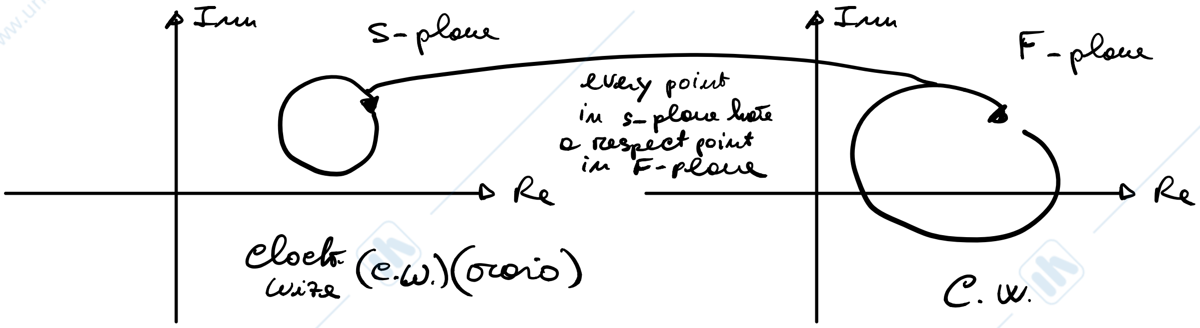
There are several steps:

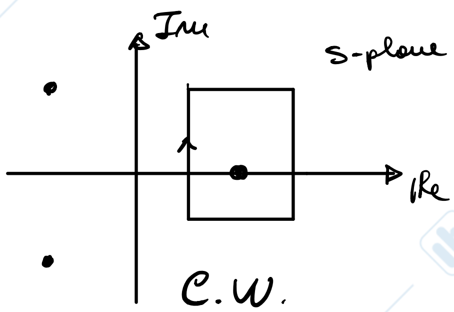
1) Mapping $F(s) = \frac{N(s)}{D(s)}$ ratio of the polynomials

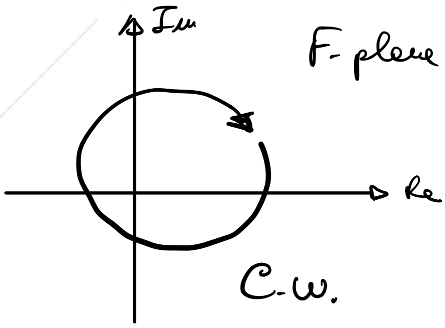



S-path $\in$ region where F(s) is analitye

2) Zeros of F(s) & s-path:

O = zeros of F(s)

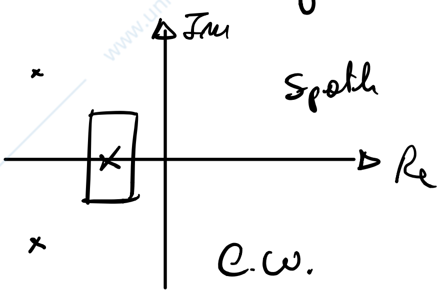


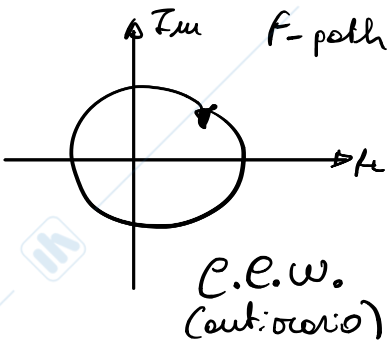




if s-path encloses Z zeros of F $\Rightarrow$ F(s) will encircle the origin Z-times

3) Poles of F(s) & s-path

$\times$ = poles of F(s)




if S-path encloses P poles of F $\Rightarrow$ F(s) will encircle the origin p-times in C.C.W.

2+3) if S-path encloses P poles and Z zeros ⟹ F(s) will encircle the origine N-time, where $N = Z - P$

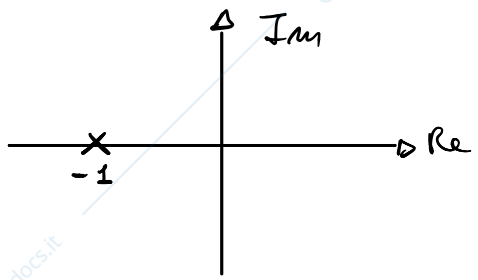
- $Z > P \to N > 0$ encirclement in c.w.
- $Z = P \to N = 0$ no encirclement
- $Z < P \to N < 0$ encirclement in c.c.w.

