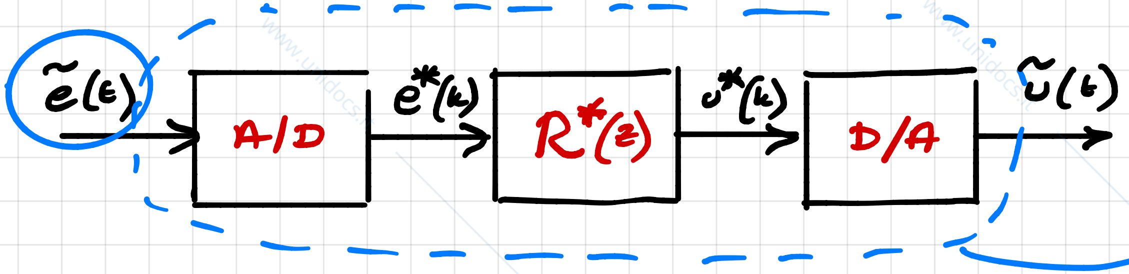
ESERCITAZIONE SU

ANALISI A TEMPO CONTINUO DI SISTEMI DI CONTROLLO DIGITALE

- RIPASSO - ANALISI A TEMPO CONTINUO DI UN CONTROLLORE DIGITALE

A BANDA LIMITATA CON $\omega_{MAX} < \frac{\omega_s}{2}$



$T > 0$

$\Psi \simeq \tilde{R}(s)$

- CONTROLLORE ANALOGICO "EQUIVALENTE"

$$\tilde{R}(s) = \frac{H_0(s)}{T} R^*(e^{sT}) = \frac{1 - e^{-sT}}{sT} R^*(e^{sT})$$

- IN $[0, \omega_s/2]$ RISULTA $H_0(s) \simeq T e^{-sT/2}$

$$\Longrightarrow \quad \tilde{R}(s) \simeq e^{-sT/2} R^*(e^{sT})$$

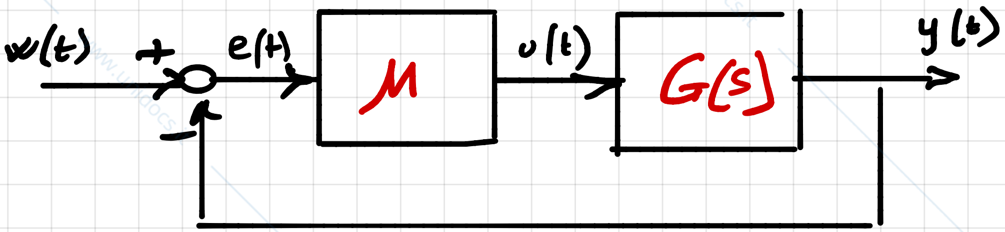
$z = e^{sT}$ TRASFORMAZIONE DI CAMPIONAMENTO

$e^{-sT/2}$: RITARDO DI DISCRETIZZAZIONE

_ Esercizio 1

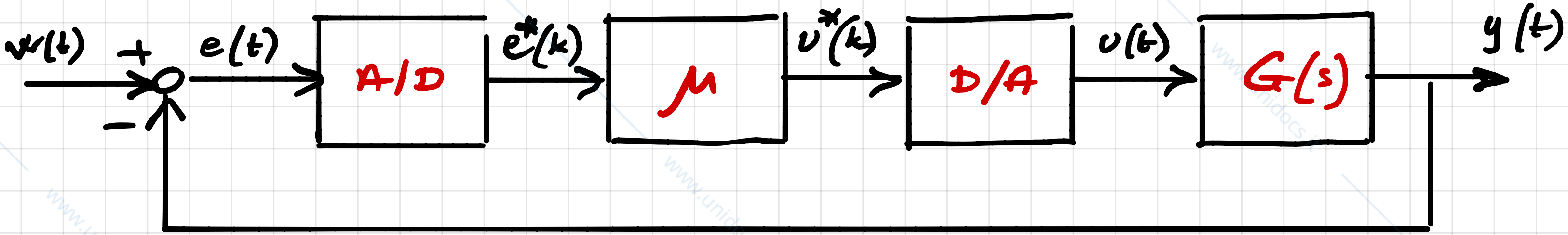
$$G(s) = \frac{1}{s(s+1)} \quad , \quad \mu = 1.5$$

