29/01/21

(1) $G(s) = \frac{20}{1-0.4s}$

1.1 $R(s) = \frac{\mu}{s}$

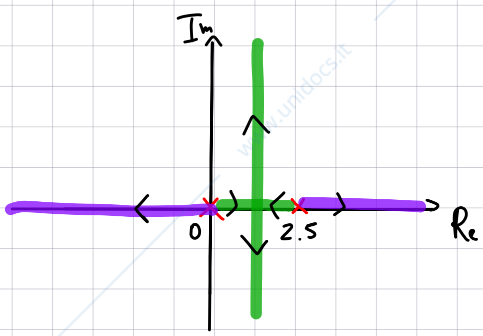
$L(s) = \frac{-\mu\, 20}{0.4\, s(s-2.5)}$

$\rho = -50\mu$

↳ polo in 2.5

LD

LI



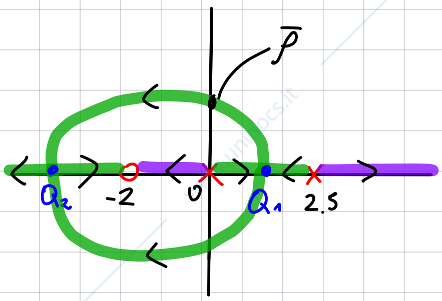
AS. STAB: $\nexists\ \rho \geq 0$ t.c. il sist. A.C. è AS. STAB.

1.2 $R(s) = \frac{\mu(1+0.5s)}{s} = \frac{\mu \cdot 0.5(s+2)}{s}$

$L(s) = \frac{\overbrace{-25\mu}^{\rho}(s+2)}{s(s-2.5)}$

LDR: $\overbrace{n=2}^{2 \text{ RAMI}}$, $m=1$, $\overbrace{\nu=1}^{1 \text{ ASINTOTO}}$

- Baricentro A.A. $x_B = \frac{1}{n}(\Sigma -p_i) = \frac{1}{2} \cdot 2.5 = 1.25$
- Baricentro asintoti: $x_A = \frac{1}{\nu}(\Sigma z_i - \Sigma p_i) = \frac{-2+2.5}{1} = 0.5$
- Orientamento asintoti: LD $180°$
  LI $0°$



Q_1 max di $\gamma(x) = -\frac{D(x)}{N^*(x)} = -\frac{s(s-2.5)}{(s+2)}$

Q_2 min di $\gamma(x)$

$\hookrightarrow \gamma'(x) = -\frac{[\overbrace{(s-2.5)+s}^{2s-2.5}](s+2) - s(s-2.5)}{(s+2)^2}$

$\gamma'(x)=0 = -(2s^2+1.5s-5-s^2+2.5s) =$

$= -(s^2+4s-5) = -(s-1)(s+5)$

$\gamma'(x) \geq 0 \rightarrow (s-1)(s+5) \leq 0$

$-5 \leq s \leq 1$

	-5		1	
$-$	MIN Q_2	$+$	MAX Q_1	$-$

AS. STAB $\Longleftrightarrow \rho > \bar{\rho}$

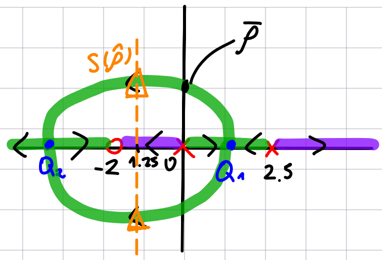
$\bar{\rho}$ - Criterio di Routh

$\varphi_{AC} = s^2 \underbrace{-2.5s + \rho s}_{s(\rho-2.5)} + 2$

$s(\underbrace{\rho - 2.5}_{})$ deve essere di segno concorde +

$\rho - 2.5 > 0 \longrightarrow \rho > \underbrace{2.5}_{\bar{\rho}} \Rightarrow \rho = -25\mu \rightarrow \bar{\mu} = -0.1$

1.3 $\hat{\mu} \begin{cases} \text{AS. STAB} \\ t_a < 4 \end{cases}$?



- AS. STAB. $\iff \hat{\mu} < \bar{\mu} = -0.1$
- $t_a < 4 \rightarrow \frac{5}{|\sigma|} < 4 \qquad |\sigma| > 1.25 \longrightarrow \hat{\sigma} = -1.25 \longrightarrow R_e(s_{DOM}(\hat{\rho})) = -1.25$

Visto che non vale la cons. del baricentro ($v=1$) utilizzo un polo con $R_e(s_i(\tilde{\rho})) < -1.25$

↳ Q_2 trovato prima in $s_{Q_2}(\tilde{\rho}) = -5$

$$\tilde{\rho} = \frac{d(-5,0) \cdot d(-5,2.5)}{d(-5,-2)} = \frac{5 \cdot 7.5}{3} = 12.5 \longrightarrow \tilde{\mu} = -0.5$$

dove $\hat{\rho} < \tilde{\rho} \Rightarrow \hat{\mu} > \tilde{\mu}$

② $T=5$

$$G^*(z)=\frac{z+1}{(z-1)(z-0.2)}$$

2.1 Vale che dai poli a t. discreto si ricavano quelli a t. continuo secondo la trasf. di camp. $z=e^{sT}$

- $z_1=1 \rightarrow s_1=T\cdot\ln z_1=0$
- $z_2=0.2 \rightarrow s_2=T\cdot\ln z_2 \cong -8.05$

2.2

$$u^*(k)=\alpha u^*(k-1)+0.5e^*(k)-0.1e^*(k-1) \longrightarrow R^*(z)=\frac{\overbrace{0.5z-0.1}^{0.5(z-0.2)}}{z-\alpha}$$

per $\alpha=0$

$\hookrightarrow L(z)=\frac{z+1}{2(z-1)z}$ $\quad \varphi_{AC}(z)=2z^2-2z+z+1=0$

$$z_{1,2}=\frac{1\pm\sqrt{1-8}}{4} \begin{cases} \frac{1}{4}+j\overbrace{\frac{\sqrt{7}}{4}}^{0.66}=z_1^* \\ \frac{1}{4}-j\frac{\sqrt{7}}{4}=z_2^* \end{cases}$$

$|z_{1,2}|\cong 0.71 \longrightarrow |z_1^*|<1 \quad |z_2^*|<1$

$\hookrightarrow$ AS. STAB.

2.3 Per $\alpha=0$

$$k_a\simeq-\frac{5}{\ln|z_{dom}|}\simeq-\frac{5}{\ln|z_{1,2}|}\cong 14.6\ s \qquad \overset{\text{t. latenza}}{K_e}=\nu_F-1=0 \qquad \overset{\text{t. continuo}}{t_a}=k_aT\simeq 73\ s$$

$$F(z)=\frac{L(z)}{1+L(z)}$$

• **PRECISIONE STATICA:** $L(z)$ ha un polo in $z=1$ → azion. integrale → $\mu_F=1$

2.4 Se $\alpha=-1 \rightarrow L(z)=\frac{0.5}{z-1}\cdot\frac{\cancel{z-0.2}}{\cancel{z+1}}\cdot\frac{\cancel{z+1}}{\cancel{z-0.2}}$ canc. critica $\Longrightarrow$ NON AS. STAB.

③ 3.1. $z_0 = L \cdot \sin(45°) = \frac{L}{\sqrt{2}}$ $\qquad p_a = 0$

EQ. CONS. MASSA (ρ cost) $\quad \frac{\partial w(x,t)}{\partial x} = 0 \longrightarrow w$ non dip. da x

$u(t) = \frac{w(t)}{\rho \cdot A}$ non dip. da x

EQ. CONS. Q. MOTO $\quad \frac{dw(t)}{dt} = -\frac{\rho A g}{L}(z^*(L,t) - z^*(0,t)) - \bar{f}\, w^2(t)$

$\bar{f} = \frac{1}{2} c_f \frac{\pi D}{\rho A^2}$

$z^*(0,t) = \underbrace{z(0)}_{\frac{L}{\sqrt{2}}} + \overbrace{\frac{p(0,t)}{\rho g}}^{p(t)}$, $\quad z^*(L,t) = \cancel{z(L)}^{0} + \overbrace{\cancel{\frac{p(L,t)}{\rho g}}}^{p_a = 0} \rightarrow 0$

MOD. DINAMICO: $\dot{w} = \frac{\rho A g}{L}\left(\frac{L}{\sqrt{2}} + \frac{p(t)}{\rho g}\right) - \bar{f} \cdot w^2(t)$

$(y = w)$

3.2. EQ. $\dot{w} = 0 \longrightarrow \bar{w} = \sqrt{\frac{\rho A g\left(\frac{L}{\sqrt{2}} + \bar{p}\right)}{L \bar{f}}}$

$\delta \dot{w}(t) = -2 \cdot \bar{f}\, \bar{w}\, \delta w(t) + \frac{A}{L}\, \delta p(t)$ $\qquad$ MODELLO LINEARIZZATO

FDT $\quad a = -2\bar{f}\bar{w}$

$b = \frac{A}{L}$

$c = 1$

$d = 0$

$G(s) = c(s-a)^{-1}b = \frac{b}{s-a} = \frac{A/L}{s + 2\bar{f}\bar{w}} = \frac{A}{2L\bar{f}\bar{w}\left(\frac{s}{2\bar{f}\bar{w}} + 1\right)} \longrightarrow \begin{cases} \mu = \frac{A}{2\bar{f}\bar{w}L} \\ \tau = \frac{1}{2\bar{f}\bar{w}} \end{cases}$

$\bar{w} = \sqrt{\frac{\rho A g\left(\frac{L}{\sqrt{2}} + \bar{p}\right)}{L \bar{f}}}$

3.3

- $t_{AA} = 5\tau$

Requisito: $t_{A.C.} \le 2\, t_{AA} = 10\tau$

$R_{PI} = \frac{K_p}{T_I}\cdot\frac{1+sT_I}{s}$ → Scelgo $T_I=\tau$ per cancellare il polo lento τ e poi scelgo K_p per soddisf. le specifiche

↳ $T_I=\tau \Rightarrow L(s) = \frac{K_p\,\mu}{\tau s}$ Ma calc. prima

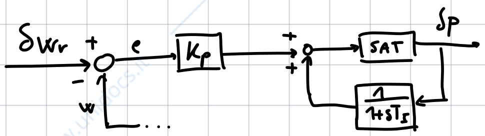
$\Rightarrow F(s) = \frac{L(s)}{1+L(s)} = \frac{K_p\mu}{\tau s + K_p\mu} = \frac{1}{\frac{\tau s}{K_p\mu}+1}$ → $\frac{\cancel{\tau}}{K_p\mu} < 2\cancel{\tau}$ → $K_p \ge \frac{1}{2\mu} = \frac{\rho\,\bar{w}\,L}{A}$ → scelta Kp

$\Rightarrow$ PROGETTO REG.: $R_{PI}(s) = \frac{\rho\,\bar{w}\,L}{A\tau}\cdot\frac{1+s\tau}{s}$

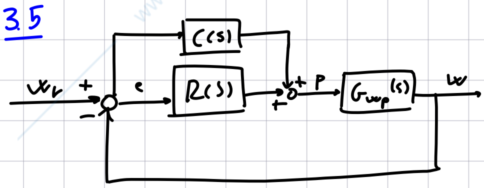
3.4

Se $p(t)$ è soggetta a saturazione → possibile fenomeno wind-up.

Schema con blocco anti-windup



3.5



STATICO solo voglio realizz.

$C(s) = \frac{1}{G_{wp}(s)} \cong \frac{1}{\mu}$

$F(s) = \frac{(C(s)+R(s))\,G_{wp}(s)}{1+R(s)\,G_{wp}(s)} \xrightarrow[s\to 0]{} 1$

28/06/11

1 $\quad L(s) = \dfrac{\rho\,(s-1)}{(s+2)(s-2)(s+4)}$

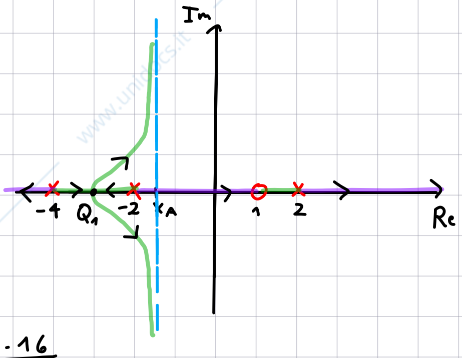
1.1 $n=3$

$m=1$

$\nu=2 \longrightarrow X_B$ si conserva

Baricentri

- $X_A = \frac{1}{2}\left(1 - \cancel{2} + \cancel{2} - 4\right) = -\frac{3}{2}$
- $X_B = \frac{1}{3}\left(-\cancel{2} + \cancel{2} - 4\right) = -\frac{4}{3}$



- ASINTOTI: 2 As. $\begin{cases} \text{LD} & 90°, 270° \\ \text{LI} & 0°, 180° \end{cases}$, $X_A = -\frac{3}{2}$

- $Q_1 = \max \gamma(x)$

$$\gamma(x) = -\frac{D(x)}{N^*(x)} = -\frac{(s^2-4)(s+4)}{s-1} = -\frac{s^3+4s^2-4s-16}{s-1}$$

$$\gamma'(x) = -\frac{[2s(s+4)+(s^2-4)](s-1)-(s^2-4)(s+4)}{(s-1)^2} = 0 \rightarrow -\left(2s^3+s^2-8s+20\right) = 0 \longrightarrow \max\ Q_1 \simeq -3$$

1.2

NON AS. STAB $\begin{cases} \rho>0 \longrightarrow \text{ho sempre poli nel semiasse dx} \\ \rho<0 \longrightarrow \text{ho sempre poli nel semiasse dx} \end{cases}$

1.3 $s=-3$

$$\rho = \frac{1\cdot 1\cdot 5}{4} = \frac{5}{4} = 1.25$$

1.4 X_B si conserva pari a $X_B = -\frac{4}{3}$

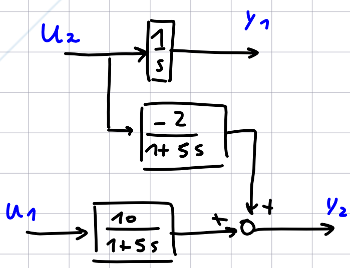
② $G(s) = \begin{array}{c|cc|c} & u_1 & u_2 & \\ & 0 & \frac{1}{s} & y_1 \\ & \frac{10}{1+5s} & \frac{-2}{1+5s} & y_2 \end{array}$

2.1

- **Poli**: $s_1 = 0$, $s_{2,3} = -\frac{1}{5}$
- **Zeri**: $\det G(s) = 0 \rightarrow -\frac{10}{s(1+5s)} = 0 \Rightarrow \det G(s) \neq 0$

non ha zeri

2.2



2.3 $\Delta(s)$

$G(s) = \begin{bmatrix} 0 & \frac{1}{s} \\ \frac{10}{1+5s} & \frac{-2}{1+5s} \end{bmatrix}$ $\tilde{G}(s) = \begin{bmatrix} \frac{1}{s} & 0 \\ -\frac{2}{1+5s} & \frac{10}{1+5s} \end{bmatrix}$

$\tilde{\Delta}(s)$ su $\tilde{G}(s) \longrightarrow \tilde{\Delta}(s) = \begin{bmatrix} 1 & 0 \\ \underbrace{+\frac{2}{10}}_{0.2} & 1 \end{bmatrix} \Rightarrow \Delta(s) = \begin{array}{c|cc|c} & x_1 & x_2 & \\ & 0.2 & 1 & u_1 \\ & 1 & 0 & u_2 \end{array}$

2.4

$G'(s) = G(s)\Delta(s) = \begin{bmatrix} 0 & \frac{1}{s} \\ \frac{10}{1+5s} & \frac{-2}{1+5s} \end{bmatrix} \begin{bmatrix} 0.2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{10}{1+5s} \end{bmatrix} \longrightarrow$ diagonale ✓

- **Regolatori**

- $R_1'(s)$ prog. su $G_1'(s) = \frac{1}{s}$

$R_1'(s) = \mu_1$ (P) $\rightarrow L_1'(s) = \frac{\mu_1}{s}$ $\omega_c = \mu_1 > 0.5$ ✓ ad.es. $\mu_1 = 0.6$

$\varphi_m = 90°$ ✓

$R'(s) = \begin{bmatrix} R_1'(s) & 0 \\ 0 & R_2'(s) \end{bmatrix}$

- $R_2'(s)$ prog. su $G_2'(s) = \frac{10}{1+5s}$

$R(s) = \Delta(s) R'(s)$

$R_2 = \mu_2 \cdot \frac{1+5s}{s}$ (PI) $\rightarrow L_2'(s) = \frac{10\mu_2}{s}$ $\omega_c = 10\mu_2 > 0.5$ ✓ ad.es. $\mu_2 = 0.06$

$\varphi_m = 90°$ ✓

③ $T = 0.1$ $\quad R^*(z) = \dfrac{3}{z-1}$

3.1 Algoritmo di controllo:

$$u^*(k) = u^*(k-1) + 3e^*(k-1)$$

3.2

$$G^*(z) = \frac{0.1}{z-0.5}$$

trasf. camp. $z = e^{sT}$

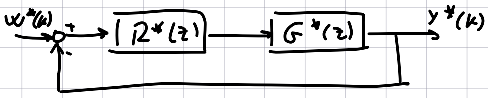
T. continuo:

