ESERCITAZIONE SULLA

DISCRETIZZAZIONE DI UN REGOLATORE ANALOGICO

- Esercizio 1 - Discretizzazione di un regolatore PI

$$R^{o}(s) = K_p\left(1 + \frac{1}{sT_i}\right)$$

- Metodo TU

$$R^{*}(z) = K_p\left(1 + \frac{T}{2T_i}\,\frac{z+1}{z-1}\right) = \frac{K_p}{2T_i}\cdot\frac{2T_i(z-1)+T(z+1)}{z-1} =$$

$$= \frac{K_p}{2T_i}\,\frac{(T+2T_i)z + (T-2T_i)}{z-1} = \frac{\beta_0 z + \beta_1}{z-1}$$

$$\begin{cases} \beta_0 = \dfrac{K_p}{2T_i}(T+2T_i) \\ \beta_1 = \dfrac{K_p}{2T_i}(T-2T_i) \end{cases}$$

- Algoritmo di controllo

$$u^{*}(k) = u^{*}(k-1) + \beta_0 e^{*}(k) + \beta_1 e^{*}(k-1)$$

- Codice software

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INPUT e
u = uold + b0 * e + b1 * eold
uold = u
eold = e
OUTPUT u
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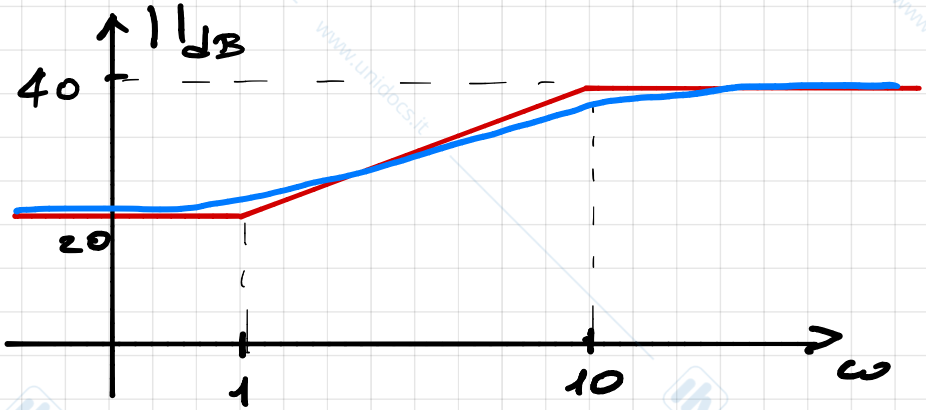
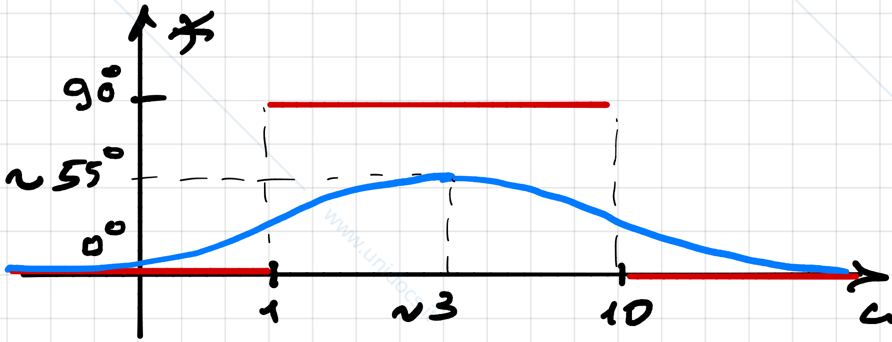
- ESERCIZIO 2 - DISCRETIZZAZIONE DI UNA RETE ANTICIPATRICE

$$R^{o}(s) = 10 \, \frac{1+s}{1+0.1s}$$

- SUPPONIAMO $\omega_c = 3$





1. SCELTA DI T
2. DISCRETIZZAZIONE CON EA, EI, TU
3. CONFRONTO

$R^{o}(j\omega)$ vs. $R^{*}(e^{j\omega T})$ (CON MATLAB)

1. SCELTA DI T

$$5\omega_c < \omega_s < 50\omega_c \qquad \omega_s = \frac{2\pi}{T} \qquad \omega_c = 3$$

$$\frac{2\pi}{50\omega_c} < T < \frac{2\pi}{5\omega_c}$$

$$\boxed{0.042 < T < 0.42}$$

- PROVIAMO CON: (A) $T = 0.1$

(B) $T = 0.5$ (!?)