- Controllo analogico (A)



1 - Confrontare i 2 schemi in termini di stabilità e prestazioni di tracking

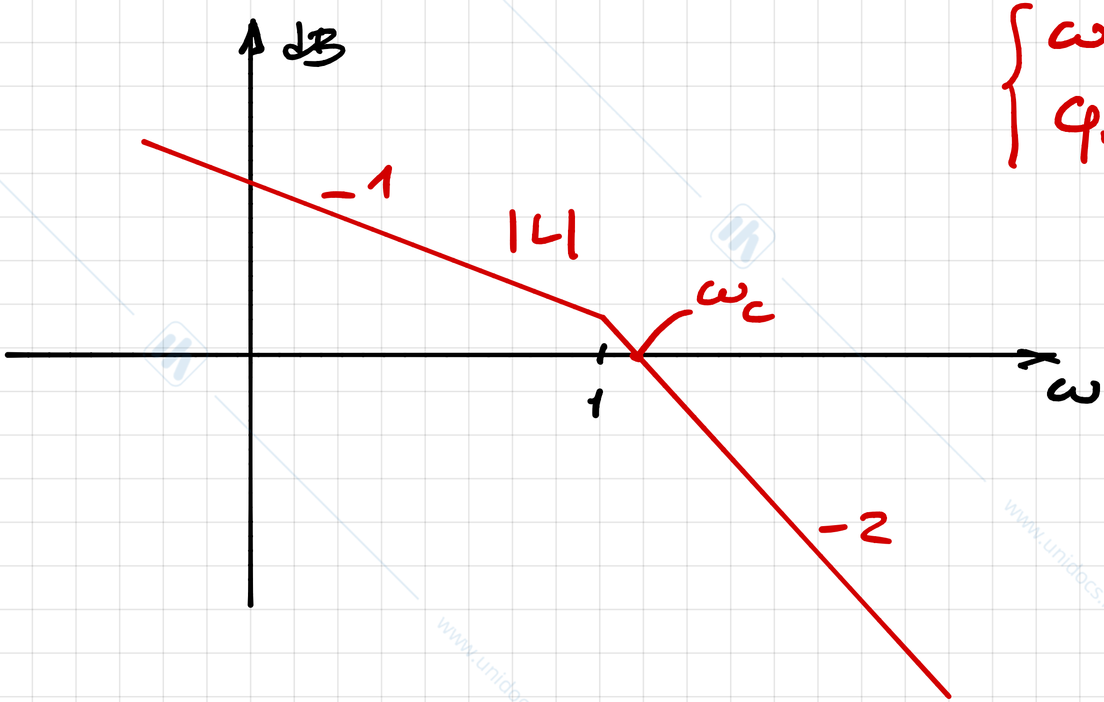
- Controllo digitale (D)



$$T = \frac{\pi}{5}$$

CONTROLLO ANALOGICO (A)

$$L(s) = \mu G(s) = \frac{1.5}{s(s+1)}$$



$$\begin{cases} \omega_c \simeq 1 \\ \varphi_m \simeq 44° > 0 \end{cases}$$

$\Downarrow$

AS. STAB.