- **Poli:** $z_1 = 0.5 \longrightarrow s_1 = \dfrac{\ln z_1}{T} \cong -7$
- **Zeri:** numero di zeri è $\underbrace{n}_{\#\text{ poli}} - 1 = 1-1=0 \rightarrow$ non ci sono zeri
- **Guadagno:** $G^*(1) = G(0) \longrightarrow G(0) = \mu_G = \dfrac{0.1}{0.5} = \dfrac{1}{5}$

$$G(s) \cong \frac{1}{5(s+7)}$$

3.3

$$L(z) = R^*(z)G^*(z) = \frac{0.3}{(z-1)(z-0.5)}$$



$$F(z) = \frac{L(z)}{1+L(z)} = \frac{0.3}{(z-1)(z-0.5)+0.3} = \frac{0.3}{\underbrace{z^2-1.5z+0.5+0.3}_{z^2-1.5z+0.8}}$$

Poli A.C. $z_{1,2} = \dfrac{1.5 \pm \sqrt{\frac{9}{4} - \frac{16}{5}}}{2}$ $\cong 0.75 + j\,0.49$, $\cong 0.75 - j\,0.49$

$\hookrightarrow |z_{1,2}| \cong 0.89 < 1 \rightarrow$ **AS. STAB.**

3.4

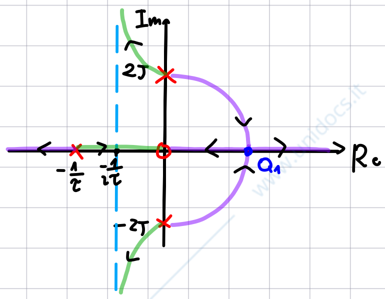
$\underset{t\text{ ass.}}{K_a} = -\dfrac{5}{\ln|z_{1,2}|} = 44.81$ $\quad \underset{t\text{. latenza}}{K_e} = \nu_F - 1 = 1$ $\quad \underset{t\text{. risposta complessivo}}{k_r} = k_e + k_a = 45.81$ $\quad t_a = k_r T = 4.581 \cong 4.6\ s$ (a t. continuo)

TVF $\displaystyle\lim_{z\to1} \cancel{(z-1)} \cdot F(z) \cdot \frac{z}{\cancel{z-1}} = \lim_{z\to1} \frac{0.3z}{z^2-1.5z+0.8} = \frac{0.3}{1-1.5+0.8} = 1$

15/07/21

1) $L(s) = \dfrac{\rho s}{(s^2+4)(s+\frac{1}{\tau})}$

1.1,2



$n=3$
$m=1$
$\nu=2$

Baricentri

- $X_A = \frac{1}{2}\left(0 - \cancel{2J} + \cancel{2J} - \frac{1}{\tau}\right) = -\frac{1}{2\tau}$
- $X_B = \frac{1}{3}\cdot\left(-\frac{1}{\tau}\right) = -\frac{1}{3\tau}$

↓

2 ASINTOTI { LD $90°, 270°$; LI $0°, 180°$

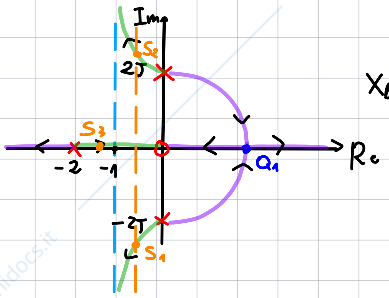
1.3 $t_a < 10$

$t_a = \dfrac{5}{|\sigma|} < 10 \quad |\sigma| > \frac{1}{2} \longrightarrow \sigma < -\frac{1}{2} \Rightarrow R_e(s_{Dom}(\rho)) = -\frac{1}{2}$

- Se $\tau = 1 \longrightarrow$ nessuna soluz. ci sono i 2 rami dai poli complessi che non superano mai l'asintoto in $-\frac{1}{2\tau} \overset{\tau=1}{=} -\frac{1}{2}$

↳ $\tau < 1$

ad es. $\bar{\tau} = \frac{1}{2}$



Ricerca S_3

X_B si conserva $\Rightarrow X_B = -\frac{2}{\cancel{3}} = \frac{1}{\cancel{3}}(\overbrace{s_1+s_2}^{-2\cdot\frac{1}{2}}+s_3) \longrightarrow S_3 = -2+1 = -1$

Regola della punteggiatura (considerando le dist. dai poli AA a s_3)

$\bar{\rho} = \dfrac{1\cdot 2\sqrt{2^2+1^2}}{1} = 2\sqrt{5} \Longrightarrow (\bar{\tau}, \bar{\rho}) = \left(\frac{1}{2}, 2\sqrt{5}\right)$

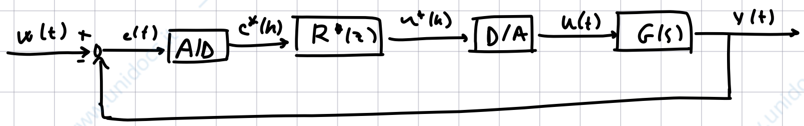
1.4 Si vede dal luogo diretto che termina nello zero in 0 per $\rho \to +\infty$ e si spiega poiché:

$D(s) + \rho N^*(s)$ dove per trovare gli zeri $N^*(s) = 0$, cioè $N^*(s) = -\dfrac{D(s)}{\rho} \xrightarrow[\rho\to+\infty]{} 0$

3

$G(s) = \frac{100}{1+10s}$ $\quad R^\circ(s) = K_p\left(1+\frac{1}{sT_i}\right)$ con $K_p=4$ $\quad T_i=10$ $\longrightarrow$ $R^\circ(s) = \frac{4(1+10s)}{10s}$

3.1



3.2 $5\omega_c < \omega_s < 50\omega_c \rightarrow \frac{2\pi}{50\omega_c} < T < \frac{2\pi}{5\omega_c}$

$L(s) = \underbrace{4\left(\frac{\cancel{1+10s}}{10s}\right)}_{R^\circ(s)} \cdot \underbrace{\frac{\cancel{100}}{\cancel{1+10s}}}_{G(s)} = \frac{40}{s} \rightarrow \omega_c = 40 \Rightarrow 0.003 < T < 0.03$

visto che è un modello approx. faccio una scelta più conservativa $T=0.005$

3.3 E I $\alpha = 1$

trasf. bilineare $\quad s = \frac{1}{T}\,\frac{z-1}{\alpha z+1-\alpha} = \frac{1}{T}\cdot\frac{z-1}{z} = \frac{200(z-1)}{z}$

$R^*(z) = R^\circ\left(\frac{200(z-1)}{z}\right) = \frac{\cancel{4}z}{10\cdot\cancel{200}_{5}(z-1)} \cdot \left(1+\frac{2000(z-1)}{z}\right) = \frac{z+2000z-2000}{50z-50} = \frac{2001z-2000}{50z-50}$

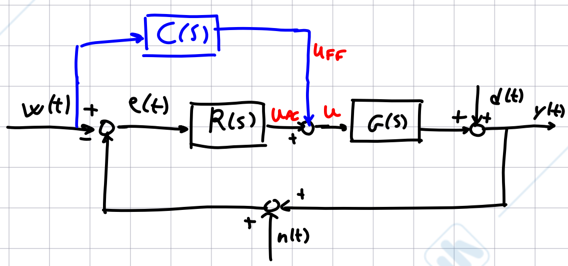
3.4 Algoritmo di controllo: $\quad u^*(k) = u^*(k-1) + \frac{2001}{50}e^*(k) - 40e^*(k-1)$

3.5
- $\omega_c^* \cong \omega_c$ poiché $\omega_s = \frac{2\pi}{T} = 400\pi >> \omega_c = 40$
- $\varphi_m^* = \underbrace{\varphi_m}_{=90°} - \underbrace{\frac{\omega_c T}{2}\cdot\frac{180°}{\pi}}_{\Delta\varphi_m \cong 5.73°} \simeq 84.27°$

<u>04/07/12</u>

① $G(s) = \dfrac{\mu}{1+s\tau}$ $R(s) = K$

1.1 Compensatore FF



Utilità:

Migliore l'inseguimento del riferimento w da parte dell'uscita y

1.2 Viene modificata la FDT tra w e y

1 Non viene mod " " " d e y

2 " " " " " " n e y

1 PRIMA FF DOPO FF

$$G_{dy}(s) = \frac{1}{1+\underbrace{L(s)}_{R(s)G(s)}} = G_{dy}(s) = \frac{1}{1+\underbrace{L(s)}_{R(s)G(s)}}$$

2

$$G_{ny} = -\frac{L(s)}{1+L(s)} = G_{ny}(s) = -\frac{L(s)}{1+L(s)}$$

non cambia l'attenuazione

1.3 FF statico

$$C^{\circ}(s) = \frac{1}{G(s)}$$ non realizz. $\longrightarrow$ $C(s) = \frac{1}{G(0)} = \frac{1}{\mu}$

ideale — STATICO

1.4 Soluzione comunque poco robusta rispetto alle incertezze:

Perturbazione μ: $\pm 10\%$

CON FF STATICO (senza perturb.)

$$G_{yw}(s) = \frac{(C(s)+R(s))G(s)}{1+R(s)G(s)} = \frac{\frac{1}{1+s\tau} + \frac{k\mu}{1+s\tau}}{1+\frac{k\mu}{1+s\tau}} = \frac{1+k\mu}{s\tau+1+k\mu} \xrightarrow{s\to 0} G_{yw}(0) = 1$$

Perturb. +10%

$$G_{yw}(s) = \frac{\left(\frac{1}{1.1\mu}+k\right)\mu}{s\tau+1+k\mu} \simeq \frac{\frac{10}{11}+k\mu}{s\tau+1+k\mu} \xrightarrow{s\to 0} G_{yw}(0) = \frac{\frac{10}{11}+k\mu}{1+k\mu} \neq 1$$

$$\text{errore} = 1 - \frac{\frac{10}{11}+k\mu}{1+k\mu} = \frac{1}{11}$$

Perturb. −10%

$$G(s) = \frac{\left(\frac{1}{0.9\mu}+k\right)\mu}{s\tau+1+k\mu} \xrightarrow{s\to 0} G_{yw}(0) = \frac{\frac{10}{9}+k\mu}{1+k\mu} \neq 1$$

$$\text{errore} = 1 - \frac{\frac{10}{9}+k\mu}{1+k\mu} = -\frac{1}{9}$$

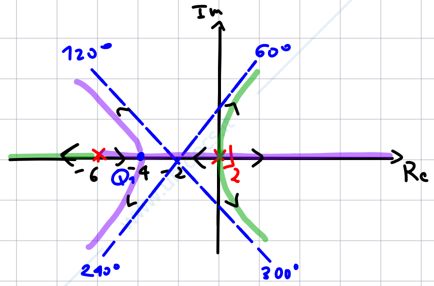
1.5 Con prefiltro vorrei che

COMP. FF — PREFILTRO

$$G_{yw}(s) = \frac{(C(s)+R(s))\cancel{G(s)}}{1+\cancel{R(s)G(s)}} = G_{yw}(s) = \frac{P(s)R(s)\cancel{G(s)}}{1+\cancel{R(s)G(s)}}$$

$$\hookrightarrow P(s) = 1 + \frac{C(s)}{R(s)} = 1 + \frac{1}{\mu k} = \frac{\mu k+1}{\mu k}$$ $\longrightarrow$ realizz. ✓ as. stab. ✓

② $L(s) = \dfrac{\rho}{s^2(s+6)}$



2.1 $n=3$, $m=0$, $\nu=3$

· Baricentri

$x_A = x_B = \frac{1}{3}(-6) = -2$

asintoti

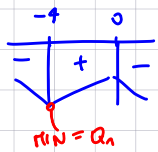
poli a.a. = poli a.c. } → si conserva x_B

· Asintoti

- $\# = 3$
- $\{60°, 180°, 300°\}$ LD
- $\{0°, 120°, 240°\}$ LI

Q_1 min di $\gamma(x) = -\dfrac{D(x)}{N(x)} = -s^3 - 6s^2$

↳ $\gamma'(x) = -3s^2 - 12s \geq 0 \rightarrow -3s(s+4) \geq 0 \rightarrow -4 < s < 0$



2.2 per $\rho > 0$ LD ha sempre una parte nel semipiano dx

per $\rho < 0$ LI ha sempre una parte nel semipiano dx } ⟹ SIST. A.C. INSTABILE

2.3 $\varphi_{ac}(s) = N(s) + D(s) = \rho + s^3 + 6s^2 = 0$ per il criterio di Routh → s ha coeff. nullo $\forall \rho$ ⟹ SIST. INSTAB.

2.4 $Q_1 = -4$

↓

$s_{c_{1,2}} = -4$

Regola punteggiatura

$$\rho = -\frac{(d(-4,0))^2 \cdot d(-4,-6)}{1} = -\frac{16 \cdot 2}{1} = -32$$

→ x_B si conserva → $x_B = -2 = (\underbrace{s_{c_1} + s_{c_2}}_{-8} + s_3) \Rightarrow s_3 = 6$

3

$G(s) = \frac{0.1}{s}$ $\quad R^0(s) = \frac{1+10s}{1+0.1s}$ $\rightarrow L(s) = R^0(s)G(s) = \frac{0.1}{s}\frac{(1+10s)}{1+0.1s}$ $\rightarrow W_c \simeq 10$ (v. p. dopo)

3.1

$\underbrace{\frac{2\pi}{50W_c}}_{0.0126} < T < \underbrace{\frac{2\pi}{5W_c}}_{0.126}$ $\rightarrow$ mi mantengo vicino al limite inf. $\Rightarrow T = 0.02$

3.2 TU $\quad \alpha = \frac{1}{2}$

$s = \frac{1}{T}\frac{z-1}{\alpha z + 1 - \alpha}$ f. bilineare

$\parallel \leftarrow$ TU, $T = \frac{2}{100}$

$100 \cdot \frac{z-1}{z+1}$

$R^*(z) = R^0\left(100\frac{z-1}{z+1}\right) = \frac{1+1000\left(\frac{z-1}{z+1}\right)}{1+10\left(\frac{z-1}{z+1}\right)} = \frac{z+1+1000z-1000}{z+1+10z-10} = \frac{1001z-999}{11z-9}$

3.3 $W = 1$ $\quad |R^0(J)| = \left|\frac{1+10J}{1+0.1J}\right| = \frac{\sqrt{1+100}}{\sqrt{1+0.1^2}} = 10$ ✓

$|R^*(e^{J\cdot 0.02})| = \left|\frac{1001e^{J\cdot 0.02}-999}{11e^{J\cdot 0.02}-9}\right| = \sqrt{(1001\cos(0.02)-999)^2+(1001\cdot\sin(0.02))^2} \cong \frac{20}{2} = 10$ ✓

3.4 Analogico

$\varphi_c = -90° - \arctg\left(\frac{1}{0.1}\right) + \arctg\left(\frac{1}{10}\right) \cong 91.37°$ $\rightarrow \varphi_m \cong 88.63°$ $\rightarrow$ no osill. $\varphi_m > 75°$

Digitale

Ci sarà un ritardo di discretizzazione $\rightarrow \Delta\varphi_m = W_c\frac{T}{2}\cdot\frac{180°}{\pi} = \frac{1}{10}\cdot\frac{180°}{\pi} \cong 5.73°$ $\quad \left(T = \frac{2}{100}\right)$

$W_c^* \simeq W_c = 10$ (T molto piccolo $\rightarrow W_s \gg 2W_c$)

$\varphi_m^* = 88.63° - 5.73° = 82.9° \rightarrow$ no osill.