- Trasformazione Bilineare

$$R^*(z) = R^0\left(\frac{1}{T}\frac{z-1}{\alpha z+1-\alpha}\right) = 10\,\frac{1+\frac{1}{T}\frac{z-1}{\alpha z+1-\alpha}}{1+\frac{0.1}{T}\frac{z-1}{\alpha z+1-\alpha}} =$$

$$= 10\,\frac{\alpha z+1-\alpha+\frac{1}{T}(z-1)}{\alpha z+1-\alpha+\frac{0.1}{T}(z-1)} =$$

$$= 10\,\frac{\left(\alpha+\frac{1}{T}\right)z+1-\alpha-\frac{1}{T}}{\left(\alpha+\frac{0.1}{T}\right)z+1-\alpha-\frac{0.1}{T}}$$

- Caso A $\quad T=0.1 \qquad \omega_s/2 = \pi/T = 10\pi \simeq 31.4$

$$\boxed{R^*(z) = 10\,\frac{(\alpha+10)z-\alpha-9}{(\alpha+1)z-\alpha}}$$

- CASO (A) $T = 0.1$

$\omega_s/2 = \pi/T = 10\pi \simeq 31.4$

$$R^*(z) = 10\,\frac{(\alpha+10)z-\alpha-9}{(\alpha+1)z-\alpha}$$

EA, $\alpha = 0$

$$R^*(z) = 10\,\frac{10z-9}{z}$$

EI, $\alpha = 1$

$$R^*(z) = 10\,\frac{11z-10}{2z-1}$$

TU, $\alpha = 1/2$

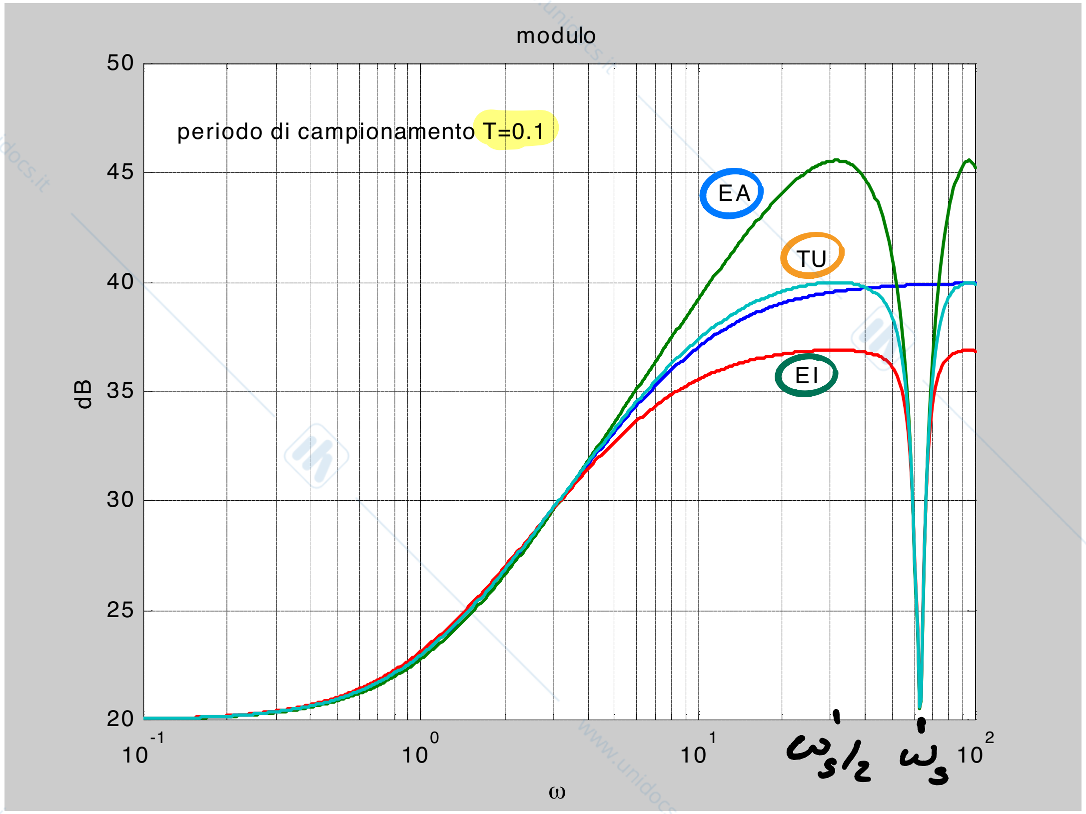
$$R^*(z) = 10\,\frac{10.5z-9.5}{1.5z-0.5}$$

