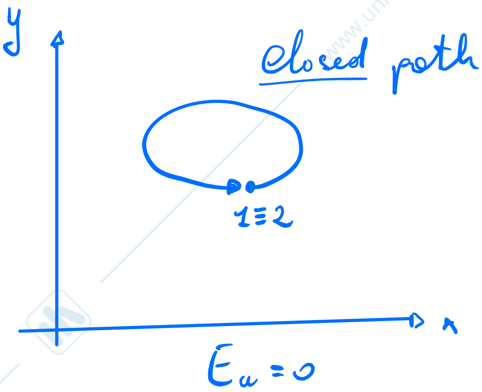
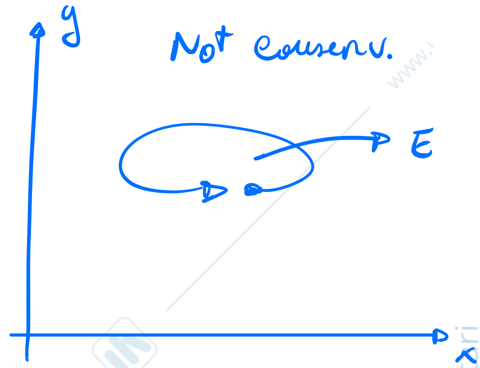
2 DOFs system under action of a positional force field (NOT conservati[ve])

$L_{1,2} = U_2 - U_1 = 0$

conservative





$$\lambda_{I,II}^2 = -\frac{k_{xx}+k_{yy}}{2} \pm \sqrt{\left(\frac{k_{xx}+k_{yy}}{2}\right)^2 - \det(K_c)} = -\gamma \pm \sqrt{\beta}$$

in conservative force field is always a Real number

$$\lambda_{I,II}^2 = -\gamma \pm \sqrt{\left(\frac{k_{xx}-k_{yy}}{2}\right)^2 + k_{xy}\cdot k_{yx}}$$

if force field is not conservative $\Rightarrow k_{xy} \neq k_{yx}$

Unlike the previous case (conservative force) now if k_{xy} and k_{yx} can have different sign so if $|k_{xy}\cdot k_{yx}| > \left(\frac{k_{xx}-k_{yy}}{2}\right)^2$ we have an Imaginary number / Complex number

So

$$\begin{cases} \beta > 0 \rightarrow \approx \text{close/similar to the conservative case} \\ \boxed{\beta < 0} \rightarrow \text{stuffs change!} \end{cases}$$

$\beta < 0$:

$$k_{xy}\cdot k_{yx} < -\left(\frac{k_{xx}-k_{yy}}{2}\right)^2$$

k_{xy} opposite sign of k_{yx} so their product $> (\ )^2$

$$|k_{xy}\, k_{yx}| > \left(\frac{k_{xx}-k_{yy}}{2}\right)^2$$

So in this case we have FLUTTER INSTABILITY

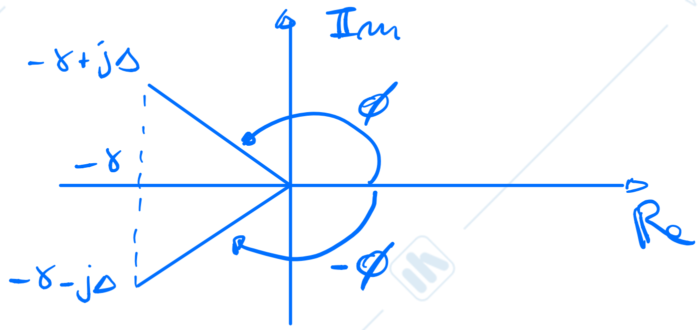
Analyze the solution if $\beta < 0$:

$$\lambda^2_{I,II} = -\gamma \pm \sqrt{\beta} = -\gamma \pm j\sqrt{-\beta} = -\gamma \pm j\Delta = M e^{\pm j\phi}$$

(K_T^+ = positive definite)