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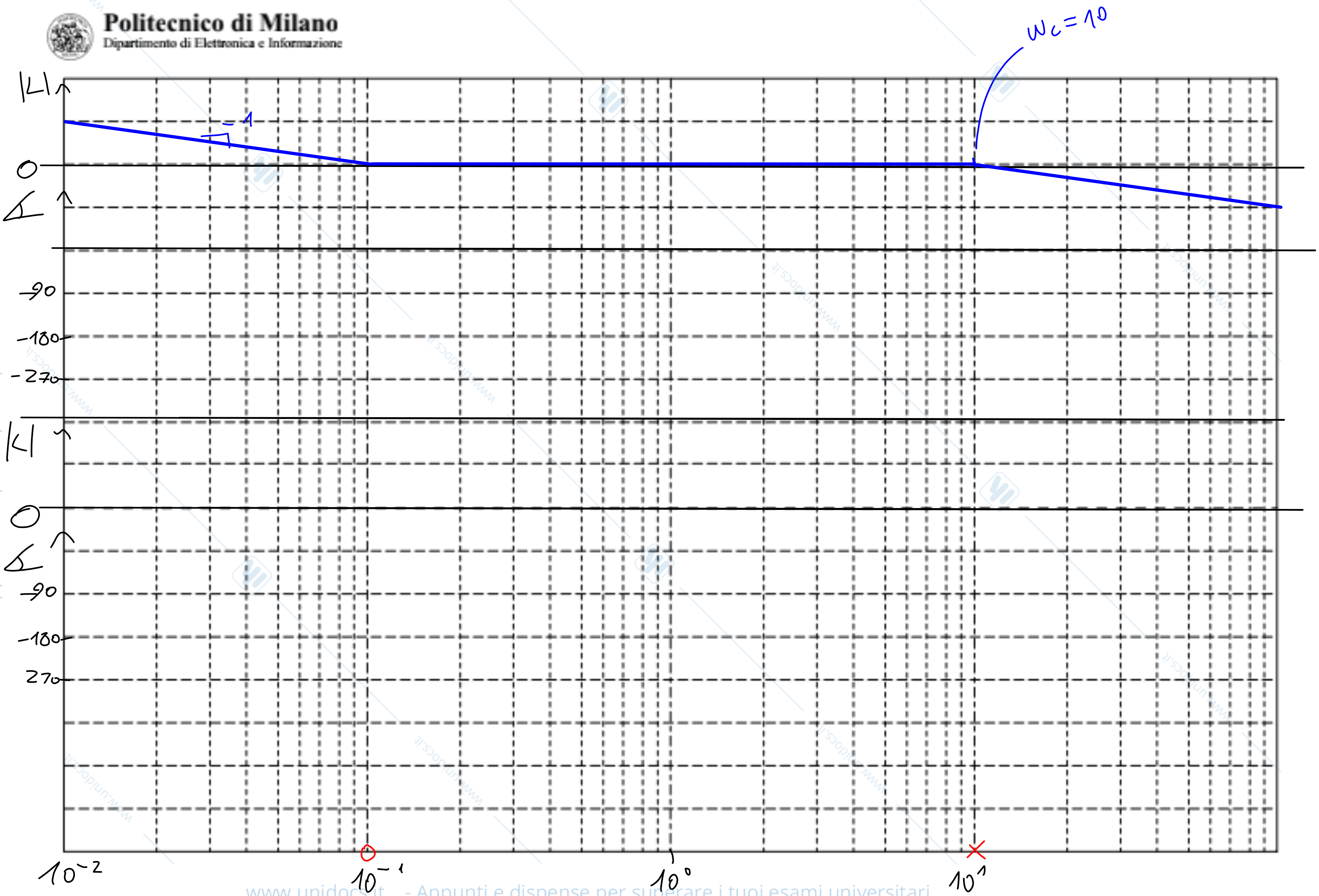


18/07/12

① $G(s) = \dfrac{1-2s}{5-2s+s^2}$ $\quad s_{1,2} = 1 \pm \sqrt{1-5} = 1 \pm 2j$

1.1 $L(s) = \dfrac{\overbrace{(-k\cdot 2)}^{\rho}\,(s-\frac{1}{2})}{s^2-2s+5}$

- $n=2$
- $m=1$
- $\nu=1$

Baricentri:

$x_A = \frac{1}{1}\left(\frac{1}{2}+1+1\right) = \frac{5}{2}$

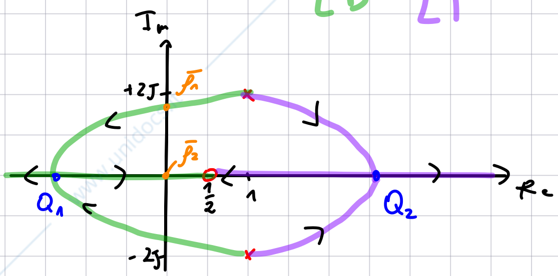
$x_B = \frac{1}{2}(+1+1) = 1$

Asintoti:

LD $180°$

LI $0°$

LD LI



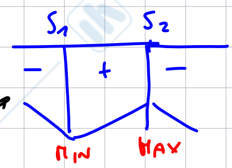
1.2 $Q_1 = \min \gamma(x)$

$Q_2 = \max \gamma(x)$

$\gamma(x) = -\dfrac{D(s)}{N^*(s)} = -\dfrac{s^2-2s+5}{s-\frac{1}{2}}$

$\gamma'(x) = -\dfrac{(2s-2)(s-\frac{1}{2}) - s^2+2s-5}{(s-\frac{1}{2})^2} = -\dfrac{\overbrace{2s^2-3s+1-s^2+2s-5}^{s^2-s-4}}{(s-\frac{1}{2})^2} \geq 0 \rightarrow \underbrace{\frac{1-\sqrt{17}}{2}}_{S_1} \leq s \leq \underbrace{\frac{1+\sqrt{17}}{2}}_{S_2}$

$s_{1,2} = \dfrac{1 \pm \sqrt{1+16}}{2}$



$\hookrightarrow Q_1 = \dfrac{1-\sqrt{17}}{2} \cong -1.56 \qquad Q_2 = \dfrac{1+\sqrt{17}}{2} \cong 2.56$

1.3 Con criterio di Routh:

$\varphi_{ac}(s) = k-2ks+s^2-2s+5 = s^2-2(k+1)s+5+k \rightarrow$ concordanza segno $\begin{cases} -2(k+1) > 0 \\ k+5 > 0 \end{cases} \rightarrow \begin{cases} k < -1 \\ k > -5 \end{cases} \rightarrow -5 < k < -1$

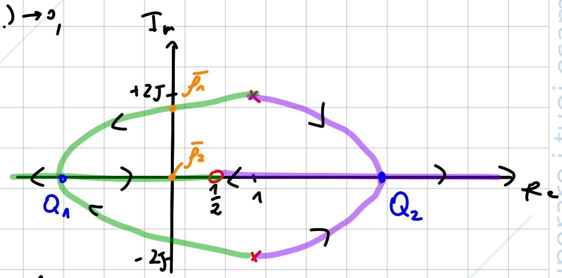
$k = -\frac{\rho}{2}$

$\underbrace{2}_{\bar{\rho}_1} < \rho < \underbrace{10}_{\bar{\rho}_2}$

Verifica: $\bar{\rho}_2 = \dfrac{(d(0,1\pm 2j))^2}{d(0,\frac{1}{2})} = \dfrac{(\sqrt{1+2^2})^2}{\frac{1}{2}} = 5\cdot 2 = 10$ ✓

$\bar{\rho}_1$ non lo verifico perchè x_B non si conserva

1.4. Vedendo l'LDR si nota che nel LD abbiamo dei ρ limite dove è garantita l'as. stab. e si nota che sia se $\rho \to 2^+$ sia se $\rho \to 10^-$ i poli a.c. avranno in entrambi i casi $Re(s_{a.c.}) \to 0$, dunque saranno poli dominanti che portano a una dinamica lenta

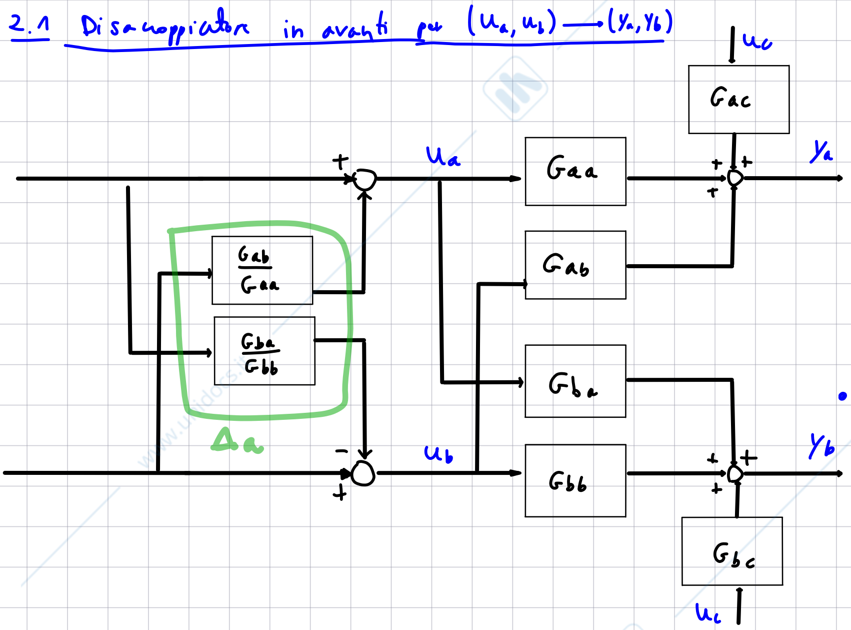


2 MIMO $\underset{IN}{(u_a, u_b, u_c)} \longrightarrow \underset{OUT}{(y_a, y_b)}$

$G_{aa} = -\frac{2}{1+2s}$ $\quad G_{ab} = 4$ $\quad G_{ac} = 1$

$G_{ba} = \frac{5}{(1+2s)(1+10s)}$ $\quad G_{bb} = \frac{3(1+s)}{1+10s}$ $\quad G_{bc} = -\frac{7}{1+10s}$

2.1 Disaccoppiatore in avanti per $(u_a, u_b) \longrightarrow (y_a, y_b)$



$$\Delta_a(s) = \begin{bmatrix} 1 & -\frac{G_{ab}}{G_{aa}} \\ -\frac{G_{ba}}{G_{bb}} & 1 \end{bmatrix}$$

- $-\frac{G_{ab}(s)}{G_{aa}(s)} = 2(1+2s) \rightarrow$ NON REALIZZ.

  ↳ STATICO $-\frac{G_{ab}(0)}{G_{aa}(0)} = 2$

- $-\frac{G_{ba}}{G_{bb}} = -\frac{5}{3(1+2s)(1+s)} \rightarrow$ DINAMICO ✓

$$\Rightarrow \Delta_a = \begin{bmatrix} 1 & 2 \\ -\frac{5}{3(1+2s)(1+s)} & 1 \end{bmatrix}$$

2.2 Si elidono solo $(1+10s)$ tra G_{ba} e G_{bb}

↳ polo negativo $\Rightarrow$ NO CANC. CRITICHE!

2.3

$$G'(s) = G(s)\Delta_a(s) = \begin{bmatrix} \underbrace{G_{aa} - \frac{G_{ab}G_{ba}}{G_{bb}}}_{G'_{aa} \,=\, \det G / G_{bb}} & 0 \\ 0 & \underbrace{G_{bb} - \frac{G_{ab}G_{ba}}{G_{aa}}}_{G'_{bb} \,=\, \frac{\det G}{G_{aa}}} \end{bmatrix}$$

$$\det G(s) = G_{aa}G_{bb} - G_{ab}G_{ba} = -\frac{6(1+s)}{(1+2s)(1+10s)} - \frac{20}{(1+2s)(1+10s)} = -\frac{6s+26}{(1+2s)(1+10s)}$$

- $G'_{aa} = -\frac{6s+26}{(1+2s)\cancel{(1+10s)}} \cdot \frac{\cancel{(1+10s)}}{3(1+s)} = -2\frac{\left(s+\frac{26}{6}\right)}{(1+2s)(1+s)} = -\frac{\frac{26}{3}\left(\frac{3}{13}s+1\right)}{(1+2s)(1+s)}$

- $G'_{bb} = +\frac{6s+26}{\cancel{(1+2s)}(1+10s)} \cdot +\frac{\cancel{1+2s}}{2} = \frac{3(s+13)}{(1+10s)} = \frac{39\left(\frac{s}{13}+1\right)}{(1+10s)}$

2.4

1. $\{(u_a, y_a), (u_b, y_b)\}$ $\lambda = \frac{\mu_{aa}\mu_{bb}}{\det G^1(0)} = \frac{-6}{-26} = \frac{3}{13}$ $\longrightarrow$ Scelta anomala $\{(u_b, y_a); (u_a, y_b)\}$

2. $\{(u_a, y_b); (u_c, y_a)\}$ $\lambda = \frac{\mu_{ba}\mu_{ac}}{\det G^2(0)} = \frac{5}{-9}$ $\longrightarrow$ scelta anomala $\{(u_a, y_a); (u_c, y_b)\}$

↳ $\det\begin{bmatrix} G_{ac} & G_{aa} \\ G_{bc} & G_{ba} \end{bmatrix} = G_{ac}G_{ba}(0) - G_{aa}G_{bc}(0) = 5 - (-2\cdot(-7)) = -9$

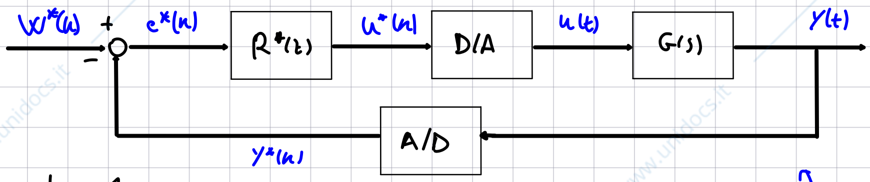
3. $\{(u_b, y_a); (u_c, y_b)\}$ $\lambda = \frac{\mu_{ab}\mu_{bc}}{\det G^3(0)} = \frac{-28}{-31} = \frac{28}{31}$ $\longrightarrow$ $\boxed{\text{Scelta naturale } \{(u_b, y_a); (u_c, y_b)\}}$

↳ $\det\begin{bmatrix} G_{ab} & G_{ac} \\ G_{bb} & G_{bc} \end{bmatrix} = G_{ab}G_{bc}(0) - G_{ac}G_{bb}(0) = -28 - 3 = -31$

Scelgo questa poiché ha meno interazione tra gli ingressi rispetto a tutti gli altri possibili accoppiamenti

(3)

3.1

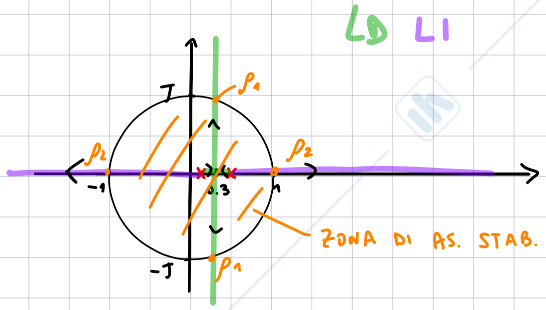


3.2 $R^*(z) = \frac{b}{z} \cdot \frac{1}{z-0.8} = \frac{b}{z(z-0.8)}$ → $R^*(z)$ str. propria, non c'è offset di mantenimento

3.3 $G^*(z) = \frac{10(z-0.8)}{(z-0.1)(z-0.5)}$ $T=2$

- poli: $\underbrace{z = e^{sT}}_{\text{TRASF. DI CAMPION.}}$
  - $z_1 = 0.1 \rightarrow s_1 = T \ln z_1 \cong -4.6$
  - $z_2 = 0.8 \rightarrow s_2 = T \ln z_2 \simeq -1.4$

- guadagno: $G(0) = G^*(1) = \frac{10 \cdot (0.2)}{0.9 \cdot 0.5} = \frac{40}{9} = 4.\overline{44}$



3.4 $L(z) = R^*(z) G^*(z) = \frac{10\,b\ \rho}{(z-0.1)(z-0.5)}$ → metodo LDR

- $n = 2$
- $m = 0$
- $\nu = 2$

→ Baricentri: $x_A = \underset{\substack{\downarrow \\ \text{si conserva}}}{x_B} = \frac{1}{2}(0.1 + 0.5) = 0.3$

Regola della punteggiatura

- $\rho_2 = -\frac{0.5 \cdot 0.9}{1} = -0.45 \rightarrow b = -0.045$
- $\rho_1 \cong \underbrace{\left(\sqrt{0.2^2 + 1}\right)^2}_{1} \cong 1 \rightarrow b = 0.1$

(ascissa $\simeq J$)

$-0.045 < b < 0.1$

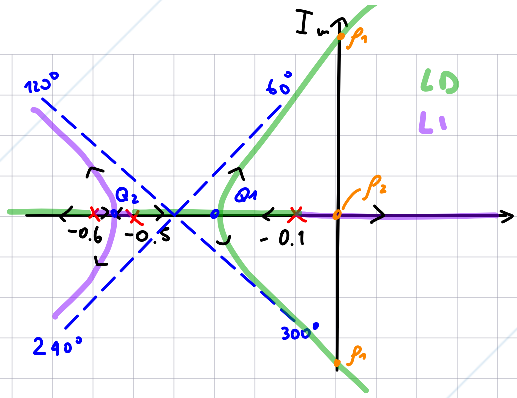
3.5 $b = 0.1$ $w^*(k) = sca^*(k)$

$F(z) = \frac{L(z)}{1+L(z)} = \frac{1}{(z-0.1)(z-0.5)+1}$

$e^*(\infty) = \lim_{z \to 1} \cancel{(z-1)} \frac{1}{(z-0.1)(z-0.5)+1} \cdot \frac{z}{\cancel{z-1}} = \frac{1}{0.9 \cdot 0.5 + 1} = \frac{20}{29} = 0.69$ → errore alto

27|06|13

(1) $L(s) = \dfrac{0.1\,\mu}{(s+0.1)(s+0.5)(1+s\tau)} = \dfrac{\overbrace{0.1\mu/\tau}^{\rho}}{(s+0.1)(s+0.5)(s+\frac{1}{\tau})}$



1.1

- $n=3$
- $m=0$
- $\nu=3$

→ **Baricentri**

$x_A = x_B = \frac{1}{3}\left(-0.1 - 0.5 - \frac{1}{\tau}\right) = -0.4$

si conserva

$\hookrightarrow \tau : \dfrac{1}{1.2-0.6} = \dfrac{10}{6} = \dfrac{5}{3}$

$\hookrightarrow \frac{1}{\tau} = 0.6$

1.2

Asintoti:

