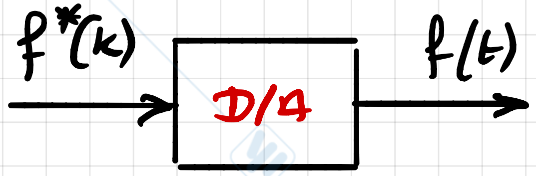
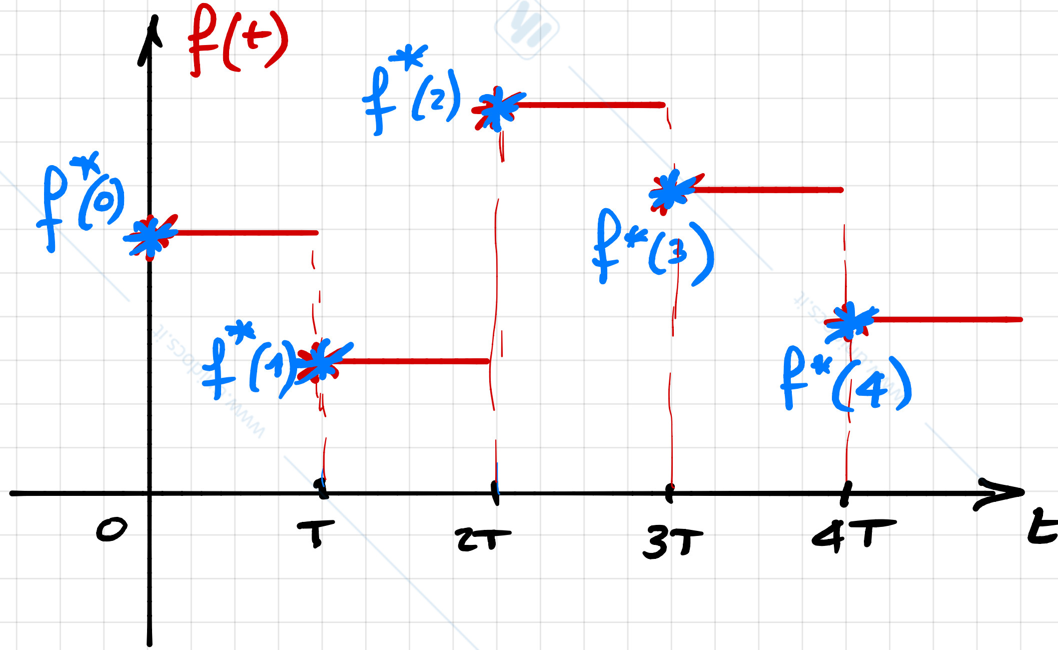
ANALISI IN FREQUENZA DEL MANTENITORE