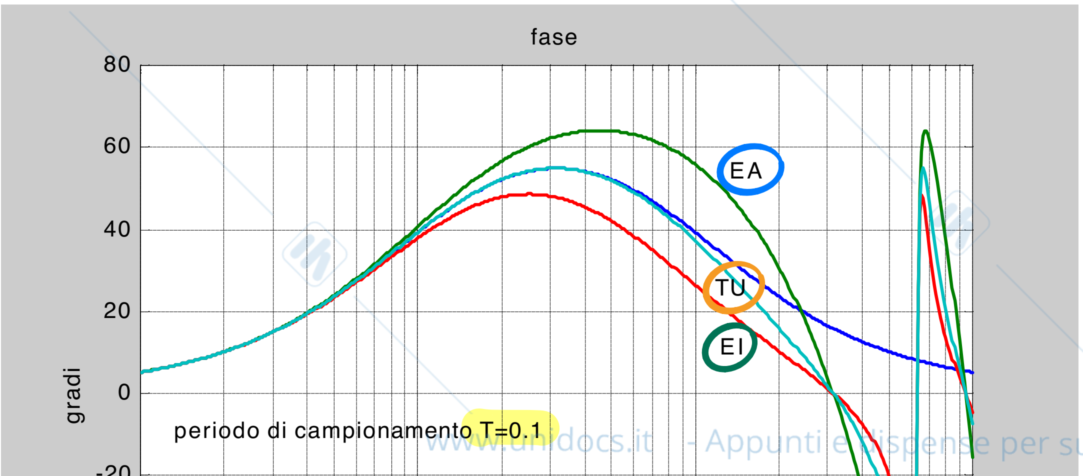
GUADAGNO STATICO

$$\mu^* = R^*(1) = 10 = R^\circ(0)$$

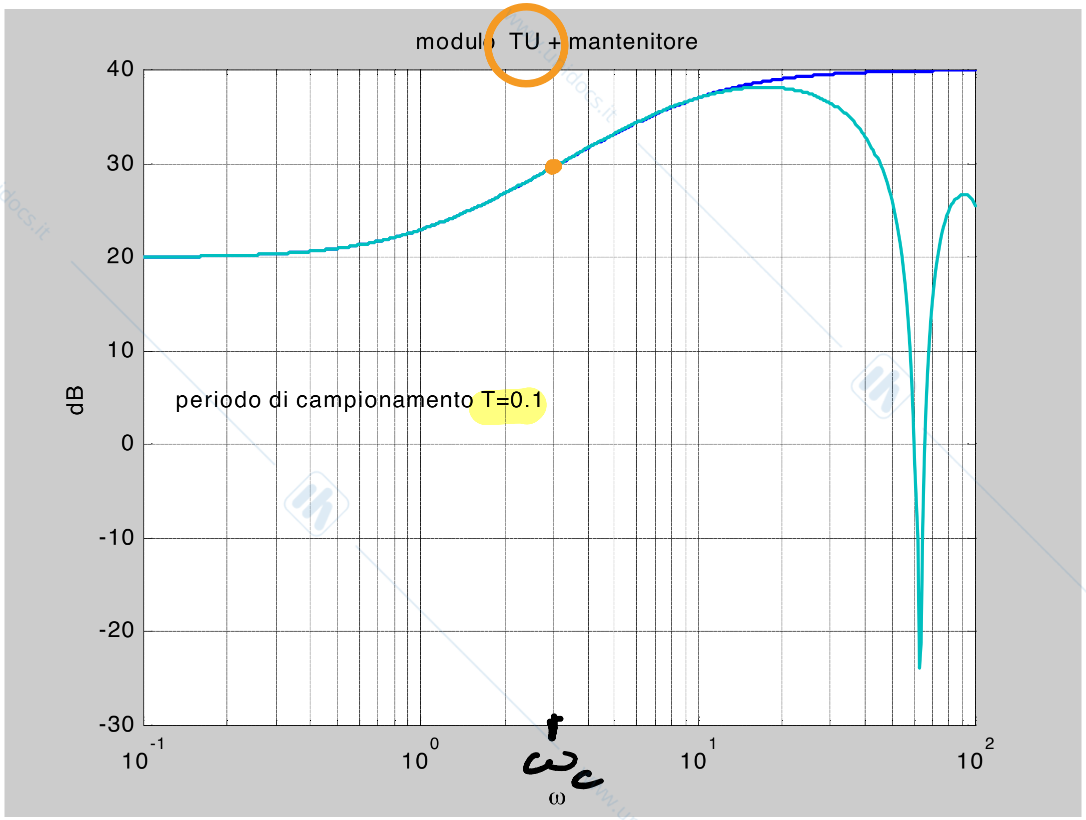
	EA	EI	TU