LD $\{60°, 180°, 300°\}$

LI $\{0°, 120°, 240°\}$

→ $Q_1 = \max \gamma(x)$

$Q_2 = \min \gamma(x)$

$\gamma(x) = -\dfrac{D(x)}{N^*(x)} = -(s+0.1)(s+0.5)(s+0.6) =$

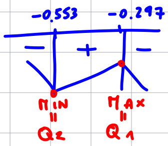
$= -\left[(s^2+0.6s+0.05)(s+0.6)\right] =$

$= -\left[s^3+0.6s^2+0.05s+0.6s^2+0.36s+0.03\right] =$

$= -\left[s^3+1.2s^2+0.41s+0.03\right]$

$\gamma'(x) = -(3s^2+2.4s+0.41) \geq 0$

$s_{1,2} = \dfrac{-1.2 \pm \sqrt{0.21}}{3} \cong -0.247,\ -0.553$



1.3 AS. STAB. $\iff \rho_2 < \rho < \rho_1$

- ρ_1 $\quad x_B$ si conserva $\rightarrow \frac{1}{3}(0+0+s_1) = -0.4 \rightarrow s_1 = -1.2$

Regola punteggiata: $\rho_1 = \dfrac{0.6 \cdot 0.7 \cdot 1.1}{1} = 0.462 \rightarrow \mu_1 = 0.077$

$\rho = 10\mu \cdot 0.6 = 6\mu$

$\rightarrow -0.005 < \mu < 0.077$

- ρ_2 $\quad$ Regola punteggiata: $(s=0)$ $\quad \rho_2 = -\dfrac{0.6 \cdot 0.5 \cdot 0.1}{1} = -0.03 \rightarrow \mu_2 = -0.005$

con $\mu_2 \doteq \max$

1.4 Voglio che $G_{yw}(s) = \dfrac{6\mu}{s^3+1.2s^2+0.41s+0.03+6\mu}$

$y(\infty) = G_{yw}(0) \cdot 1 = \dfrac{6\mu}{0.03+6\mu} \cong 0.72$

② **2.1** Serve per progettare direttamente 2 regolatori indip. (1,2) su 2 FDT $G(s)$ separate, quindi serve per ottenere una matrice $G(s)$ diagonale

2.2

$$G(s)=\begin{bmatrix} \frac{8(1+s)}{1+10s} & 2 \\ -4 & \frac{4}{1+s} \end{bmatrix} \longrightarrow \det G(s) = \frac{32}{1+10s} + 8 = \frac{40(1+2s)}{1+10s}$$

$$\Delta_i(s) = \frac{G_{11}G_{22}}{\det G}\begin{bmatrix} 1 & -\frac{G_{12}}{G_{11}} \\ -\frac{G_{21}}{G_{22}} & 1 \end{bmatrix} = \frac{\cancel{32}^{\,4}}{\cancel{(1+10s)}}\cdot\frac{\cancel{(1+10s)}}{\underset{5}{\cancel{40}}(1+2s)}\begin{bmatrix} 1 & -\frac{1+10s}{4(1+s)} \\ \frac{1}{1+s} & 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{5(1+2s)} & -\frac{1+10s}{5(1+s)(1+2s)} \\ \frac{4}{5(1+s)(1+2s)} & \frac{4}{5(1+2s)} \end{bmatrix}$$

2.3

- Poli: $-\frac{1}{10}, -1$ → tutti con $Re(p)<0$
- Zeri: $\det G(s)=0 \longrightarrow s=-\frac{1}{2} < 0$

$\Longrightarrow$ NO CANC. CRITICHE

2.4 $\lambda = \frac{\mu_{11}\mu_{22}}{\det G(0)} = \frac{32}{40} = \frac{4}{5} \longrightarrow$ scelta naturale $\{(u_1,y_1);(u_2,y_2)\}$

2.5

- $R_2(s)$ su G_{22}
- $R_1(s)$ su $G_{11}^* = G_{11} - \frac{G_{21}G_{12}R_2}{1+R_2G_{22}}$

(3) $G(s) \longrightarrow \tau_{max} = 20 \quad R^0(s) = \frac{-2(1+4s)}{s}$ $\xrightarrow{t.c.}$ SIST. A.C. 2 VOLTE + VELOCE $\longrightarrow \tau_{AC} = \frac{\tau_{MAX}}{2} = 10$

$\longrightarrow \varphi_m \geq 75° \longrightarrow \omega_c \simeq \frac{1}{\tau_{AC}} = 0.1$

3.1. $\underbrace{\frac{2\pi}{50\omega_c}}_{1.256} \leq T \leq \underbrace{\frac{2\pi}{5\omega_c}}_{12.56}$ $\longrightarrow$ Visto che prenderò delle approx. per il regolatore scelgo T vicino al limite inferiore

$\longrightarrow \boxed{T = 2}$

3.2 TRASF. BILINEARE $s = \frac{1}{T} \cdot \frac{z-1}{\alpha z + 1 - \alpha}$ $\xrightarrow[T=2]{TU \ \alpha = \frac{1}{2}}$ $s = \frac{z-1}{z+1}$ $\quad R^*(z) = R^0\left(\frac{z-1}{z+1}\right) = \frac{-2(z+1+4z-4)}{z-1} = \frac{-2(5z-3)}{z-1}$

• Algoritmo di controllo: $u^*(k) = u^*(k-1) - 10e^*(k) + 6e^*(k-1)$

3.3 $R^*(e^{J\omega T}) \overset{T=2}{=} \frac{-2(5e^{J2\omega}-3)}{e^{J2\omega}-1}$ $\longrightarrow$

$$\left| R(e^{J2\omega}) \right| = 2 \frac{\sqrt{\underbrace{(5\cos(2\omega)-3)^2}_{\to 4} + \underbrace{25\sin^2(2\omega)}_{\to 0}}}{\sqrt{\underbrace{(\cos(2\omega)-1)^2}_{\to 0} + \underbrace{\sin^2(2\omega)}_{\to 0}}} \xrightarrow[\omega \to 0]{} \infty$$

$$\angle R^*(e^{J2\omega}) = \underbrace{-180°}_{\text{x chè } \mu_R < 0} + \underbrace{\angle(5\cos(2\omega) - 3 + J5\sin(2\omega))}_{\to \angle 2 = 0°} - \underbrace{\angle(e^{J2\omega} - 1)}_{\sim \angle J2\omega = 90°, \ \omega \to 0} = -270°$$

(+90° se per $\mu_r < 0$ si prende conv. +180°)

3.4 $R^*(z)$ NON è strett. propria $\longrightarrow$ c'è ritardo di mantenimento $\tau_n = \tau_c = 0.1T = 0.2$

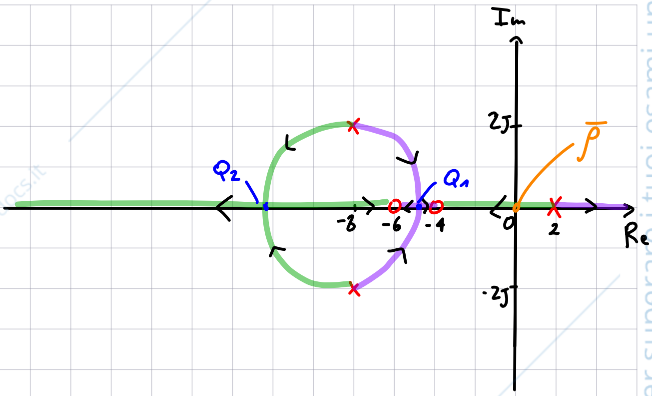
↳ Ritardo mantenitore + Ritardo campion. $\Longrightarrow$ peggiora φ_m

• $\omega_c^* \cong \omega_c \quad (\omega_s >> 2\omega_{max})$

• $\Delta\varphi_m = \underbrace{-\omega_c}_{0.1} \cdot \underbrace{\left(0.1T + \frac{T}{2}\right)}_{1.2} \cdot \frac{180°}{\pi} = -6.875°$

15/07/13

1 $L(s) = \dfrac{\rho(s+6)(s+4)}{(s-2)(s^2+16s+68)}$ $\quad s_{1,2} = -8 \pm 2J$



1.1

$n = 3$

$m = 2$

$\nu = 1$

• Baricentri

$X_A = (-6-4-8-8+2) = -24$

$X_B = (-8-8+2) = -14$

↳ LD 180°

LI 0°

1.2.

LI → NON è AS.STAB. nessun ρ → ci sono sempre dei poli A.C. sul semipiano dx.

LD → AS.STAB $\Longleftrightarrow \rho > \bar{\rho}$

↳ $S(\bar{\rho}) = 0$ → Regola della punteggiatura: $\bar{\rho} = \dfrac{2 \cdot \left(\sqrt{8^2+2^2}\right)^2}{4 \cdot 6} = \dfrac{136}{24} = \dfrac{17}{3}$

1.3 Il sistema A.C. non avrà poli dominanti c.c. poiché per $\rho \to +\infty$ il polo + veloce avrà $\tau \to \frac{1}{4} = 0.25$, e per ρ grande (non così grande da non avere poli c.c.) vale sicuramente (anche troppo ampia come ipotesi) che $Re(p_{cc}) < -8 \Rightarrow \tau(p_{cc}) < \frac{1}{8} = 0.125$ ← polo + veloce di $\tau = 0.25$

1.4 Se $\nu=1$ significa che avremo un asintoto che va all'∞ sull'asse a 180° per LD e sull'asse a 0° per LI

Essendo che m rami finiscono negli zeri e questi zeri sono con $Re(s)<0$ → significa che: per $\rho \to +\infty$ (ρ grande) essendo l'asintoto l'asse reale avrò sempre dei poli A.C. con $Re(p)_{A.C.}<0$ che tenderanno agli zeri negativi in a.a.

$\Longrightarrow$ con ρ abbastanza grande posso stabilizzare il sist. A.C.

② $G(s) = \dfrac{2e^{-24s}}{s}$

2.1 $R(s) = \mu_R \longrightarrow L(s) = \dfrac{2\mu_R\, e^{-24s}}{s}$

→ Il ritardo peggiora il margine di fase φ_m → Può pregiudicare la stab. del sistema

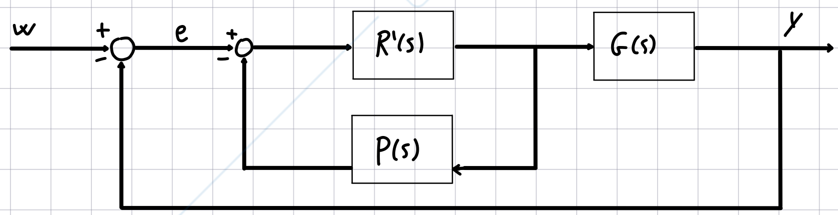
per la stab. $(\varphi_m = 180° - |\varphi_c| > 0)$
e $\mu_R > 0$

in questo caso:
- $\omega_c = 2\mu_R$
- $\varphi_c = \left| -90° - \omega_c \cdot 24 \cdot \dfrac{180°}{\pi} \right| < 180°$

$$\Longrightarrow \quad 180° - 90° - \frac{48\mu_R \cdot 180°}{\pi} > 0$$

$$\frac{1}{2} - \frac{48\mu_R}{\pi} > 0 \longrightarrow \mu_R < \frac{\pi}{96}$$

2.2



$R'(s)$ sulla basi di $G'(s) = \dfrac{2}{s}$

$(P(s) = G'(s)(1 - e^{-24s}))$

$G(s) = G'(s)e^{-24s}$

$P(s) + G(s) = G'(s)$

↳ Mi basta un semplice $R'(s) = \mu_R^*$ dove:
- $\varphi_m = 90°$
- $\omega_c = 2\mu_R^*$ → ad es. $\mu_R^* = 10$ → $\omega_c = 20$

2.3

$$G_{yw}(s) = \frac{\overbrace{\frac{R'(s)}{1+R'(s)P(s)} \cdot G(s)}^{L(s)}}{1 + L(s)} = \frac{\frac{R'(s)G'(s)e^{-\tau s}}{\cancel{1 + R'(s)P(s)}}}{\frac{1 + R'(s)P'(s) + R'(s)G'(s)e^{-\tau s}}{\cancel{1 + R'(s)P(s)}}} = \frac{R'(s)G'(s)e^{-\tau s}}{1 + R'(s)G'(s) - \cancel{R'(s)G'(s)e^{-\tau s}} + \cancel{R'(s)G'(s)e^{-\tau s}}} = \overbrace{\frac{R'(s)G'(s)}{1 + R'(s)G'(s)}}^{\text{FDT RAZIONALE}} \cdot \overbrace{e^{-\tau s}}^{\text{RITARDO}}$$

2.4

$$e^{-24s} \cong \frac{1-12s}{1+12s} \quad \text{Approx. Padé}$$

$$R(s) = \frac{R'(s)}{1+R'(s)G'(s)\left(1-\frac{1-12s}{1+12s}\right)} = \frac{10}{1+\frac{20}{\cancel{s}}\left(\frac{\cancel{1}+12\cancel{s}-\cancel{1}+12\cancel{s}}{1+12s}\right)} = \frac{10(1+12s)}{1+12s+20\cdot 24} = \frac{10(1+12s)}{12s+481} \approx \frac{1}{48}\frac{(1+12s)}{\left(1+\frac{s}{40}\right)}$$

3

$$G(s) = \frac{10}{s+2} \qquad T = 0.1$$

3.1 $R^*(z)$ da algoritmo $\longrightarrow$ $R^*(z) = \dfrac{0.8z-0.6}{z-1}$

3.2 • Metodo dello scalino

$$\begin{cases} A+B=0 & B=-5 \\ 2A=10 & \rightarrow A=5 \end{cases}$$

$$G(s) = \frac{10}{s+2} \longrightarrow Y(s) = \frac{G(s)}{s} = \frac{10}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} = \frac{5}{s} - \frac{5}{s+2} = \mathcal{L}\big(\underbrace{5-5e^{-2t}}_{y(t)}\big)$$

$$y^*(k) = y(kT) = 5\left(1-e^{-2kT}\right)$$

$$Y^*(z) = \mathcal{Z}\left[y^*(k)\right] = 5\left(\frac{z}{z-1} - \frac{z}{z-\underbrace{\lambda}_{\lambda=e^{-2T}}}\right) = 5\left(\frac{\overbrace{z(z-\lambda)-z(z-1)}^{\cancel{z^2}-\lambda z-\cancel{z^2}+z}}{(z-1)(z-\lambda)}\right) = 5\frac{z(1-\lambda)}{(z-1)(z-\lambda)}$$

$$G^*(z) = Y^*(z)\cdot\frac{z-1}{z} = 5\frac{(1-\lambda)}{z-\lambda} = 5\,\frac{1-e^{-2T}}{z-e^{-2T}} = 5\,\frac{1-e^{-0.2}}{z-e^{-0.2}} \cong \frac{0.9}{z-0.82}$$

• Significato di $G^*(z)$

$G^*(z)$ sarebbe la FDT che riassume la serie $\xrightarrow{u^*(k)} \boxed{D/A} \xrightarrow{u(t)} \boxed{G(s)} \xrightarrow{y(t)} \boxed{A/D} \xrightarrow{y^*(k)}$

(tralasciando il ritardo del mantenitore e filtro anti-aliasing)

3.3 $L(z) = R^*(z)G^*(z) = \dfrac{0.9\,(0.8z-0.6)}{(z-1)(z-0.82)}$

$$\varphi_{AC} = z^2 - 1.82z + 0.82 + 0.72z - 0.54 = z^2 - 1.1z + 0.28 = 0$$

$$z_{1,2} = \frac{1.1 \pm \sqrt{0.09}}{2} \quad \begin{cases} 0.7 < 1 \\ 0.4 < 1 \end{cases} \Rightarrow \text{il sist. A.C. è AS. STAB.}$$

3.4.

$$L(z) = R^*(z)G^*(z) = \frac{0.9\,(0.8z-0.6)}{(z-1)(z-0.82)} \qquad F(z) = \frac{L(z)}{1+L(z)} = \frac{0.9\,(0.8z-0.6)}{z^2-1.1z+0.28}$$

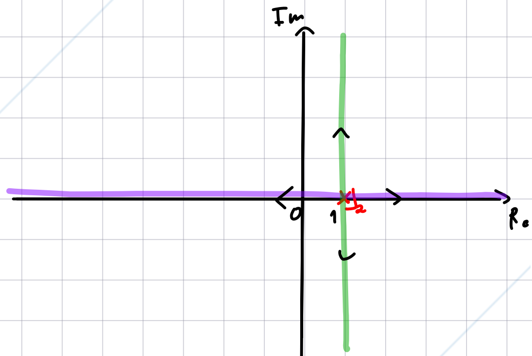
- PRECISIONE STATICA: Polo in $z=1$ → azione integrale ⟹ $\mu_F = 1$
- PRECISIONE DINAMICA:
  - $K_\ell = \nu_F - 1 = 0$ + latenza
  - $k_a \simeq -\dfrac{5}{\ln|\underbrace{z_{dom}}_{0.7}|} \cong 14$

