bode Criterion
  - Root - Locus
- Dynamic modeling of actuator
  - Hydraulic actuator
  - Electrical actuator (DC motor, AC brushless motor)
  - Pneumatic actuator
- Modern Control (Time domain)
  - Controllability, observability - state mod
  - pole placement

ADVENTAGES

- improve performances: accuracy, repeatability, fast response, better dynamic behaviour
- economical side → less cost
- safety

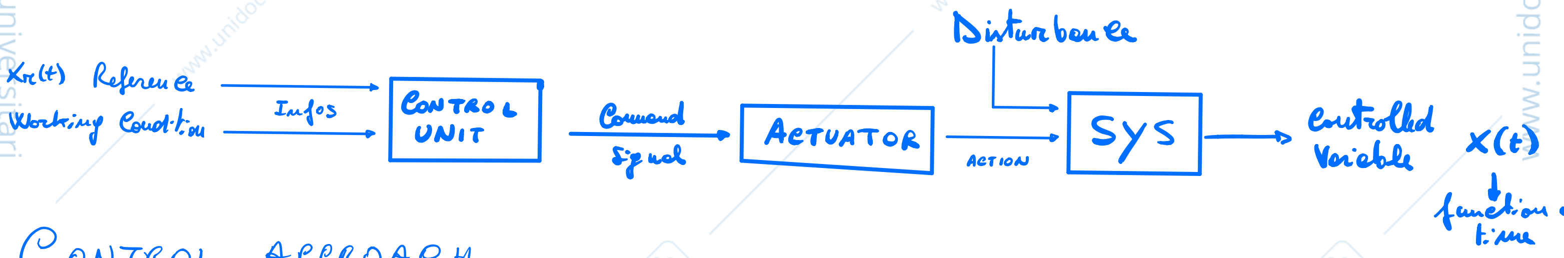
DEFINITION OF CONTROL DYNAMIC SYSTEM



NOTE:
one of the main gols of controll is to let the output variable less depending from uncontrollable variable

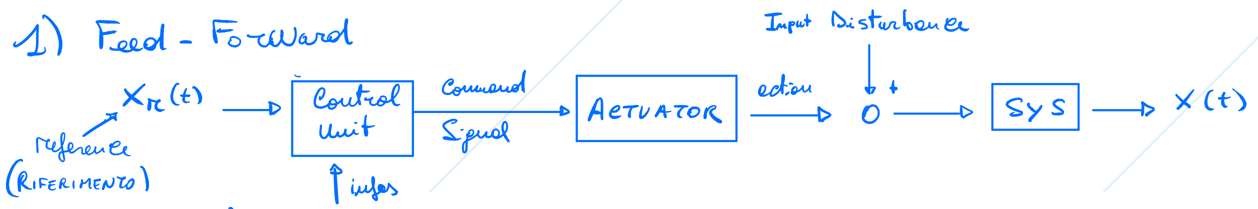
SISO = singol input (control variable) → singol output

MAIN ELEMENTS OF A CONTROLLED SYS



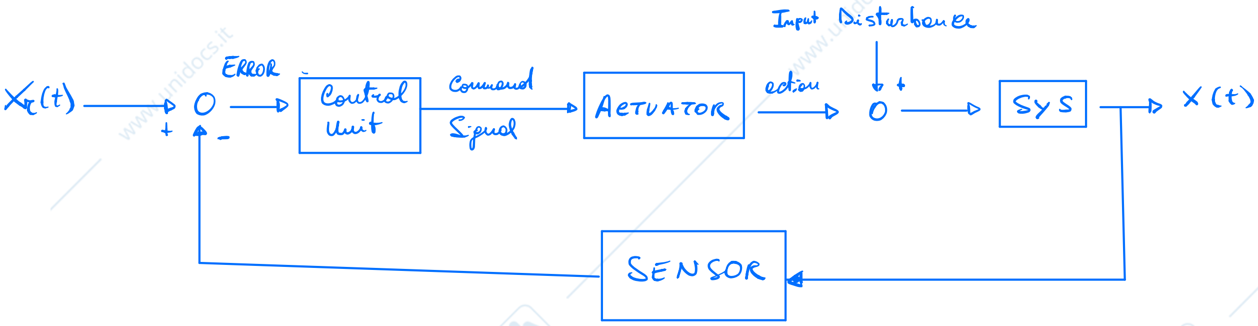
CONTROL APPROACH

1) Feed - Forward



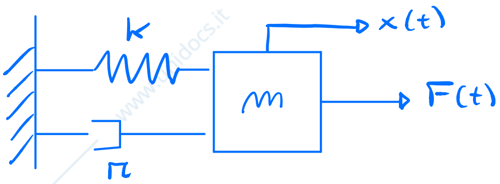
general operating information
- model of the sys.
- environmental parameters

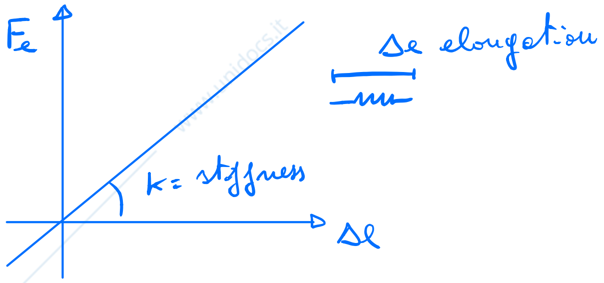
2) Feed - back



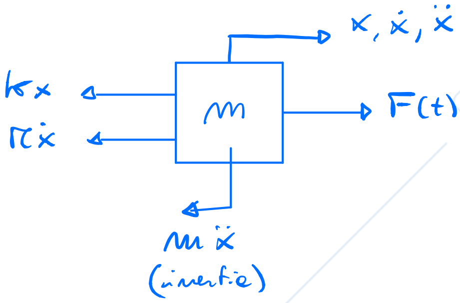
EXAMPLE : single degree of freedom system

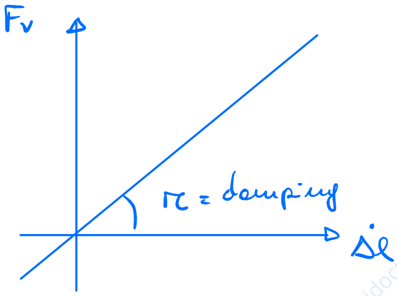
Elastic Force





Viscous Force





ORDINARY DIFFERENTIAL EQUATION

$$m\ddot{x} + r\dot{x} + kx = F(t) = F_0 \cos \Omega t$$

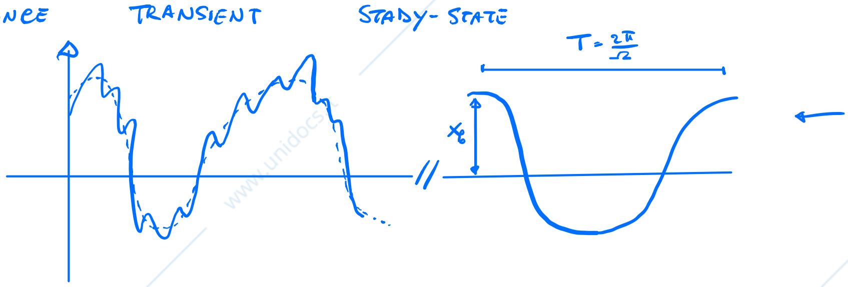
└→ driving frequency

$\Omega = 2\pi/T$ = circular frequency (PULSAZIONE) $[rad/s]$

$f = 1/T$ = frequency (FREQUENZA) $[1/s]$

SOLUTION :

$$\underset{\text{RESPONCE}}{X(t)} = \underset{\text{TRANSIENT}}{X_G(t)} + \underset{\text{STADY-STATE}}{X_p(t)} = \underbrace{e^{-\frac{r}{2\omega}t}\left(A\cos\omega_d t + B\sin\omega_d t\right)} + \underbrace{X_{p_0}\cos(\Omega t + \varphi)}$$



after a certain period we observe just the stady-state responce

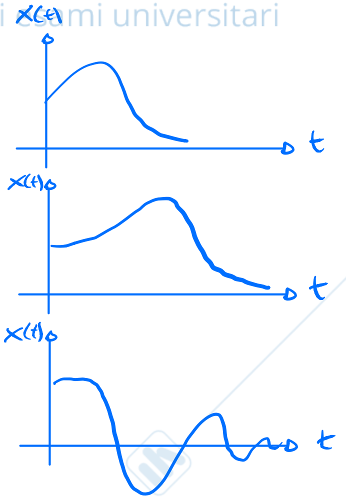
DEFINITIONS

- $\omega_0 = \sqrt{\frac{k}{m}}$ $\left[\frac{rad}{s}\right]$ → natural frequency
- $r_{cr} = 2m\omega_0$ → critical damping
- $h = \frac{r}{r_{cr}} = \frac{r}{2m\omega_0}$ → damping ratio
- $\omega_d = \omega_0\sqrt{1-h^2}$ (if $h<1$)