- ANALISI IN FREQUENZA DELLO ZOH





$$f(t) = f^*(k), \quad kT \leq t < (k+1)T$$

T PERIODO DI MANTENIMENTO

$\omega_H = \frac{2\pi}{T}$ PULS. DI MANTENIMENTO

- DETERMINARE I LEGAMI TRA

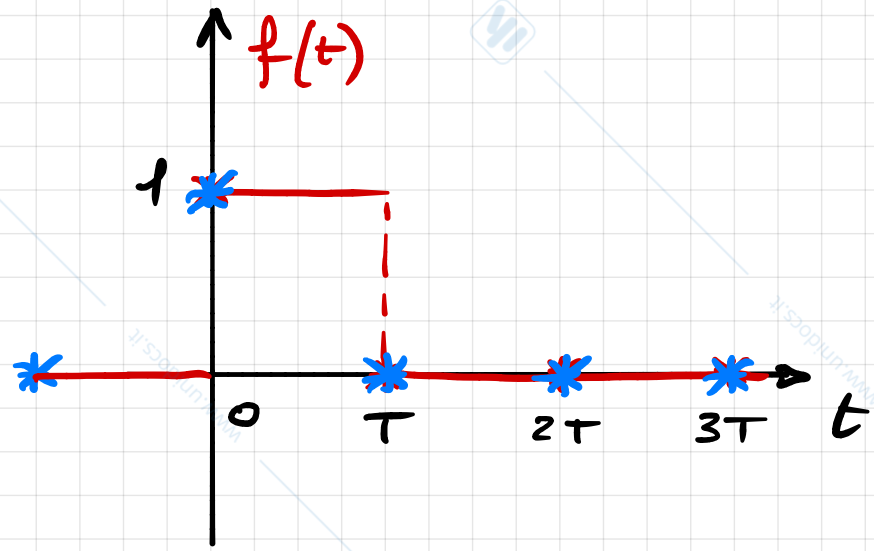
$$F^*(e^{j\theta}) = \mathcal{F}^*[f^*(k)] \quad \text{E} \quad F(j\omega) = \mathcal{F}[f(t)]$$

$$F^*(z) = \mathcal{Z}[f^*(k)] \quad \text{E} \quad F(s) = \mathcal{L}[f(t)]$$

- Risposta all'impulso

$$f^*(k) = imp^*(k) = \begin{cases} 0 & k \neq 0 \\ 1 & k = 0 \end{cases}$$

$$F^*(z) = 1$$

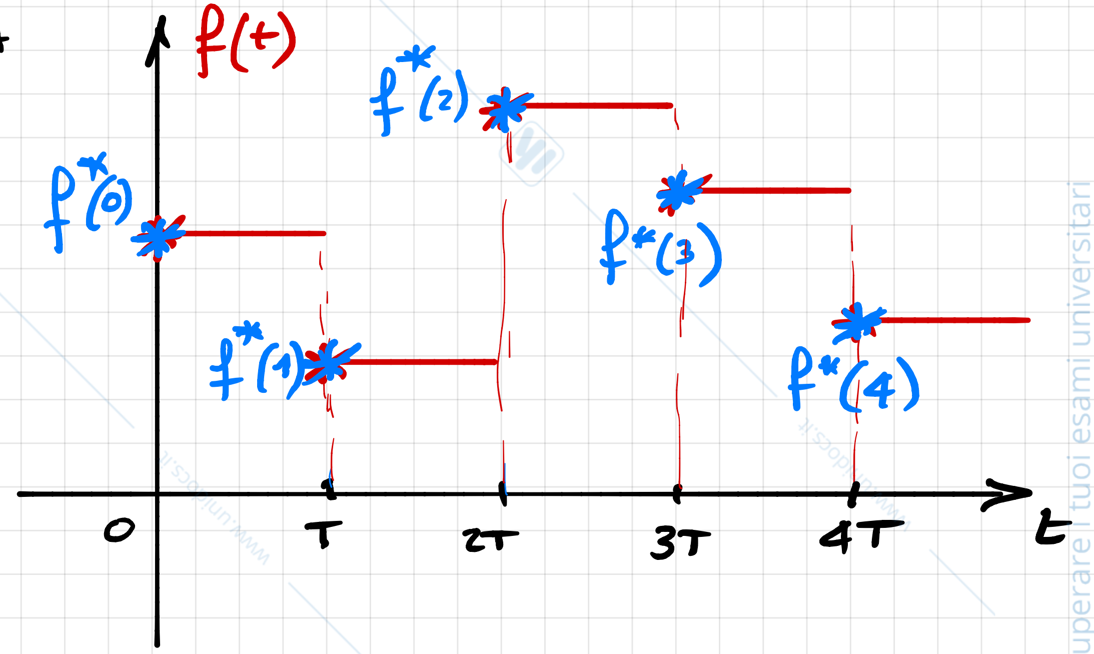


$$f(t) = sca(t) - sca(t-T) = h_0(t)$$

$$F(s) = \frac{1}{s} - \frac{e^{-sT}}{s} = \frac{1-e^{-sT}}{s} = H_0(s)$$

- RISPOSTA A $f^*(k)$ GENERICA

$f^*(k)$



$$f(t) = f^*(0)h_0(t) + f^*(1)h_0(t-T) + f^*(2)h_0(t-2T) + \dots = \sum_{k=0}^{\infty} f^*(k)h_0(t-kT)$$

