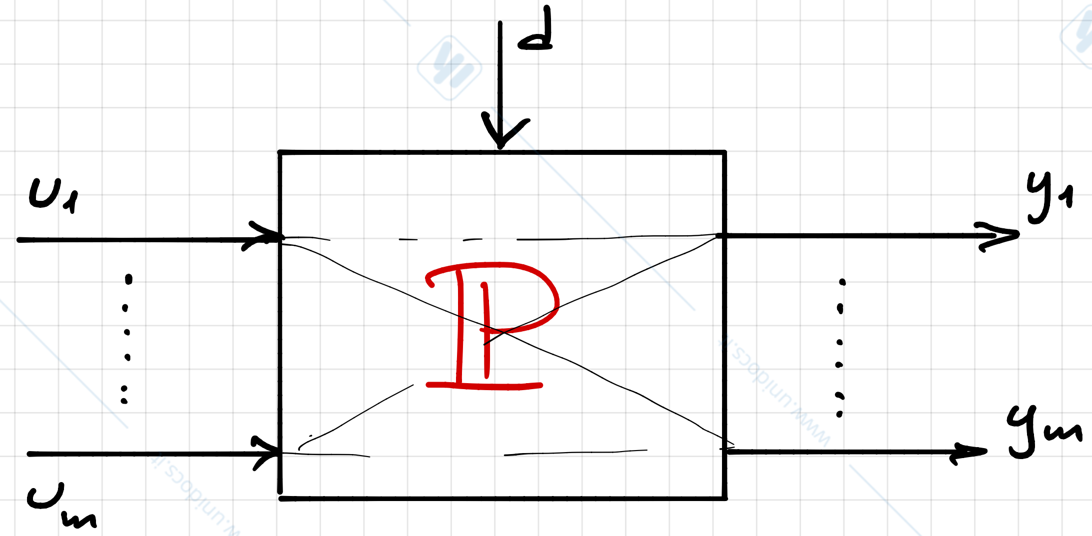
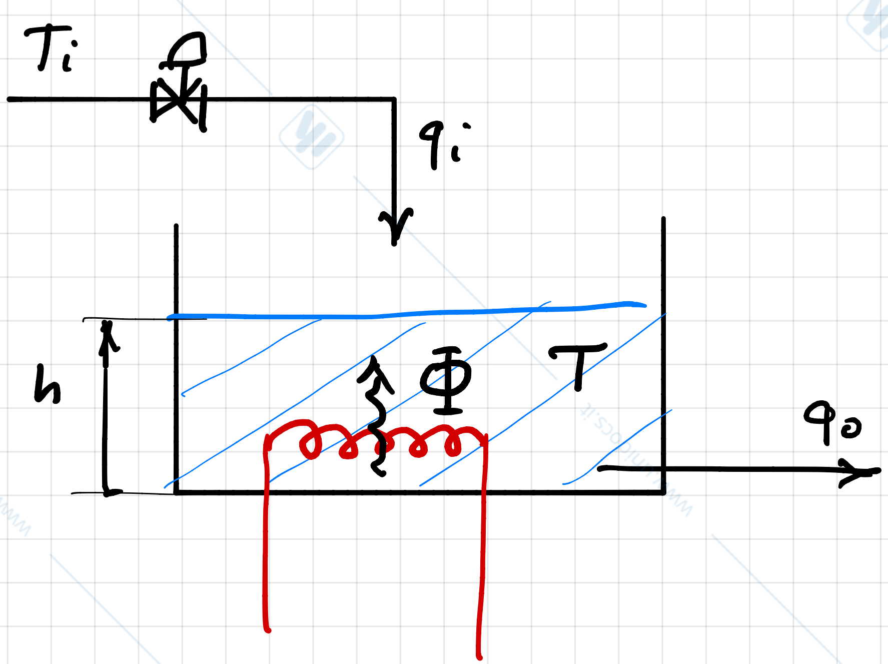
CONTROLLO MULTIVARIABILE