$\Longrightarrow$ SISTEMA DISCRETAMENTE SMORZATO

$$t_a \simeq \frac{500}{\varphi_m \omega_c} \simeq 11 \quad , \quad e(\infty) = 0 \quad , \quad \Delta \simeq 0.21$$

- CONTROLLO DIGITALE (D)

$T = \frac{\pi}{5} \Longrightarrow \omega_s = \frac{2\pi}{T} = 10$ $\qquad R^*(z) = \mu = 1.5$

$\omega_s/2 = 5$

$$\tilde{R}(s) = \frac{H_0(s)}{T} R^*(e^{sT}) = \frac{1.5(1-e^{-sT})}{sT} = \frac{1.5(1-e^{-s\pi/5})}{s\pi/5}$$

D. BODE DIFFICILI DA TRACCIARE

$$\tilde{L}(s) = \tilde{R}(s)G(s) = \frac{5}{\pi}\frac{(1-e^{-s\pi/5})}{s}\frac{1.5}{s(s+1)} = \frac{7.5(1-e^{-s\pi/5})}{\pi s^2(s+1)}$$

- PONENDO $H_0(s) \simeq Te^{-sT/2}$ SI OTTIENE:

$$\tilde{R}(s) \simeq e^{-sT/2}\mu = 1.5e^{-s\pi/10}$$

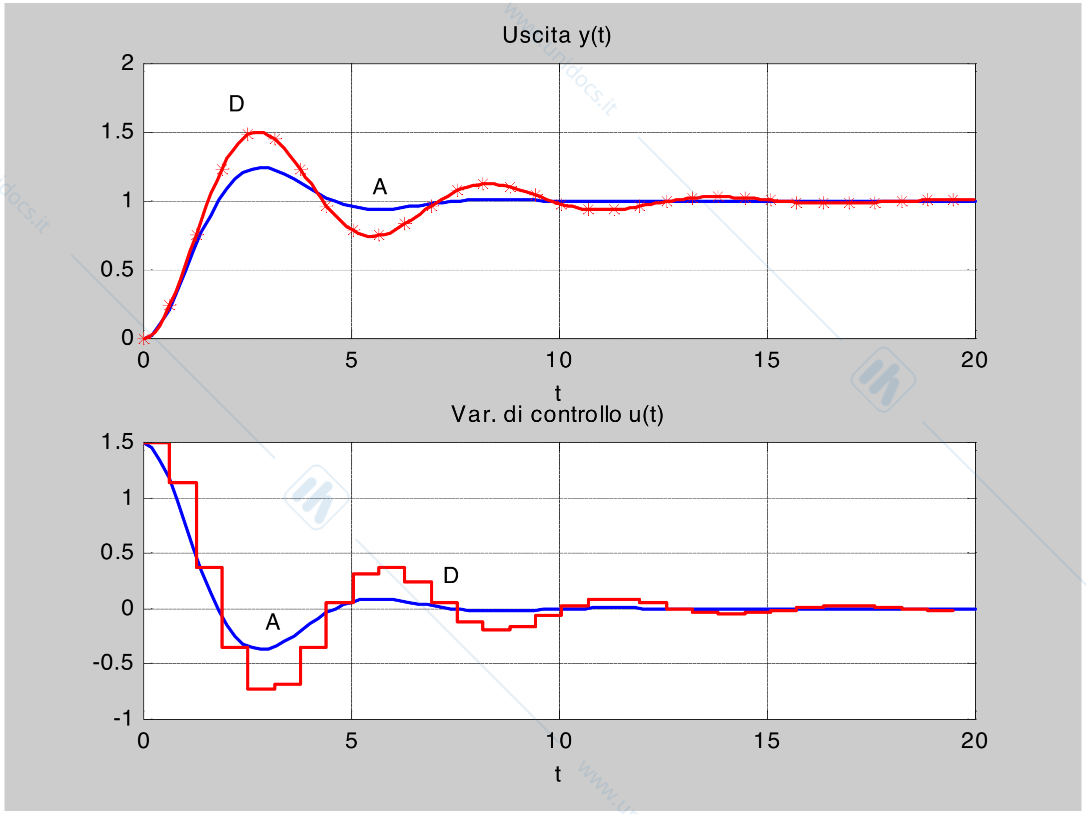
$$\tilde{L}(s) \simeq e^{-s\pi/10}\frac{1.5}{s(1+s)}$$

$$\begin{cases} \tilde{\omega}_c \simeq \omega_c \simeq 1 \\ \tilde{\varphi}_m \simeq \varphi_m - \tilde{\omega}_c \frac{T}{2}\frac{180^\circ}{\pi} \simeq 44^\circ - 18^\circ = 26^\circ \end{cases}$$

AS. STAB.

