Stability of n-th dofs sys.

$$[M]\underline{\ddot{x}} + [R]\underline{\dot{x}} + [K]\underline{x} = \underline{F}(\underline{x}, \underline{\dot{x}})$$

static $\longrightarrow$ $[K]\underline{x}_0 = \underline{F}(\underline{x}_0, 0)$

x_0 = solution (one of the possible)

linearization $\longrightarrow$ $\underline{x}_D = \underline{x} - \underline{x}_0$

$\underline{\dot{x}}_D = \underline{\dot{x}}$

$\underline{\ddot{x}}_D = \underline{\ddot{x}}$

this 2 terms are equal

Substitut $\longrightarrow$ $[M]\ddot{x}_D + [R_S]\dot{x}_D + [K_S]x_D + \cancel{[K_S]x_0} = \cancel{\underline{F}(x_0, 0)} + \frac{\partial F}{\partial x}\Big|_0 x_D + \frac{\partial F}{\partial \dot{x}}\Big|_0 \dot{x}_D - [K_F]x_D - [R_F]\dot{x}_D$

linearize sys: $[M]\ddot{x}_D + \underbrace{([R_S]+[R_F])}_{[R_T]}\dot{x}_D + \underbrace{([K_S]+[K_F])}_{[K_T]}x_D = \underline{0}$ $\longrightarrow$ SOLUTION OF FREE MOTION

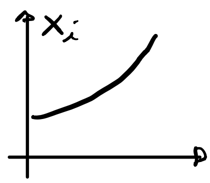
(order of the system: $2n$ $\rightarrow$ when n is the number of dofs)

Sufficient condition for stability:

- if $[M]$, $[R_T]$, $[K_T]$ are positive definite <u>and</u> symmetric $\Rightarrow$ ASIMPTOTICALLY STABLE SOLUTION

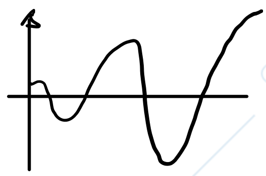
  (<u>NOTE</u>: if the overall energy decrese, the amplitude decrese!)

- if $[K_T]$ is <u>not</u> positive definite $\Rightarrow$ STATIC DIVERGENCE



at least one coordinate

- if $[R_T]$ is not positive definite $\Rightarrow$ DINAMIC INSTABILITY



at least 1 coordinate

- if $[K_T]$ is not symmetric $\Rightarrow$ <u>POSSIBLE</u> FLUTTER INSTABILITY

  (in case of 2 dofs we can also inspect the sign of the term: $k_{xy} k_{yx}$)

- if $[R_T]$ is not symmetric $\Rightarrow$ R_T can be not dissipative $\Rightarrow$ POSSIBLE INSTABILITY

Continue with a more numerical analysis of the possible instability

$$\begin{cases} [M]\ddot{\underline{x}}_D + \underbrace{([R_S]+[R_F])}_{[R_T]}\dot{\underline{x}}_D + \underbrace{([k_S]+[k_F])}_{[k_T]}\underline{x}_D = \underline{0} \\ \underline{z} = \begin{Bmatrix} \dot{\underline{x}}_D \\ \underline{x}_D \end{Bmatrix} (2n,1) = \text{state vector} \quad ; \quad \dot{\underline{z}} = \begin{Bmatrix} \ddot{\underline{x}}_D \\ \dot{\underline{x}}_D \end{Bmatrix} \\ [M]\dot{\underline{x}}_D - [M]\dot{\underline{x}}_D = 0 \end{cases}$$

$\rightarrow$ SOLUTION OF FREE MOTION

$$\begin{bmatrix} [M] & [0] \\ [0] & [M] \end{bmatrix} \begin{Bmatrix} \ddot{\underline{x}}_D \\ \dot{\underline{x}}_D \end{Bmatrix} + \begin{bmatrix} [R_T] & [k_T] \\ -[M] & [0] \end{bmatrix} \begin{Bmatrix} \dot{\underline{x}}_D \\ \underline{x}_D \end{Bmatrix} = 0$$

