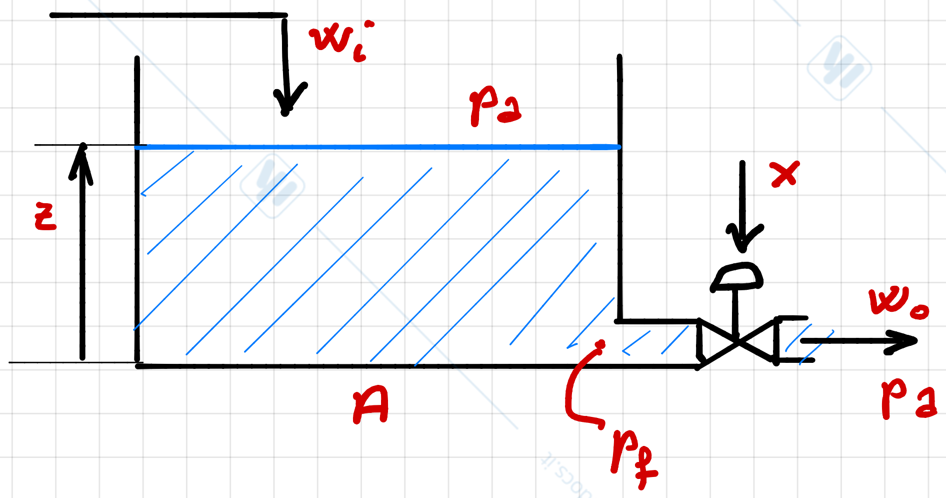
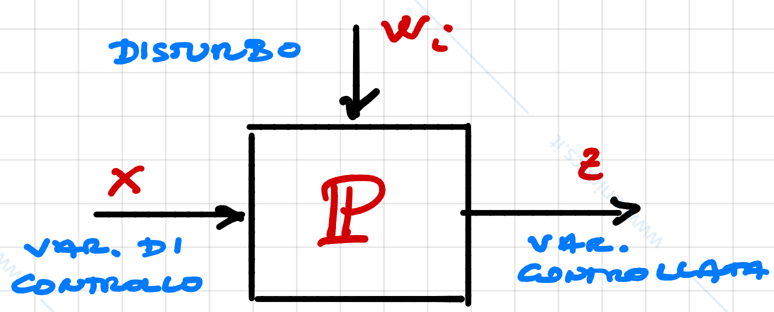
CONTROLLO DI LIVELLO IN UN SERBATOIO

- MODELLO



ρ DENSITÀ
g ACC. GRAVITÀ
p_a PRESSIONE ATMOSFERICA



- SERBATOIO: $\dot{z}(t) = \frac{1}{\rho A}(w_i(t) - w_o(t))$

$p_f(t) = p_a + \rho g z(t)$ STEVINO

- VALVOLA: $w_o(t) = k A_v \eta(x(t)) \sqrt{\rho (p_f(t) - p_a)} =$

$= k A_v \eta(x(t)) \rho \sqrt{g z(t)} =$

$= \varphi \rho \eta(x(t)) \sqrt{z(t)}$

$\varphi = k A_v \sqrt{g}$

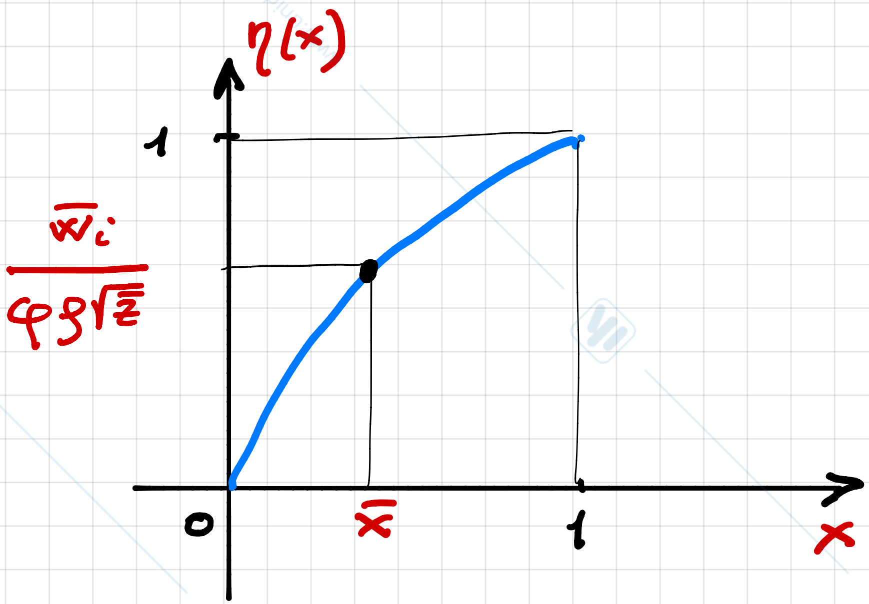
$\Longrightarrow$ $\dot{z}(t) = \frac{1}{\rho A} w_i(t) - \frac{\varphi}{A} \eta(x(t)) \sqrt{z(t)}$ NON LINEARE