$\Longrightarrow$ SISTEMA MENO SMORZATO!

$\tilde{t}_s \simeq 19$, $e(\infty) = 0$, $\tilde{\Delta} \simeq 0.43$


Uscita y(t)
D
A
2
1.5
1
0.5
0
0
5
10
15
20
t
Var. di controllo u(t)
1.5
1
0.5
0
-0.5
-1
D
A
0
5
10
15
20
t

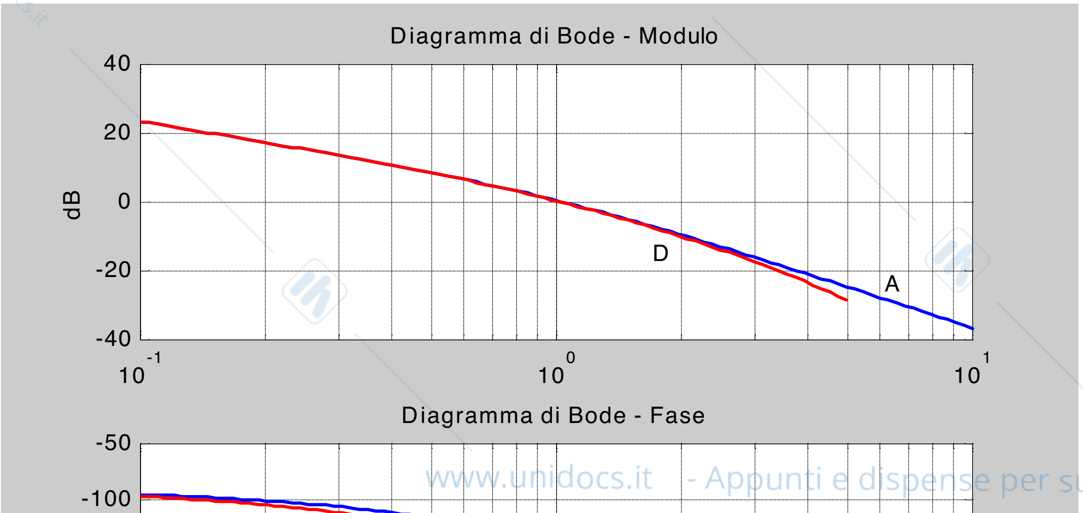


Diagramma di Bode - Modulo
40
20
0
-20
-40
dB
D
A
10^-1
10^0
10^1
Diagramma di Bode - Fase
-50
-100

2. CALCOLARE MAX T PER CUI SI PERDE LA STABILITÀ

$$\tilde{\varphi}_m = \varphi_m - \Delta\varphi_m \qquad (\varphi_m = 44^\circ)$$