$$\begin{bmatrix} [M] & [0] \\ [0] & [M] \end{bmatrix}^{-1} \begin{bmatrix} [M] & [0] \\ [0] & [M] \end{bmatrix} \dot{\underline{z}}_D + \begin{bmatrix} [M] & [0] \\ [0] & [M] \end{bmatrix}^{-1} \begin{bmatrix} [R_T] & [k_T] \\ -[M] & [0] \end{bmatrix} \underline{z}_D = 0$$

$$\begin{bmatrix} M^{-1} & 0 \\ 0 & M^{-1} \end{bmatrix} \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \dot{\underline{z}}_D + \begin{bmatrix} M^{-1} & 0 \\ 0 & M^{-1} \end{bmatrix} \begin{bmatrix} R_T & k_T \\ -M & 0 \end{bmatrix} \underline{z}_D = 0$$

So we obtain:

$$\begin{bmatrix} [I] & [0] \\ [0] & [I] \end{bmatrix} \dot{\underline{z}}_D + \begin{bmatrix} M^{-1}R_T & M^{-1}k_T \\ -[I] & [0] \end{bmatrix} \underline{z}_D = 0$$

we can neglect the identity matrix:

$$\dot{\underline{Z}}_D = \begin{bmatrix} M^{-1}R_T & M^{-1}K_T \\ -[I] & [0] \end{bmatrix} \underline{Z}_D$$

$$\boxed{\dot{\underline{Z}}_D = [A]\,\underline{Z}_D} \longrightarrow$$ 2n dofs first order, ordinary differential equation

$[A]$ → STATE MATRIX

$$\underline{Z}_D(t) = \underline{\bar{Z}}\,e^{\lambda t}$$ → solution of the free-motion (so also the solution of the homogeneous eq.)

$$\dot{\underline{Z}}_D(t) = \lambda\,\underline{\bar{Z}}\,e^{\lambda t}$$ if we substitut we obtain:

$$\lambda\,\underline{\bar{Z}}\,e^{\lambda t} = [A]\,\underline{\bar{Z}}\,e^{\lambda t}$$

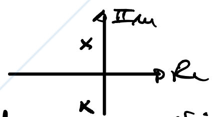
$$\lambda\,\underline{\bar{Z}} = [A]\,\underline{\bar{Z}}$$

$$\boxed{\left([A] - [I]\lambda\right)\underline{Z} = 0} \longrightarrow$$ eigenvalue problem

We can have 2n value of λ_i where λ_i can be real number or complex conjugate number

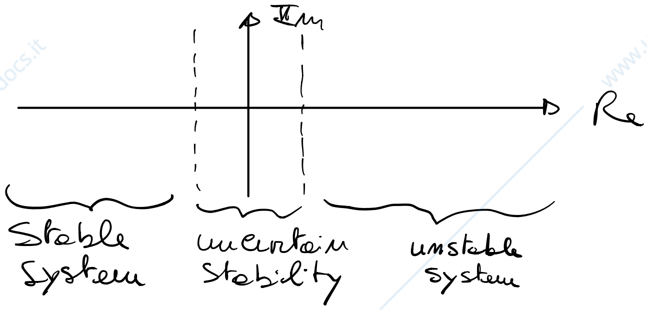
If all $Re(\lambda_i) < 0 \Longrightarrow$ asimptotic stability

If we have $Re(\lambda_i)$ strictly negative but very close to the imaginary axes



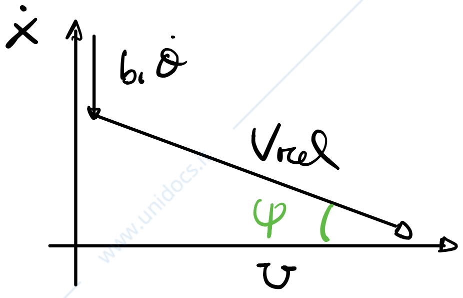
we can be satisfy from a matematical point of view but in reality we can't say that the real (not-linear) system is stable.