$h<1$

$h=1$

$h>1$



TRANSIEN

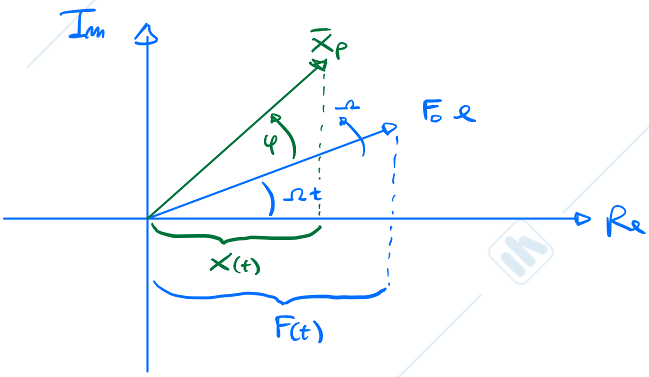
The solution is:

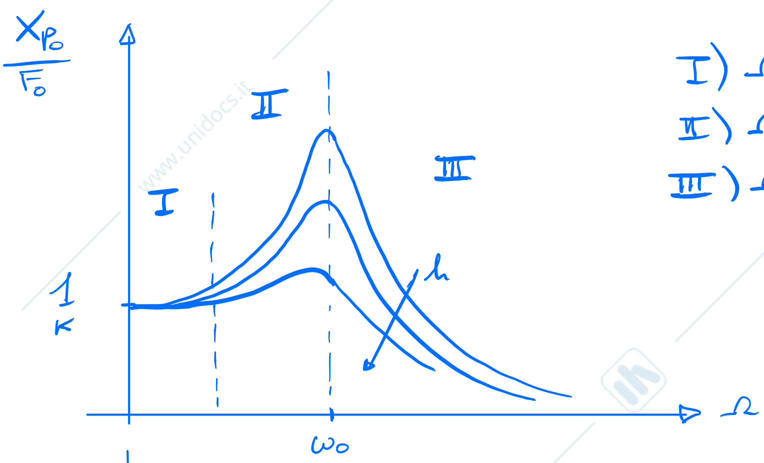
$$x_{p_0} = \frac{F_0}{\sqrt{(k-m\Omega^2)^2+(r\Omega)^2}}$$

$$\tan\varphi = \frac{r\Omega}{k-m\Omega^2}$$

$$X(t) = x_{p_0}\cos(\Omega t+\varphi) = Re\left(\bar{x}_p e^{i\Omega t}\right) = Re\left(x_p e^{i\varphi} e^{i\Omega t}\right)$$

$$F(t) = F_0\cos\Omega t = Re\left(F_0 e^{i\Omega t}\right)$$

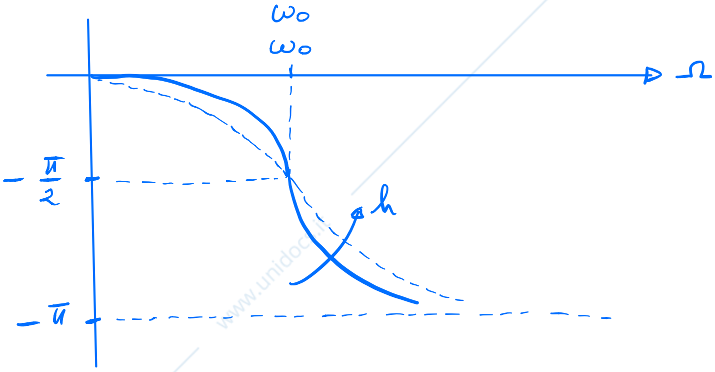




I) $\Omega \ll \omega_0$ RIGID AREA

II) $\Omega \approx \omega_0$ RESONANCE AREA

III) $\Omega > \omega_0$ SEISMIC AREA



APPLICATION OF SUPERPOSITION PRINCIPLE ⟷ LINEAR SYSTEM

→ Principio di sovrapposizione degli effetti

$$F(t) = \sum_m^{\infty} Re\left(\bar{F}_m e^{i\Omega_m t}\right)$$

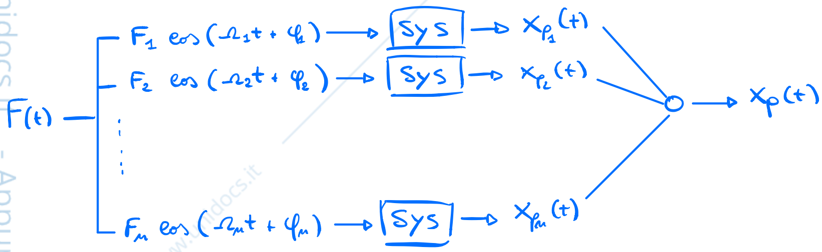
↑ periodic

$$\Omega_m = m\Omega_0$$

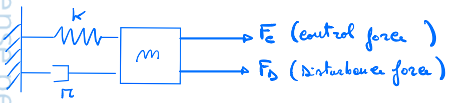
$$X(t) = \sum_m^{\infty} Re\left(\bar{x}_{p_m} e^{i\Omega_m t}\right)$$

$$\dot{x}(t) = \sum_m^{\infty} Re\left(i\Omega_m \bar{x}_{p_m} e^{i\Omega_m t}\right)$$

FOURIER SERIES :



• Now examin the feed-forward approach (FF) (Considering like reference the motion $x_r(t)$)



$$\Longrightarrow m\ddot{x} + r\dot{x} + kx = F_c + F_D(t)$$

Reference motions = $\underbrace{\left.\begin{matrix} x_r(t) \\ \dot{x}_r(t) \\ \ddot{x}_r(t) \end{matrix}\right\}}_{\text{known}}$ → it is the motion that we want to ask to the system

CONSIDER THE REQUIRED FORCE $F_c(t)$

$$m\ddot{x}_r + r\dot{x}_r + kx_r = F_c(t)$$

→ but is the actuator able to provide a force like this?

$$x_r(t) = \sum_m Re\left(\bar{x}_{p_{r,m}} e^{i\Omega_m t}\right)$$

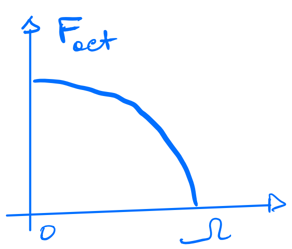
$$\dot{x}_r(t) = \sum_m Re\left(i\Omega_m \bar{x}_{p_{r,m}} e^{i\Omega_m t}\right)$$

$$\ddot{x}_r(t) = \sum_m Re\left(-\Omega_m^2 \bar{x}_{r,m} e^{i\Omega_m t}\right)$$

Substituting

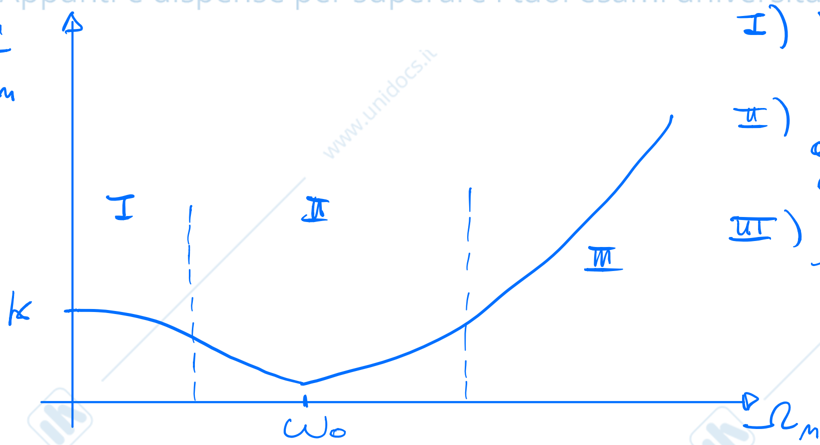
$$\boxed{\left(-\Omega_m^2 m + i\Omega_m r + k\right)\bar{x}_{r,m} = F_{c,m}}$$

PERFORMANCE IN ACTUATOR



$$\left|\bar{F}_{c,m}\right| = F_{c,m} = \sqrt{\underbrace{\left(k - m\Omega_m^2\right)^2}_{Re} + \underbrace{\left(r\Omega_m\right)^2}_{Im}} \cdot \left|\bar{x}_{r,m}\right|$$

$$\frac{\text{required force}}{\text{required displacement}} = \frac{F_{e,m}}{X_{r,m}}$$