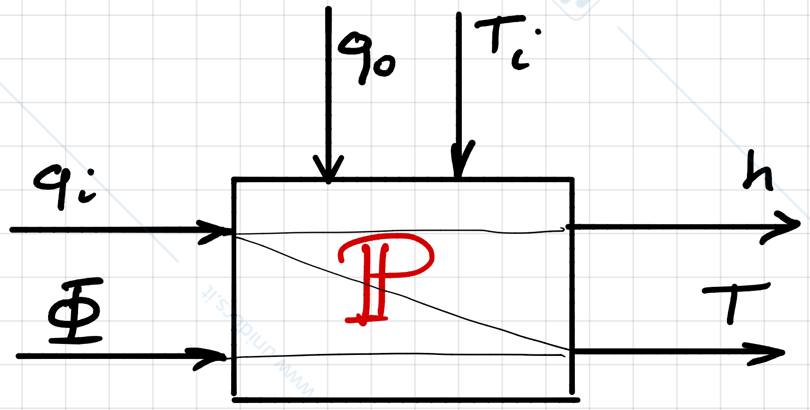
- Sistemi di Controllo MIMO (multi-input multi-output)



- Problema: controllo congiunto di y_i mediante u_j
- Difficoltà: possibili interazioni

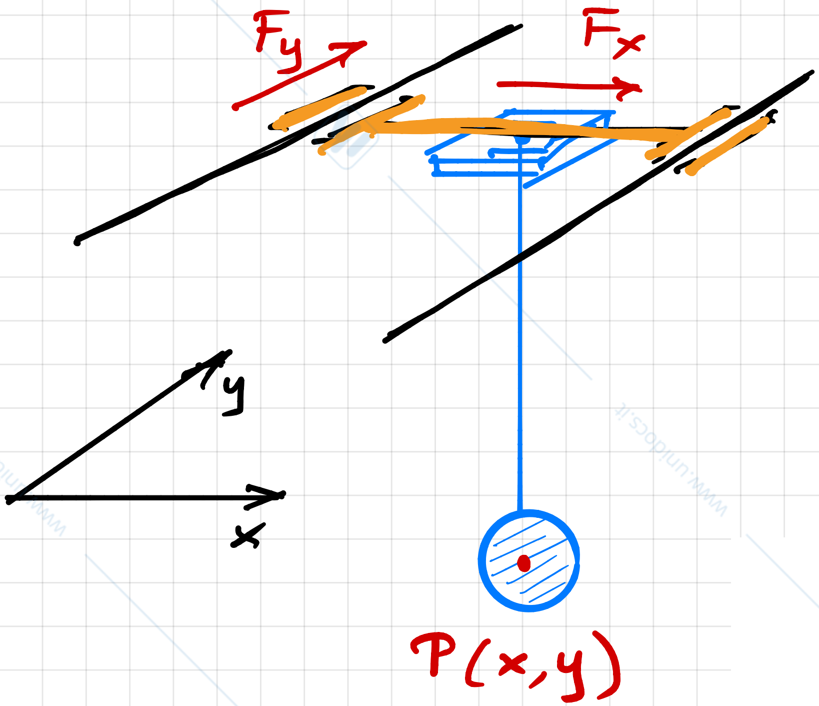
\- ESEMPIO 1 – CONTROLLO DI LIVELLO E TEMPERATURA

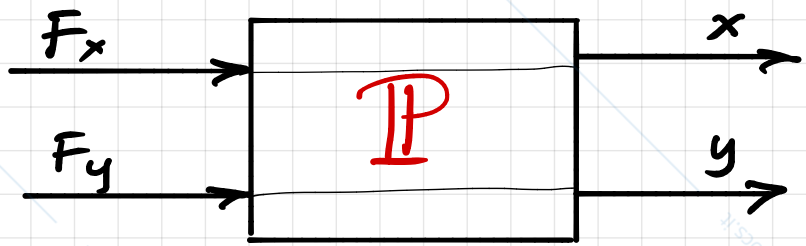




INTERAZIONE "PARZIALE"