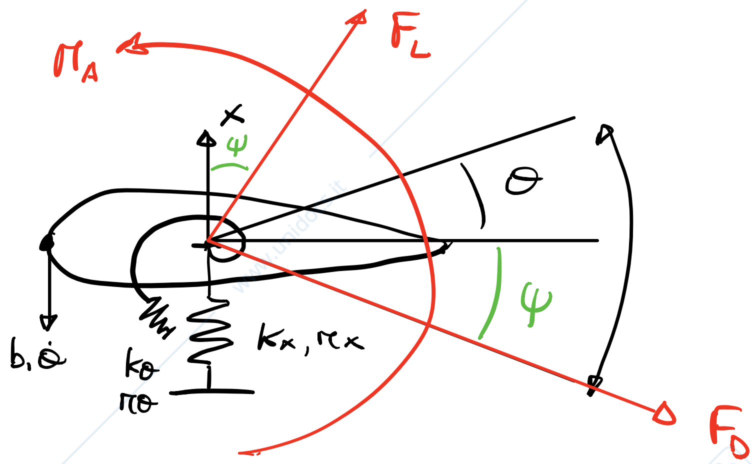
At the beginning we have a not-linear sys and we want to linearize in order to approximately analyse the real behaviour of the sys, but we have to distinguish 3 cases depending from the value of



CHIEDERE PROF

TWO DOFS SYS PROFILE IN A FLUID FLOW





$$\alpha = \theta + \varphi = \theta + \frac{\dot{x} - b_1\dot{\theta}}{U}$$

$$x)\quad m\ddot{x} + r_x\dot{x} + k_x x = F_L\cos\varphi - F_D \operatorname{sen}\varphi$$

$$\theta)\quad J\ddot{\theta} + r_\theta\dot{\theta} + k_\theta\theta = M_A$$

imposing : $q = \frac{1}{2} S\rho$

$$x)\quad m\ddot{x} + r_x\dot{x} + k_x x = q\,C_L(\alpha)\,V_{rel}^2\cos\varphi - q\,C_D(\alpha)\,V_{rel}^2 \operatorname{sen}\varphi$$

$$\theta)\quad = q\,B\,C_M(\alpha)\,V_{rel}^2$$

assume $\alpha_0 = 0$ (because simmetric profile)

$$\begin{cases} '' = q\left(C_{L_0} + K_{L_0}\alpha + \dots\right)\left(U^2 + \dots\right)\left(1 - \dots\right) - q\left(C_{D_0} + K_{D_0}\alpha + \dots\right)\left(U^2 + \dots\right)\dfrac{\dot{x} - b_1\dot{\theta}}{U} \\ '' = q\,B\left(C_{M_0} + K_{M_0}\alpha + \dots\right)\left(U^2 + \dots\right) \end{cases}$$

but $C_{D0}, k_{M0}, k_{D0} = 0$

$$\begin{cases} u = q k_{L0}\left(\theta + \dfrac{\dot x - b_1\dot\theta}{U}\right)U^2 - q C_{D0} U^2 \dfrac{\dot x - b_1\dot\theta}{U} \\ u = q B k_{M0}\left(\theta + \dfrac{\dot x - b_1\dot\theta}{U}\right)U^2 \end{cases}$$

$$\begin{cases} u = q k_{L0} V^2 \theta + qU(k_{L0} - C_{D0})\dot x - qUb_1(k_{L0} - C_{D0})\dot\theta \\ u = qBk_{M0} U^2\theta + qBk_{M0}U\dot x - qBb_1U\dot\theta \end{cases}$$

matrix expression:

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}\begin{Bmatrix} \ddot x \\ \ddot\theta \end{Bmatrix} + \begin{bmatrix} r_x + qU(C_{D0} - k_{L0}) & qUb_1(k_{L0} - C_{D0}) \\ -qBk_{M0}U & r_\theta + qBb_1Uk_{M0} \end{bmatrix}\begin{Bmatrix} \dot x \\ \dot\theta \end{Bmatrix} + \begin{bmatrix} k_x & -qk_{L0}U^2 \\ 0 & k_\theta - qBk_{M0}U^2 \end{bmatrix}\begin{Bmatrix} x \\ \theta \end{Bmatrix} = \underline{0}$$