- EQUILIBRIO

$$\overline{w_i} = \overline{w_0} = \varphi \rho \eta(\bar{x}) \sqrt{\bar{z}}$$

- FISSATI $\bar{z}$, $\overline{w_i}$, $\bar{x}$ DEVE ESSERE TALE CHE:

$$\eta(\bar{x}) = \frac{\overline{w_i}}{\varphi \rho \sqrt{\bar{z}}} \implies \bar{x} = \eta^{-1}\left(\frac{\overline{w_i}}{\varphi \rho \sqrt{\bar{z}}}\right), \quad \bar{x} \in [0,1]$$



- LINEARIZZAZIONE INTORNO ALL'EQUILIBRIO

MODELLO NON LINEARE

$$\dot{z}(t) = \frac{1}{\rho A} w_i(t) - \frac{\varphi}{A} \eta(x(t)) \sqrt{z(t)}$$

EQUILIBRIO $\bar{z}, \bar{w}_i, \bar{x}$

$$\delta z(t) = z(t) - \bar{z} \quad , \quad \delta x(t) = x(t) - \bar{x} \quad , \quad \delta w_i(t) = w_i(t) - \bar{w}$$

$$\delta \dot{z}(t) = \alpha \delta z(t) + \beta \delta x(t) + \gamma \delta w_i(t)$$

MODELLO LINEARIZZATO

$$\left. \begin{cases} \alpha = -\dfrac{\varphi \eta(\bar{x})}{2A\sqrt{\bar{z}}} < 0 \\ \beta = -\dfrac{\varphi \sqrt{\bar{z}}}{A} \dfrac{d\eta(\bar{x})}{dx} < 0 \\ \gamma = \dfrac{1}{\rho A} > 0 \end{cases} \right.$$

DIPENDONO DALLO STATO DI EQUILIBRIO

- CALCOLO FdT

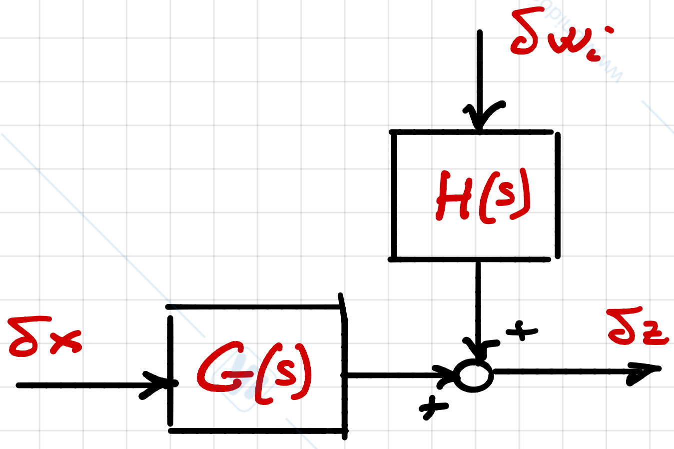
MODELLO LINEARIZZATO

$$\delta\dot{z}(t) = \alpha\,\delta z(t) + \beta\,\delta x(t) + \gamma\,\delta w_i(t)$$

$$Z(s) = \mathcal{L}[\delta z(t)] \;,\; X(s) = \mathcal{L}[\delta x(t)] \;,\; W_i(s) = \mathcal{L}[\delta w_i(t)]$$

$$(s-\alpha)Z(s) = \beta X(s) + \gamma W_i(s)$$

$$\Rightarrow \boxed{Z(s) = \underbrace{\frac{\beta}{s-\alpha}}_{G(s)} X(s) + \underbrace{\frac{\gamma}{s-\alpha}}_{H(s)} W_i(s)}$$

