Control of Industrial Robots

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SOLUTION

Control of Industrial Robots
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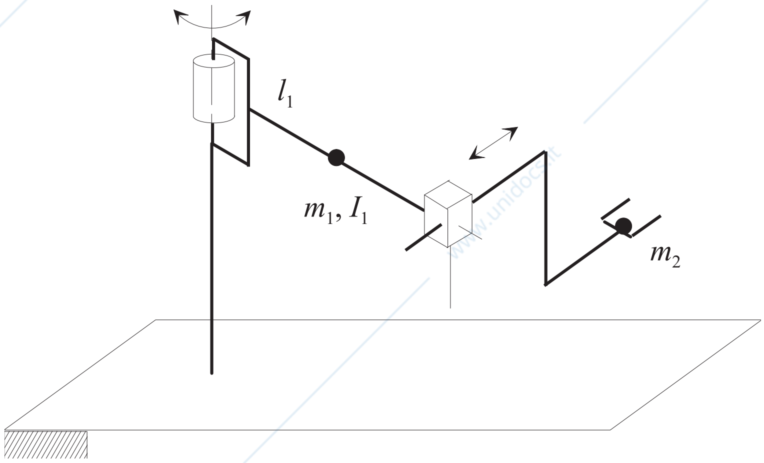
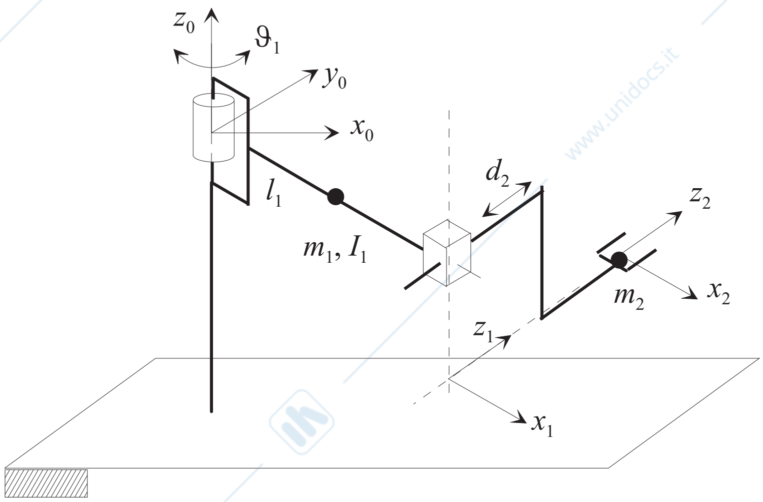
EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

Denavit-Hartenberg frames can be defined as sketched in this picture:



Computations of the Jacobians:

Link 1

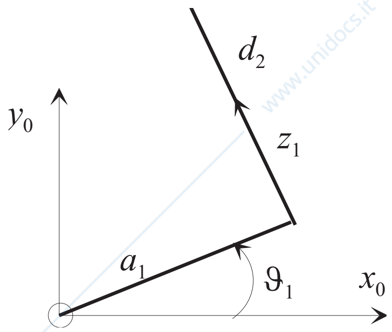
$$\mathbf{J}_P^{(l_1)} = \begin{bmatrix} \mathbf{j}_{P_1}^{(l_1)} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_0 \times (\mathbf{p}_{l_1} - \mathbf{p}_0) & \mathbf{0} \end{bmatrix} = \begin{bmatrix} -l_1 s_1 & 0 \\ l_1 c_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{J}_O^{(l_1)} = \begin{bmatrix} \mathbf{j}_{O_1}^{(l_1)} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_0 & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Link 2

$$\mathbf{J}_P^{(l_2)} = \begin{bmatrix} \mathbf{j}_{P_1}^{(l_2)} & \mathbf{j}_{P_2}^{(l_2)} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_0 \times (\mathbf{p}_{l_2} - \mathbf{p}_0) & \mathbf{z}_1 \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - d_2 c_1 & -s_1 \\ a_1 c_1 - d_2 s_1 & c_1 \\ 1 & 0 \end{bmatrix}$$

For the above computations, we can make reference to the following picture:



and to the following auxiliary vectors:

$$\mathbf{p}_{l_1} = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}, \mathbf{p}_{l_2} = \begin{bmatrix} a_1 c_1 - d_2 s_1 \\ a_1 s_1 + d_2 c_1 \\ \star \end{bmatrix}, \mathbf{p}_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}, \mathbf{z}_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}$$