4) given the S-path; from N # of +/- (c.w./c.c.w.) encirclement of the origin by F(s) ⟹ we know the difference between Z and P inside the S-path $(N = Z - P)$

5) $F(s) = 1 + G(s) H(s)$ $\left(\text{remember: } L = \frac{G}{1+GH}\right)$

$F(s) = 0 \to$ origine of F-plane

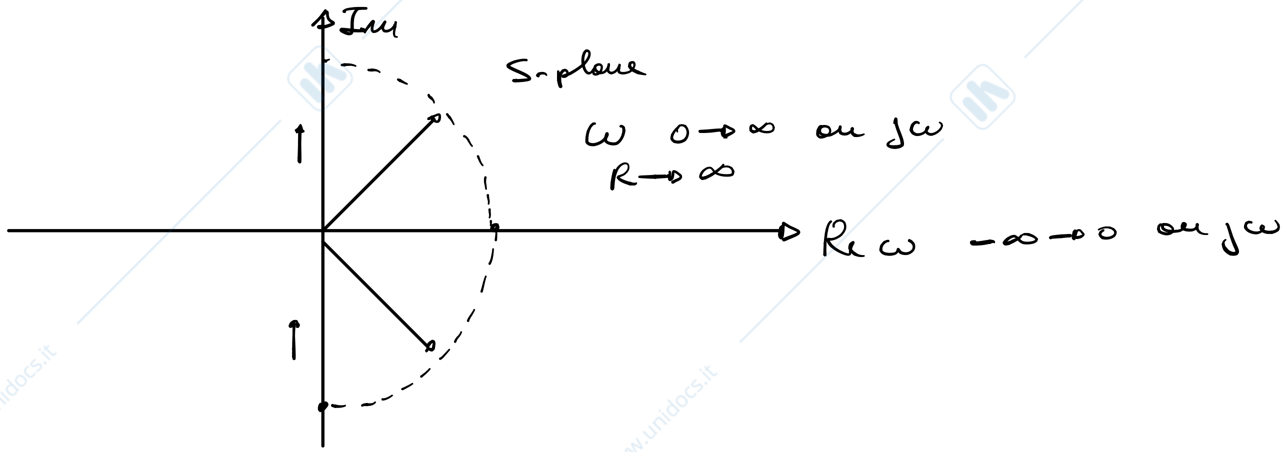
$1 + GH = 0 \to GH = -1 \to$ origine become $(-1, 0)$

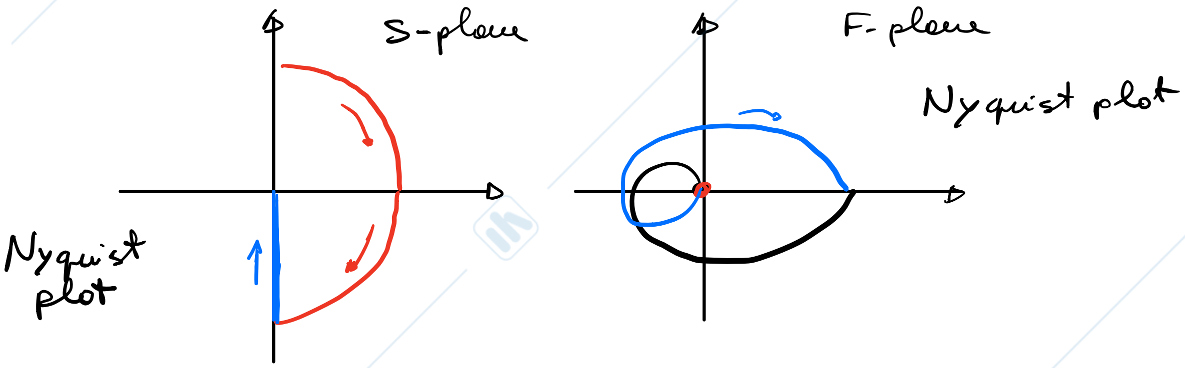


# 6) Niquist Path

NOTE: moving from non linear sys to linear one we can assume the stability of the linear sys because equilibrium position is already included into the linearization procedure