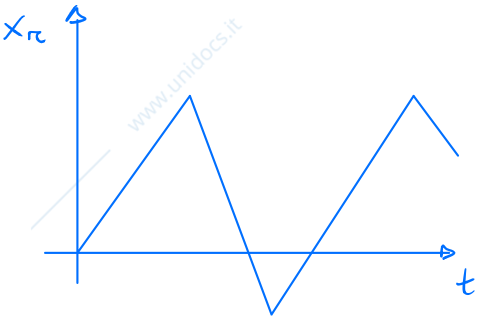
I) the system work well

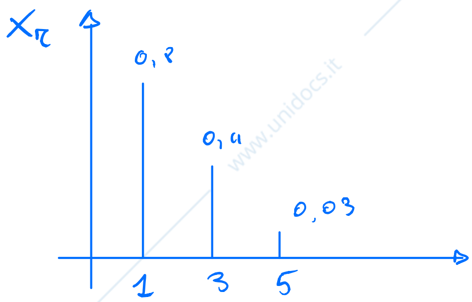
II) there is some uncertency because we are in resonance

III) We need a big force even for a low displacement

EXAMPLE

$m = 10\,kg$ $\quad K = 1000\ N/m$ $\quad r = 1\ Ns/m$

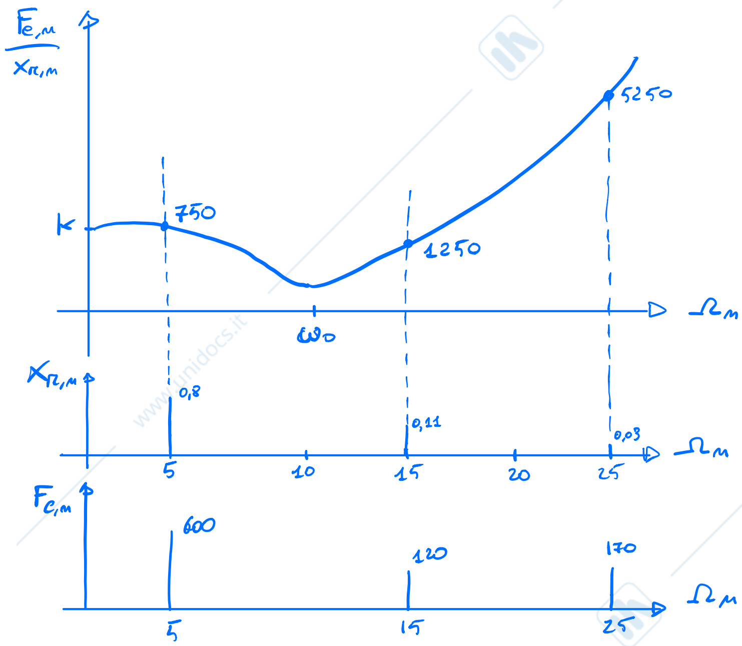




$$\frac{8}{\pi^2} = 0,8$$

$$\frac{8}{3^2\pi^2} = 0,11$$

$$\frac{8}{5^2\pi^2} = 0,03$$



→ DISPLACEMENT

→ ACTUATOR FORCE THAT IS REQUIRED

So if I know the displacement I could calculate the required force and "the other way around" (VICEVERSA)

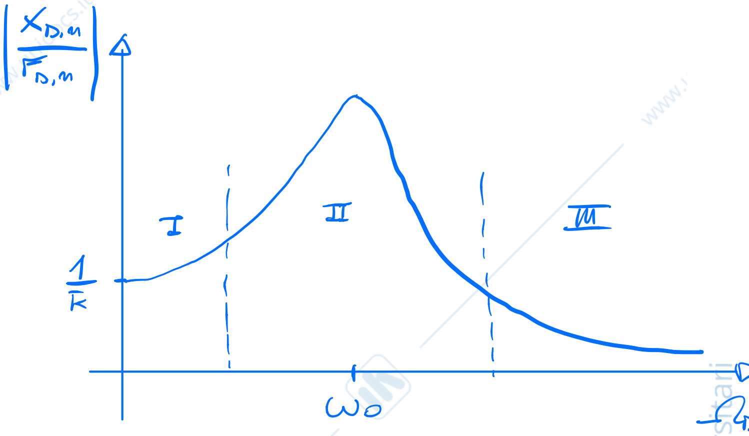
CONSIDER THE DISTURBANCE FORCE $F_D(t)$

$$m\ddot{x}_R + r\dot{x}_R + Kx_R = F_D(t)$$

$$F_D(t) = \sum_m Re\left(\bar{F}_{D,m}\, e^{i\Omega_{D,m} t}\right)$$

$$m\ddot{x} + r\dot{x} + Kx = F_{D,m}\cos(\Omega_{D,m} t + \varphi_{D,m})$$

$$|\bar{X}_{D,m}| = \frac{F_{D,m}}{\sqrt{(k - m\Omega_{D,m}^2)^2 + (r\Omega_{D,m})^2}}$$



Now we can focus on the frequency range of the disturbance

So the 2 main point of this analisys are:
1) find the actuator force
2) focus on the frequency of the disturbance

• Now examin the feed-back approach (FB) : Unlike the FF approach the FB approach has the porpouse to adjust the output if it is "wrong"

$$m\ddot{x} + r\dot{x} + kx = F_e + F_D(t) = k_p(x_r - x_c) + r_D(\dot{x}_r - \dot{x}) + F_D(t)$$

$$m\ddot{x} + (r + r_D)\dot{x} + (k + k_p)x = \underbrace{k_p x_r(t) + r_D \dot{x}_r(t)}_{\text{equivalent force } = F_e} + F_D(t)$$

[So the consequence of this approach is (adding a new damping and a new spring) to control the output and to modificate the result if is required]

• Response to F_e (Tracking)

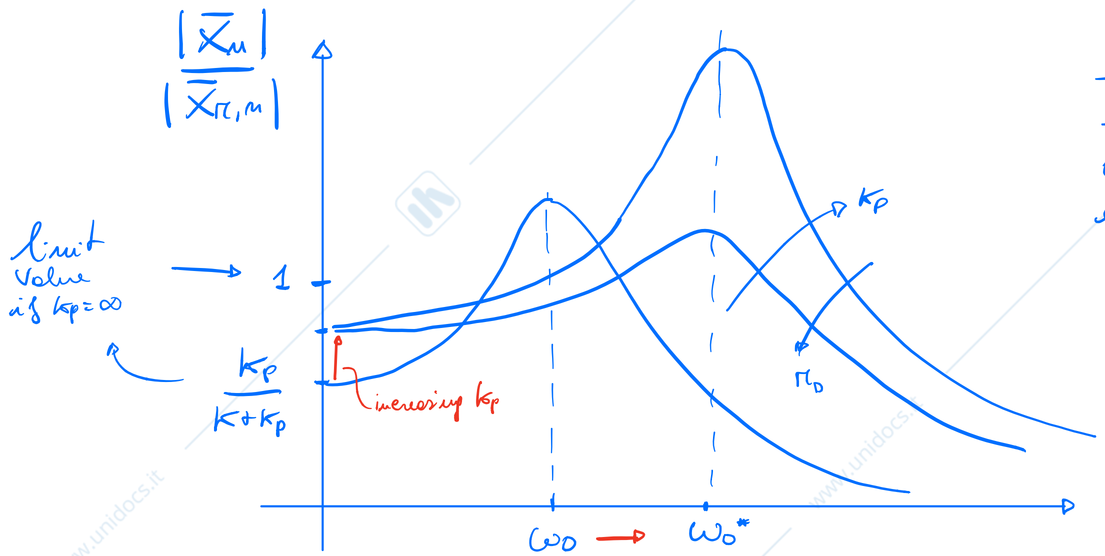
n-th component

$$(-m\Omega_n^2 + i\Omega_n r + k)\bar{X}_n = k_p \bar{x}_{r,n} + r_D i\Omega_n \bar{x}_{r,n}$$

Portion of F_e : $F_{e,n} = \sqrt{k_p^2 + (\Omega_n r_D)^2}\;|\bar{X}_{r,n}|$

$$|\bar{X}| = x_n = \frac{\sqrt{k_p^2 + (r\Omega_n)^2}}{\sqrt{(k + k_p - m\Omega_n^2)^2 + ((r + r_D)\Omega_n)^2}}$$

(so we have modify the curve)