0.7 (calc. prima con y_{nc})

1/07/14

① $G(s) = \frac{10}{(s-1)^2}$

1.1 $R(s) = \mu \longrightarrow L(s) = \frac{10\mu}{(s-1)^2}$



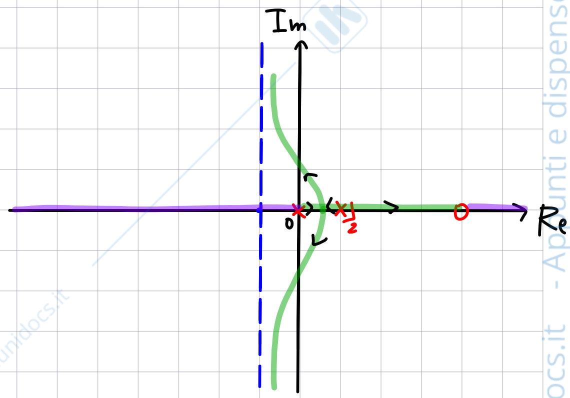
$n = 2$
$m = 0$
$\nu = 2$ → si conserva

Baricentri: $X_A = X_B = \frac{2}{2} = 1$

Asintoti:
LD $90°, 270°$
LI $0°, 180°$

SEMPRE INSTABILE ⟹ sia in LD che in LI ci saranno sempre dei poli A.C. (con $Re(s)>0$)

1.2 $R(s) = \frac{\mu(1+s\tau)}{s} \longrightarrow L(s) = \frac{10\mu(1+s\tau)}{s(s-1)^2}$



$n = 3$
$m = 1$
$\nu = 2$ → si conserva

Baricentri:
$X_A = \frac{1}{2}\left(\frac{1}{\tau}+2\right) = \frac{1}{2\tau}+1$
$X_B = \frac{1}{3}(+2) = \frac{2}{3}$

Asintoti:
LD $90°, 270°$
LI $0°, 180°$

↳ per aspirare alla as. stab. devo almeno avere che $X_A < 0$

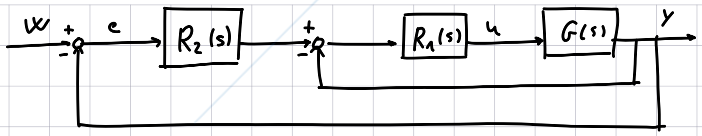
⟹ $\frac{1}{2\tau}+1 < 0 \to \tau > -\frac{1}{2} \to$ ad es. $\tau = -\frac{1}{4} \to X_A = -1$

IL SIST RIMANE SEMPRE INSTABILE

1.3
1. $\varphi_{AC}(s) = (s-1)^2 + 10\mu = s^2 - 2s + 1 + 10\mu$ → X Criterio di Routh (s ha coeff. < 0) ⟹ SIST. INSTABILE

2. $\varphi_{AC}(s) = s(s-1)^2 - \frac{10\mu}{4}s + 10\mu = s^3 - 2s^2 + s\left(1 - \frac{10\mu}{4}\right) + 10\mu$ → X Criterio Routh ⟹ SIST. INSTABILE

1.4.



- $R_1(s)$ progettato per stab. $G(s)$
- $R_2(s)$ prog. per soddisf. le specifiche

(2) $P(s) = \dfrac{b}{s+a}$, $a>0$, $b>0$ $\quad G(s) = \dfrac{1}{s}$ $\quad R(s) = \mu \rightarrow L(s) = \dfrac{\mu}{s} \rightarrow \begin{cases} \omega_c = \mu \\ \varphi_m = 90° \end{cases}$

2.1. $P(s)$ potrebbe servire in questo caso per diminuire il t. assest. t_a o per migliorare la moderazione

- SENZA PREFILTRO

$G_{yw}(s) = \dfrac{\mu}{s+\mu} \rightarrow t_a = \dfrac{5}{\mu}$ $\quad G_{uw} = \dfrac{s\mu}{s+\mu} \xrightarrow[s \to +\infty]{} u(0) = \mu \rightarrow \mu \uparrow \; \omega_c \uparrow \; t_a \downarrow \; u(0) \uparrow$ effetto negativo

- CON PREFILTRO

$G_{yw}(s) = P(s) \cdot \dfrac{\mu}{s+\mu} = \dfrac{b}{s+a} \cdot \dfrac{\mu}{s+\mu} = \dfrac{b\mu}{(s+a)(s+\mu)} \rightarrow$ se $b=a$ $\quad F(s) \xrightarrow[s\to 0]{} F(0) = \dfrac{a\mu}{a\mu} = 1 \rightarrow$ OTTIMA PRECISIONE STATICA

$\rightarrow$ prendo $P(s) = \dfrac{a}{s+a} \Rightarrow u(0) = \lim\limits_{s\to+\infty} \dfrac{sa\mu}{(s+a)(s+\mu)} = 0$ moderazione migliorata

t_a peggiora ulteriormente se $a<\mu$

2.2 $w(t) = \text{sca}(t)$

- SENZA PREFILTRO

$G_{uw} = \dfrac{s\mu}{s+\mu} \xrightarrow[s \to +\infty]{} u(0) = \mu$

$\longrightarrow$ migliora la mod. con il prefiltro (non c'è un salto discontinuo di u in 0)

- CON PREFILTRO

$u(0) = \lim\limits_{s\to+\infty} \dfrac{sa\mu}{(s+a)(s+\mu)} = 0$

2.3 $G_{yw}(s) = \dfrac{a}{s+a} \cdot \dfrac{\mu}{s+\mu} \longrightarrow$ se $a<\mu \Rightarrow t_a = \dfrac{5}{a} \uparrow$ sale

2.4. $P(s)\,R(s) = C(s) + R(s)$

$C(s) = R(s)\,\underbrace{(P(s)-1)}_{\frac{\cancel{a}-s-\cancel{a}}{s+a}} = -\dfrac{\mu s}{s+a}$ realizz. ✓ stabile ✓

③ $R^{\circ}(s) = \frac{0.2}{s}$ $T = 2$

3.1 TRASF. BILINEARE $s = \frac{1}{T}\,\frac{z-1}{\alpha z + 1 - \alpha} \xrightarrow[T=2]{\text{TU } \alpha = \frac{1}{2}} s = \frac{z-1}{z+1}$

$R^{*}(z) = R^{\circ}\left(\frac{z-1}{z+1}\right) = 0.2\,\frac{z+1}{z-1}$

3.2 $G^{*}(z) = \frac{0.4}{z-0.5} \longrightarrow$ poli $z = e^{sT} \longrightarrow s = \frac{1}{T}\ln z \cong -0.346$

guadagno $G(0) = G^{*}(1) = \frac{4}{5}$

$G(s) = \frac{4}{5}\cdot\frac{1}{(s+0.346)}$

3.3 $L(z) = R^{*}(z)\,G^{*}(z) = \frac{0.08(z+1)}{(z-0.5)(z-1)}$

$\varphi_{Ac}(z) = z^2 - 1.5z + 0.5 + 0.08z + 0.08 = z^2 - 1.42z + 0.58 \longrightarrow z_{1,2} \cong 0.71 \pm j0.275 \longrightarrow |z_{1,2}| \cong 0.761 < 1 \Rightarrow$ AS. STAB.

3.4. $F(z) = \frac{L(z)}{1+L(z)} = \frac{0.08(z+1)}{z^2 - 1.42z + 0.58}$

VALORE REGIME

$\lim_{z\to 1} \cancel{(z-1)}\,\frac{0.08(z+1)}{z^2-1.42z+0.58}\,\frac{z}{\cancel{z-1}} = \frac{0.16}{0.16} = 1$

k_l

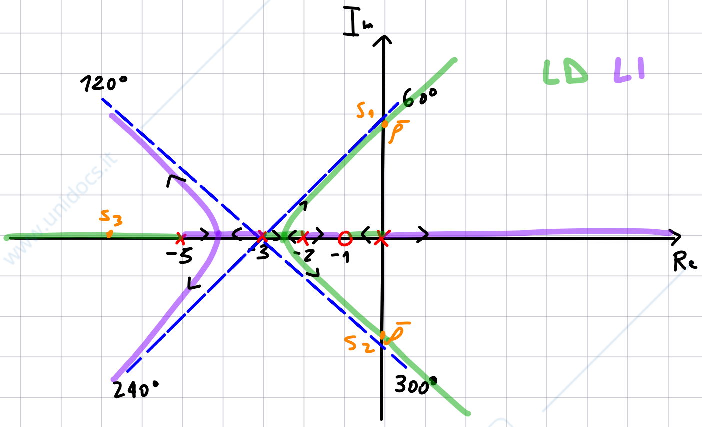
$k_l = V_r \cdot 1 = 0$

k_a

$k_a = -\frac{5}{\ln|z_{1,2}|} \cong 18.3$

17/07/14

1) $L(s) = \dfrac{\rho(s+1)}{s(s+2)(s+3)(s+5)}$



1.1,2 $n = 4$ Baricentri

$m = 1$ $\quad X_A = \frac{1}{3}(1-2-3-5) = -3$

$\nu = 3$ $\quad X_B = \frac{1}{4}(-2-3-5) = -\frac{10}{4} = -\frac{5}{2}$

si conserva

3 Asintoti → $\begin{cases} LD & 60°, 180°, 300° \\ LI & 0°, 120°, 240° \end{cases}$

1.3

AS. STAB. $\Longleftrightarrow$ $\rho < \bar{\rho}$ ⟶ $\rho > 0$ non basta solo suff.

↳ potrebbero esserci poli a.c. nel semipiano Re dx

1.4 $\bar{\rho}$

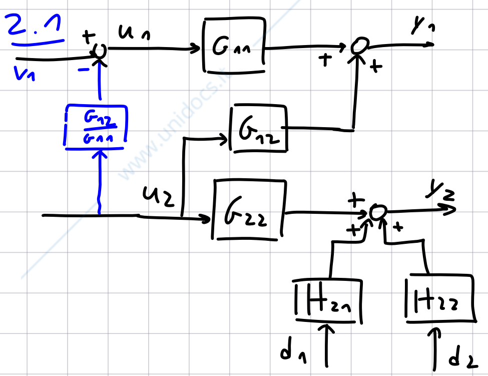
→ Sfrutto la conservazione di X_B e trovo s_3: $\quad X_B = -\frac{5}{2} = \frac{(\cancel{s_1+s_2}^{\,0}+s_3)}{4}$ ⟶ $s_3 = -10$

→ Uso la regola della punteggiatura e trovo $\bar{\rho}$

$$\bar{\rho} = \frac{5 \cdot 7 \cdot 8 \cdot 10}{9} = \frac{2800}{9} \simeq 311$$

2

2.1



2.2 Disaccoppiatore in avanti (basta un compens. tra v_1 e u_2)

$$\Delta_A = \begin{bmatrix} 1 & -\frac{G_{12}}{G_{11}} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{5(1+3s)}{(1+s)(1+4s)} \\ 0 & 1 \end{bmatrix}$$

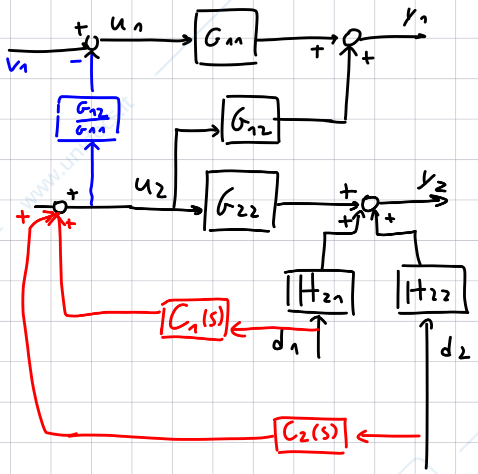
VERIFICA CHE NON AVVENGANO CANC. CRITICHE:

Poli: $-\frac{1}{3}, -\frac{1}{4}, -1 < 0$ ✓

Zeri: $\det G(s) = 0 \rightarrow \frac{0.24}{1+3s} = 0 \rightarrow$ non ci sono proprio zeri ✓

2.3
- $v_2 = u_2 \rightarrow u_2 = 1$ sempre quindi anche regime ($u_2(t)$ con $t \gg$)
- $y_{2\infty} = \lim_{s\to 0} \cancel{s}\, \underbrace{G_{y_2 u_2}(s)}_{G_{22}} \cdot \frac{1}{\cancel{s}} = 0.6 \longrightarrow$ Il disaccoppiatore non agisce sul disturbo u_2 per y_2
- $U_1(s) = \cancel{V_1}(s) - \frac{G_{12}}{G_{11}} \cdot V_2(s) \longrightarrow u_{1\infty} = \lim_{s\to 0} s\, \frac{-G_{12}}{G_{11}} \cdot \frac{1}{s} = -5$
- $Y_1(s) = \underbrace{\left(-\frac{G_{12}}{\cancel{G_{11}}} \cdot \cancel{G_{11}} + G_{12}\right)}_{=0} U_1(s) \qquad y_{1\infty} = 0 \longrightarrow$
  - A MENO DI INCERTEZZE su $G(s)$
  - Il disaccoppiatore fa il suo dovere, rendendo indipendente y_1 da $u_2 = v_2$

2.4



Voglio che

$$Y_{2d}(s) = G_{d21}(s)\,D_1(s) + G_{d22}(s)\,D_2(s) = 0$$

$$\begin{cases} G_{d21} = C_1(s)\,G_{22}(s) + H_{21}(s) = 0 \xrightarrow{\text{STATICO}} C_1(s) = -\frac{\mu_{H21}}{\mu_{G22}} = \frac{1}{0.6} = \frac{5}{3} \\ G_{d22} = C_2(s)\,G_{22}(s) + H_{22}(s) = 0 \xrightarrow[\text{STATICO}]{} C_2(s) = -\frac{\mu_{H22}}{\mu_{G22}} = -1 \end{cases}$$

$$\Longrightarrow C(s) = \begin{bmatrix} 0 & 0 \\ \frac{5}{3} & -1 \end{bmatrix}$$

3

3.1 MANTENITORE $\longrightarrow$ Convertitore D/A: Si occupa della conversione del segnale digitale in analogico, creando una funzione cost. a tratti generata dai valori digitali mantenuti costanti per un certo tempo di mantenimento
(nel caso del mant. di ordine $\emptyset$)

3.2 $u(t) = u^*(k)$ per $kT_M + \underbrace{\tau_M}_{\text{OFFSET}} \le t \le (k+1)T_M + \tau_M$

t. mantenimento

3.3 $H_0(s) = \frac{1 - e^{-sT}}{s}$: la trasf. di Laplace di una serie di scalini traslati di T che può rappr. una funz. cost. a tratti moltiplicati per l'opportuno valore in k.

Approx di $H_0 \simeq T e^{-\frac{sT}{2}}$

↳ poiché dai grafici di Bode si vede che è un filtro PB con $B \simeq [0, \overbrace{\frac{\omega_m}{2}}^{T = \frac{2\pi}{\omega_m}})$
e dalla fase si vede che nel tratto fino a $\frac{\omega_m}{2}$ ha un andamento approx. come un ritardo di $\frac{T}{2}$

3.4.

$f^*(k) = a^k$, $a = -2$, $k \geq 0$

$$F(z) = Z(f^*(k)) = \sum_{k=0}^{\infty} f^*(k)\, z^{-k} = \sum_{k=0}^{\infty} (\overbrace{a z^{-1}}^{|a z^{-1}| < 1})^k = \frac{1}{1 - a z^{-1}} = \frac{z}{z-a} = \frac{z}{z+2}$$

30/06/15

1 **1.1** $L(s) = \dfrac{4\mu\,(s+1)}{(s-1)(s+2)(s+4)}$ — ρ

Baricentri

$n = 3$

$m = 1$

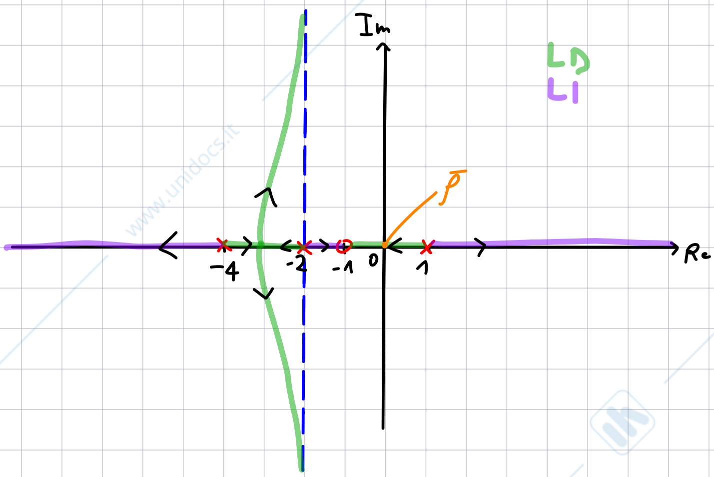
$\nu = 2 \longrightarrow$

$x_A = \frac{1}{2}(\cancel{1}-\cancel{2}-4+\cancel{1}) = -2$

$x_B = \frac{1}{3}(1-2-4) = -\frac{5}{3}$

Si considera

Asintoti $\begin{cases} \text{LD} & 90°, 270° \\ \text{LI} & 0°, 180° \end{cases}$



per $\rho > \bar\rho \iff$ AS. STAB.