$$\boxed{F(s)} = \sum_{k=0}^{\infty} f^*(k)\,\mathcal{L}[h_0(t-kT)] = \sum_{k=0}^{\infty} f^*(k)e^{-sTk}H_0(s) = \boxed{H_0(s)F^*(e^{sT})}$$

"FDT" DELLO ZOH

- CON $s = j\omega \implies \boxed{F(j\omega) = H_0(j\omega)F^*(e^{j\omega T})}$

"R.I.F." DELLO ZOH

- STUDIO DI $H_0(j\omega)$

$$\boxed{H_0(j\omega)} = \frac{1-e^{-j\omega T}}{j\omega} = e^{-j\omega T/2}\underbrace{\left(\frac{e^{j\omega T/2}-e^{-j\omega T/2}}{2j}\right)}_{\text{sen}\frac{\omega T}{2}}\frac{2}{\omega} =$$

$$= \boxed{T e^{-j\omega T/2}\underbrace{\frac{\text{sen}\,\omega T/2}{\omega T/2}}_{\text{sinc}\frac{\omega T}{2}}}$$

$$|H_0(j\omega)| = T\left|\frac{\text{sen}\,\omega T/2}{\omega T/2}\right|$$

$$\measuredangle H_0(j\omega) = -\frac{\omega T}{2} + \underbrace{\measuredangle\left(\frac{\text{sen}\,\omega T/2}{\omega T/2}\right)}_{= \begin{cases} 0, & \text{sen}\frac{\omega T}{2} > 0 \\ \pm\pi, & \text{sen}\frac{\omega T}{2} < 0 \end{cases}}$$

(in radianti)