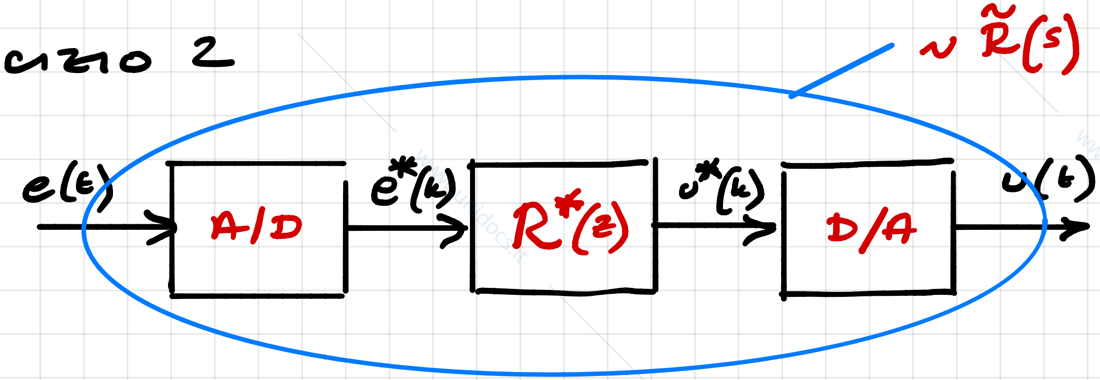
$$\tilde{\omega}_c \simeq 1$$

$$\Delta\varphi_m = \tilde{\omega}_c \frac{T_{MAX}}{2} \frac{180^\circ}{\pi} = 44^\circ \implies T_{MAX} = \frac{88\pi}{180} \simeq 1.54$$

_NOTA: NON È UNA VALUTAZIONE PRECISA!

È UNA STIMA PER ECCESSO (VEDI ANALISI A TEMPO DISCRETO)

- Esercizio 2



$T > 0$

$R^*(z)$: $\quad u^*(k) = u^*(k-1) + c\,e^*(k-1)$, $\quad c > 0$

1. RICAVARE $\tilde{R}(s)$
2. VALUTARE $|\tilde{R}(j\omega)|$, CALCOLANDO POI $|\tilde{R}(j0)|$ E $\left|\tilde{R}\left(j\frac{\omega_s}{2}\right)\right|$
3. RIPETERE 1 E 2 CON L'APPROX $H_0(s) \simeq T e^{-sT/2}$

(1)

$$u^*(k) = u^*(k-1) + c\,e^*(k-1)$$

$$U^*(z) = \frac{1}{z} U^*(z) + \frac{c}{z} E^*(z)$$

$$U^*(z)\left(1 - \frac{1}{z}\right) = \frac{c}{z} E^*(z) \implies R^*(z) = \frac{c/z}{1 - 1/z} = \frac{c}{z-1}$$

$$\tilde{R}(s) = \frac{H_0(s)}{T} R^*(e^{sT}) = \frac{1-e^{-sT}}{sT} \frac{c}{e^{sT}-1} = c\,\frac{e^{-sT}}{sT}$$

($1-e^{-sT} = e^{-sT}(e^{sT}-1)$)

(2)