The higher k_p the lower error we have because error $= 1 - \frac{k_p}{k+k_p}$

$$\omega_0 = \left|\frac{k + k_p}{m}\right|$$

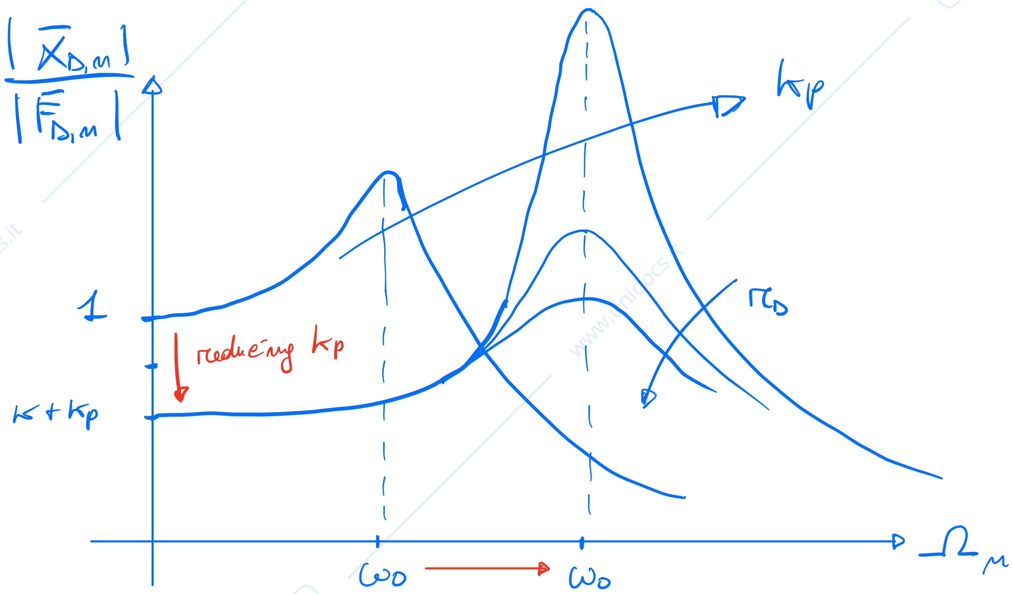
$$h = \frac{r + r_D}{2m\omega_0} = \frac{r + r_D}{2\sqrt{m(k + k_p)}}$$

Response to $F_D(t)$

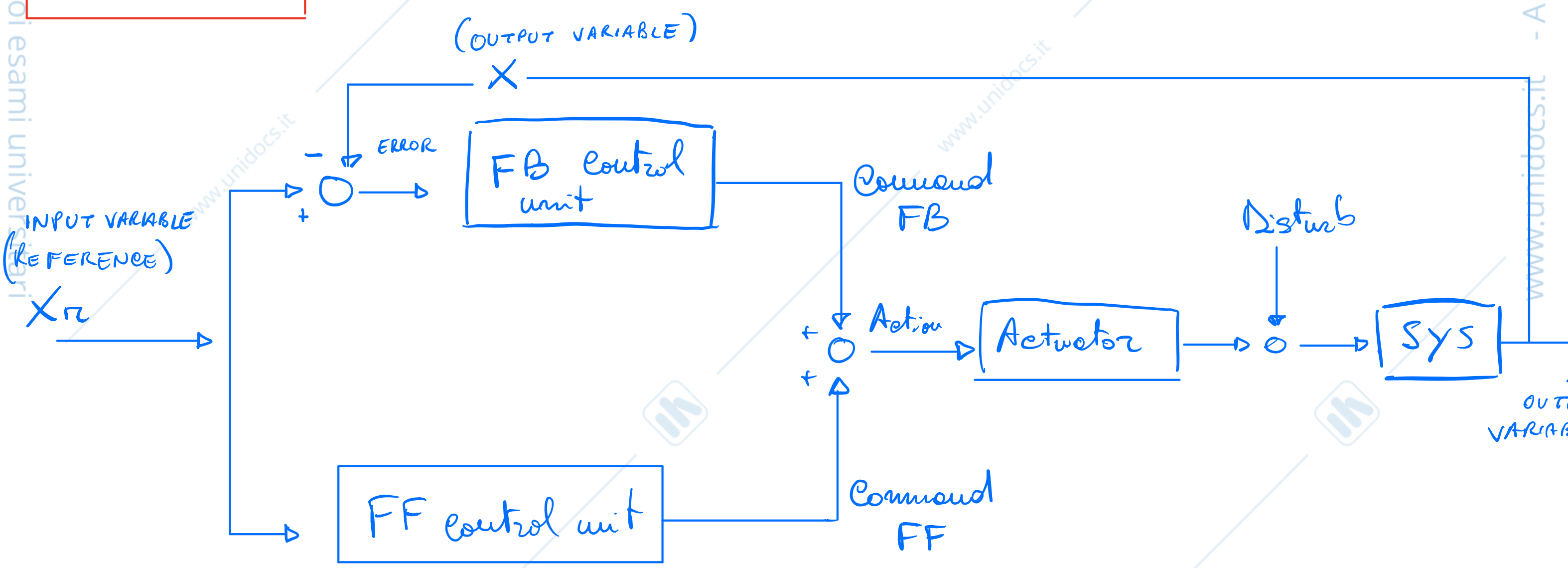
$$(M+M_D)\dot{x} + (k+k_p)x = F_D(t)$$

With Fourier n-th component

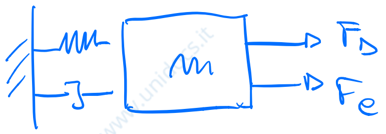
$$\left|\bar{X}_{D,n}\right| = \frac{\left|F_{D,n}\right|}{\sqrt{\left(k+k_p-M\Omega_n^2\right)^2+\left((M+M_D)\Omega_n\right)^2}}$$



$\boxed{FF + FB}$



Corresponding equations



$$m\ddot{x} + r\dot{x} + kx = F_e + F_D(t)$$

$$m\ddot{x} + r\dot{x} + kx = \underbrace{m\ddot{x}_r + r\dot{x}_r + kx_r}_{F_{e,FF}} + \underbrace{k_p(x_r - x) + r_D(\dot{x}_r - \dot{x})}_{F_{e,FB}} + F_D(t)$$

$$F_D(t) = \sum_m Re\left(F_{D,m}\, e^{i\Omega_{D,m}t}\right)$$

Including a new variable:

DEVIATION FROM THE REFERENCE: $\tilde{x} = x - x_r = -e$ (MOTION, REFERENCE, ERROR)

$$\dot{\tilde{x}} = \dot{x} - \dot{x}_r$$
$$\ddot{\tilde{x}} = \ddot{x} - \ddot{x}_r$$

So:

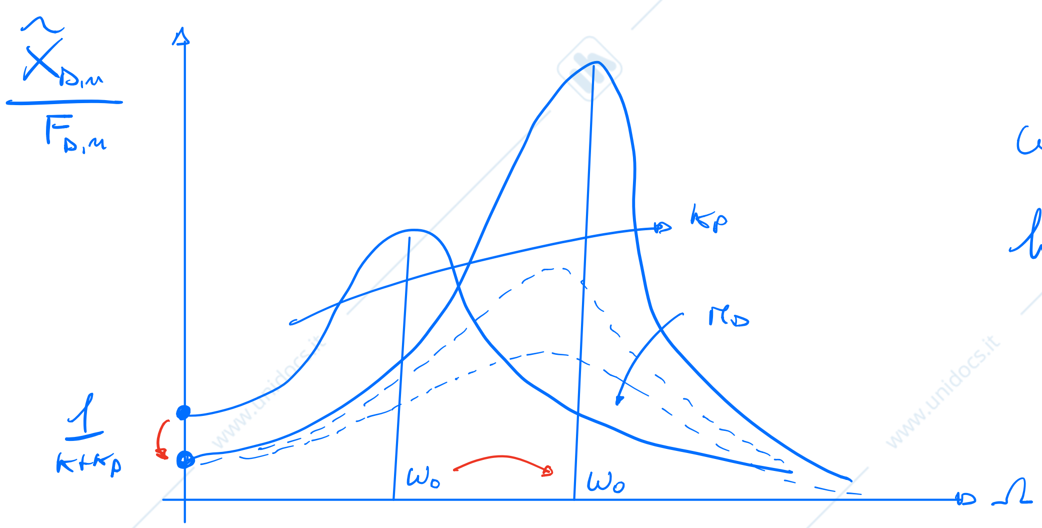
$$m\ddot{\tilde{x}} + r\dot{\tilde{x}} + k\tilde{x} = -k_p\tilde{x} - r_D\dot{\tilde{x}} + F_D(t)$$

$$m\ddot{\tilde{x}} + (r + r_D)\dot{\tilde{x}} + (k + k_p)\tilde{x} = F_D(t)$$

We can use Fourier Series introducing generic element $\tilde{X}_{D,m}$ for each m-th components and $F_{D,m}$ in the same way

$$\left[-m\Omega_{D,m}^2 + i(r + r_D)\Omega_{D,m} + (k + k_p)\right]\tilde{X}_{D,m}\cancel{e^{i\Omega_{D,m}t}} = F_{D,m}\cancel{e^{i\Omega_{D,m}t}}$$