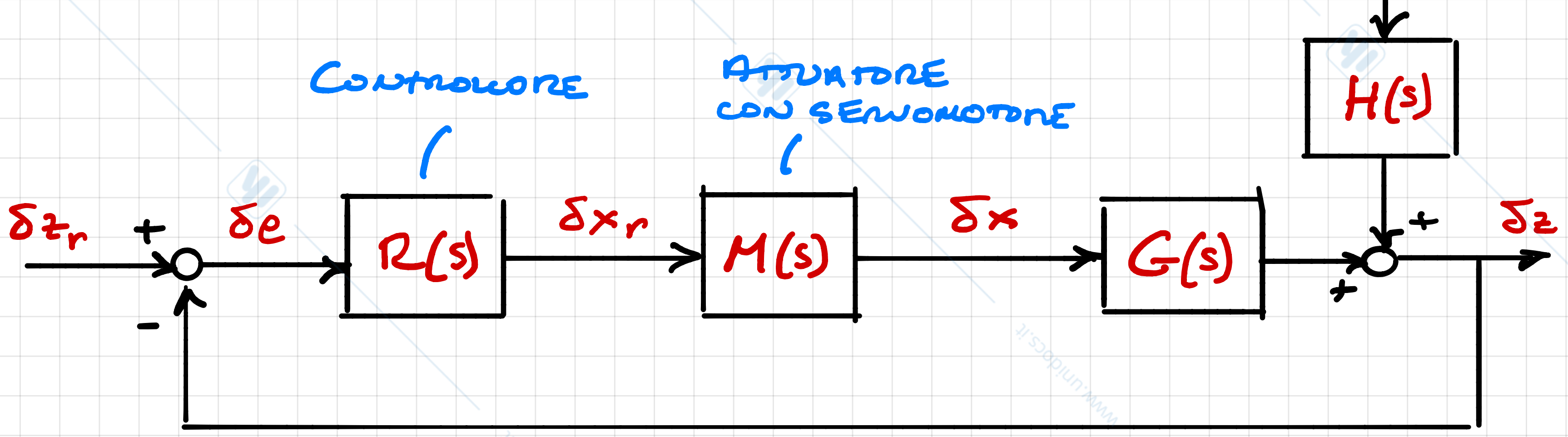


$$G(s) = \frac{\beta}{s-\alpha} = \frac{\mu_G}{1+s\tau}$$

$$H(s) = \frac{\gamma}{s-\alpha} = \frac{\mu_H}{1+s\tau}$$

$$\begin{cases} \tau = -\dfrac{1}{\alpha} = \dfrac{2A\sqrt{\bar{z}}}{\varphi\eta(\bar{x})} > 0 \\ \mu_G = -\dfrac{\beta}{\alpha} = -\dfrac{2\bar{z}}{\eta(\bar{x})}\dfrac{d\eta(\bar{x})}{dx} < 0 \\ \mu_H = -\dfrac{\gamma}{\alpha} = \dfrac{2\sqrt{\bar{z}}}{\rho\varphi\eta(\bar{x})} > 0 \end{cases}$$

- Schema sistema di controllo



$$G(s)=\frac{\mu_G}{1+s\tau}\ ,\quad H(s)=\frac{\mu_H}{1+s\tau}\ ,\quad M(s)=\frac{1}{1+s\tau_M}\ ,\quad \tau_M << \tau$$