POLO	0	0.5	0.333
ZERO	0.9	0.909	0.905

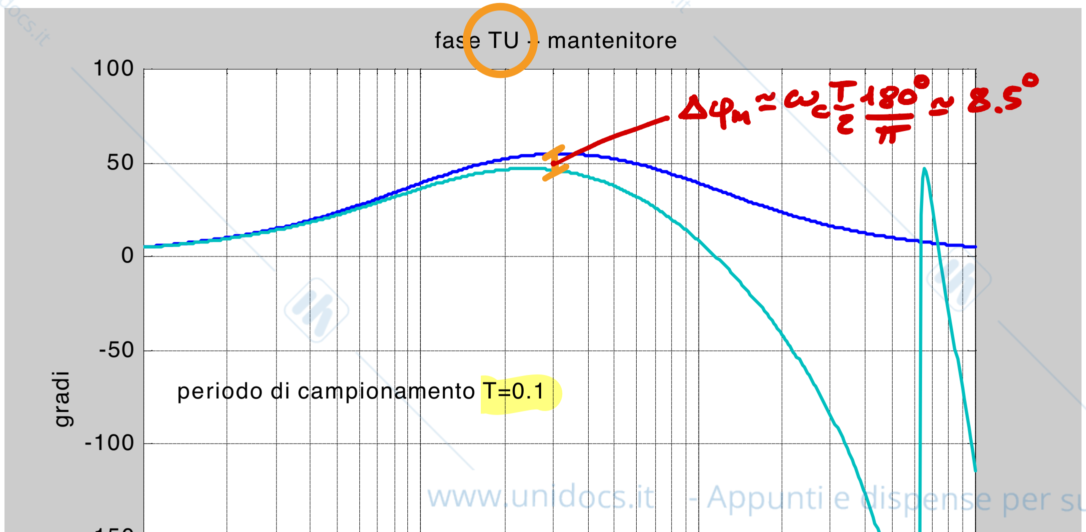
Confronto $R^{o}(j\omega)$ vs. $R^{*}(e^{j\omega T})$