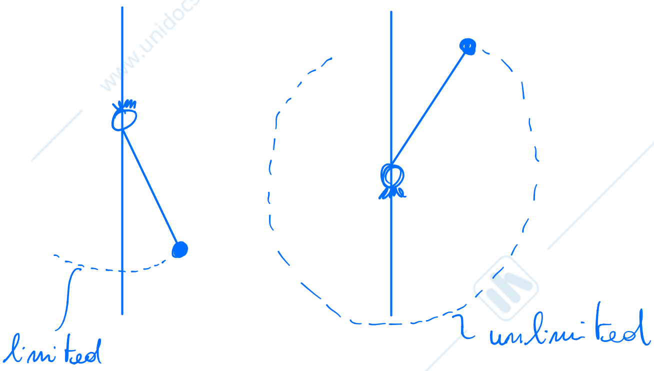
$$\left|\tilde{X}_{D,m}\right| = \frac{F_{D,m}}{\sqrt{\left(k + k_p - m\Omega_{D,m}^2\right)^2 + \left((r + r_D)\Omega_{D,m}\right)^2}}$$



$$\omega_0 = \sqrt{\frac{k + k_p}{m}}$$

$$h = \frac{r + r_D}{2\sqrt{m(k + k_p)}}$$

STABILITY OF EQUILIBRIUM POSITION OF THE SYSTEM



In a mathematical point of view:

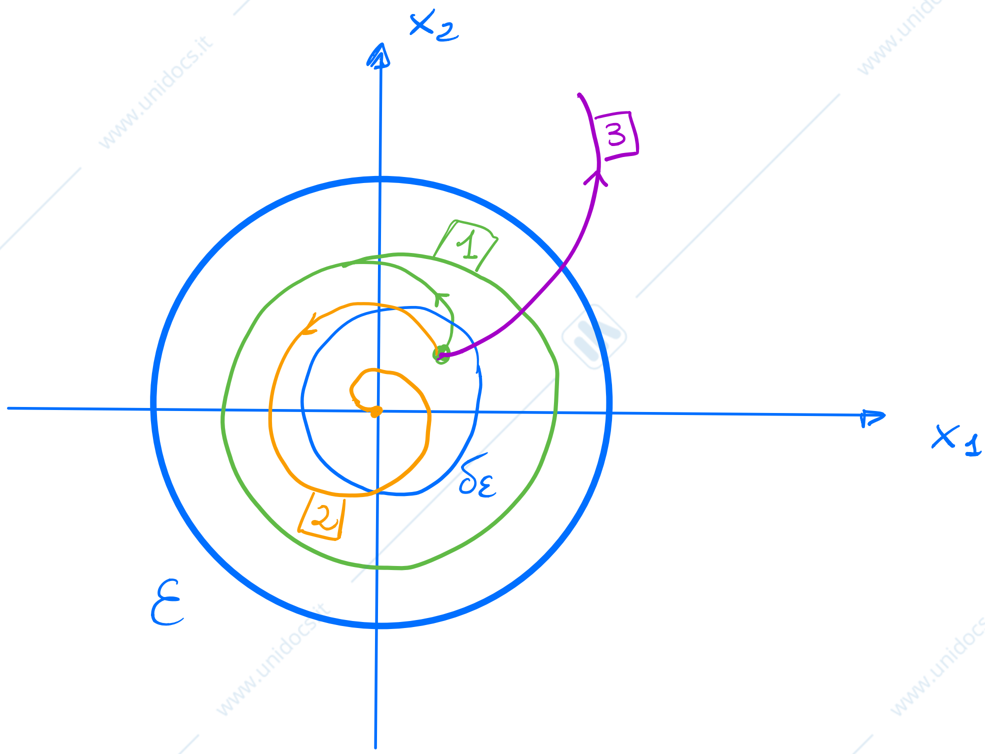
$m\ddot{x} = f(\dot{x}, x) \longrightarrow$ <u>static equilibrium position</u> $\Rightarrow \dot{x} = 0$ and $\ddot{x} = 0 \Rightarrow$

$\Rightarrow 0 = f(0, x) \longrightarrow x_0$ (more than 1 solution!)

$x_\Delta = x - x_0 \longrightarrow m\ddot{x}_\Delta = f(\dot{x}_\Delta, x_\Delta)$

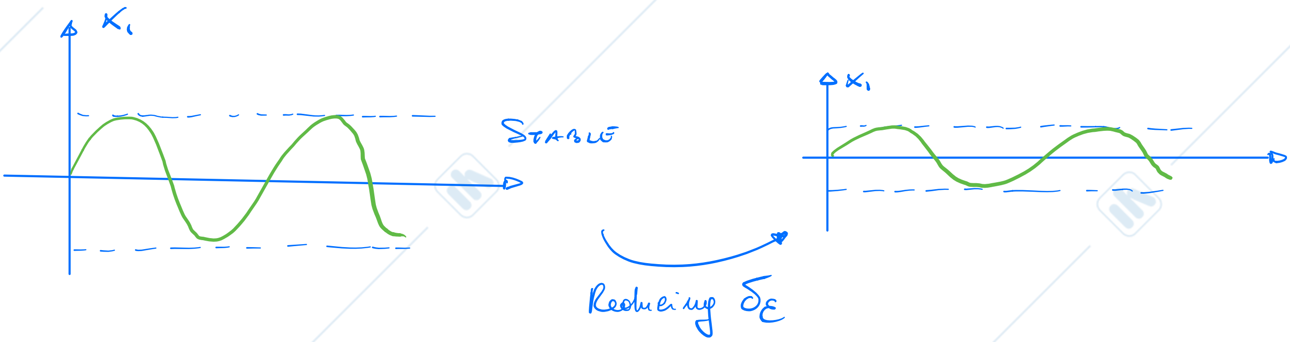
Define state vector $\underline{x}_\Delta = \begin{Bmatrix} x_\Delta \\ \dot{x}_\Delta \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$

<u>State plane (piano delle fasi)</u>

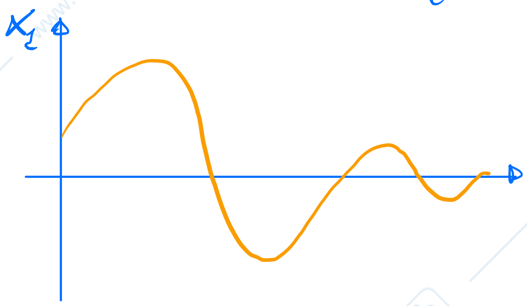


1) define a boundary $\mathcal{E}$ (small as we like):

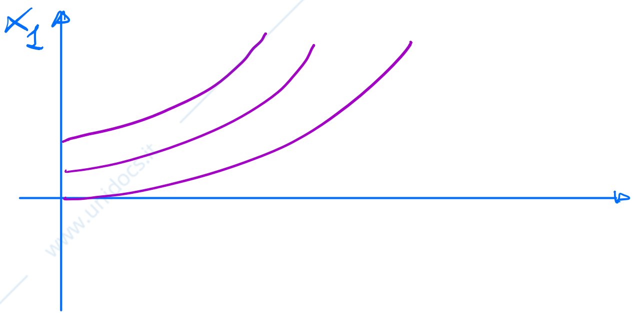
IF a boundary $\delta_{\mathcal{E}}$ for initial conditions $\exists$, and when $\| \underline{X}_{\Delta,0} \| < \delta_{\mathcal{E}}$, and $\| \underline{X}(t) \| < \mathcal{E}$ $\forall t > 0$ THEN equilibrium position is STABLE



2) if 1) holds, and $\lim_{t \to \infty} \| \underline{x} \| = 0$ $\xRightarrow{\text{Then}}$ the equilibrium position is ABSINTOTICALLY STABLE



3) if not 1) $\Longrightarrow$ equilibrium position is ANSTABLE



$m\ddot{x} = f(\dot{x}, x)$ Non linear system $\Rightarrow$ to solve this we have to:

find static equilibrium $f(0,x)=0 \longrightarrow$ eq. pos. (s) x_0

$x_D = x - x_0$; $\dot{x}_D = \dot{x}$; $\ddot{x}_D = \ddot{x}$

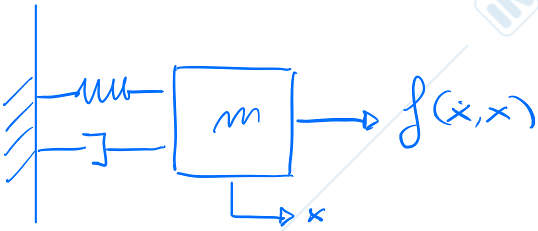
$$m\ddot{x}_D = f(\dot{x}_D, x_D + x_0) = \text{LINEAR} = f(0,x_0) + \left.\frac{\partial f}{\partial \dot{x}}\right|_0 \dot{x}_D + \left.\frac{\partial f}{\partial x}\right|_0 x_D$$

$$m\ddot{x}_D - \left.\frac{\partial f}{\partial \dot{x}}\right| \dot{x}_D - \left.\frac{\partial f}{\partial x}\right| x_D = 0$$

LIN. SYS	NOT LIN. SYS
Asintotically Stable	Asintotically Stable
Unstable	Unsta
Stable	?