$R(s)$ da progettare

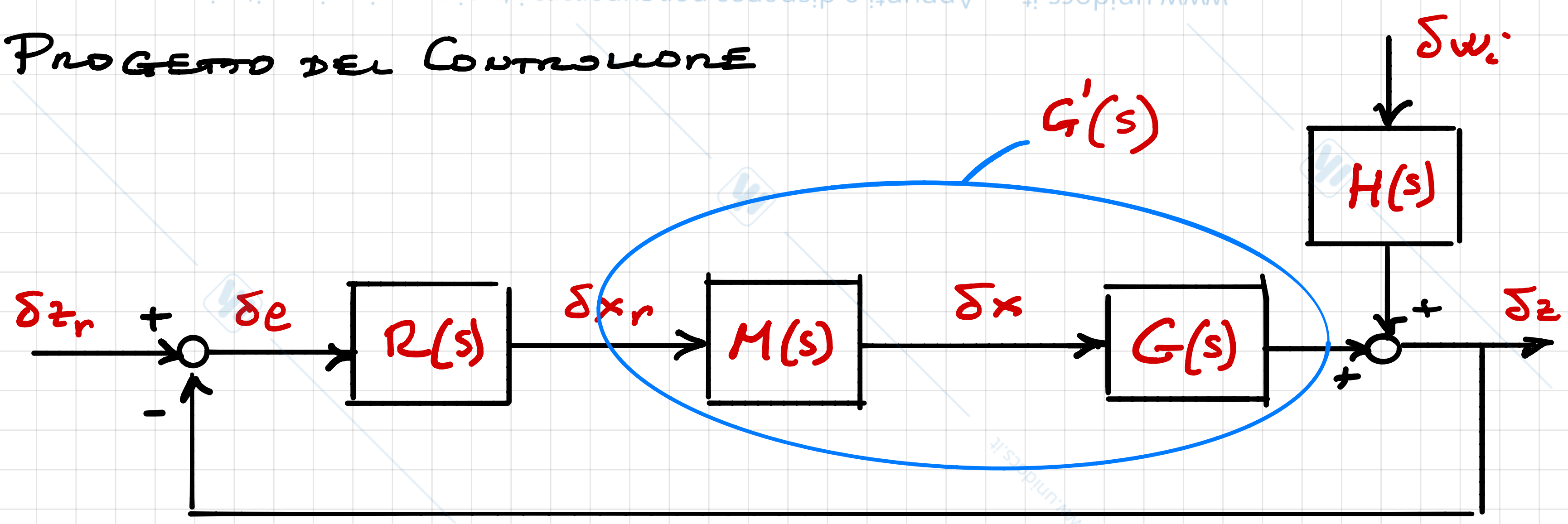
- CASO PARTICOLARE - VALVOLA LINEARE

$$\eta(x) = x \qquad \bar{x} = \frac{\bar{w}_i}{\varphi \rho \sqrt{\bar{z}}}$$

$$\begin{cases} \tau = \dfrac{2A\sqrt{\bar{z}}}{\varphi \bar{x}} = \dfrac{2A\rho\bar{z}}{\bar{w}_i} \\ \mu_G = -\dfrac{2\bar{z}}{\bar{x}} = -\dfrac{2\bar{z}^{3/2}\varphi\rho}{\bar{w}_i} \\ \mu_H = \dfrac{2\sqrt{\bar{z}}}{\varphi\rho\bar{x}} = \dfrac{2\bar{z}}{\bar{w}_i} \end{cases}$$

- NOTA: . AL CRESCERE DI $\bar{w}_i$ DIMINUISCONO $\tau, |\mu_G|, \mu_H$
  . AL CRESCERE DI $\bar{z}$ AUMENTANO $\tau, |\mu_G|, \mu_H$