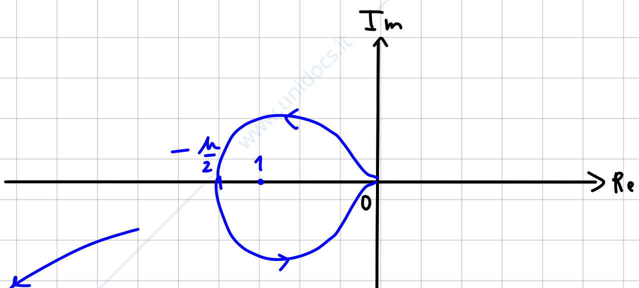
dove per la regola della puntegg.: $\bar\rho = \dfrac{1 \cdot 2 \cdot 4}{1} = 8 \longrightarrow \bar\mu = 2$

$(s=0)$

$\Longrightarrow$ per $\mu > 2$

1.2. Per $\mu > 0$

$\begin{cases} |L(0)| = -\frac{\mu}{2} \\ |L(\infty)| = 0 \end{cases}$ $\qquad \begin{cases} \angle L(0) = \angle -\frac{\mu}{2} = -180° \\ \angle L(\infty) = 0° \end{cases}$



$P = 1 \longrightarrow$ Per 1 giro antiorario intorno a -1 $(N=1)$

$\Rightarrow -\frac{\mu}{2} < -1 \longrightarrow \mu > 2$

1.3 Baricentro poli in anello chiuso è:

$$X_B(\rho) = \frac{1}{n}\sum_{i=1}^{n} \underbrace{s_i(\rho)}_{\text{poli A.C.}}$$

$\hookrightarrow X_B = -\frac{5}{3}$ si conserva e coincide con X_B A.A. poichè $V = n-m = 2$

1.4 Per $\mu \to +\infty$, i poli A.C del LD diventano c.c. con uno smorzamento sempre + basso ⇒ maggiore sovraelong.

(l'altro polo → allo zero in -1 e quindi si cancella con esso)

2

2.1 $A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ $D = 0$

$$G(s) = C\underbrace{(sI-A)^{-1}}B = \frac{1}{(s+1)(s+2)}\begin{bmatrix} s+2 & s+2 \\ s+2 & -s \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \frac{1}{(s+1)(s+2)}\begin{bmatrix} s+2 & 2(s+2) \\ s+2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & \frac{2}{s+1} \\ \frac{1}{s+1} & \frac{2}{(s+1)(s+2)} \end{bmatrix}$$

$$(sI-A) = \begin{bmatrix} s+1 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$\hookrightarrow (sI-A)^{-1} = \frac{1}{(s+1)(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s+1 \end{bmatrix}$$

2.2 Canc. critiche

Poli $G(s)$: $-1, -2 < 0 \, V$

Zeri $G(s)$: $\det G(s) = 0 \quad \frac{2-2(s+2)}{(s+1)^2(s+2)} = \frac{-2\cancel{(s+1)}}{(s+1)^{\cancel{2}}(s+2)} = 0 \to$ No zeri

} cond. suff.

$$\Delta_a(s) = \begin{bmatrix} 1 & -\frac{G_{12}}{G_{11}} \\ -\frac{G_{21}}{G_{22}} & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ \underbrace{-\frac{s+2}{2}}_{\text{non realizz.}} & 1 \end{bmatrix}$$

2.3

$$\lambda = \frac{\mu_{11}\mu_{22}}{\det G(0)} = -\frac{1}{1} = -1 << 0$$ → scelta "anomala" accoppiamenti:

$\{(y_1, u_2); (y_2, u_1)\}$ ma con interazioni

③ 3.1 Sistema a segnali campionati significa ricavare FDT discreta $G^*(z)$ che rappresenti il sottosistema mantenitore - G(s) - campionatore

$$u^*(k) \to \boxed{D/A} \xrightarrow{u(t)} \boxed{G(s)} \xrightarrow{y(t)} \boxed{A/D} \xrightarrow{y^*(k)}$$

3.2. $T = 8$

$G(s) = \frac{25}{1+50s}$

Metodo gradino

$$\begin{cases} 50A + B = 0 \to B = -1250 \\ A = 25 \end{cases}$$

$$Y(s) = \frac{25}{s(1+50s)} = \frac{A}{s} + \frac{B}{1+50s} = \frac{25}{s} - \frac{25}{s+\frac{1}{50}} = \mathcal{L}\left(25(1-e^{-\frac{t}{50}})\right), \; t \geq 0$$

$$y^*(k) = y(kT) = 25(1-e^{-\frac{kT}{50}})$$

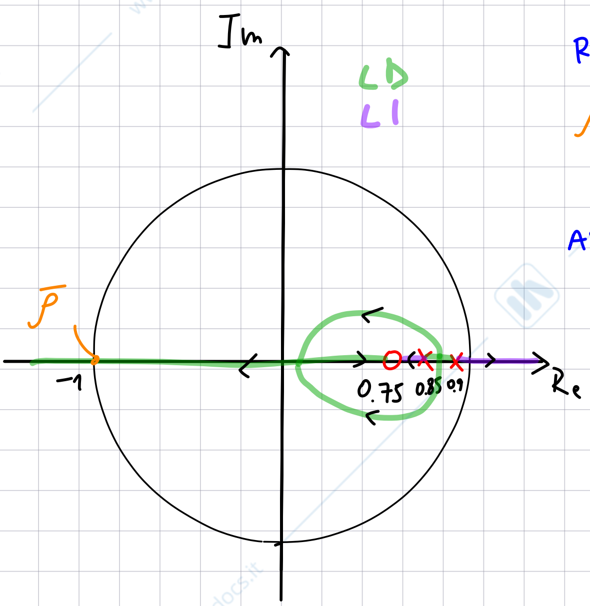
$$Y^*(z) = Z[y^*(k)] = 25\left(\frac{z}{z-1} - \frac{z}{z-e^{-\frac{T}{50}}}\right) = \frac{25z(1-e^{-\frac{T}{50}})}{(z-1)(z-e^{-\frac{T}{50}})}$$

$$G^*(z) = Y^*(z) \cdot \frac{z-1}{z} = 25\,\frac{1 - e^{-\frac{T}{50}}}{z - e^{-\frac{T}{50}}} \cong \frac{3.75}{z - 0.85}$$

$T \leftarrow 8$

3.3.

$$R^*(z) = \mu\,\frac{(0.2z - 0.15)}{z - 0.9} \longrightarrow L(z) = \frac{\overbrace{0.75\mu}^{\rho}(z - 0.75)}{(z-0.9)(z-0.85)}$$



Reg. puntgd.

$$\bar{\rho} = \frac{1.9 \cdot 1.85}{1.75} \cong 2$$

AS. STAB. $\Longleftrightarrow \rho < \bar{\rho} \quad \rightarrow \mu < \bar{\mu} = \frac{8}{3} \simeq 2.67$

3.4.

$$e_\infty = \lim_{z \to 1} (\cancel{z-1})\,\frac{1}{1 + L(z)} \cdot \frac{z}{\cancel{z-1}} = \lim_{z \to 1} \frac{z(z-0.9)(z-0.85)}{0.75(z-0.75)} = 0.08$$

↳ Usando approx $\tilde{R}(s) = e^{-\frac{sT}{2}}\, R^*(e^{sT}) \longrightarrow \tilde{L}(s) = \tilde{R}(s)G(s) \longrightarrow \omega_c : \ |\tilde{L}(j\omega_c)| = 1$

20/07/15

1 $L(s) = \dfrac{\rho(s-1)}{(s+3)^3}$

1.1 Analisi con φ_{AC}

$\bar{s} = -3 + j\omega$

$$\varphi_{AC} = \rho(s-1) + (s+3)^3 = \rho(-4+j\omega) - j\omega^3 = 0$$

↳ $-4\rho + j\rho\omega - j\omega \neq 0 \quad \forall \rho$ reale $\Longrightarrow \bar{s}$ non è un polo A.C.

1.2.3

$n = 3$

$m = 1$

$\nu = 2$ → si conserva

Baricentri

- $x_A = \frac{1}{2}(-1-3-3-3) = -5$
- $x_B = \frac{1}{3}(-3-3-3) = -3$

ASINTOTI

LD $90°, 270°$

LI $0°, 180°$

- da $s=0$

$$\bar{\rho}_2 = \frac{27}{1} = 27$$

- da cons. x_B in A.C.

$$\frac{1}{3}(\cancel{s_1+s_2}+s_3) = -3 \rightarrow s_3 = -9$$

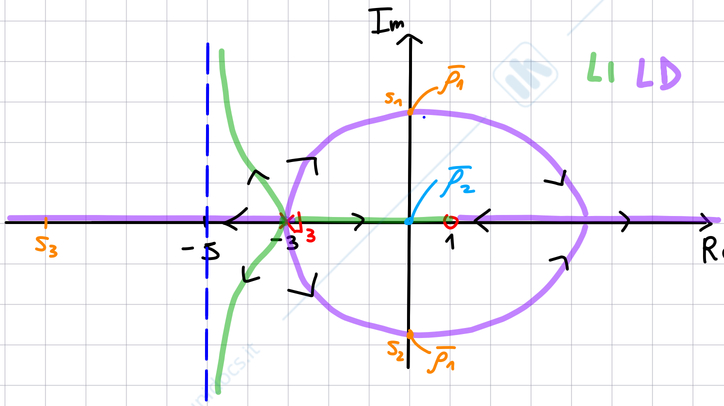
$$\bar{\rho}_1 = \frac{-6^3}{10} = -21.6$$

AS. STAB.

$\Updownarrow$

$\bar{\rho}_1 < \rho < \bar{\rho}_2$

$-21.16 < \rho < 27$



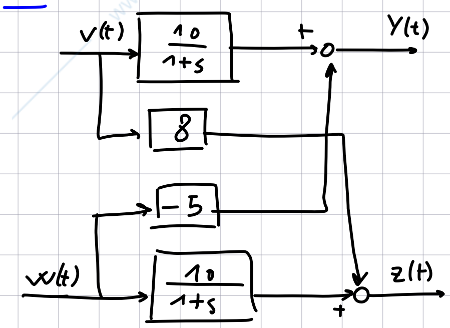
②

$$Y(s) = -5W(s) + \frac{10}{1+s}V(s)$$

$$Z(s) = 8V(s) + \frac{10}{1+s}W(s)$$

$$\longrightarrow G(s) = \begin{array}{c|c} & \begin{array}{cc} v & w \end{array} \\ \hline \begin{array}{c} y \\ z \end{array} & \begin{bmatrix} \frac{10}{1+s} & -5 \\ 8 & \frac{10}{1+s} \end{bmatrix} \end{array}$$

2.1



2.2 Contr. Decentralizzato

$$\lambda = \frac{\mu_{yv}\,\mu_{zw}}{\det G(0)} \qquad G(0) = \begin{array}{c|c} & \begin{array}{cc} v & w \end{array} \\ \hline \begin{array}{c} y \\ z \end{array} & \begin{bmatrix} 10 & -5 \\ 8 & 10 \end{bmatrix} \end{array} \longrightarrow \det G(0) = 100 + 40 = 140$$

$\hookrightarrow \lambda = \frac{100}{140} = \frac{5}{7} \simeq 1 \longrightarrow$ scelta "naturale" (v, y); (w, z) ✓

2.3

- $R_z(s) = \frac{0.2(1+s)}{s} \longrightarrow L_z(s) = R_z(s) \cdot G_{zw}(s) = \frac{2}{s} \longrightarrow \begin{cases} \omega_c = 2 \\ \varphi_m = 90° \end{cases}$
- R_y progettato su $G^*_{yv}(s) = G_{yv} - \frac{R_z(s)\,G_{zv}(s)\,G_{yw}(s)}{1 + R_z(s)\,G_{zw}(s)} = \frac{10}{1+s} + \frac{\frac{8\,(1+s)}{\cancel{s}}}{\frac{s+2}{\cancel{s}}} = \frac{10(s+2) + 8(s+1)^2}{(s+1)(s+2)} =$

$$= \frac{10(s+2)+8(s+1)^2}{(s+1)(s+2)} = \frac{10s+20+8s^2+16s+8}{\underbrace{(s+1)(s+2)}_{s^2+3s+2}} = \frac{8s^2+26s+28}{(s+1)(s+2)}$$

$$s^2 + \underbrace{\frac{26}{8}}_{2\omega_n\cdot\xi} s + \underbrace{\frac{28}{8}}_{\omega_n^2} \rightarrow \begin{cases} \omega_n \cong 1.87 \\ \xi \cong 0.87 \end{cases}$$

$$|G^*_{yv}(0)| = 14 \simeq 23dB$$

con $L_y(s) = G_{yw}(s) \rightarrow \omega_c \simeq 20$

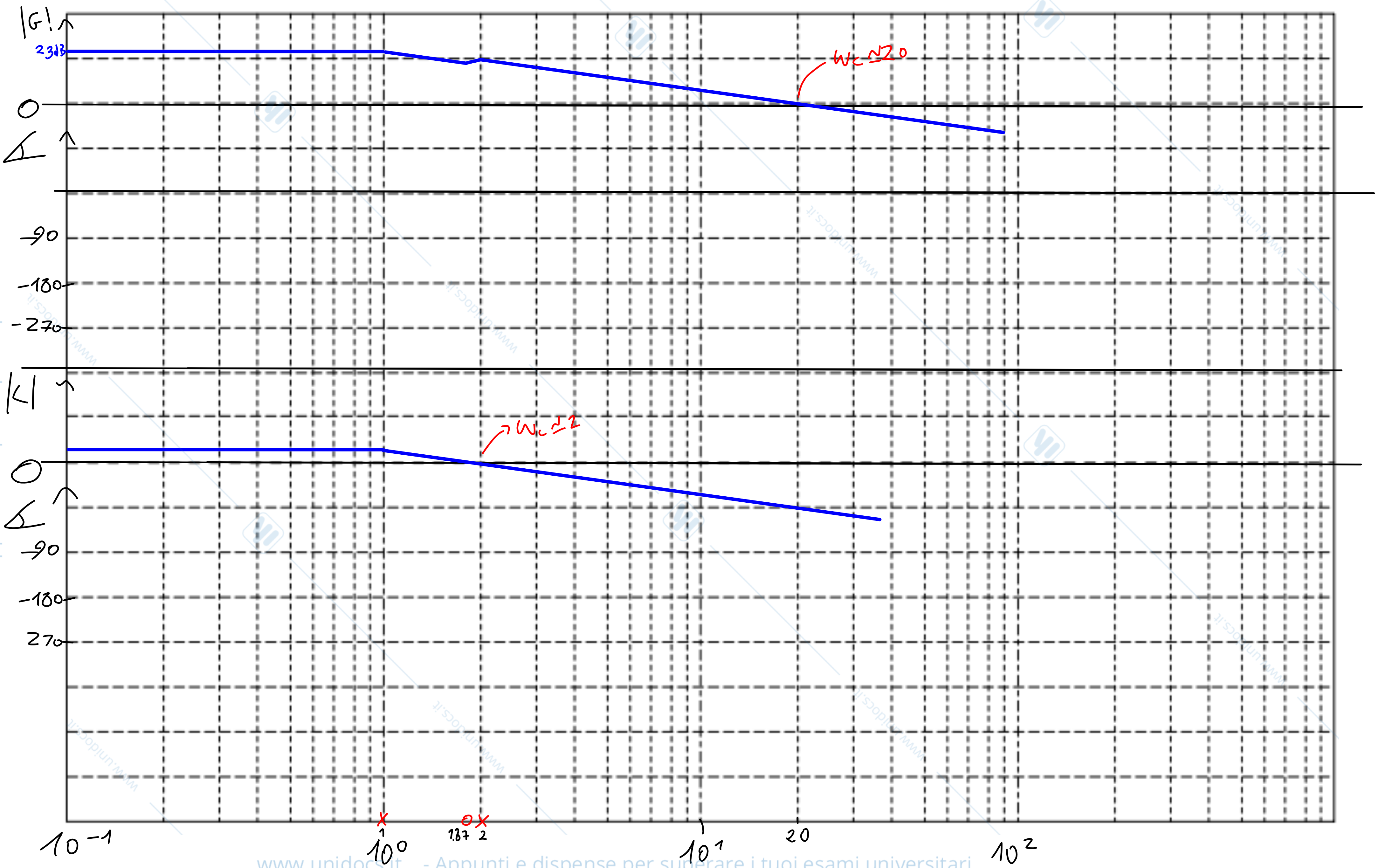
$$\varphi_c = 0° \underbrace{- \arctg\left(\frac{\omega_c}{2}\right)}_{-84.3°} \underbrace{- \arctg\left(\frac{\omega_c}{1}\right)}_{-87.13°} + \underbrace{\arctg\left(\frac{2\omega_c\omega_n\xi}{\omega_n^2-\omega_c^2}\right)}_{\sim 0} \cong -171.5°$$

$\varphi_m = 8.5°$

$\rightarrow$ Scelgo $R_y = \dfrac{\frac{1}{7}(2+s)}{8s^2+26s+28} \rightarrow L_y(s) = \dfrac{2}{s+1}$ $\begin{cases} \omega_c \simeq 2 \\ \varphi_m = 116.6 \end{cases}$

$$\varphi_L = -\arctg\left(\frac{\omega_c}{1}\right) = -63.4°$$

2.4 INCONVENIENTI:
- Calcoli più complicati
- Regolatori più complic. per non avere limit. di prestazioni


|G|
23dB
0
$\omega_c \simeq 20$
-90
-180
-270
|L|
0
$\omega_c \simeq 2$
-90
-180
270
10^{-1}
1
10^0
1.87
2
10^1
20
10^2

3 **3.1** $L(s) = \frac{0.5}{s(1+0.1s)}$ $\rightarrow \omega_c \simeq 0.5$

↳ Scelta T

$$\underbrace{\frac{2\pi}{50\omega_c}}_{\sim 0.25} \leq T \leq \underbrace{\frac{2\pi}{5\omega_c}}_{\sim 2.5}$$

→ con $T = 0.4$ facciamo una scelta + prudente e robusta rispetto a eventuali approx.