$\longrightarrow$ (We want to have an asintotically stable system)

APPLICATION TO SINGLE DEGREE OF FREDOM SYSTEM



$$m\ddot{x} + r\dot{x} + kx = f(\dot{x}, x)$$

1) static eq. position $kx = f(0,x) \longrightarrow x_0 \Rightarrow x_D = x - x_0$; $\dot{x}_D = \dot{x}$; $\ddot{x}_D = \ddot{x}$

2) Linearization $m\ddot{x}_D + r\dot{x}_D + kx_D + \cancel{kx_0} = f(0,\cancel{x_0}) + \left.\frac{\partial f}{\partial \dot{x}}\right|_0 \dot{x}_D + \left.\frac{\partial f}{\partial x}\right|_0 x_D + \ldots$

$$\ddot{x}_D + \underbrace{\frac{1}{m}\left(r - \left.\frac{\partial f}{\partial \dot{x}}\right|_0\right)}_{p} \dot{x}_D + \underbrace{\frac{1}{m}\left(k - \left.\frac{\partial f}{\partial x}\right|_0\right)}_{q} x_D = 0$$

$$\ddot{x}_D + p\dot{x}_D + q x_D = 0$$

$x_D = Ae^{\lambda t}$; $\dot{x}_D = \lambda A e^{\lambda t}$; $\ddot{x}_D = \lambda^2 A e^{\lambda t}$

$$(\lambda^2 + p\lambda + q) A e^{\lambda t} = 0 \Rightarrow \underbrace{\lambda^2 + p\lambda + q = 0}_{\text{CARATTERISTIC EQUATION}} \Rightarrow \boxed{\underbrace{\lambda_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}}_{\text{SOLUTIONS}}}$$

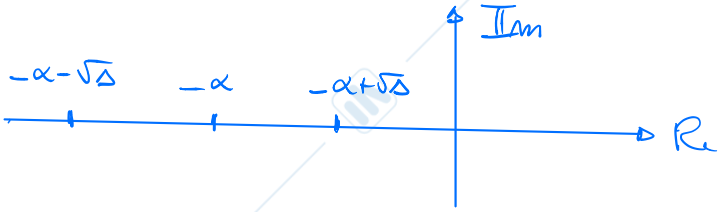
Now analyze the different solution depending from p and q:

(I) $p>0$; $q>0$ $\qquad \lambda_{1,2} = -\alpha \pm \sqrt{\Delta}$

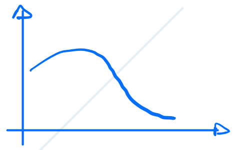
Ia) $\sqrt{\Delta} < \alpha$ if $\Delta > 0$

$\lambda_1 = -\alpha - \sqrt{\Delta} < 0$

$\lambda_2 = -\alpha + \sqrt{\Delta} < 0$ (because $\sqrt{\Delta} < \alpha$)

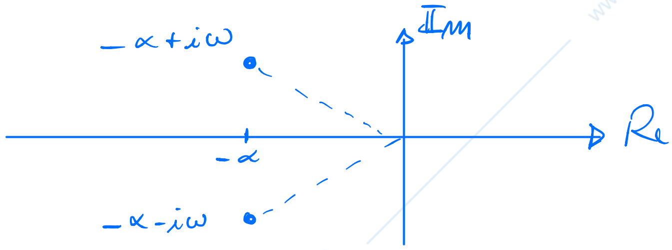


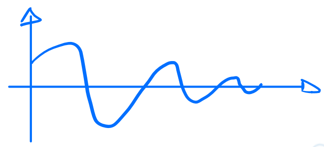
$$\boxed{x_h(t) = A e^{\lambda_1 t} + B e^{\lambda_2 t}}$$



Asymptotically Stable

Ib) if $\Delta < 0$ $\qquad \sqrt{\Delta} = \pm i \sqrt{-\Delta} = \pm i\omega \Rightarrow \lambda_{1,2} = -\alpha \pm i\omega$





Asymptotically Stable

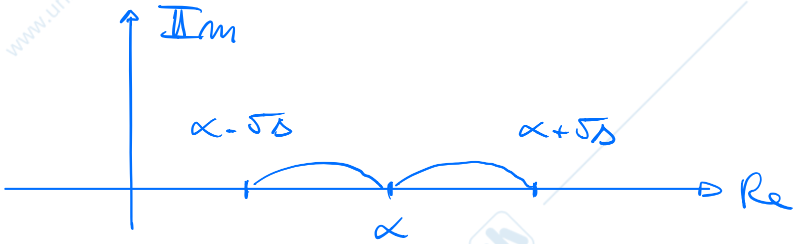
(II) $p<0$; $q>0$ $\qquad \lambda_{1,2} = \alpha \pm \sqrt{\Delta}$

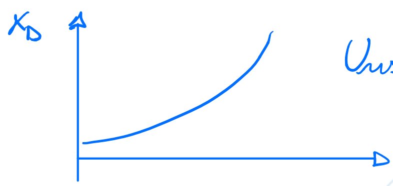
IIa) $\Delta > 0$; $\sqrt{\Delta} < |\alpha|$

$\lambda_1 = \alpha + \sqrt{\Delta} > 0$

$\lambda_2 = \alpha - \sqrt{\Delta} > 0$

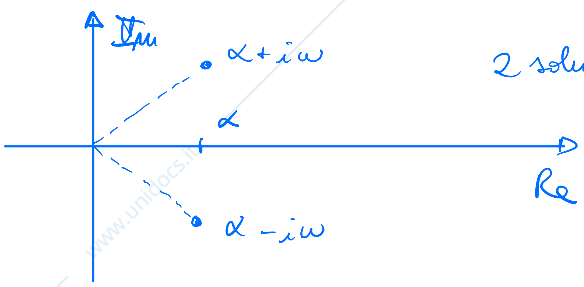
2 positive exponential



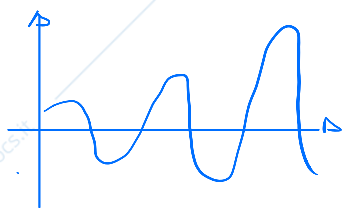


Unstable

IIb) $\Delta < 0$; $\pm\sqrt{\Delta} = \pm i\sqrt{-\Delta} = \pm i\omega$; $\lambda_{1,2} = \alpha \pm i\omega$



2 solution Re > 0

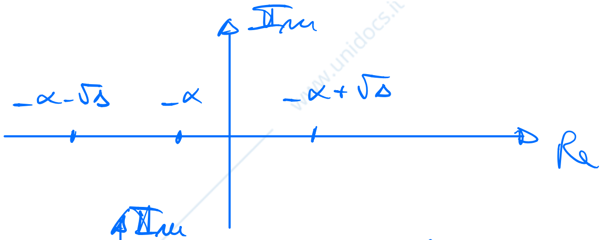


Dinamic Instability

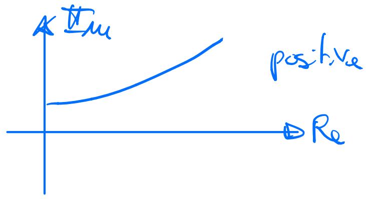
III $\rho < 0$; $p > 0$ $\lambda_{1,2} = -\alpha \pm \sqrt{\Delta}$ $\sqrt{\Delta} > \alpha$

$\lambda_1 = -\alpha + \sqrt{\Delta}$

$\lambda_2 = -\alpha - \sqrt{\Delta}$

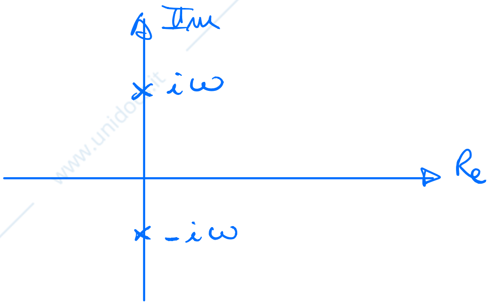


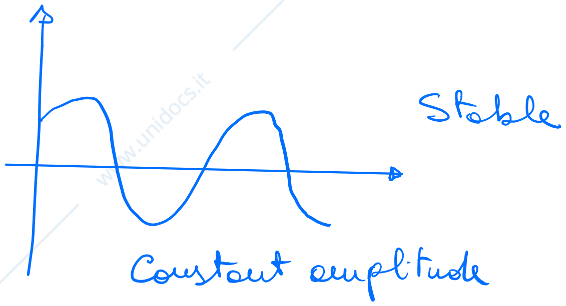
$\Rightarrow$ $\lambda_1 > 0$, $\lambda_2 < 0$