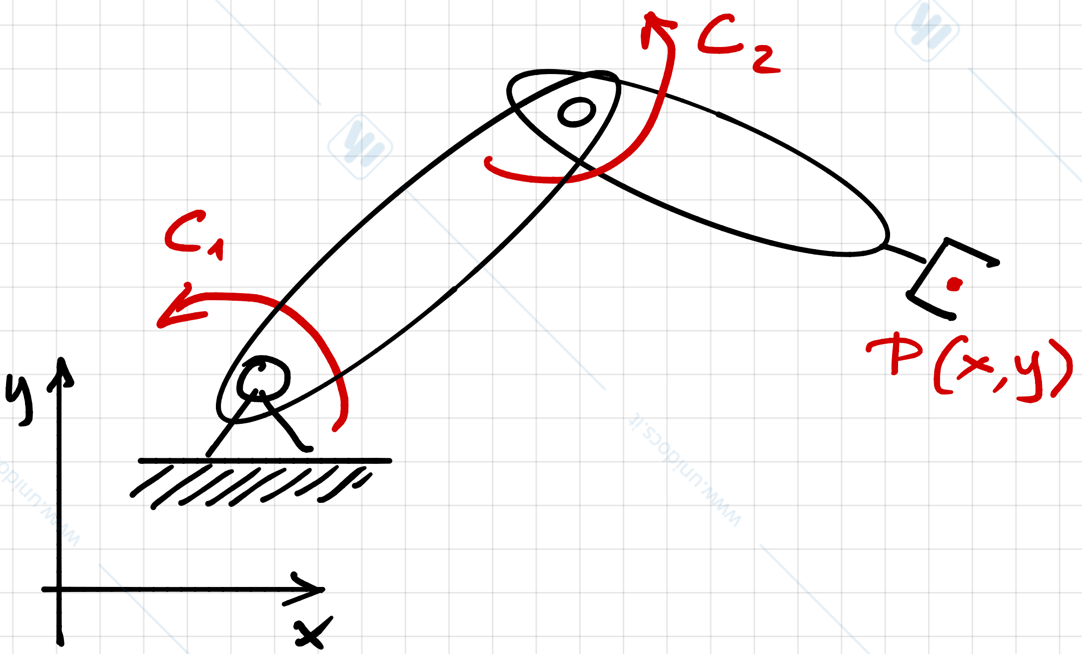
- ESEMPIO 2 - CONTROLLO DEL MOTO DI UN CARRO-PONTE

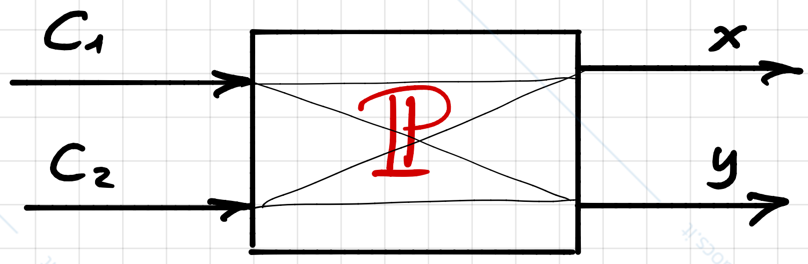




INTERAZIONE NULLA

– ESEMPIO 3 – Controllo di un Braccio Robotico





INTERAZIONE "COMPLETA"

- MATRICE DI TRASFERIMENTO (RICHIAMI)

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \qquad u(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix} \qquad y(t) = \begin{bmatrix} y_1(t) \\ \vdots \\ y_p(t) \end{bmatrix}$$

- MATRICE DI TRASFERIMENTO

$$G(s) = C(sI - A)^{-1}B + D = \begin{bmatrix} G_{11}(s) & \cdots & G_{1m}(s) \\ \vdots & & \vdots \\ G_{p1}(s) & \cdots & G_{pm}(s) \end{bmatrix}$$