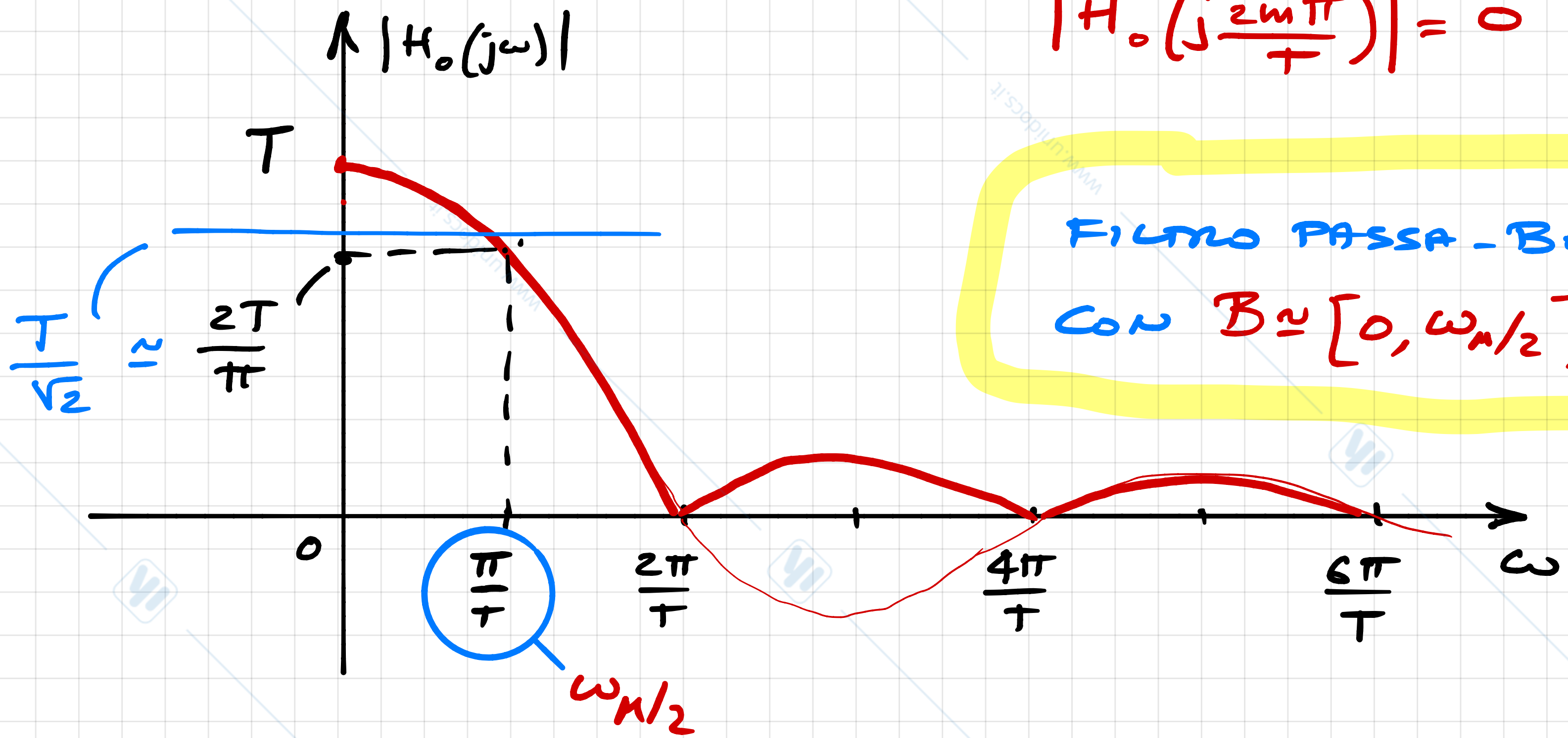
- STUDIO DI $|H_0(j\omega)|$

$$|H_0(j\omega)| = T \left| \frac{\text{sen}\, \omega T/2}{\omega T/2} \right|$$

$$|H_0(j0)| = T$$

$$\left|H_0\left(j\frac{\pi}{T}\right)\right| = \frac{2T}{\pi}$$

$$\left|H_0\left(j\frac{2m\pi}{T}\right)\right| = 0$$



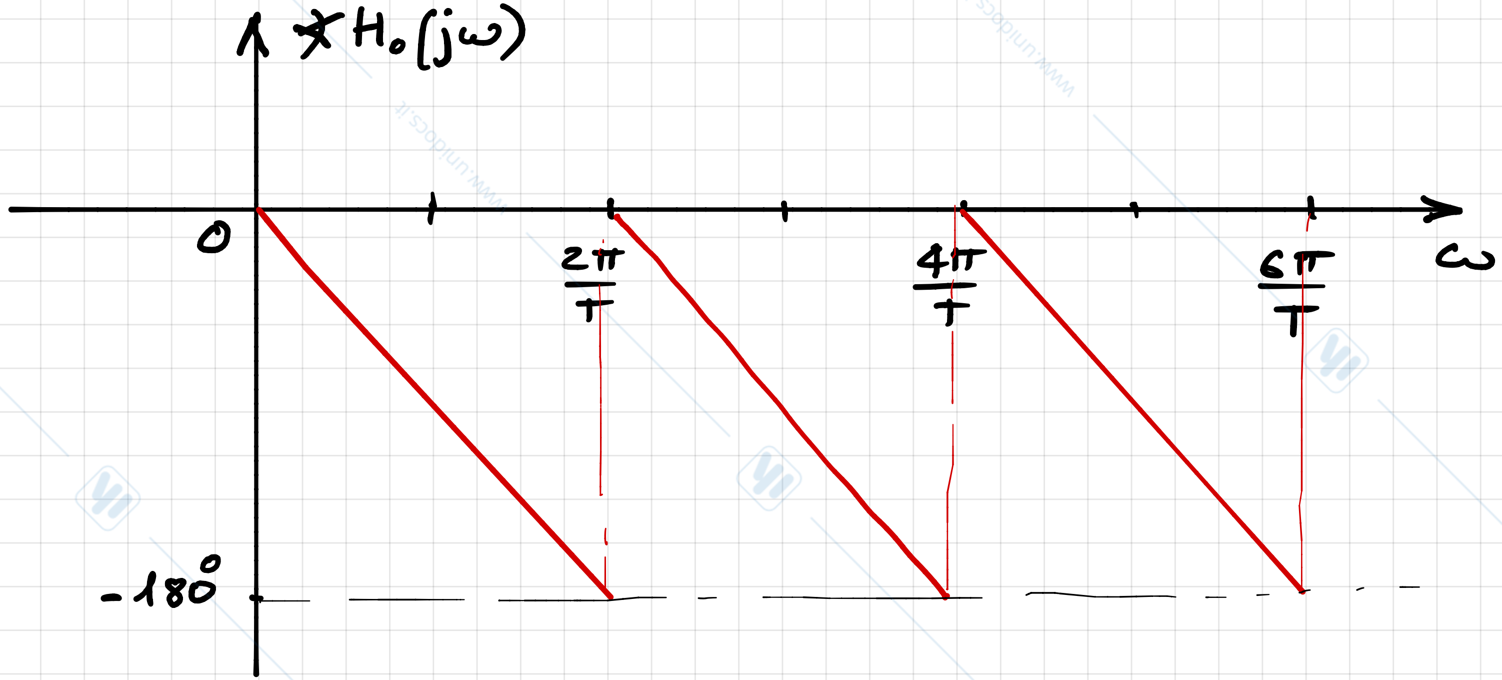
FILTRO PASSA-BASSO

CON $B \simeq [0, \omega_M/2]$

– STUDIO DI $\measuredangle H_0(j\omega)$

(IN GRADI)

$$\measuredangle H_0(j\omega) = -\frac{\omega T}{2}\frac{180^\circ}{\pi} + \underbrace{\measuredangle\left(\frac{\operatorname{sen}\omega T/2}{\omega T/2}\right)}_{=\begin{cases} 0^\circ, & \operatorname{sen}\frac{\omega T}{2} > 0 \\ -180^\circ, & \operatorname{sen}\frac{\omega T}{2} < 0 \end{cases}}$$