asimptotic stability $\rightarrow Re(\lambda)$ or $Re(p) < 0$




- $0 \rightarrow \infty$ : polar plot of F(s)
- $R \rightarrow \infty$ in the neighbourhood of the F-origin
- $-\infty \rightarrow 0$ the complex conjugate of the polar plot of F(s)

7)

$$L = \frac{G}{1+GH} = \frac{N_G D_H}{N_G N_H + D_G D_H} \longrightarrow \text{poles of } L$$

$$1+GH = \frac{N_G N_H - D_G D_H}{D_G D_H} \begin{array}{l} \longrightarrow \text{zeros of } 1+GH \\ \longrightarrow \text{poles of } 1+GH \end{array}$$

$$GH = \frac{N_G N_H}{D_G D_H} \longrightarrow \text{poles of } GH$$

our final goals are poles of L

- if there are z zeros of 1+GH in the $\overbrace{\text{S-path}}^{\text{Nyquist-Path}}$, and P poles of 1+GH in the $\underbrace{\text{S-path}}_{\text{Nyquist Path}}$ → GH will encircle (−1, 0) N times, ($N = Z - P$)

- if there are $\boxed{Z\text{-poles of } L}$ in the Ny-path and, and P poles of GH in the Nyq.-path → GH will encircle in (−1,0) N times

Remind: in order to have L asymptotically stable $\overset{\text{necessary and sufficient Condition}}{\Longleftrightarrow}$ no poles of L Re > 0 ⟹ $\boxed{Z=0}$

CONDITION FOR ASYMPTOTICALLY STABILITY:

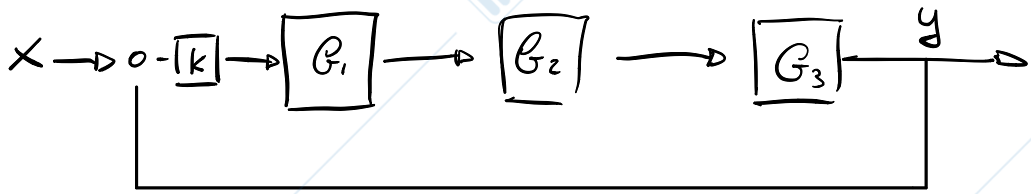
$N = Z - P$, for asymptotic stability ⟹ $Z = 0$

- if a closed loop function (L) is asymptotically stable, and if GH has P poles Re > 0 ⟹ the Nyquist plot of GH <u>must</u> encircle the point (−1, 0) N times, with $N = -P$ in C.C.W. (we need to know just the number of poles of GH)

- if GH has no poles Re ≥ 0 (so GH is asymp. stable) ⟹ L (closed loop fu) will be asymp. stable if and only if GH makes no turn around (−1, 0)

Thanks to this procedure we can analize the stability of the sys. befor closing the loop, just looking at the GH plot!