- PROGETTO DEL CONTROLLORE



$$G'(s) = G(s)M(s) = \frac{\mu_G}{(1+s\tau)(1+s\tau_M)}$$

$\mu_G < 0$

$\tau_M << \tau$

PI

$$R(s) = \mu_R \frac{1+s\tau}{s} = -\frac{K_p}{T_i}\frac{1+sT_i}{s}$$

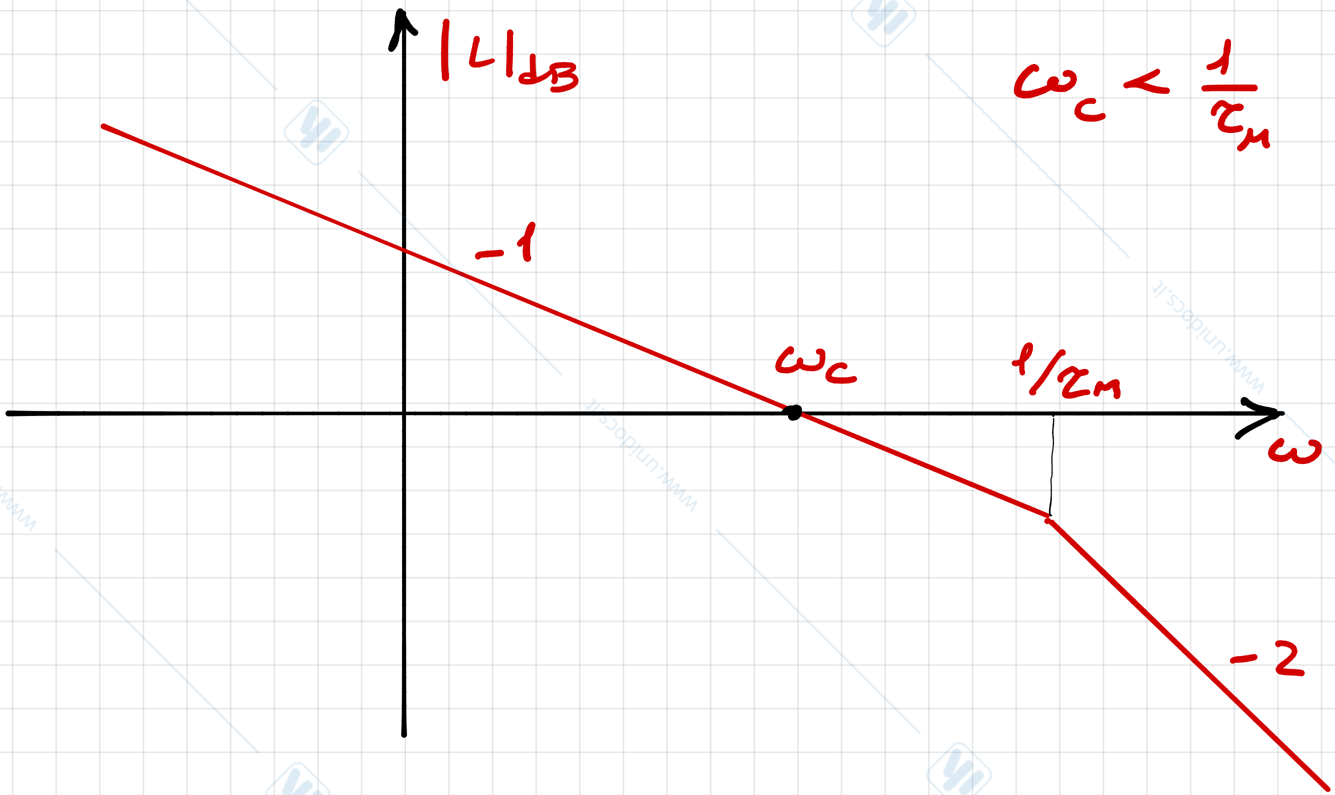
$T_i = \tau$

$-\frac{K_p}{T_i} = \mu_R < 0$

$$L(s) = R(s)G'(s) = \underbrace{-\frac{K_p}{T_i}\mu_G}_{>0}\frac{1}{s(1+s\tau_M)}$$

RESTA DA TARARE K_p

- LINEE GUIDA SULLA TARATURA DI K_p



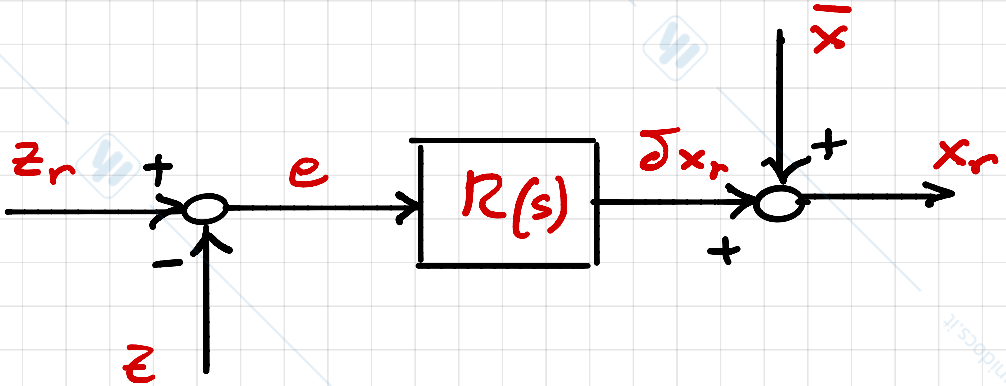
$\omega_c < \frac{1}{\tau_M}$

$\omega_c \simeq \frac{|K_p \mu_G|}{\tau}$

$\varphi_m = 90° - \arctg \omega_c \tau_M$

- AUMENTANDO K_p :
  - AUMENTA ω_c
  - DIMINUISCE φ_m
  - PEGGIORA LA MODERAZIONE (ATTENZIONE! $|\delta x|$ È LIMITATO)