Confronto $R^o(j\omega)$ vs. $R^*(e^{j\omega T})\,\frac{H_0(j\omega)}{T}$





- CASO (B) $T = 0.5$

$$\omega_s/2 = \pi/T = 2\pi \simeq 6.28$$

$$R^*(z) = 10 \, \frac{(\alpha+2)z - \alpha - 1}{(\alpha+0.2)z - \alpha + 0.8}$$

EA

$\alpha = 0$

$$R^*(z) = 10 \, \frac{2z-1}{0.2z+0.8}$$

EI

$\alpha = 1$

$$R^*(z) = 10 \, \frac{3z-2}{1.2z-0.2}$$

TU

$\alpha = 1/2$

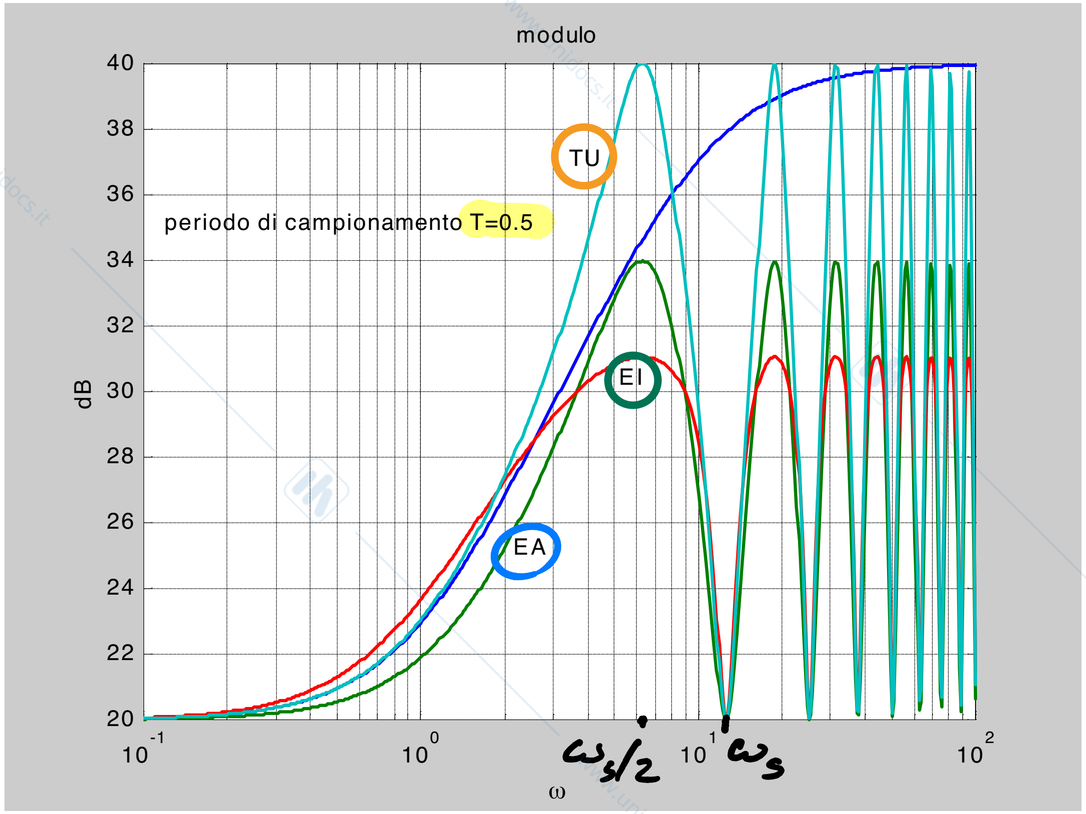
$$R^*(z) = 10 \, \frac{2.5z-1.5}{0.7z-0.3}$$