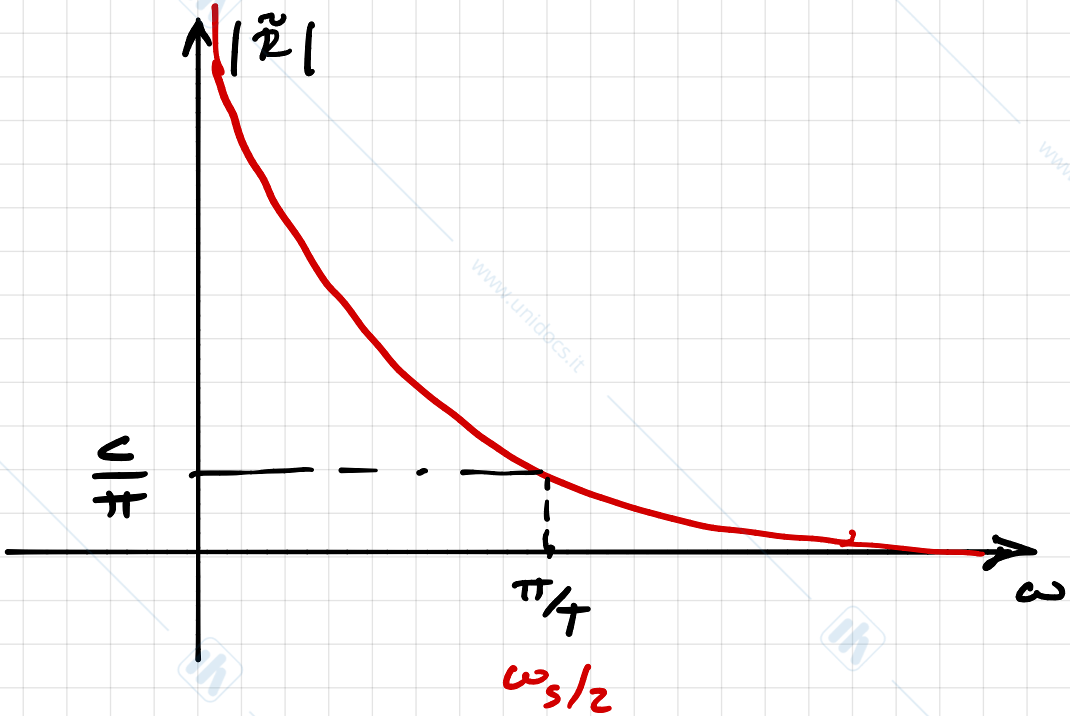
$$\tilde{R}(s) = c\,\frac{e^{-sT}}{sT}$$

$$|\tilde{R}(j\omega)| = \frac{c}{T}\left|\frac{e^{-j\omega T}}{j\omega}\right| = \frac{c}{T\omega}$$

- IN PARTICOLARE:

$$|\tilde{R}(j0)| = \infty$$

$$\left|\tilde{R}\left(j\frac{\pi}{T}\right)\right| = \frac{c}{\pi}$$



(3)

$$H_0(s) \simeq T e^{-sT/2}$$

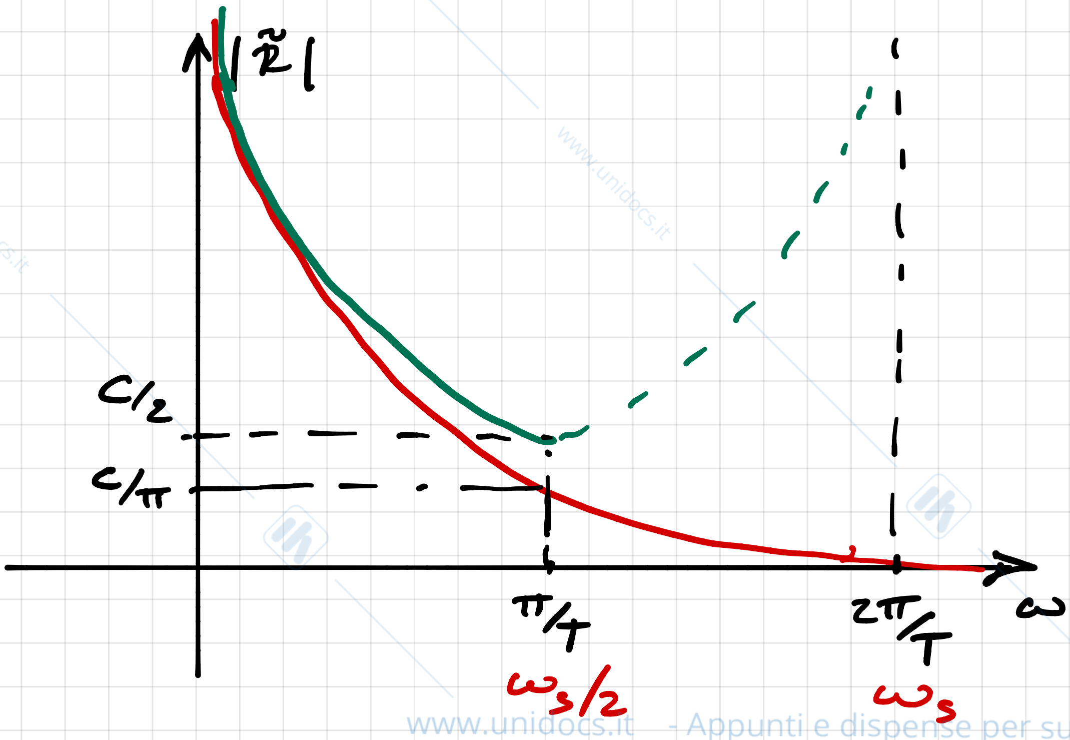
$$\tilde{R}(s) \simeq e^{-sT/2} R^*(e^{sT}) = e^{-sT/2} \frac{c}{e^{sT}-1}$$