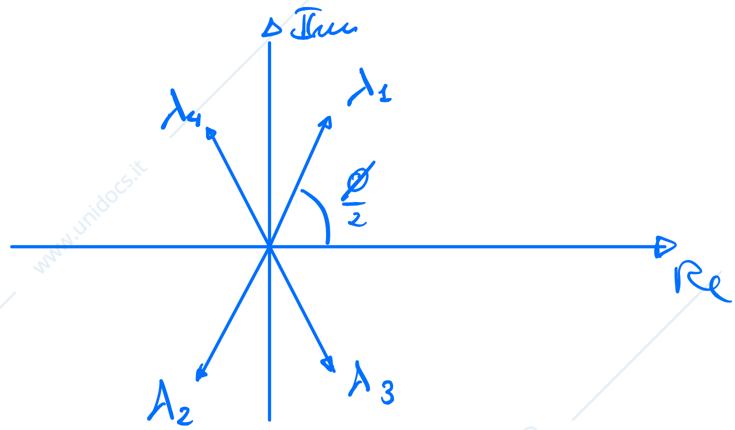
$$M = \sqrt{\gamma^2 + \Delta^2}$$

$$\tan \varphi = \frac{\Delta}{\gamma}$$



$$\lambda_{1,2} = \pm\sqrt{M}\, e^{j\phi/2}$$

$$\lambda_{3,4} = \pm\sqrt{M}\, e^{-j\frac{\phi}{2}}$$



So $\lambda_i = \pm\alpha \pm j\omega$

Solution of the equation of motion

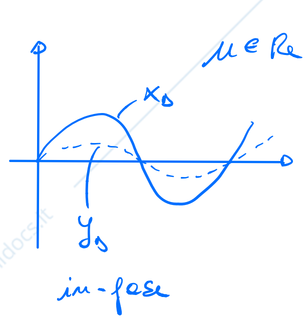
$$(\lambda^2 + k_{xx})\, x_0 + k_{xy}\, y_0 = 0 \qquad (1^{st}\ eq.)$$

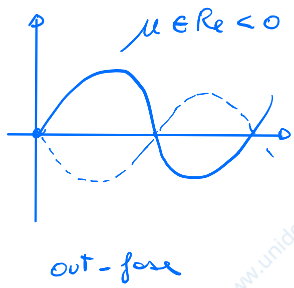
$$\frac{y_0}{x_0} = -\frac{\lambda^2 + k_{xx}}{k_{xy}} = -\frac{\lambda_I^2 + k_{xx}}{k_{xy}} = -\frac{\gamma + j\Delta + k_{xx}}{k_{xy}} = -\frac{k_{xx} + \gamma}{k_{xy}} - j\frac{\Delta}{k_{xy}}$$

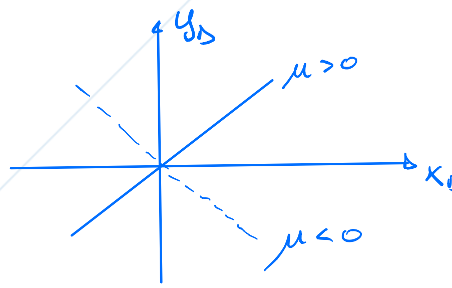
$$\frac{y_0}{x_0} = \bar{\mu}_I = \mu_I e^{j\psi_I}$$

We can have:

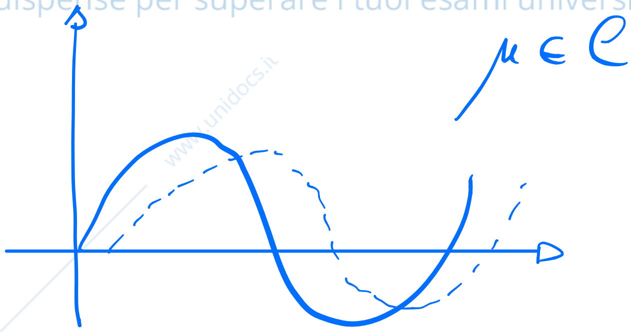
A) μ = Real Number







B) μ = complex number



$$\left(\frac{y_D}{x_D}\right)_{II} = -\frac{\gamma - j\Delta + k_{xx}}{k_{xy}} = -\frac{\gamma + k_{xx}}{k_{xy}} + j\frac{\Delta}{k_{xy}} = \mu_{II}$$