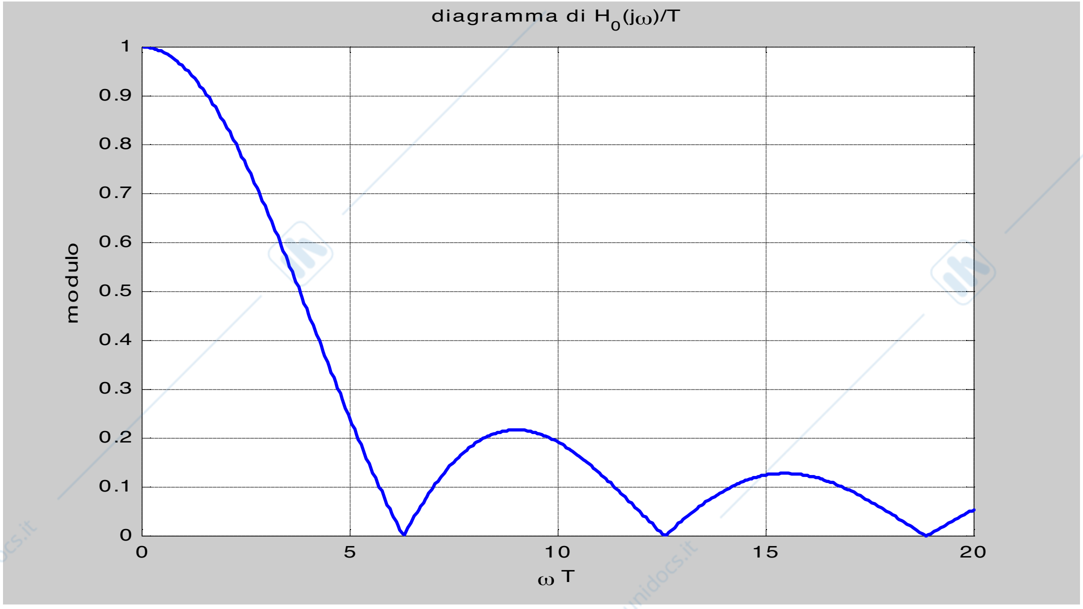


diagramma di $H_0(j\omega)/T$
modulo
ω T
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20

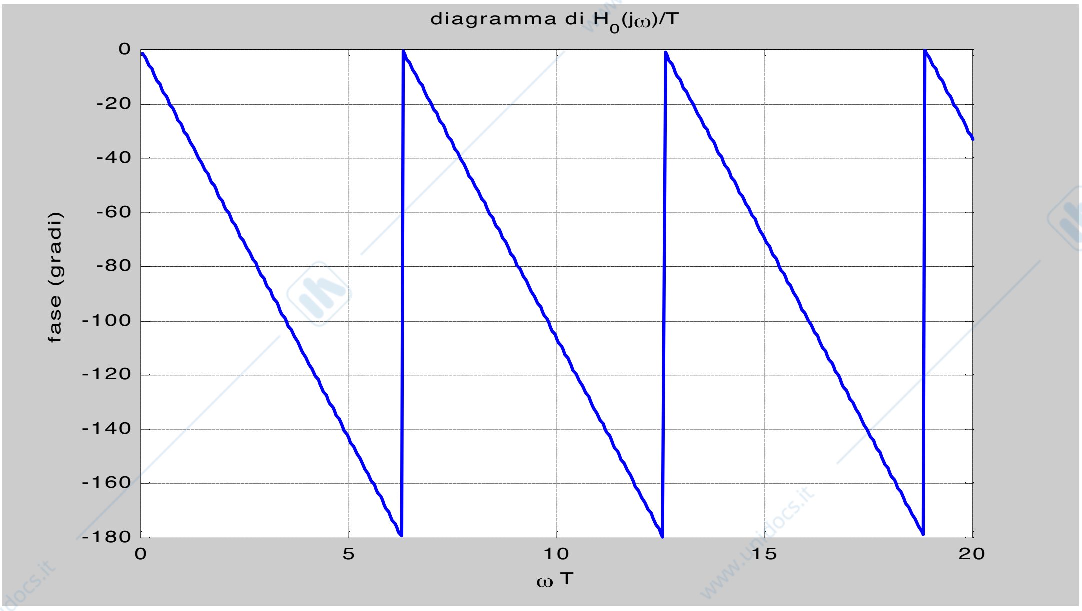


diagramma di $H_0(j\omega)/T$
fase (gradi)
ω T
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
0
5
10
15
20