MATRICE $p \times m$

$$Y(s) = G(s)U(s)$$ CON $x(0) = 0$

$$Y_i(s) = \sum_{j=1}^{m} G_{ij}(s)U_j(s)$$

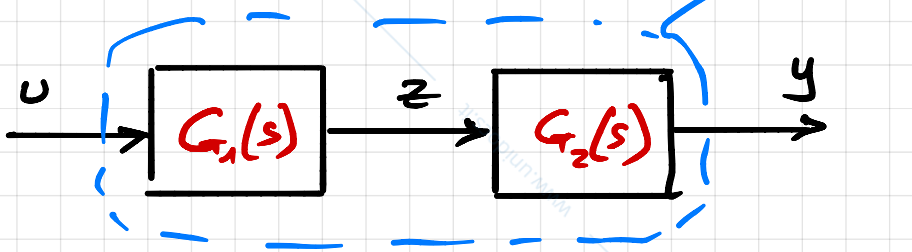
$$G_{ij}(s) = \frac{Y_i(s)}{U_j(s)}$$ CON $U_k(s) = 0,\ k \neq j$, $x(0) = 0$

DESCRIVE L'EFFETTO DI $u_j(t)$ SU $y_i(t)$

- SCHEMI A BLOCCHI MIMO

$$Y(s) = \underbrace{G(s)}_{p \times m} U(s)$$

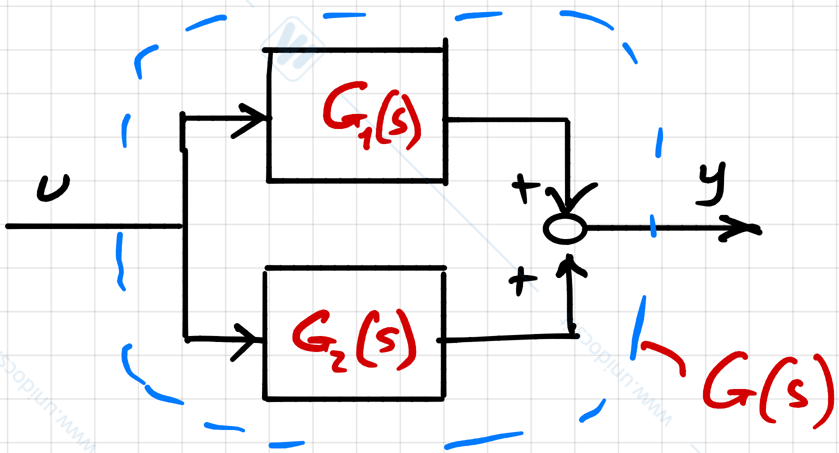
- BLOCCHI IN SERIE



$$Y(s) = G_2(s) Z(s) = \underbrace{G_2(s) G_1(s)}_{G(s)} U(s)$$

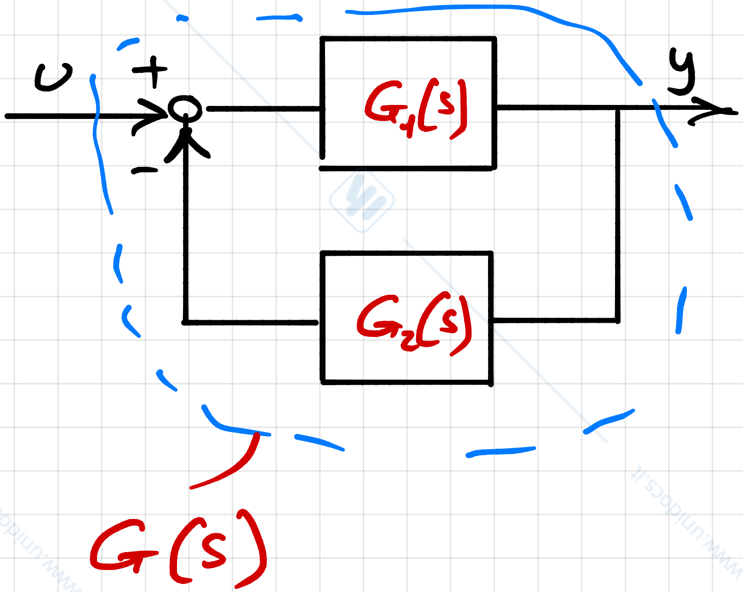
L'ORDINE È IMPORTANTE!