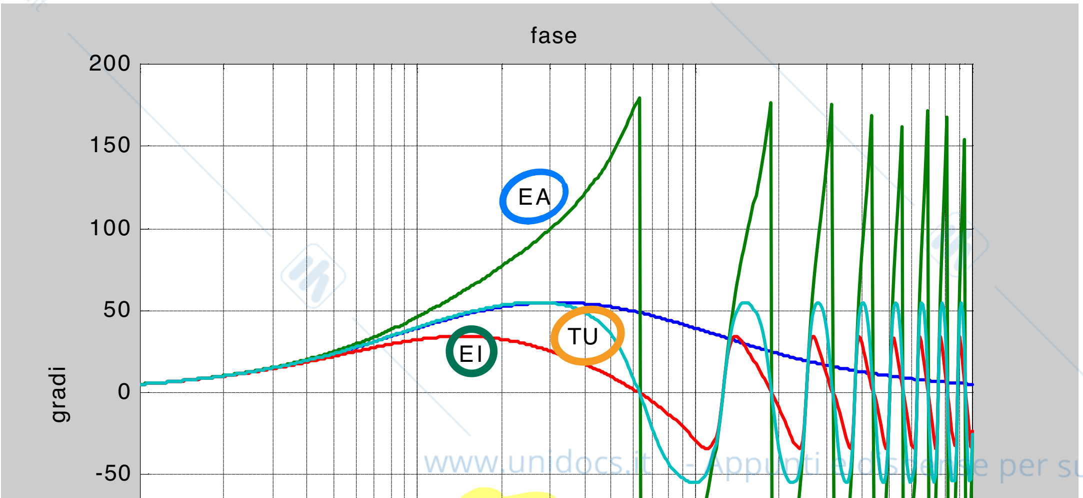
GUADAGNO STATICO

$$\mu^* = R^*(1) = 10 = R^\circ(0)$$

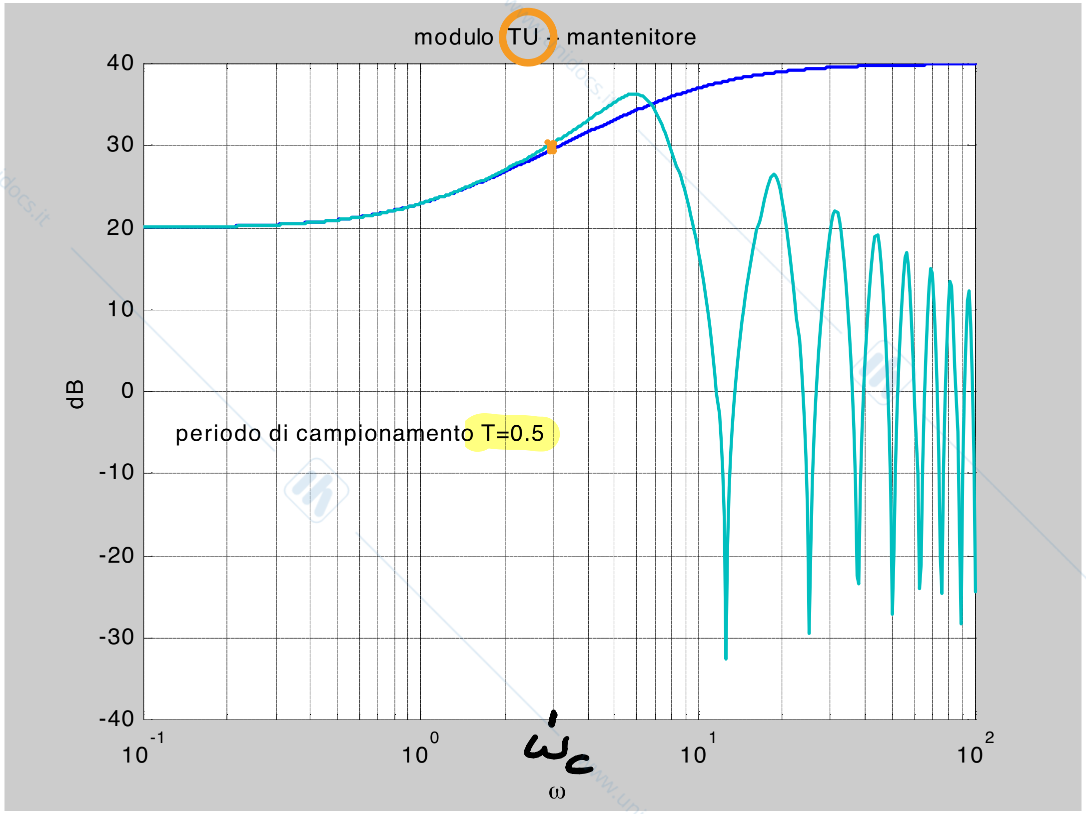
	EA	EI	TU
POLO	-4	0.167	-0.429
ZERO	0.5	0.667	0.6