3.2 $R^0(s) = \frac{1}{1+0.1s}$

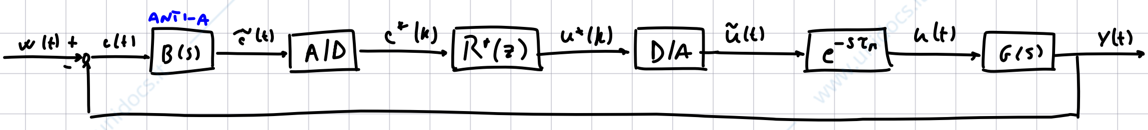
Con TU → $\alpha = \frac{1}{2}$ TRASF. CAMP. $s = \frac{1}{T} \frac{z-1}{\alpha z + 1 - \alpha} \overset{\alpha = \frac{1}{2},\ T = 0.4}{=} 5 \cdot \frac{z-1}{z+1}$

$$R^*(z) = R^0\left(5 \frac{z-1}{z+1}\right) = \frac{z+1}{z+1+0.5z-0.5} = \frac{z+1}{3.5z+0.5} = \frac{2}{7} \frac{z+1}{z+\frac{1}{7}}$$

• Algoritmo di controllo

$$u^*(k) = -\frac{u^*(k-1)}{7} + \frac{2}{7} c^*(k) + \frac{2}{7} c^*(k-1)$$

3.3 NOTA: $R^*(z)$ NON strett. proprio → c'è un ritardo nella conv. D/A di $\tau_M = \tau_c = 0.3$ (blocco a parte)



3.4.

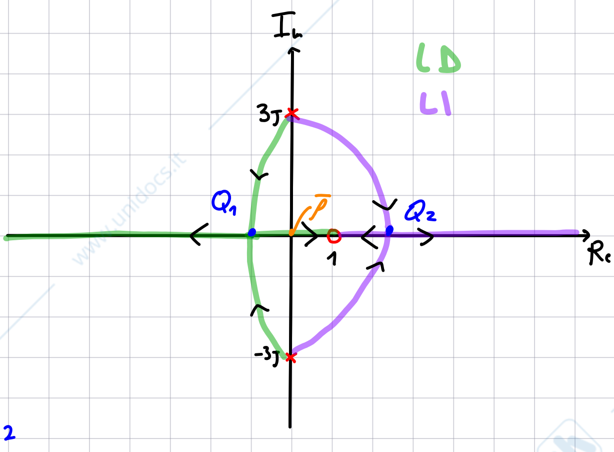
ANALOGICO $\varphi_c = -90° - \arctg\left(\frac{\overset{0.5}{\omega_c}}{10}\right) \cong -92.86° \rightarrow \varphi_m \cong 87.14°$

DIGITALE $\omega_c^* \cong \omega_c = 0.5$ (per scelta T) → a causa delle conversioni D/A, A/D avremo dei ritardi e un peggioramento di φ_m pari a:

$$\Delta\varphi_m = +\omega_c \frac{T}{2} \frac{180°}{\pi} + \omega_c \tau_m \frac{180°}{\pi} = +14.32° \rightarrow \varphi_m^* = \varphi_m - \Delta\varphi_m = 72.82° \rightarrow \left(\xi = \frac{\varphi_m}{100} \cong 0.73\right)$$

15/01/21

① $L(s) = \rho \dfrac{(s-1)}{s^2+9}$



1.1 $n=2$, $m=1$, $\nu=1$

• Baricentri

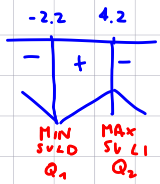
$X_A = (-1 + \cancel{3j} - \cancel{3j}) = -1$

$X_B = \frac{1}{2}(\cancel{-3j} + \cancel{3j}) = 0$

1.2 $\gamma(x) = -\dfrac{D(x)}{N^*(x)} = -\dfrac{s^2+9}{s-1}$

$\gamma'(x) = -\dfrac{2s^2-2s-s^2-9}{(s-1)^2} \geq 0 \rightarrow s^2-2s-9 \leq 0$

$s_{1,2} = 1 \pm \sqrt{1+9}$ ~ 4.2, ~ -2.2



1.3 AS.STAB $\Longleftrightarrow \rho < \overline{\rho}$

dove con la regola della punteggiatura: $(s=0)$ $\overline{\rho} = \dfrac{3 \cdot 3}{1} = 9$

• Per $\rho = 1 < 9$ (AS.STAB.) (Il margine di guadagno è il max fattore moltipl. ρ di $L(s)$ prima di perdere AS.STAB)

$\Downarrow$

$k_m = \overline{\rho} = 9$

Infatti: $L(s) = \dfrac{(s-1)}{s^2+9}$ $\quad k_m = \dfrac{1}{a}$ dove $a = |L(j \cdot \omega_\pi)|$ con ω_π: $\angle L(j\omega_\pi) = \pi$

$\hookrightarrow \angle L(j\omega_\pi) = \arctg\left(\frac{\omega_\pi}{-1}\right) - \cancel{\arctg}\,0 \;) = 180°$

$j^2\omega_\pi^2 + 9$

$-\omega_\pi^2 + 9$

$\hookrightarrow \omega_\pi = -tg(180°) = 0$

$\hookrightarrow |L(j\omega_\pi)| = |L(j\,0)| = \frac{1}{9}$

$\Rightarrow k_m = 9 < 10$

(2)

2.1 $R^*(z) = \dfrac{0.4}{z(z-1)}$

Prestazioni statiche: Polo integrale $\Rightarrow$ assicura un'ottima precisione statica in risposta allo scalino

2.2 (EA) $\alpha = 0$

TRASF. BILIN.
$s = \dfrac{1}{T}\,\dfrac{z-1}{\alpha z+1-\alpha} = \dfrac{z-1}{T} \rightarrow z = sT+1$

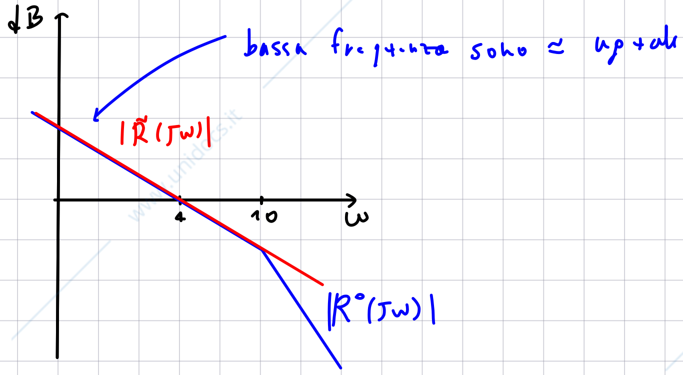
$\hookrightarrow R^\circ(s) = R^*(sT+1) = \dfrac{0.4}{Ts(sT+1)} \overset{T=0.1}{=} \dfrac{4}{(0.1s+1)s}$

2.3

$\tilde{R}(s) = e^{-s\frac{T}{2}} R^*(e^{sT}) = e^{-s\frac{T}{2}} \dfrac{0.4}{e^{sT}(e^{sT}-1)} = e^{-\frac{3}{2}sT}\dfrac{0.4}{(e^{sT}-1)} \simeq \dfrac{4e^{-0.15s}}{s}$ (non ho ritardo di mantenimento $\rightarrow R^*(z)$ è strett. propria)

$\hookrightarrow$ per $s \to 0$ ($\omega \to 0$) avrei $e^{sT}-1 \sim sT$

$R^\circ(s) = R^*(sT+1) = \dfrac{4}{(0.1s+1)s}$



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③ Var. di controllo: $w_1(t), x(t)$

Var. controllate: $z_1(t), w_2(t)$

Dati

- $p_a = 0$
- Valvola lineare $\eta(x) = x$ apertura stelo
- K, A_v, ρ, g
- coeff. attrito $\bar{f} = 0$

3.1

Hp. valvola con accumulo trasc.

SERBATOIO: • Cons. massa: $\dot{m} = w_i(t) - w_s(t) \xrightarrow{m = \rho A z_f} \dot{z}_1 = \frac{1}{\rho A}(\underbrace{w_1(t)}_{\text{var. controllo}} - \underbrace{w_s(t)}_{W_2})$

all'uscita del serbatoio

• Altezza di carico: $z_s^* = z_f + \cancel{\frac{p_a}{\rho g}}^{0} = z_f = z_1$

• Pressione p_f: $\cancel{\frac{p_a}{\rho}} + g z_f = \frac{p_f}{\rho} \rightarrow p_f = \rho g z_f = \rho g z_1$

Bernoulli

VALVOLA: $w_v = K A_v \times \sqrt{\rho(p_f - p_2)}$

portata uscita valvola; lineare; $p_2(0,t)$

$$p_2 = \rho g z_1 - \frac{w_2^2(t)}{K^2 A_v^2 \times \rho}$$

CONDOTTA: • Cons. massa: $w_v(t) = w_2(t) = K A_v \times \sqrt{\rho^2 g z_1 - \rho p_2}$

• Cons. Q. moto: $\dot{w}_2(t) = -\frac{\rho A_c g}{L}(\underbrace{z_2^*(L,t)}_{\cancel{\frac{p_a}{\rho g}} + 0} - \underbrace{z_2^*(0,t)}_{\frac{p_2}{\rho g} + z_2}) - \cancel{\bar{f}}\, w_2^2(t) = \frac{\rho A_g}{L} \cdot z_2 + \frac{A}{L} p_2$

$$\Rightarrow \begin{cases} \dot{z}_1 = \frac{1}{\rho A}(w_1 - w_2) \\ \dot{w}_2 = \frac{\rho A g z_2}{L} + \frac{\rho A_c g z_1}{L} - \frac{A_c w_2^2}{L K^2 A_v^2 x^2 \rho} = \frac{\rho A g}{L}(z_1 + z_2) - \frac{A_c w_2^2}{L K^2 A_v^2 x^2 \rho} \end{cases}$$

MODELLO DINAMICO

3.2.

- EQUILIBRIO:

$$\begin{cases} \bar{w}_1 = \bar{w}_2 \\ \bar{w}_2 = \rho A_v \bar{x} K \sqrt{g(\bar{z}_1 + \bar{z}_2)} \end{cases}$$

- MODELLO LINEARIZZ.

POSITIVI: $\alpha_1, \beta_1, \beta_3$

DIP. EQ.: β_2, β_3

$$\begin{cases} \delta \dot{z}_1 = \frac{1}{\rho A}(\delta w_1 - \delta w_2) = \overbrace{\frac{1}{\rho A}}^{\alpha_1 > 0} \delta w_1 \overbrace{- \frac{1}{\rho A}}^{\alpha_2 < 0} \delta w_2 \\ \delta \dot{w}_2 = \underbrace{\frac{\rho A_c g}{L}}_{\beta_1 > 0} \delta z_1 \underbrace{- \frac{2 A_c \bar{w}_2}{L K^2 A_v^2 \bar{x}^2 \rho}}_{\beta_2 < 0} \delta w_2 \underbrace{+ \frac{2 A_c \bar{w}_2^2}{L K^2 A_v^2 \bar{x}^3 \rho}}_{\beta_3 > 0} \delta x \end{cases}$$

3.3

$$A = \begin{array}{cc} \delta z_1 & \delta w_2 \\ \left[\begin{array}{cc} 0 & \alpha_2 \\ \beta_1 & \beta_2 \end{array}\right] \end{array} \begin{array}{c} \delta \dot{z}_1 \\ \delta \dot{w}_2 \end{array} \qquad B = \begin{array}{cc} \delta w_1 & \delta x \\ \left[\begin{array}{cc} \alpha_1 & 0 \\ 0 & \beta_3 \end{array}\right] \end{array} \begin{array}{c} \delta \dot{z}_1 \\ \delta \dot{w}_2 \end{array} \qquad C = I_{2\times 2} \qquad D = 0$$

$$G(s) = \cancel{C}^{\,I}(sI - A)^{-1}B$$

$$(sI - A) = \begin{bmatrix} s & -\alpha_2 \\ \beta_1 & s-\beta_2 \end{bmatrix} \qquad (sI-A)^{-1} = \frac{1}{\underbrace{s(s-\beta_2)-\alpha_2\beta_1}_{s^2-\beta_2 s-\alpha_2\beta_1}} \begin{bmatrix} s-\beta_2 & \alpha_2 \\ \beta_1 & s \end{bmatrix}$$

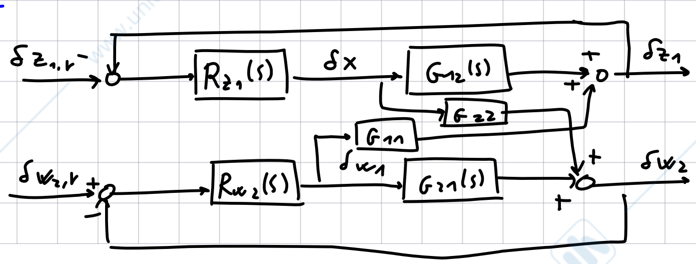
$$G(s) = \frac{1}{s^2-\beta_2 s-\alpha_2\beta_1} \begin{bmatrix} s-\beta_2 & \alpha_2 \\ \beta_1 & s \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 \\ 0 & \beta_3 \end{bmatrix} = \frac{1}{s^2-\beta_2 s-\alpha_2\beta_1} \begin{bmatrix} \alpha_1(s-\beta_2) & \alpha_2\beta_3 \\ \beta_1\alpha_1 & s\beta_3 \end{bmatrix}$$

3.4

$$G(0) = \frac{1}{-\alpha_2\beta_1} \begin{bmatrix} -\alpha_1\beta_2 & \alpha_2\beta_3 \\ \beta_1\alpha_1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{\alpha_1\beta_2}{\alpha_2\beta_1} & -\frac{\beta_3}{\beta_1} \\ -\frac{\alpha_1}{\alpha_2} & 0 \end{bmatrix} \rightarrow \det G(0) = -\frac{\alpha_1\beta_3}{\alpha_2\beta_1}$$

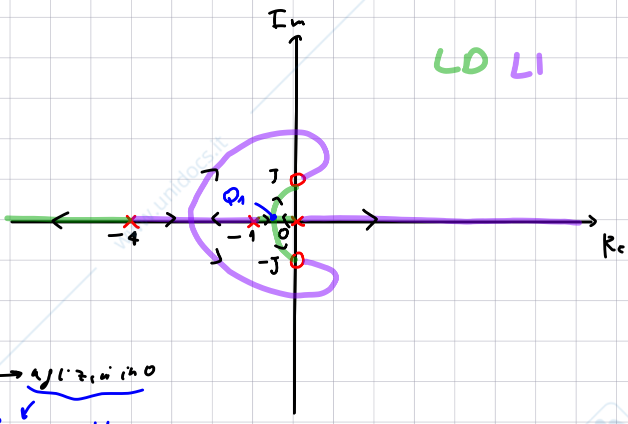
$$\lambda = \frac{\frac{\alpha_1\beta_2}{\alpha_2\beta_1}\cdot 0}{-\frac{\alpha_1\beta_3}{\alpha_2\beta_1}} = 0 \longrightarrow \text{scelta acc. } (x, z_1);\ (w_1, w_2)$$

3.5



20/07/20

① $L(s) = \rho \dfrac{(s^2+1)}{(s+1)(s+4)s}$



1.1 $n=3$, $m=2$, $\nu=1$

Baricentri

$x_A = 1\,(\cancel{j}-\cancel{j}-1-4-0) = -5$

$x_B = \frac{1}{3}(-1-4-0) = -\frac{5}{3}$

A.S. STAB $\Longleftrightarrow$ certamente $\rho>0$

per ρ troppo grande i poli A.C. $\to$ agli zeri in 0

non sarebbe quindi as. stab.