- BLOCCHI IN PARALLELO



$$Y(s) = \underbrace{\left(G_1(s) + G_2(s)\right)}_{G(s)} U(s)$$

- Blocchi in retroazione



$$Y(s) = G_1(s)\left(U(s) - G_2(s)\,Y(s)\right)$$

$$\left(I + G_1(s)G_2(s)\right)Y(s) = G_1(s)\,U(s)$$

$$Y(s) = \underbrace{\left(I + G_1(s)G_2(s)\right)^{-1} G_1(s)}_{G(s)}\,U(s)$$

L'ordine è importante

- POLI E ZERI DI UN SISTEMA MIMO (CASO $m=p$)

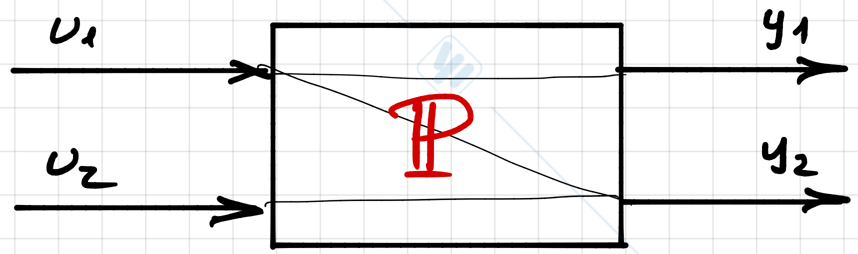
$G(s)$ MATRICE DI TRASFERIMENTO $m \times m$

- POLI: TUTTE LE RADICI DEI DENOMINATORI DI $G_{ij}(s)$
- ZERI: TUTTE LE RADICI DI $\det G(s) = 0$

ASINTOTICA STABILITÀ $\Longleftrightarrow$ TUTTI I POLI HANNO $Re < 0$

- DISACCOPPIAMENTO DI SISTEMI "TRIANGOLARI"