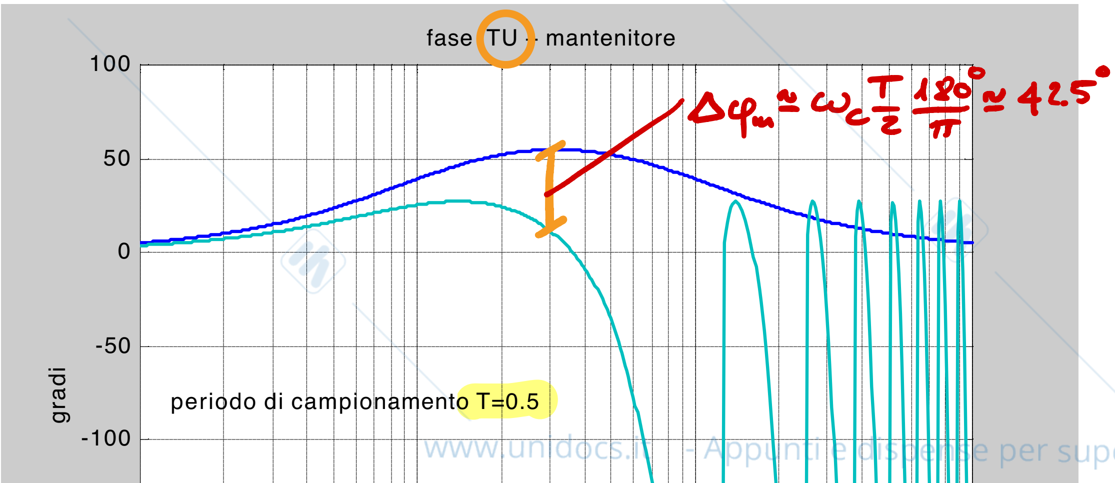
Confronto $R^{\circ}(j\omega)$ vs. $R^{*}(e^{j\omega T})$