## Example



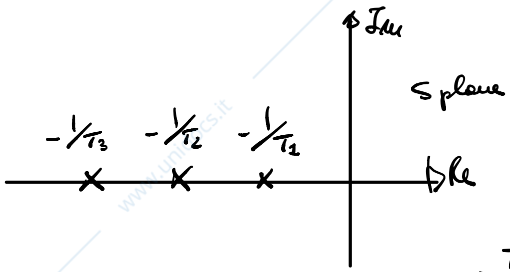

$$G = k\,G_1 G_2 G_3 = k \prod_{i=1}^{3} \frac{1}{1+T_i s}$$

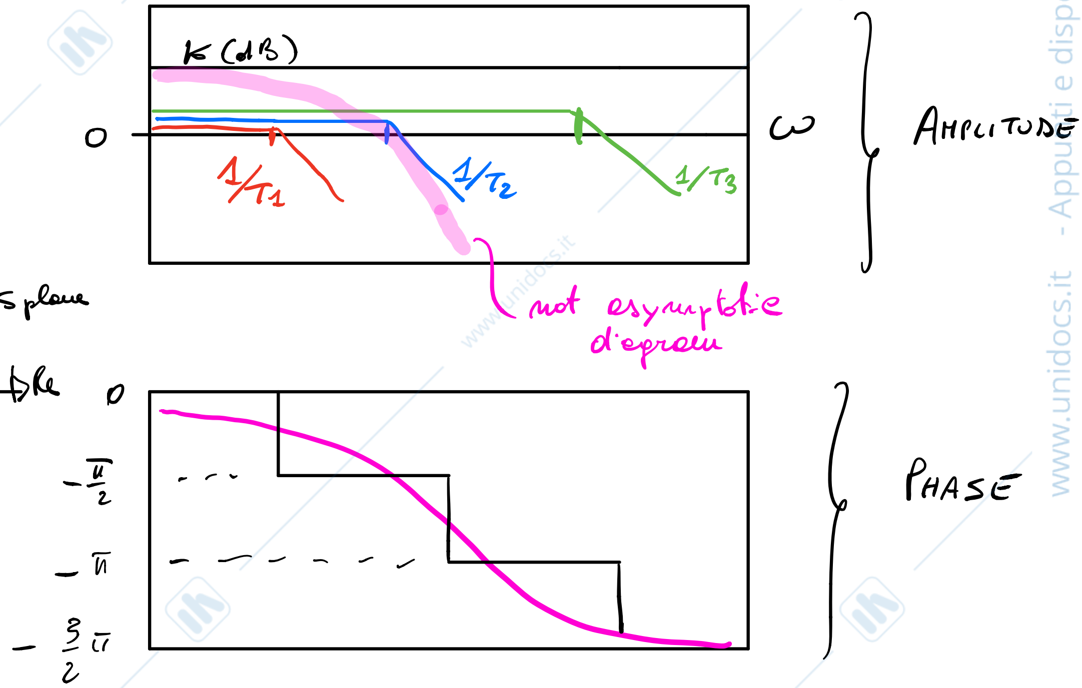
$$H = 1$$

$$L = \frac{G}{1+GH}$$

$$G_i = \frac{1}{1+T_i s}$$

Bode diagram (it is not mandatory if we have the numerical value)