$\bar{\mu}_I$ and $\bar{\mu}_{II}$ are conjugate complex number

mode shape : $\underline{\phi}_I = \begin{Bmatrix} 1 \\ \mu_I e^{j\psi_I} \end{Bmatrix}$

$$\underline{\phi}_{II} = \begin{Bmatrix} 1 \\ \mu_{II} e^{j\psi_{II}} \end{Bmatrix}$$

The solution vector is :

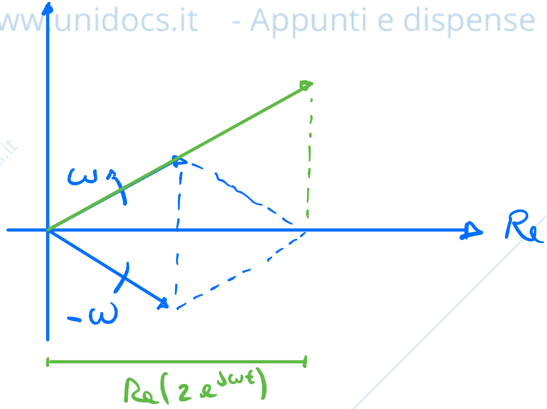
$$\underline{z}(t) = \begin{Bmatrix} x_D(t) \\ y_D(t) \end{Bmatrix} = \sum_{k=1,4} C_k \underline{z}_k e^{\lambda_k t}$$

$$= C_1 \underline{z}_1 e^{\alpha t} e^{j\omega t} + C_2 \underline{z}_2 e^{-\alpha t} e^{j\omega t} + C_3 \underline{z}_3 e^{-\alpha t} e^{j\omega t} + C_4 \underline{z}_4 e^{\alpha t} e^{j\omega}$$

Just for the first row:

$$x_D(t) = C_1 e^{\alpha t} e^{j\omega t} + C_3 e^{\alpha t} e^{-j\omega t} + C_2 e^{-\alpha t} e^{-j\omega t} + C_4 e^{-\alpha t} e^{j\omega t}$$

$$x_D(t) = e^{\alpha t} \underbrace{\left(C_1 e^{j\omega t} + C_3 e^{-j\omega t}\right)}_{C_1 \text{ and } C_3 \text{ must be complex conjugate}} + e^{-\alpha t}\left(C_4 e^{j\omega t} + C_2 e^{-j\omega t}\right)$$