The inertia matrix can be computed now:

$$\begin{aligned} \mathbf{B}(\mathbf{q}) &= m_1 \mathbf{J}_P^{(l_1)^T} \mathbf{J}_P^{(l_1)} + I_1 \mathbf{J}_O^{(l_1)^T} \mathbf{J}_O^{(l_1)} + m_2 \mathbf{J}_P^{(l_2)^T} \mathbf{J}_P^{(l_2)} + \\ &= m_1 \begin{bmatrix} l_1^2 & 0 \\ 0 & 0 \end{bmatrix} + I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + m_2 \begin{bmatrix} a_1^2 + d_2^2 & a_1 \\ a_1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \end{aligned}$$

where:

$$\begin{aligned} b_{11} &= m_1 l_1^2 + I_1 + m_2 \left(a_1^2 + d_2^2 \right) \\ b_{12} &= m_2 a_1 \\ b_{22} &= m_2 \end{aligned}$$

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

   The only derivative in the Christoffel symbols which is different from zero is:

$$\frac{\partial b_{11}}{\partial q_2} = 2m_2 d_2$$

   therefore

$$\begin{array}{cc} c_{111} = 0 & c_{211} = -\frac{1}{2}\frac{\partial b_{11}}{\partial q_2} = -m_2 d_2 \\ c_{112} = c_{121} = \frac{1}{2}\frac{\partial b_{11}}{\partial q_2} = m_2 d_2 & c_{212} = c_{221} = 0 \\ c_{112} = 0 & c_{222} = 0 \end{array}$$

   The matrix of the Coriolis and centrifugal terms is thus:

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

   where:

$$\begin{aligned} c_{11} &= c_{111}\dot{q}_1 + c_{112}\dot{q}_2 = m_2 d_2 \dot{d}_2 \\ c_{12} &= c_{121}\dot{q}_1 + c_{122}\dot{q}_2 = m_2 d_2 \dot{\vartheta}_1 \\ c_{21} &= c_{211}\dot{q}_1 + c_{212}\dot{q}_2 = -m_2 d_2 \dot{\vartheta}_1 \\ c_{22} &= c_{221}\dot{q}_1 + c_{222}\dot{q}_2 = 0 \end{aligned}$$

3. Check that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

   We have that:

$$\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} 2m_2 d_2 \dot{d}_2 & 0 \\ 0 & 0 \end{bmatrix} - 2\begin{bmatrix} m_2 d_2 \dot{d}_2 & m_2 d_2 \dot{\vartheta}_1 \\ -m_2 d_2 \dot{\vartheta}_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2m_2 d_2 \dot{\vartheta}_1 \\ 2m_2 d_2 \dot{\vartheta}_1 & 0 \end{bmatrix}$$

   which is a skew-symmetric matrix.

4. Cite one case in robotics where the property that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric is used.

   This property is used for example in the proof of stability of the PD + gravity compensation controller.

EXERCISE 2

[1] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

1. Consider the generation of a position trajectory in the Cartesian space. Select as an initial point $\mathbf{p}_i = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$ and as a final point $\mathbf{p}_f = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$. Write the expression of a segment connecting the initial and the final points, parameterized with the natural coordinate.

   The general expression of the segment is:

$$\mathbf{p}(s) = \mathbf{p}_i + \frac{s}{\|\mathbf{p}_f - \mathbf{p}_i\|}(\mathbf{p}_f - \mathbf{p}_i)$$

   Since:

$$\mathbf{p}_f - \mathbf{p}_i = \begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix}, \|\mathbf{p}_f - \mathbf{p}_i\| = 5$$

   we have:

$$\mathbf{p}(s) = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} + \frac{s}{5}\begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix}$$

2. Assume a travel time $T = 2s$. Design a trajectory, which covers the path determined in the previous step, using a cubic dependence on time.

   We need to find the coefficients of the polynomial:

$$s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

   with the boundary conditions:

$$\begin{array}{ll} s(0) = 0 & s(2) = 5 \\ \dot{s}(0) = 0 & \dot{s}(2) = 0 \end{array}$$

   From the conditions at $t = 0$, we easily obtain $a_0 = a_1 = 0$. From the conditions at $t = 2$ we obtain the linear system:

$$\begin{aligned} 4a_2 + 8a_3 &= 5 \\ 4a_2 + 12a_3 &= 0 \end{aligned}$$

   and then:

$$\begin{aligned} a_2 &= 15/4 \\ a_3 &= -5/4 \end{aligned}$$

   Then the expression of the cubic polynomial is:

$$s(t) = \frac{15}{4}t^2 - \frac{5}{4}t^3$$

3. Compute the maximum linear velocity of the end effector along the trajectory designed in the previous step. Check whether this maximum value exceeds a maximum admissible velocity of 2 m/s. In case, explain (without going through the computations) how you would modify the trajectory generation.