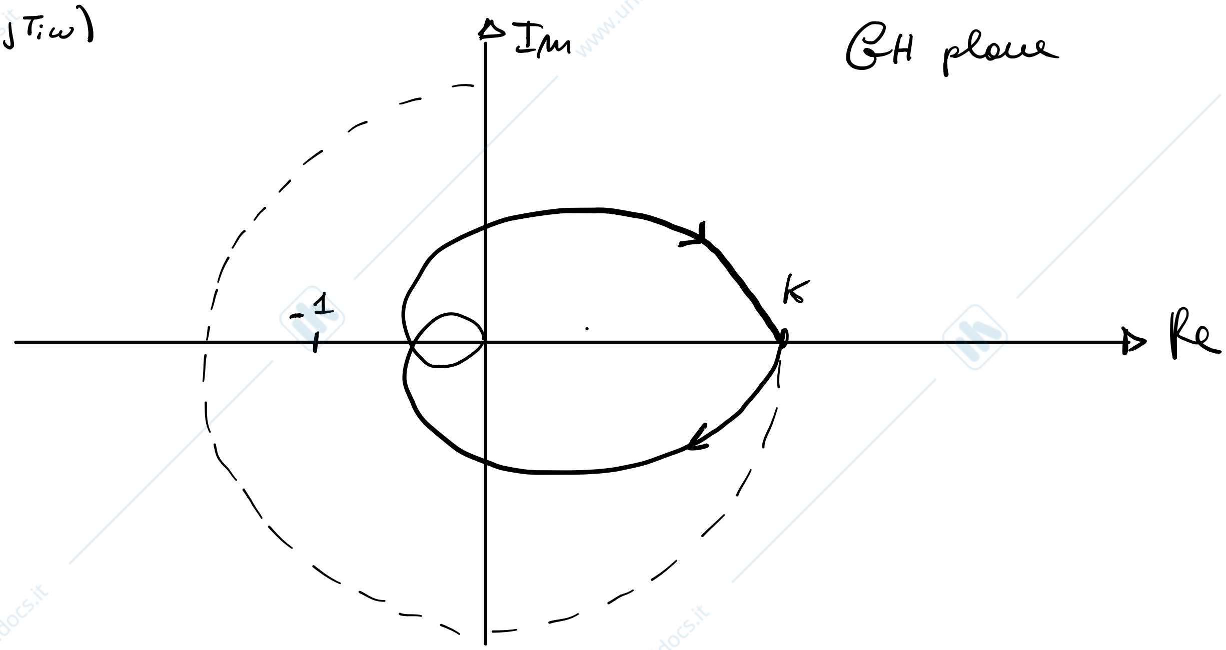


# Nyquist-Plot

$$GH = \frac{K}{\pi(1+jT_i\omega)}$$

$$N = Z - P$$




N is obtain from the inspection of Nyquist plot

Z is what we want to find

P is obtain from the inspection of GH

In our case:

$N = 0$

$Z = ?$

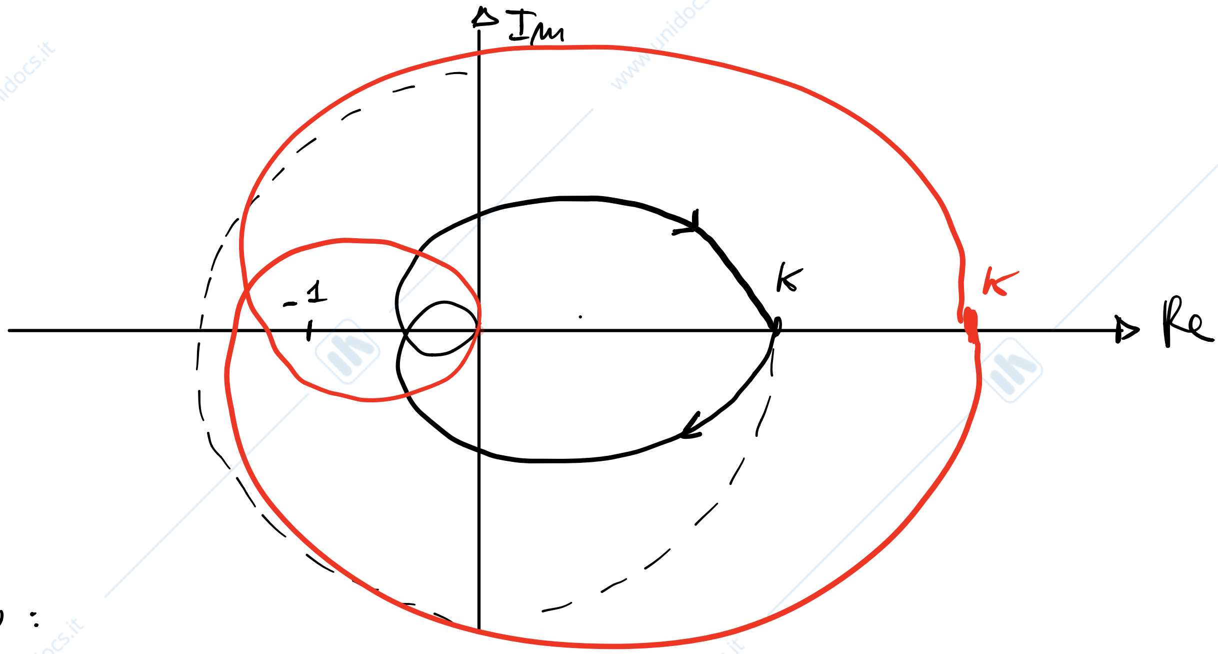
$P = 0$ because GH is asymptotically stable

so $0 = Z - 0 \Rightarrow Z = 0$

$L = \frac{G}{1+GH}$ has no poles (Z) Re > 0

$\Rightarrow$ L is asymp. stable

if $k$ increase:




Now:

$N = Z - P$

$2 = Z - 0 \Rightarrow Z = 2 \Rightarrow L$ has 2 poles $R > 0$