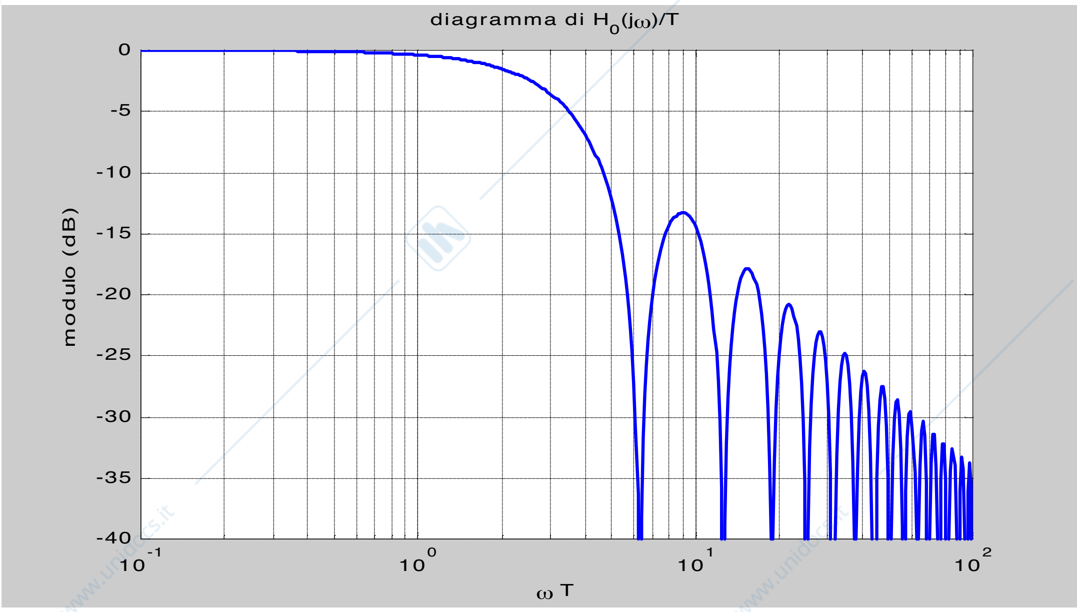


diagramma di $H_0(j\omega)/T$
modulo (dB)
ω T
0
-5
-10
-15
-20
-25
-30
-35
-40
10^{-1}
10^{0}
10^{1}
10^{2}

- APPROSSIMAZIONE DELLO ZOH

- IN $\left[0, \frac{\omega_m}{2}\right]$: $\begin{cases} |H_0(j\omega)| \simeq T \\ \measuredangle H_0(j\omega) = -\omega \frac{T}{2} \end{cases}$

$$\Longrightarrow \boxed{H_0(s) \simeq T e^{-sT/2}}$$

- ANALISI IN FREQUENZA DEL CONTROLLORE $R^*(z)$

$$e^*(k) \longrightarrow \boxed{R^*(z)} \longrightarrow u^*(k)$$

$$U^*(z) = R^*(z)\,E^*(z)$$

$$E^*(e^{j\vartheta}) = \mathcal{F}^*[e^*(k)]$$

$$U^*(e^{j\vartheta}) = \mathcal{F}^*[u^*(k)]$$

$$U^*(e^{j\vartheta}) = R^*(e^{j\vartheta})\,E^*(e^{j\vartheta})$$

RISPOSTA IN FREQUENZA

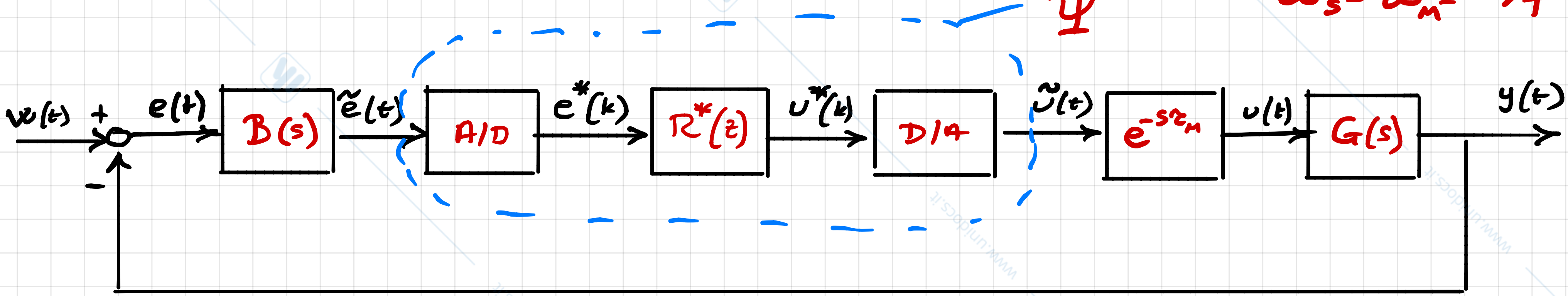
(FUNZIONE SIMMETRICA E PERIODICA, PERIODO 2π)