   The maximum linear velocity corresponds to the maximum value of the derivative of function $s(t)$:

$$\|\dot{\mathbf{p}}\|_{\max} = \dot{s}_{\max} = \dot{s}(1) = \frac{30}{4} - \frac{15}{4} = 3.75$$

   Since the obtained maximum velocity is higher than the maximum admissible value, the time law has to be scaled, by suitably extending the positioning time.

4. Suppose that the manipulator is kinematically redundant for the task of end effector positioning. Write the expression of the inverse kinematics based on the *weighted* pseudo-inverse matrix and explain what is the optimization problem solved with this approach. Also explain what might be the reason to use the weighted pseudo-inverse instead of the standard pseudo-inverse matrix.

   The expression of the inverse kinematics is:

$$\dot{\mathbf{q}} = \mathbf{J}_W^{\sharp}\dot{\mathbf{p}}$$

   where $\mathbf{J}_W^{\sharp}$ is the weighted pseudo-inverse matrix of the Jacobian, defined as:

$$\mathbf{J}_W^{\sharp} = \mathbf{W}^{-1}\mathbf{J}^T\left(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}\right)^{-1}$$

   This solution solves an optimization problem, where the cost function to be minimized is:

$$g(\dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{W}\dot{\mathbf{q}}$$

   subject to the constraint:

$$\dot{\mathbf{p}} - \mathbf{J}\dot{\mathbf{q}} = 0$$

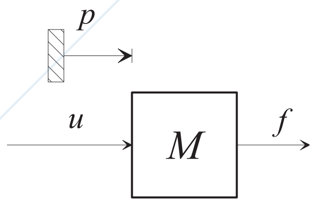
   If $\mathbf{W}$ is diagonal, it can be used to relatively weigh the joint velocities (a large W_i corresponds to a small $\dot{q}_i$).

EXERCISE 3

1. Define the mechanical impedance of a system and explain what is the purpose of an impedance controller.

   The mechanical impedance is defined as the dynamical relation that is established between force and velocity (or displacement) for a mechanical system. The admittance is the reciprocal of the impedance. The impedance control aims at making the system take on a desired mechanical impedance, like a generalized mass-spring-damper system.

2. Consider now a simple mass as in this picture:



Write the expression of an (explicit) impedance controller that can assign a prescribed and complete impedance relation.

To completely assign the impedance relation, we need to measure the interaction force. If this is the case, a suitable control law is:

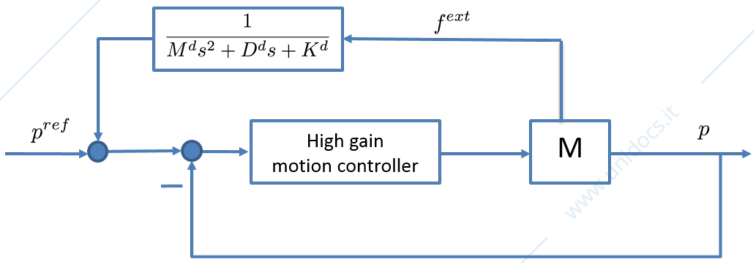
$$u = \frac{M}{M_d}(-k_1 p - k_2 v + f) - f$$

In this case, the following impedance relation is obtained:

$$M_d a + k_2 v + k_1 p = f$$

3. Still making reference to a single degree of freedom mechanism, sketch the block diagram of an admittance controller. What is the assumption that must be enforced on the motion control system in order to claim that the prescribed impedance is actually achieved?

The block diagram is sketched in the picture:



In order to have the prescribed impedance assigned, we need to assume a high bandwidth position controller, so that it guarantees that the output of the admittance controller is correctly tracked.

4. The admittance controller can be used to implement one of the possible collaborative modes between the robot and the human. Explain what is this mode and cite also the other collaborative modes allowed by the safety standards.

The admittance control allows to implement the manual guidance. Other admissible collaborative modes are the protective stop, the speed and separation monitoring and the power and force limitation.