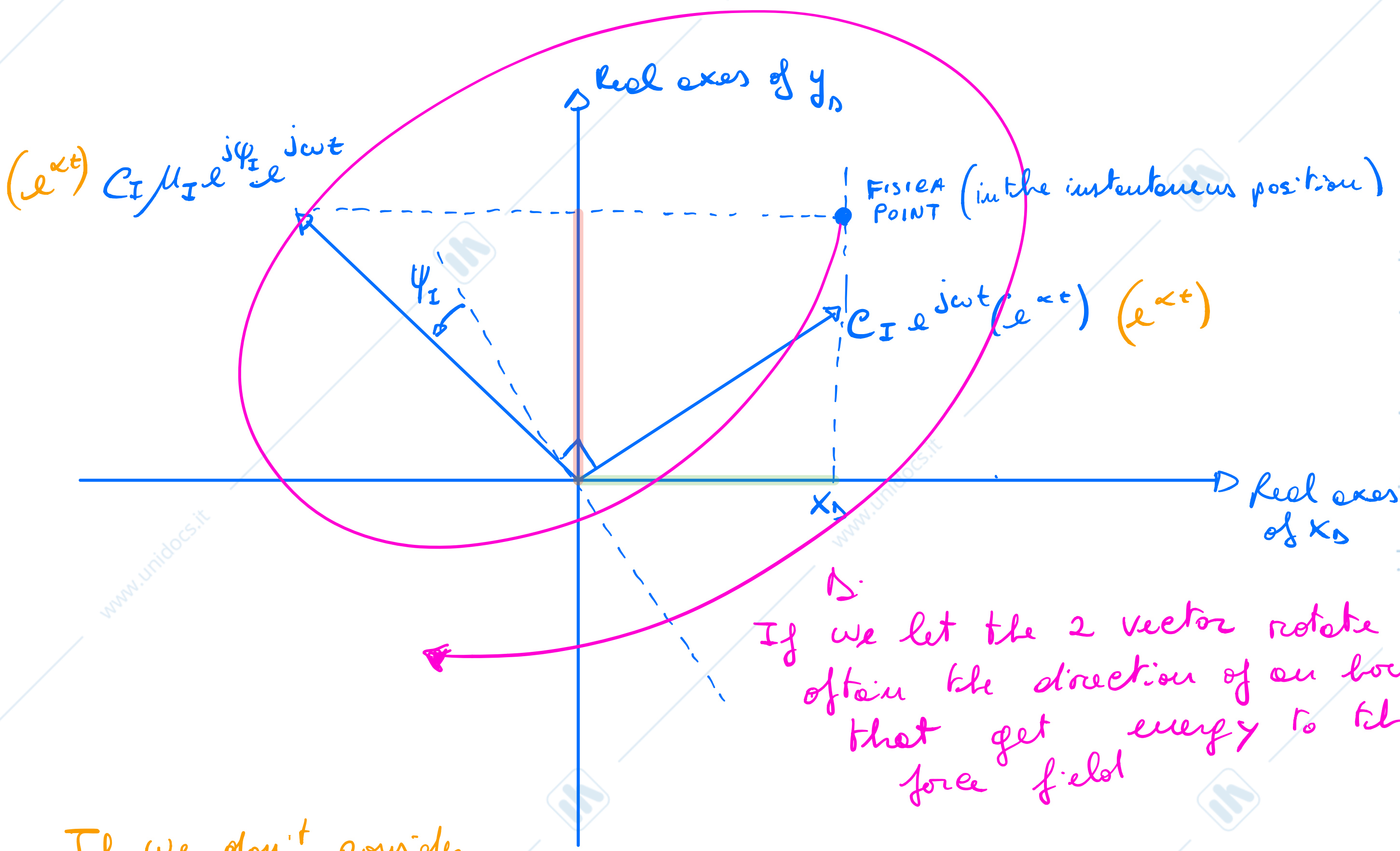


$$\Rightarrow e^{j\omega t} + e^{-j\omega t} = 2\cos\omega t$$

$$\underline{Z}(t) = \begin{Bmatrix} x_D(t) \\ y_D(t) \end{Bmatrix} = e^{\alpha t}\left(Re\left(c_I z_I e^{j\omega t}\right)\right) + e^{-\alpha t}\left(Re\left(c_{II} z_{II} e^{j\omega t}\right)\right)$$