ANALISI A TEMPO CONTINUO DEI SISTEMI DI CONTROLLO DIGITALE

- SISTEMI DI CONTROLLO DIGITALE - ANALISI A T. CONTINUO

$T_s = T_M = T$

$\omega_s = \omega_M = 2\pi/T$



- È POSSIBILE CALCOLARE LA "R.I.F." $\tilde{R}(j\omega)$ DI Ψ ?

- IN CASO AFFERMATIVO CI SI RICONDUCE A UN SISTEMA RETROAZIONATO A T. CONTINUO CON

$$L(s) = B(s)\tilde{R}(s)e^{-s\tau_M}G(s)$$

$\Longrightarrow$ STRUMENTI CLASSICI DI ANALISI DI STABILITÀ, PRESTAZIONI, ...

- STUDIO DI Ψ

$$\tilde{e}(t) \longrightarrow \boxed{A/D} \xrightarrow{e^*(k)} \boxed{R^*(z)} \xrightarrow{u^*(k)} \boxed{D/A} \longrightarrow \tilde{u}(t)$$

Ψ

SPETTRI $\quad E(j\omega) \quad E^*(e^{j\theta}) \quad U^*(e^{j\theta}) \quad U(j\omega)$

- CAMPIONATORE $\quad E^*(e^{j\omega T}) = \frac{1}{T} E_s(j\omega) \; , \; E_s(j\omega) = \sum_{h=-\infty}^{+\infty} E(j(\omega + h\omega_s))$

- CONTROLLORE $\quad U^*(e^{j\theta}) = R^*(e^{j\theta}) E^*(e^{j\theta})$

- MANTENITORE $\quad U(j\omega) = H_0(j\omega) U^*(e^{j\omega T})$

$$\Longrightarrow \quad U(j\omega) = \frac{H_0(j\omega)}{T} R^*(e^{j\omega T}) E_s(j\omega)$$

- APPROSSIMAZIONE

$$U(j\omega) = \frac{H_0(j\omega)}{T} R^*(e^{j\omega T}) E_s(j\omega)$$

- SE $\tilde{e}(t)$ È A BANDA LIMITATA CON $\omega_{MAX} < \frac{\omega_s}{2}$:

- $E_s(j\omega) = E(j\omega) \quad , \quad \omega \in [0, \omega_s/2]$ PER ASSENZA DI ALIASING
- $\frac{H_0(j\omega)}{T} E_s(j\omega) \simeq 0 \quad , \quad \omega > \omega_s/2$ PER L'AZIONE FILTRANTE DI $H_0(j\omega)$

$$\Longrightarrow \quad U(j\omega) \simeq \underbrace{\frac{H_0(j\omega)}{T} R^*(e^{j\omega T})}_{\tilde{R}(j\omega)} E(j\omega) \quad , \forall \omega$$

- IN TERMINI DI FDT:

$$U(s) = \tilde{R}(s) E(s) \quad , \quad \boxed{\tilde{R}(s) = \frac{H_0(s)}{T} R^*(e^{sT}) = \frac{1-e^{-sT}}{sT} R^*(e^{sT})}$$

- ULTERIORE APPROSSIMAZIONE

$$\tilde{R}(s) = \frac{H_0(s)}{T} R^*(e^{sT})$$

- IN $[0, \omega_s/2]$ RISULTA:

$$H_0(s) \simeq T e^{-sT/2}$$

$$\Longrightarrow \boxed{\tilde{R}(s) \simeq e^{-sT/2} R^*(e^{sT})}$$

RITARDO DI DISCRETIZZAZIONE