$x_D(t) = A e^{\lambda_1 t} + B e^{\lambda_2 t}$

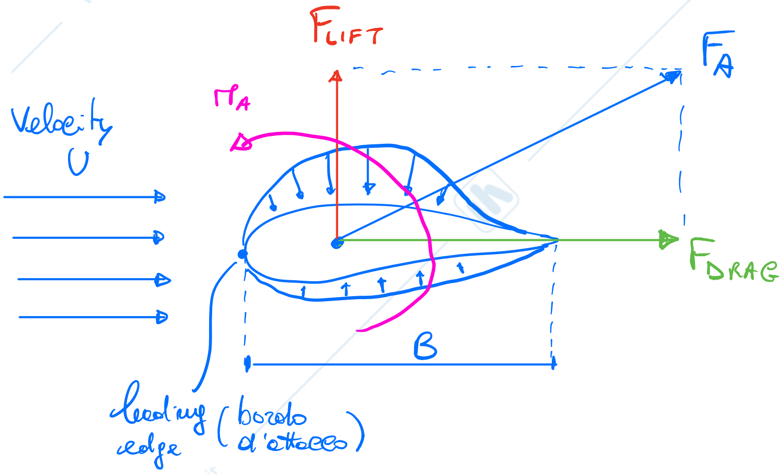
IV $\rho = 0$; $p > 0$ $\lambda_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} = \pm\sqrt{-q} = \pm i\omega$

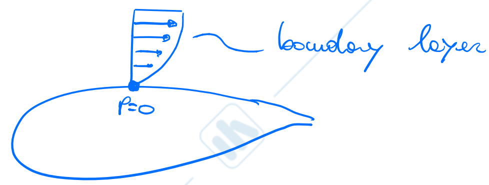


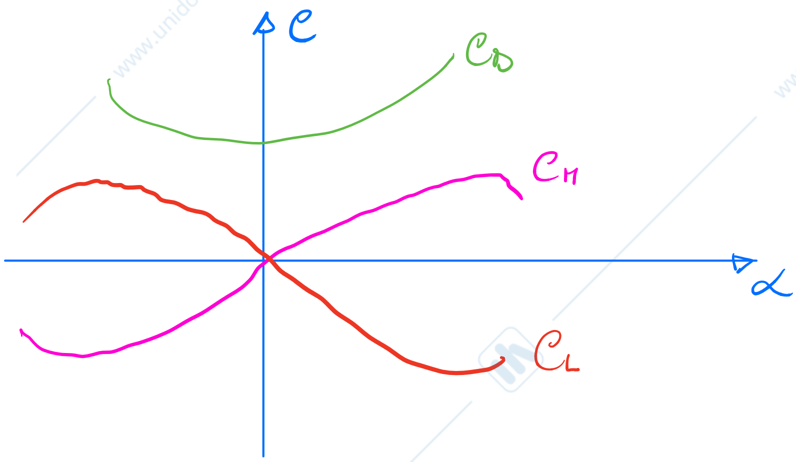


$x_D(t) = A \cos\omega t + B \operatorname{sen}\omega t$

FLUID - BODY INTERACTION







$$F_D = \frac{1}{2} \rho \, S \, C_D(\alpha) \, U^2$$

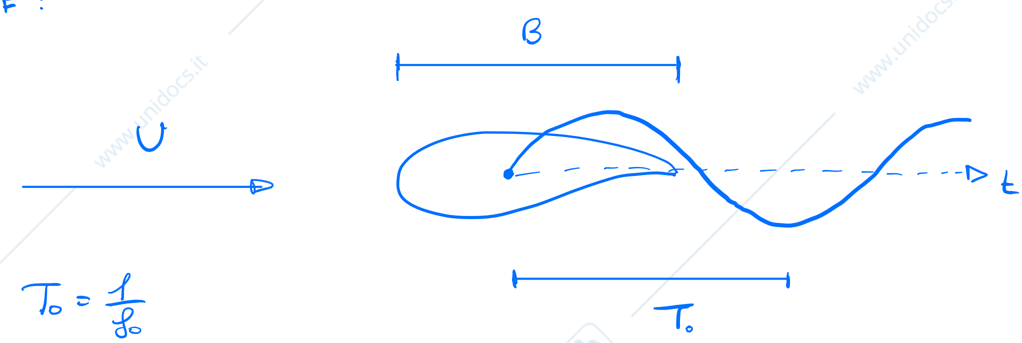
reference surface — Drag coefficient — velocity of the fluid

$$F_L = \frac{1}{2} \rho \, S \, C_L(\alpha) \, U^2$$

$$M_A = \frac{1}{2} \rho \, S \, B \, C_M(\alpha) \, U^2$$

lenght of the body in the fluid — moment coefficient

If :

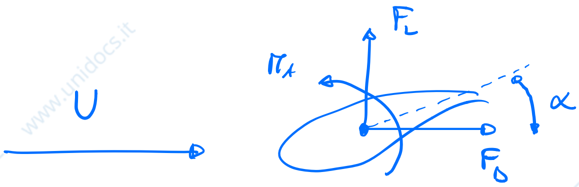


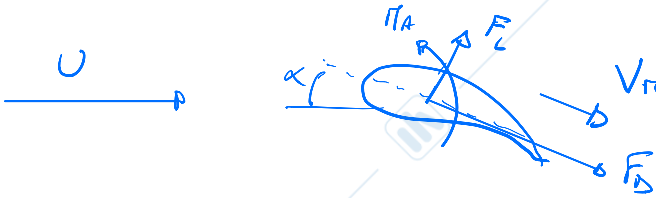
$$T_0 = \frac{1}{f_0}$$

$$\frac{T_0}{B/U} = \frac{U}{fB} = V^* = \text{reduced speed} > 10 \div 15$$

if $V^* > 10 \div 15 \Longrightarrow$ we can use coeff. determinated in Steady State

IF STADY STATE CASE

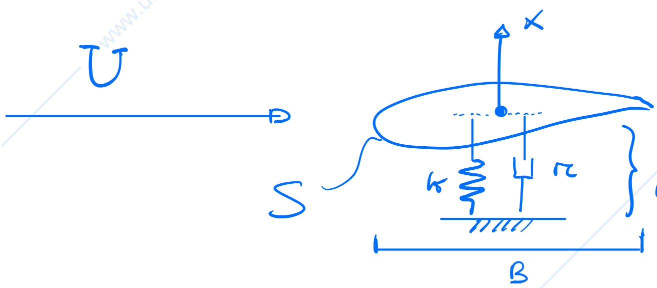




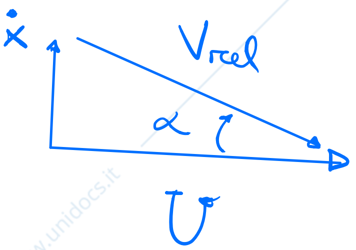
V_{rel} = relative fluid flow ⇒

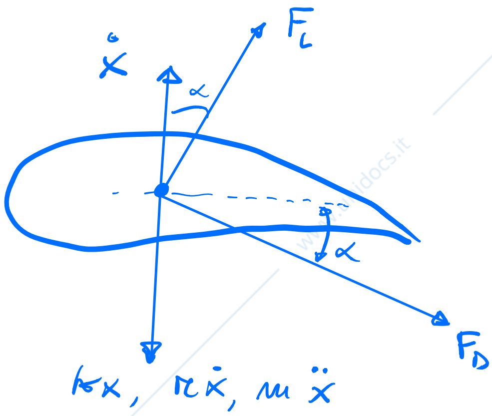
⇒ F_D is orientated like the Relative fluid flow!