$m=2$

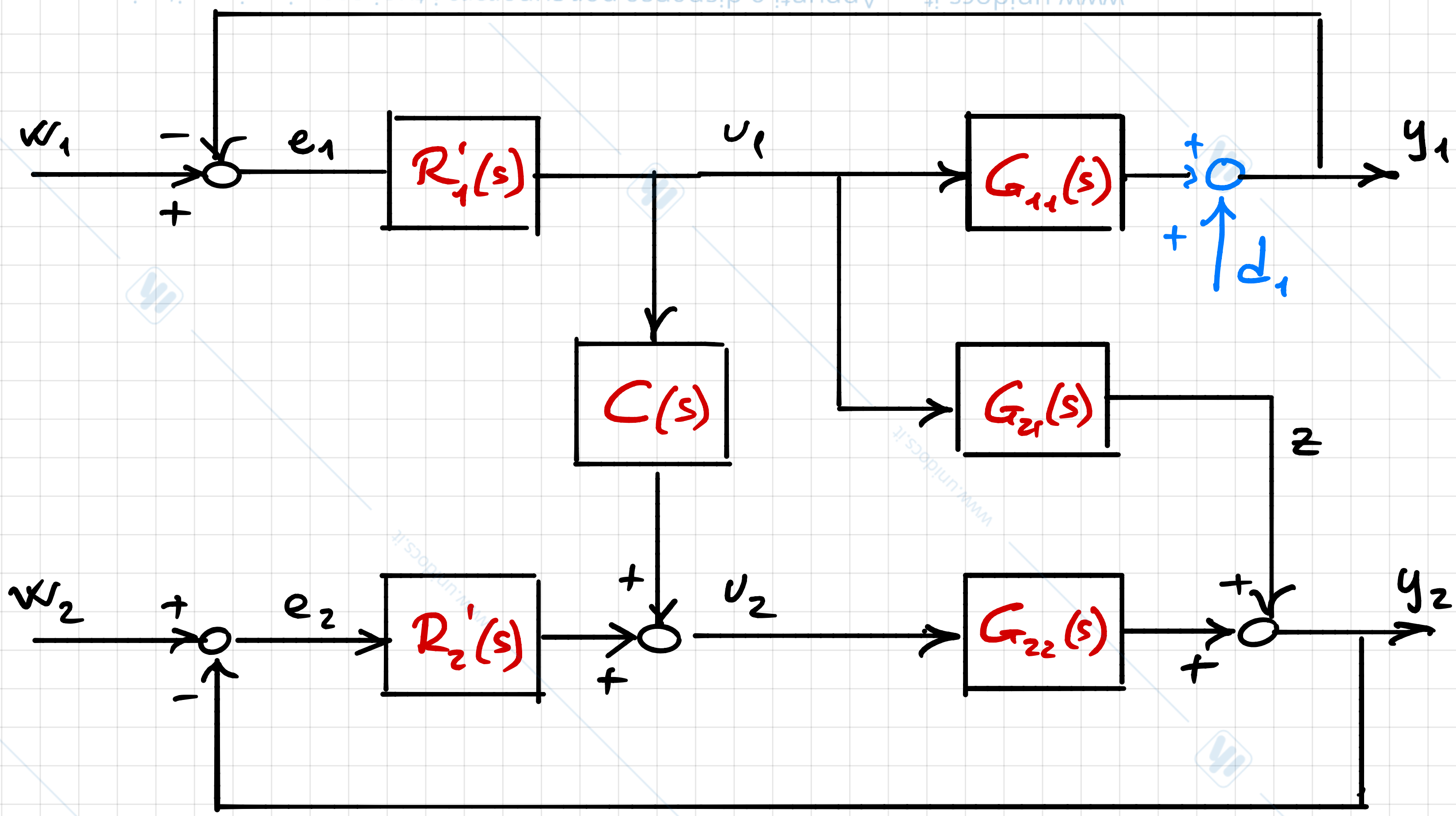


$$G(s) = \begin{bmatrix} G_{11}(s) & 0 \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

MATRICE TRIANGOLARE (INTERAZIONE PARZIALE)

$$\begin{cases} Y_1(s) = G_{11}(s)U_1(s) \\ Y_2(s) = G_{21}(s)U_1(s) + G_{22}(s)U_2(s) \end{cases}$$

- CI SI PUÒ RICONDURRE AL PROGETTO DI 2 SISTEMI DI CONTROLLO SISO ?



1. PROGETTO DI $R_1'(s)$ SU $G_{11}(s)$

2. PROGETTO DI $R_2'(s)$ SU $G_{22}(s)$ + COMPENSAZIONE IN A.A. DEL "DISTURBO" u_1 CON $C(s) = -\dfrac{G_{21}(s)}{G_{22}(s)}$

- ESEMPIO

$$G_{11}(s) = \frac{10(1+s)}{(1+10s)(1+2s)}$$

$$G_{21}(s) = \frac{4}{1+2s}$$

$$G_{22}(s) = \frac{1+5s}{(1+10s)(1+s)}$$

- SPECIFICHE (PER ENTRAMBI GLI ANELLI)

$e(\infty) = 0$ CON RIF. A SCALINO

$\omega_c \simeq 5$

$\varphi_m \geq 60°$

- PROGETTO

$$R_1'(s) = \frac{1+10s}{s} \Rightarrow L_1'(s) = R_1'(s) G_{11}(s) = \frac{10(1+s)}{s(1+2s)}$$

$\omega_{c1} \simeq 5$

$\varphi_{m1} \simeq 84°$

$$R_2'(s) = \frac{10(1+s)}{s} \Rightarrow L_2'(s) = R_2'(s) G_{22}(s) = \frac{10(1+5s)}{s(1+10s)}$$