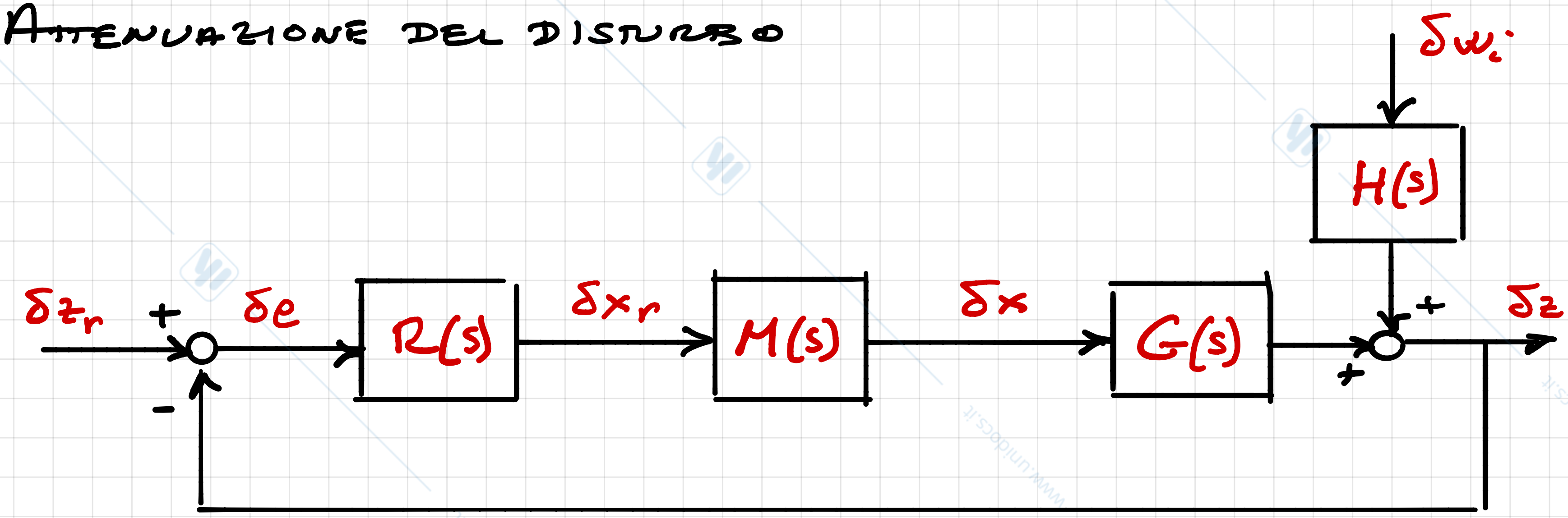
- IMPLEMENTAZIONE DEL CONTROLLORE



$$x_r = \bar{x} + \delta x_r$$

- NOTA $\boxed{\delta e} = \delta z_r - \delta z = (z_r - \bar{z}) - (z - \bar{z}) = z_r - z = \boxed{e}$