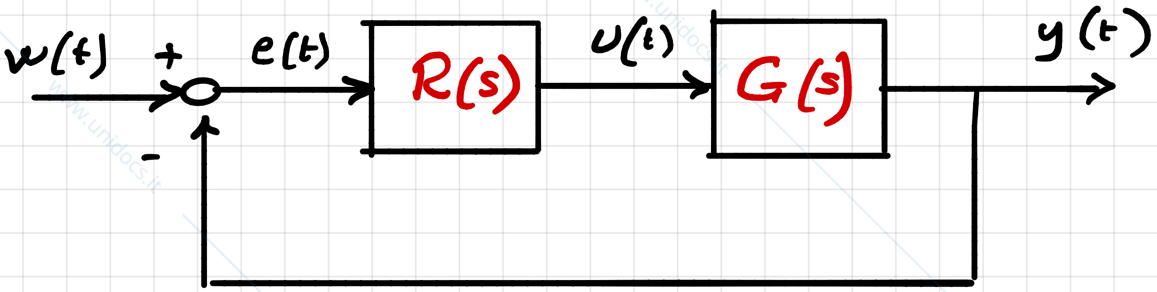


Confronto $R^0(j\omega)$ vs. $R^*(e^{j\omega T})\dfrac{H_0(j\omega)}{T}$





- ESERCIZIO 3



$$G(s) = 50$$

$$R(s) = \frac{\mu}{s}$$

1. SCEGLIERE μ IN MODO DA AVERE INSEGUIMENTO PRECISO DI $w(t) = \text{sen}(10t)$

2. PER UN SISTEMA DI CONTROLLO DIGITALE SCEGLIERE T

3. CON IL METODO DI TUSTIN RICAVARE $R^*(z)$ + ALGORITMO DI C.

4. DIFFERENZE DI PRESTAZIONI TRA $R(s)$ E $R^*(z)$ IN RISPOSTA A $w(t) = \text{sca}(t)$

5. VALUTARE EFFETTI DI: . TEMPO DI ELABORAZIONE Δ
   - FILTRO ANTI-ALIASING $B(s)$

1 - OCCORRE $\omega_c > 10$

$$L(s) = R(s)\,G(s) = \frac{50\mu}{s} \Longrightarrow \omega_c = 50\mu$$

$$50\mu > 10 \Longrightarrow \mu > 0.2 \qquad \text{p.e. } \boxed{\mu = 0.4}$$

$$L(s) = \frac{20}{s} \; - \begin{cases} \omega_c = 20 \\ \varphi_m = 90° \end{cases}$$

2 - CONVIENE SCEGLIERE:

$$\frac{2\pi}{50\omega_c} < T < \frac{2\pi}{5\omega_c}, \quad \omega_c = 20 \Longrightarrow 0.006 < T < 0.06$$

$$\text{p.e. } \boxed{T = 0.01}$$

$$\frac{\omega_s}{2} \simeq 314$$

③ $R^{\circ}(s) = \frac{\mu}{s}$

- METODO DI TUSTIN $\boxed{R^{*}(z) = R^{\circ}\left(\frac{2}{T}\frac{z-1}{z+1}\right) = \frac{\mu T}{2}\frac{z+1}{z-1}}$

p.e. $\mu = 0.4$, $T = 0.01$ $\Longrightarrow R^{*}(z) = 2 \cdot 10^{-3}\frac{z+1}{z-1}$

- ALGORITMO DI CONTROLLO

$$\boxed{u^{*}(k) = u^{*}(k-1) + 2 \cdot 10^{-3}\left(e^{*}(k) + e^{*}(k-1)\right)}$$

④ - PRESTAZIONI STATICHE IDENTICHE $e(\infty) = 0$

- PRESTAZIONI DINAMICHE:

$\omega_c \simeq \omega_c^{*} = 20$ VISTO CHE $\omega_c \ll \omega_s/2$

$$\begin{cases} \varphi_m = 90^{\circ} \\ \varphi_m^{*} \simeq 90^{\circ} - \omega_c \frac{T}{2}\frac{180^{\circ}}{\pi} = 90^{\circ} - \overbrace{0.1\,\frac{180^{\circ}}{\pi}}^{\sim 6^{\circ}} \simeq 84^{\circ} \end{cases}$$

$\Longrightarrow$ PRESTAZIONI QUASI IDENTICHE

5

_ TEMPO DI ELABORAZIONE $\Delta \leq T$

_ DATO CHE $R^*(z)$ NON È STR. PROPRIA, INTRODUCE UN OFFSET DI MANTENIMENTO $\tau_M = \Delta$ E QUINDI UN RITARDO $e^{-s\tau_M}$

CASO PEGGIORE $\Delta = T$

$0.2 \frac{180°}{\pi} \simeq 11°$

$\Longrightarrow$ ω_c^* INVARIATA

φ_m^* RIDOTTO DI $\omega_c^* \Delta \frac{180°}{\pi}$

_ FILTRO ANTI-ALIASING $B(s) = \frac{1}{1+s/\omega_B}$, $\omega_B \gg \omega_c$

_ LA FUNZIONE D'ANELLO CONTIENE ANCHE $B(s)$

$\Longrightarrow$ ω_c^* QUASI INVARIATA

φ_m^* RIDOTTO DI $\arctg \frac{\omega_c^*}{\omega_B}$

_ NOTA: LA SOMMA DI QUESTI EFFETTI PUÒ PORTARE AD AVERE $\varphi_m^* < 75°$

PRESENZA DI OSCILLAZIONI