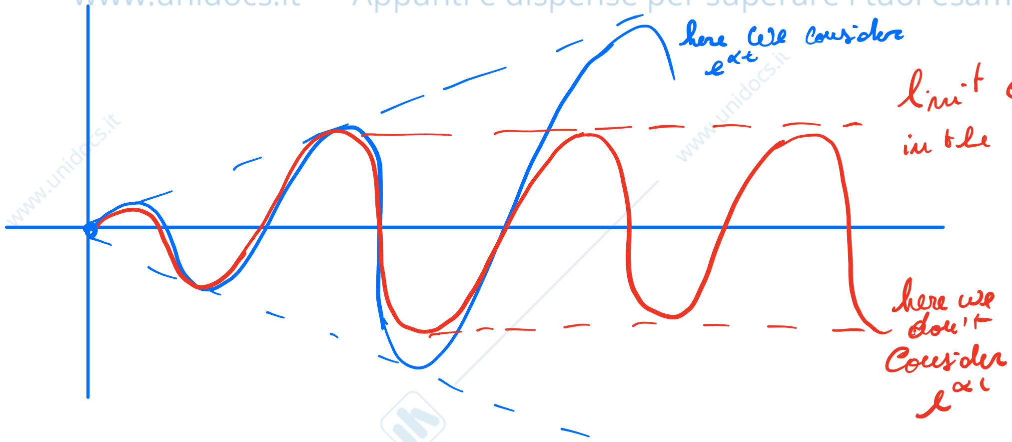
$$Z(t) = \begin{Bmatrix} e^{\alpha t} Re\left(c_I \cdot 1 e^{j\omega t}\right) + e^{-\alpha t} Re\left(c_{II} \cdot 1 e^{j\omega t}\right) \\ e^{\alpha t} Re\left(c_I \mu_I e^{j\psi_I} e^{j\omega t}\right) + e^{-\alpha t} Re\left(c_{II} \mu_{II} e^{j\psi_{II}} e^{j\omega t}\right) \end{Bmatrix}$$

Grafic (qualitative) representation of the expanding solution



If we let the 2 vector rotate we obtain the direction of ou bod that get energy to the force field

If we don't consider $(e^{\alpha t})$ we have a constant amplitude
If we consider it the amplitude increase!

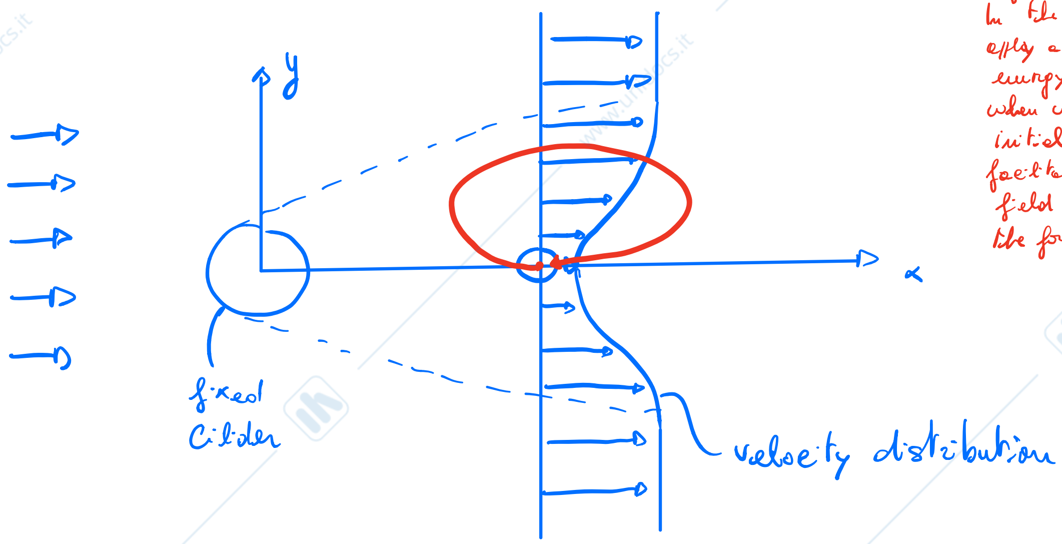


limit cycle in the real system

↳ even if it is a limit amplitude it is unstable

REAL EXAMPLE

Drag Force $F_D = \frac{1}{2} \rho S C_D V^2(x,y)$ → this force depend from velocity



If we want to move inside the "
In the beginning we have to
apply a force so we are putting
energy into the system, then
when we come back to the
initial position our motion
facilitated from the force
field so we get energy from
the force field!