- ATTENUAZIONE DEL DISTURBO



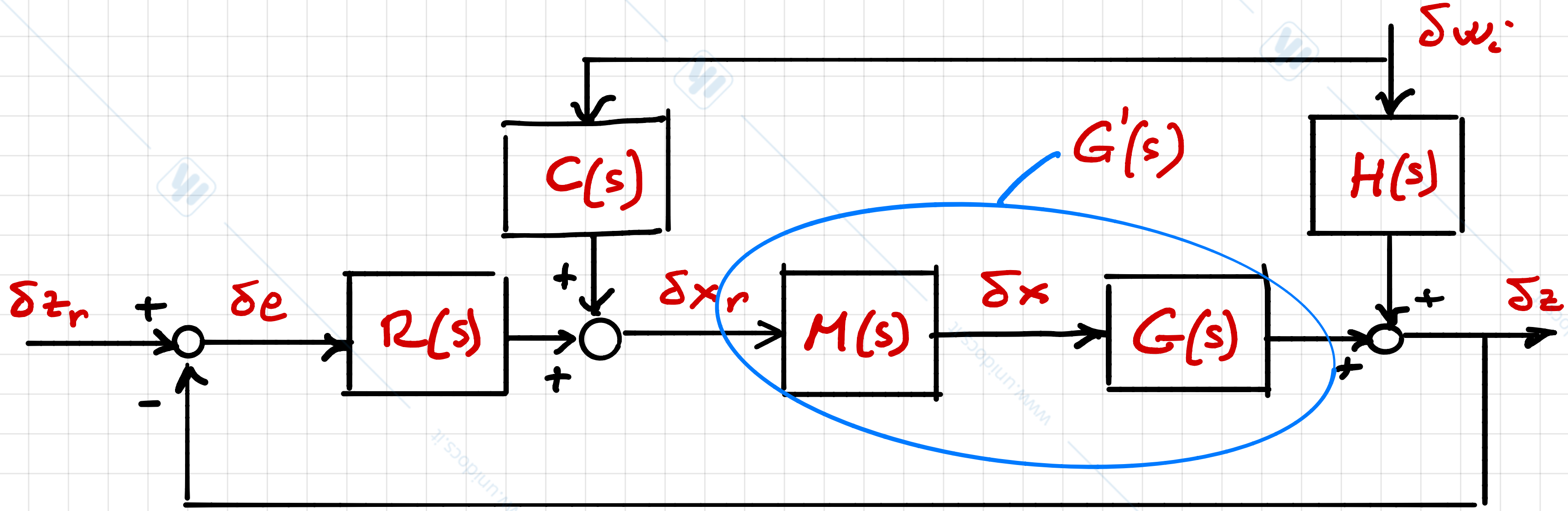
$$G_{zw_i}(s) = \frac{H(s)}{1+L(s)} = H(s)S(s)$$

CONTIENE POLO "LENTO" CON COST. TEMPO τ

PASSA - ALTO CON $B_s \simeq [\omega_c, \infty)$

$\Longrightarrow$ IL TRANSITORIO IN RISPOSTA AL DISTURBO δw_i È COMUNQUE LENTO

- PROGETTO DEL COMPENSATORE IN A.A.



COMP. IDEALE $C^{o}(s) = -\dfrac{H(s)}{G'(s)} = -\dfrac{\mu_H}{\mu_G}(1+s\tau_M)$ NON REALIZZABILE

COMP. STATICO $C(s) = -\dfrac{\mu_H}{\mu_G} = \dfrac{1}{\varphi g\sqrt{\bar{z}}} > 0$

ATTENZIONE!
NON ROBUSTO

(A MENO DI USARE UN COMP. ADATTATIVO
CON GUADAGNO CHE SI ADATTA
IN FUNZIONE DI $\bar{z}$)

- Esempio Numerico

- Dati

$A = 1\ [m^2]$

$A_v = 14 \cdot 10^{-4}\ [m^2]$

$k = \frac{\sqrt{2}}{C_r} = 4 \qquad (C_r \simeq 0.35) \implies \varphi = k A_v \sqrt{g} \simeq 1.75 \cdot 10^{-2}$

$\rho = 1000\ [kg/m^3]$

$g = 9.8\ [m/s^2]$

$\tau_M = 3\ [s]$

- Equilibrio

$\bar{z} = 1.3\ [m]$

$\bar{w}_i = 10\ [kg/s]$

$\implies$ valvola lineare $\bar{x} = \frac{\bar{w}_i}{\varphi \rho \sqrt{\bar{z}}} = 0.5$

- Parametri modello linearizzato

$\tau = 260\ [s]$

$\mu_G = -5.2\ [m]$

$\mu_M = 0.26\ [ms/kg]$

- SPECIFICHE DI PROGETTO

- CON $\delta z_r = 0.3 \, \mathrm{sca}(t)$

(a) $e(\infty) = 0$

(b) $t_a \leq 60$ [s] $\Longrightarrow$ $\omega_c \geq \frac{5}{36} \simeq 0.14$

(c) $\varphi_m \geq 60°$