EXAMPLE : Translating body (profile) under fluid action



to modeling the wind we use a spring and a damper





(F_D is orientated like the relative speed)

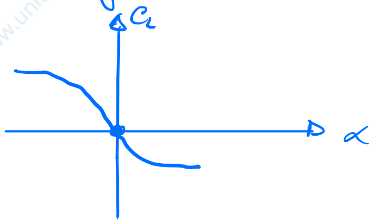
$$-m\ddot{x} - r\dot{x} - kx - F_D \sin\alpha + F_L \cos\alpha = 0$$

$$(*) \quad \boxed{m\ddot{x} + r\dot{x} + kx = F_L \cos\alpha - F_D \sin\alpha} = f(\dot{x})$$

$$\tan\alpha = \frac{\dot{x}}{U} \approx \alpha$$

Static condition

substituting in (*) $\Rightarrow$ $kx = F_L$ if $C_L \neq 0$ in $\alpha = 0$ $\Rightarrow$ $x_0 = \dfrac{F_{L0}}{k}$



but in our case $\alpha = 0 \Rightarrow C_L = 0 \Rightarrow x_0 = 0$

Linearization

$$m\ddot{x} + r\dot{x} + kx = \frac{1}{2}\rho S C_L(\alpha) V_{rel}^2 \cos\alpha - \frac{1}{2}\rho S C_D(\alpha) V_{rel}^2 \,\mathrm{sen}\,\alpha =$$

imposing $\frac{1}{2}\rho S = q$

we can eliminate this terms because we want a linear equation

$$= q\Big[C_{L0} + \underbrace{\frac{dC_L}{d\alpha}\Big|_0}_{k_L}(\alpha - \alpha_0) + \dots\Big]\Big[U^2 + \cancel{\dot{x}^2}\Big]\Big[1 - \cancel{\frac{\alpha^2}{2}} + \dots\Big] - q\Big[C_{D0} + \frac{dC_D}{d\alpha}\Big|_0(\alpha - \alpha_0) + \dots\Big]\Big[U^2 + \cancel{\dot{x}^2}\Big]$$

$$= q C_{L0} U^2 + q k_L \frac{\dot{x}}{U} U^2 - q C_{D0} U^2 \frac{\dot{x}}{U} - \cancel{q k_D \alpha U^2 \alpha}$$

is not linear term because we have α^2

$$= q C_{L0} U^2 + q(k_L - C_{D0}) U \dot{x}$$

So:

$$\boxed{m\ddot{x} + r\dot{x} + kx = \overbrace{q C_{L0} U^2}^{F_{L0} = k x_0} + q U (k_L - C_{D0})\dot{x}}$$

$$m\ddot{x} + r\dot{x} + kx = kx_0 + qU(k_L - C_{D0})\dot{x}$$

$$m\ddot{x} + r\dot{x} + k\underbrace{(x - x_0)}_{x_D} = 〃$$

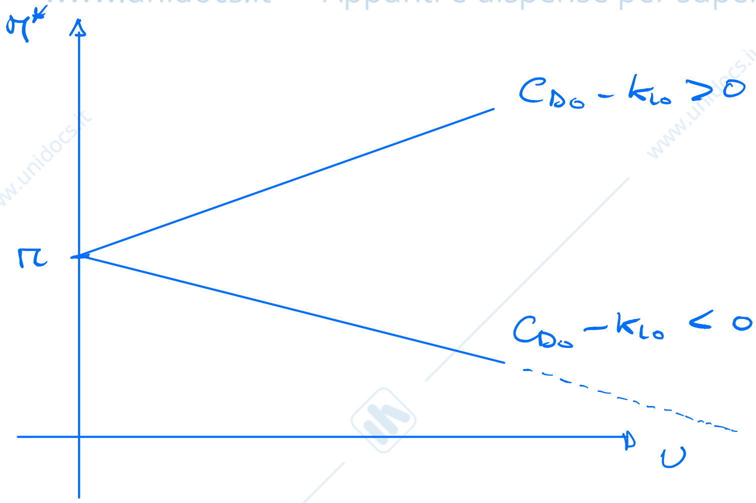
with $C_{L0} \neq 0$ $\boxed{k x_0 = F_{L0}}$ we impose

$x_D = x - x_0$
$\dot{x}_D = \dot{x}$
$\ddot{x}_D = \ddot{x}$

$$m\ddot{x}_D + r\dot{x}_D + kx_D = qU(k_L - C_{D0})\dot{x}_D$$

$$m\ddot{x}_D + \underbrace{\big[r + qU(C_{D0} - k_L)\big]}_{r^*}\dot{x}_D + kx_D = 0$$

omogeneus equation

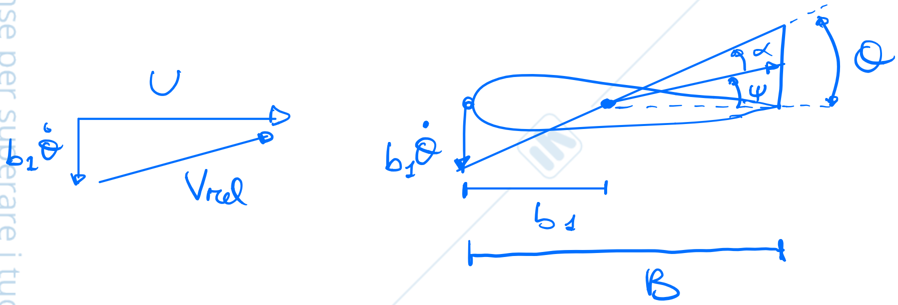


in the airfoil we have no instability regarding vertical motion

unstable profile

EXAMPLE : ROTATING PROFILE

$$J\ddot{\theta} + r_\theta \dot{\theta} + k_\theta \theta = M_A$$



α = angle of attack $= \theta - \psi$

$$\alpha = \theta - \frac{b_1 \dot{\theta}}{U}$$

$$J\ddot{\theta} + r_\theta \dot{\theta} + k_\theta \theta = \underbrace{\frac{1}{2}\rho S}_{q} B \, C_M(\alpha) \, V_{rel}^2$$

$$= qB\Big[\underbrace{C_{M0}}_{C_{M0}=0} + \overbrace{\frac{dC_{M0}}{d\alpha}\Big|_0}^{K_{M0}}(\alpha - \alpha_0) + \dots\Big]\Big[U^2 + (b_1\dot{\theta})^2\Big]$$

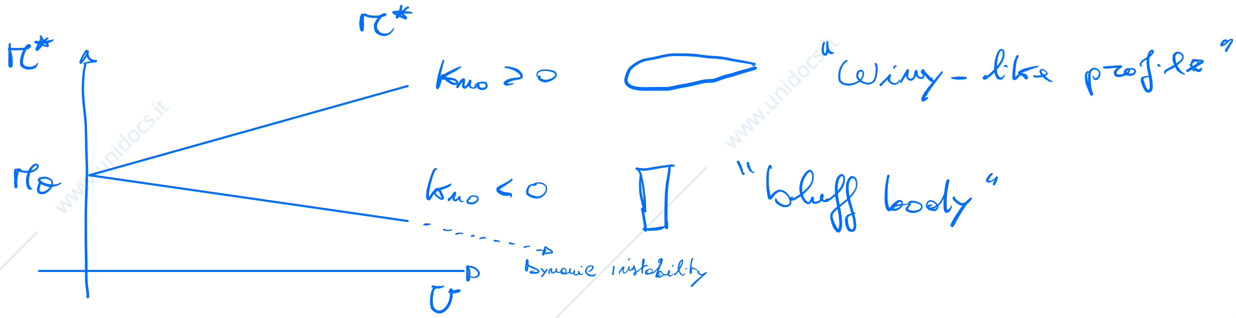
$$= qB\,K_{M0}\left(\theta - \frac{b_1 \dot{\theta}}{U}\right)U^2$$

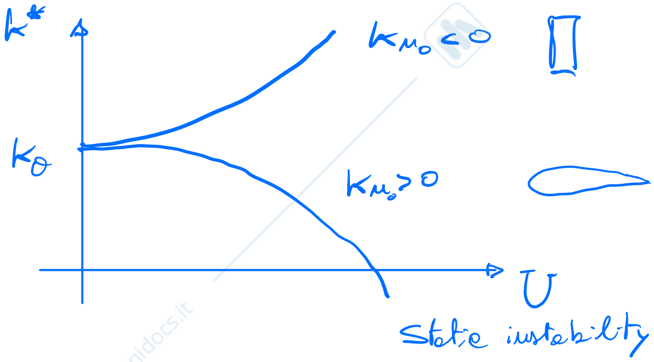
So:

$$J\ddot{\theta} + r_\theta \dot{\theta} + k_\theta \theta = qBK_{M0}U^2\theta - qBK_{M0}b_1U\dot{\theta}$$

$$J\ddot{\theta} + \Big[r_\theta + qBb_1K_{M0}U\Big]\dot{\theta} + \Big[k_\theta - qBK_{M0}U^2\Big]\theta = 0$$

omogeneus equation


$k_{M_0} > 0$
"Wing-like profile"
$k_{M_0} < 0$
"bluff body"
Dynamic instability
U


k^*
k_θ
$k_{M_0} < 0$
$k_{M_0} > 0$
U
Static instability