1.2. $\to Q_1 \cong -0.39$

$\hookrightarrow$ Q_1 è il max di $\gamma(x) = -\dfrac{\overbrace{(x+1)(x+4)x}^{x^3+5x^2+4x}}{x^2+1}$

$$\gamma'(x) = -\frac{(3x^2+10x+4)(x^2+1) - 2x(x^3+5x^2+4x)}{(x^2+1)} = -\frac{x^4+x^2-10x-4}{(x^2+1)^2} = -\frac{K(s)}{M(s)}$$

$\gamma'(Q_1) = 0 \cong K(-0.39) = 0.03$ ✓

$$\gamma''(x) = \frac{-6x^5+30x^4+16x^3+20x^2+18x-10}{(x^2+1)^4}$$

$\gamma''(-0.39) < 0 \longrightarrow$ MAX ✓

• ρ in corrisp. di -0.39

$$\bar{\rho} = \frac{0.39 \cdot 0.61 \cdot 3.61}{(0.39^2+1)} \cong 0.745$$

② 2.1

Valuto 3 accopp. principali e derivo le conclusioni: (metodo RGA ho i poli con $Re(p)<0$)

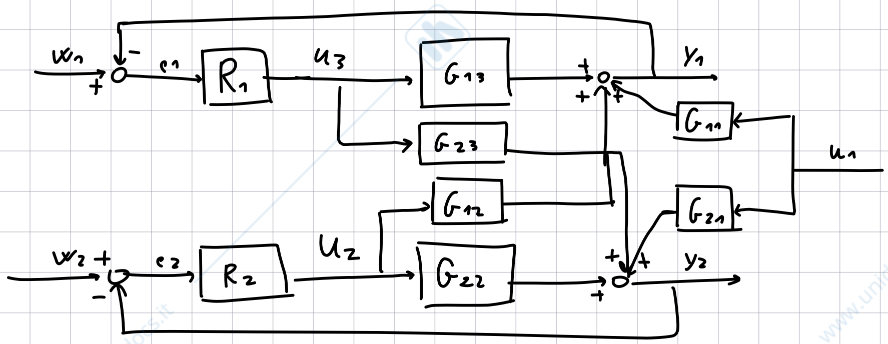
1: $\{u_1, y_1\}; \{u_2, y_2\}$ $\quad G^1(s) = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \rightarrow G^1(0) = \begin{bmatrix} -0.4 & 0.2 \\ 1 & 0.5 \end{bmatrix}$ $\quad \lambda = \dfrac{-0.2}{-0.2-0.2} = \dfrac{1}{2}$ → scelta (indifferente) $\{u_1, y_1\}; \{u_2, y_2\}$ con interazione

2: $\{u_1, y_1\}; \{u_3, y_2\}$ $\quad G^2(s) = \begin{bmatrix} G_{11} & G_{13} \\ G_{21} & G_{23} \end{bmatrix} \rightarrow G^2(0) = \begin{bmatrix} -0.4 & 0.8 \\ 1 & -1 \end{bmatrix}$ $\quad \lambda = \dfrac{0.4}{0.4-0.8} = -1$ → scelta $\{u_3, y_1\}; \{u_1, y_2\}$ con interazione

3: $\{u_3, y_1\}; \{u_2, y_2\}$ $\quad G^3(s) = \begin{bmatrix} G_{13} & G_{12} \\ G_{23} & G_{22} \end{bmatrix} \rightarrow G^3(0) = \begin{bmatrix} 0.8 & 0.2 \\ -1 & 0.5 \end{bmatrix}$ $\quad \lambda = \dfrac{0.4}{0.4+0.2} \cong 0.67$ → scelta $\boxed{\{u_3, y_1\}; \{u_2, y_2\}}$ ← interazione minore (+ vicino a 1)

migliori accopp.

2.2



2.3 • R_1 prog. su $G_{13} = \dfrac{0.8(1+2s)}{1+s}$

• R_2 prog. su $G^*_{22} = G_{22} - \dfrac{R_1 G_{12} G_{23}}{1 + R_1 G_{13}}$

③ $G(s) = \dfrac{10}{(1+10s)(1+s)}$

3.1 $L(s) = R(s)G(s) = \dfrac{0.6}{s(1+s)}$

a) $e_\infty = \lim_{s\to 0} \cancel{s} \dfrac{1}{1+L(s)} \cdot \dfrac{1}{\cancel{s}} = \lim_{s\to 0} \dfrac{s(1+s)}{s(1+s)+0.6} = 0$ ✓ (garantito dall'integratore di R(s))

b) $\omega_c \cong 0.6$ (visto che $0.6 < 1$) ✓

puls. polo

c) $\varphi_c = -90° - \text{arctg}\left(\dfrac{\omega_c}{1}\right) \simeq 121° \longrightarrow \varphi_m = 180° - 121° = 59°$ ✓

d) $t_a = \dfrac{5}{\frac{\varphi_m}{100}\omega_c} \cong 14$ ✓

caso $\varphi_m < 75°$

3.2 • Scelta T

$\underbrace{\dfrac{2\pi}{50\omega_c}}_{\simeq 0.21} \leq T \leq \underbrace{\dfrac{2\pi}{5\omega_c}}_{\simeq 2.1} \longrightarrow T = 0.5$ (mi tengo vicino al margine inferiore)

TV $\quad \alpha = \frac{1}{2}$ $\quad$ TRASF. BILINEARE $\quad s = \dfrac{1}{T} \dfrac{z-1}{\alpha z + 1 - \alpha} \overset{T,\,\alpha}{=} 4\,\dfrac{z-1}{z+1}$

$$R^*(z) = R\left(4\frac{z-1}{z+1}\right) = \frac{0.06\left(\frac{(z+1)+40z-40}{\cancel{(z+1)}}\right)}{\frac{4(z-1)}{\cancel{(z+1)}}} = \underbrace{\frac{0.06}{4}}_{0.015} \frac{(41z-39)}{z-1} = \frac{0.615z - 0.585}{z-1}$$

3.2 a) Polo integrale $z=1$ ✓

b) $\omega_c^* = \omega_c$ (vista la scelta di T piccolo)

c) $\Delta\varphi_m = \omega_c^* \dfrac{T}{2} \cdot \dfrac{180°}{\pi} \cong 8.6° \longrightarrow \varphi_m^* = \varphi_m - \Delta\varphi_m \simeq 50.4°$ (anche se $R^*(z)$ non è strett. propria e ci potrebbe essere un ritardo di mantenimento $\tau_m < T$ che peggiora φ_m^*)

d) $t_a = \dfrac{5}{\frac{\varphi_m^*}{100} \cdot \omega_c^*} = 16.53$ ✓

④ **Dati**: • $p_i = cost$

• valvola lineare $\eta(x) = x$

• $\rho, g, p_a, k, A_v, L, A, h$

INGRESSI: $\omega(t), x(t)$ USCITE: $w(t)$

4.1

• **P**: $H = \frac{p_0 - p_i}{\rho g} = \alpha \omega^2 - \beta w^2 \longrightarrow p_0 = \underset{\text{inizio condotta}}{p(0,t)} = p_i + \rho g (\alpha \omega^2 - \beta w^2)$

Hp. no accumulo nella pompa: $\underset{\text{uscita pompa}}{w_0} = w$

• **C**: • Cons. massa: $\underset{\text{a fine condotta}}{w_c} = w_0 = w$

• Cons. Q. moto: $\dot{w} = -\frac{\rho A g}{L}\left(\underbrace{z^*(L,t)}_{h + \frac{p_v}{\rho g}} - \underbrace{z^*(0,t)}_{\frac{p_0}{\rho g} = \frac{p_i}{\rho g} + \alpha\omega^2 - \beta w^2}\right) - \underset{\text{coeff. d'attrito}}{\bar{f}} w^2$

?

• **V**: Hp. no accumulo: $\underset{\text{uscita valvola}}{w_{ov}} = w_c = w$

• Modello: $w = K A_v x \sqrt{\rho(p_v - p_a)} \longrightarrow p_v = \frac{w^2}{k^2 A_v^2 x^2 \rho} + p_a$

$$\Rightarrow \dot{w} = -\frac{\rho A g}{L}\left(h + \frac{w^2}{k^2 A_v^2 x^2 \rho^2 g} + \frac{p_a}{\rho g} - \frac{p_i}{\rho g} - \alpha \omega^2 + \beta w^2\right) - \bar{f} w^2 =$$

$$= -\left(\underbrace{\frac{A}{A_v^2 k^2 x^2 \rho L}}_{\varphi(x(t))} \underbrace{+ \beta + \bar{f}}_{c_1}\right) w^2 + \underbrace{\frac{\alpha \rho A g}{L}}_{c_2} \omega^2 \underbrace{- \frac{\rho A g h}{L} - \frac{p_a A}{L} + \frac{p_i A}{L}}_{c_3}$$

4.2. EQUILIBRIO

$$0 = -(\varphi(\bar{x}(t)) + c_1)\bar{w}^2 + c_2 \bar{\omega}^2 + c_3$$

$$\bar{w} = \sqrt{\frac{c_2\bar{\omega}^2+c_3}{\varphi(\bar{x}(t))+c_1}}$$

MODELLO LINEARIZZATO

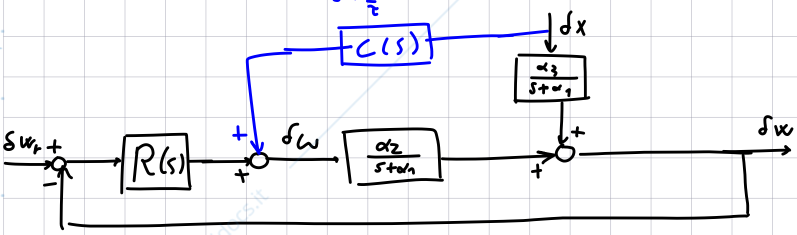
$$\delta\dot{w} = \underbrace{-2(\varphi(\bar{x}(t))+c_1)\bar{w}}_{\alpha_1}\,\delta w \underbrace{+2c_2\bar{\omega}}_{\alpha_2}\,\delta\omega \underbrace{- \bar{w}^2 \frac{d\varphi(\bar{x}(t))}{d\bar{x}}}_{\alpha_3}\,\delta x$$

4.3.

$\mathcal{L}$ $\hookrightarrow$ $sW(s) = -\alpha_1 W(s) + \alpha_2 \Omega(s) + \alpha_3 X(s)$

$$W(s) = \underbrace{\frac{1}{s+\alpha_1}}_{\frac{1}{s+\frac{1}{\tau}}}\left(\alpha_2 \Omega(s) + \alpha_3 X(s)\right)$$

con $\tau = \frac{1}{\alpha_1}$



$$C(s) = -\frac{\alpha_3}{\alpha_2}$$

STATICO
SENNO' NON REALIZZ.

05/07/16

① **1.1** $L(s) = \dfrac{K}{\tau^2 (s+\frac{1}{\tau})^2 s}$ — ρ

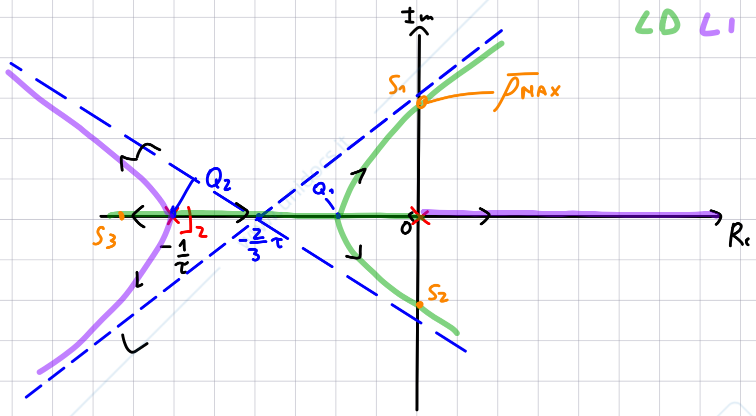
$n=3$ **Baricentri**

$m=0$ $X_A = X_B = \frac{1}{3}\left(-\frac{1}{\tau} - \frac{1}{\tau} - 0\right) = -\frac{2}{3\tau}$

$V=3$ si conserva

Asintoti LD $\{60°, 180°, 300°\}$

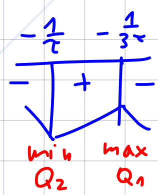
LI $\{0°, 120°, 240°\}$



$Q_2 = -\frac{1}{\tau}$

Q_1 max su LD di $\gamma(x) = -(s+\frac{1}{\tau})^2 \cdot s = -\left(s^3 + \frac{2}{\tau}s^2 + \frac{1}{\tau^2}s\right)$

↳ $\gamma'(x) = -\left(3s^2 + \frac{4}{\tau}s + \frac{1}{\tau^2}\right) \geq 0$ $\quad s_{1,2} = \dfrac{-\frac{2}{\tau} \pm \sqrt{\frac{4}{\tau^2} - \frac{3}{\tau^2}}}{3} =$ $-\frac{1}{\tau} \rightarrow Q_2$; $-\frac{1}{3\tau} \rightarrow Q_1$



1.2 - Trovo S_3 dalla cons. del baricentro:

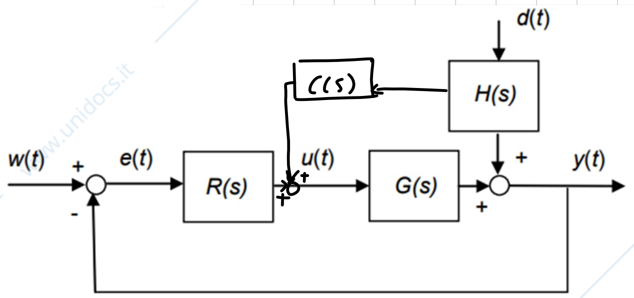
$\frac{1}{\cancel{3}}(\cancel{s_1} + \cancel{s_2} + s_3) = -\frac{2}{\cancel{3}\tau} \rightarrow s_3 = -\frac{2}{\tau}$

- Regola punt. gg.: $\rho_{MAX} = \dfrac{\left(\frac{1}{\tau}\right)^2 \cdot \frac{2}{\tau}}{1} = \frac{2}{\tau^3}$ $\xrightarrow{\rho_{MAX} = \frac{K_{MAX}}{\tau^2}}$ $K_{MAX} = \frac{2}{\tau}$ AS STAB. $\iff 0 \leq K < \frac{2}{\tau}$

1.3 Polo dominante A.A $s = -\frac{1}{\tau}$

Polo dom. con minore cost. di temp. è in $Q_1 = -\frac{1}{3\tau}$ $\Big\}$ $3\tau > \tau$ ↳ A.C. 3 volte + lento

1.4



• IDEALE

$$C^{\circ}(s) = -\frac{H(s)}{G(s)} = +5(1+s\tau)$$ NON REALIZZ.!

↳ STATICO $C(s) = -\frac{\mu_H}{\mu_G} = 5$

2 2.1 $R^*(z) = \frac{\beta}{z}$

2.2 TRASF. BILINEARE $s = \frac{1}{T} \cdot \frac{z-1}{\alpha z + 1 - \alpha} \xrightarrow{EA \; \alpha=0, \; T=1} s = z-1 \rightarrow z = s+1 \rightarrow R^{\circ}(s) = R^*(s+1) = \frac{\beta}{s+1}$ REALIZZ. ✓ (polinop.) ✓

2.3. • Verifica su scelta T con ω_c desiderato

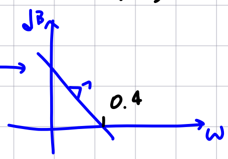
$$\underbrace{\frac{2\pi}{50\omega_c}}_{0.31} \leq T \leq \underbrace{\frac{2\pi}{5\omega_c}}_{3.1}$$ $T = 1$ ✓

↳ $\omega_c^* \simeq \omega_c = 0.4$

sist. discreto

Sist. analogico Considero $\hat{R}(s) \overset{\text{approx.}}{\cong} e^{-s\frac{T}{2}} R^*(e^{sT}) = e^{-s\frac{T}{2}} \frac{\beta}{e^{sT}} = \beta e^{-\frac{3}{2}Ts}$

$L(s) \cong \tilde{R}(s)\, G(s) = \frac{2\beta}{s} e^{-1.6s} \rightarrow$



$\Rightarrow \omega_c = 2\beta \rightarrow \beta = 0.2$

2.4.

- $\varphi_c = -90° - \omega_c \cdot 1.6 \cdot \frac{180°}{\pi} \cong -126.67° \longrightarrow \varphi_m \cong 53.3°$

- $c(\infty) = \lim_{s \to 0} \cancel{s} \frac{s}{s + 0.4e^{-1.6s}} \cdot \frac{1}{\cancel{s}} = 0$

$\Delta\varphi_m \cong 12.6°$

2.5

$$R^\circ(s) = \frac{\beta}{s+1} \overset{\beta = 0.2}{=} \frac{0.2}{s+1}$$

(ω_c rimane invariata)
$\omega_c = 0.4$

$$L(s) = R^\circ(s)\,G(s) = \frac{0.4\,e^{-0.1s}}{s(s+1)}$$

- $\varphi_c = -90° - \arctan\left(\frac{\omega_c}{1}\right) - \omega_c \cdot 0.1 \cdot \frac{180°}{\pi} \cong -114.1° \longrightarrow \varphi_m \cong 65.9°$
- $c(\infty) = \lim_{s \to 0} \cancel{s} \frac{s(s+1)}{s(s+1) + 0.4e^{-0.1s}} \cdot \frac{1}{\cancel{s}} = 0$ (c'è sempre az. integrale)