Compact notation

$$[M]\underline{\ddot x} + [R_T(U)]\underline{\dot x} + [K_T(U)]\underline{x} = \underline{0}$$

if $r_{11} < 0$ DYNAMIC (VERTICAL) INSTABILITY

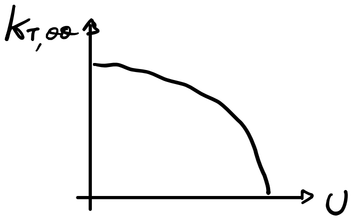
if $r_{22} < 0$ " (TORSIONAL) INSTABILITY

if $K_{T22} < 0$ STATIC (TORSIONAL DIVERGENCE) INSTABILITY

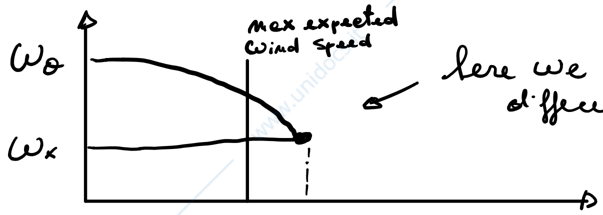
because of K_T is not symmetric $\Rightarrow k_{x\theta}\, k_{\theta x} = 0$ (*) $\Rightarrow$
(The product of undiagonal terms)

$\Rightarrow$ so K_T is not a <u>DIRECT</u> cause of FLATTER

R_T not symm $\longrightarrow$ can be not dissipative



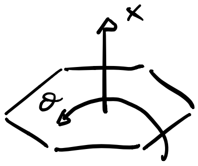
Wing like profile

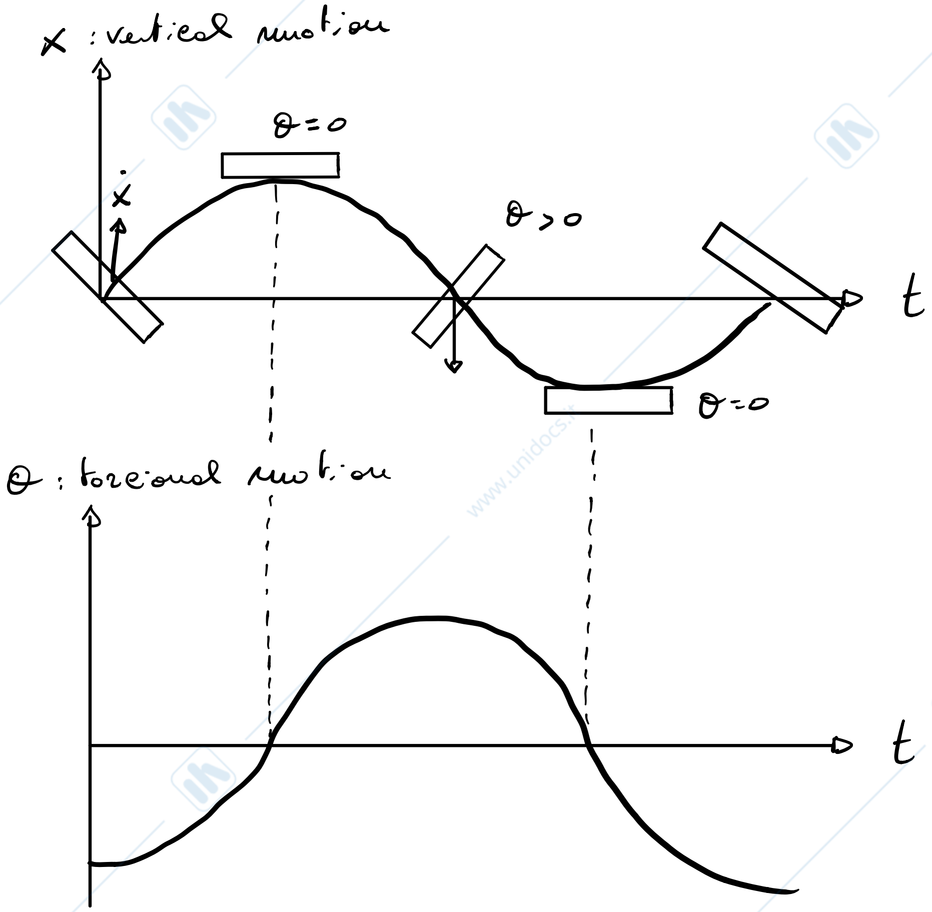


here we set the 2 frequency, originaly different, to the same value

U : FLUTTER VELOCITY

(Remember: flutter condition if $\underbrace{k_{xy}\, k_{yx} < -\left(\frac{k_{xx}-k_{yy}}{2}\right)^2}_{\text{in this case is not valid}}$)





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