$\omega_{c2} \simeq 5$

$\varphi_{m2} \simeq 89°$

$$C^{\circ}(s) = -\frac{G_{21}(s)}{G_{22}(s)} = -\frac{4(1+10s)(1+s)}{(1+2s)(1+5s)}$$ COMP. DINAMICO

$$\bar{C}(s) = -\frac{G_{21}(0)}{G_{22}(0)} = -4$$ COMP. STATICO

- SIMULAZIONI

$$w_1(t) = sca(t)$$

$$d_1(t) = sca(t-3)$$

$$w_2(t) = sca(t-5)$$

1 SENZA $C(s)$

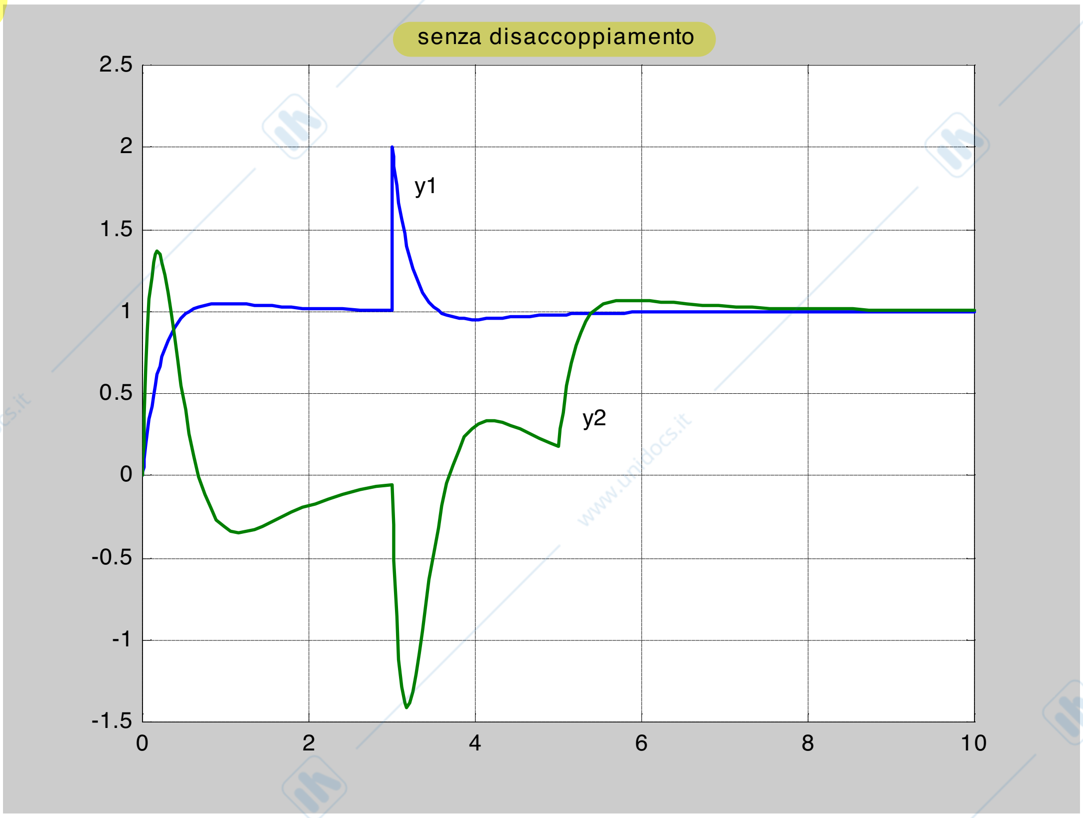
2 CON $C(s)$

- $C^{\circ}(s)$ DISACCOPPIAMENTO DINAMICO
- $\bar{C}(s)$ DISACCOPPIAMENTO STATICO

CONTROLLO DI SISTEMA MIMO TRIANGOLARE

$w_1(t) = sca(t)$, $d_1(t) = sca(t-3)$, $w_2(t) = sca(t-5)$

1.



2.