$$|\tilde{R}(j\omega)| \simeq \left| e^{-j\omega T/2} \frac{c}{e^{j\omega T}-1} \right| = \frac{c}{|e^{j\omega T}-1|} = \frac{c}{\sqrt{2(1-\cos\omega T)}}$$

- IN PARTICOLARE:

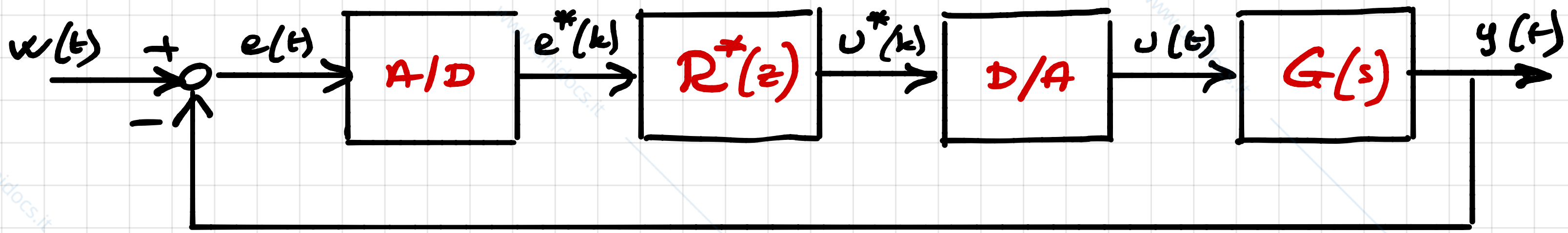
$$|\tilde{R}(j0)| = \infty$$

$$\left|\tilde{R}\left(j\frac{\pi}{T}\right)\right| = \frac{c}{2}$$



DISCRETA APPROSSIMAZIONE IN $[0, \omega_s/2]$

- ESERCIZIO 3



$R^*(z) = \frac{\mu}{z}$, $\mu > 0$, $G(s) = \frac{1}{s}$, $T > 0$ DATO

1. VALUTARE μ_{MAX} PER CUI IL SISTEMA DI CONTROLLO È ASINTOTICAMENTE STABILE

(1)

$$\tilde{R}(s) \simeq e^{-sT/2} R^*(e^{sT}) = e^{-sT/2} \frac{\mu}{e^{sT}} = \mu e^{-\frac{3T}{2}s}$$

$$\tilde{L}(s) = \tilde{R}(s) G(s) = \frac{\mu}{s} e^{-\frac{3T}{2}s}$$

RITARDO

$$\omega_c = \mu$$

$$\varphi_c = -90° - \omega_c \frac{3T}{2} \frac{180°}{\pi} = -90° - \frac{3\mu T}{2} \frac{180°}{\pi} = -90° - \frac{270°}{\pi} \mu T$$

$$\varphi_m = 180° - |\varphi_c| = 90° - \frac{270°}{\pi} \mu T$$

AS. STAB $\Leftrightarrow \varphi_m > 0° \Longleftrightarrow \frac{270°}{\pi} \mu T < 90° \Longleftrightarrow \mu T < \frac{\pi}{3}$

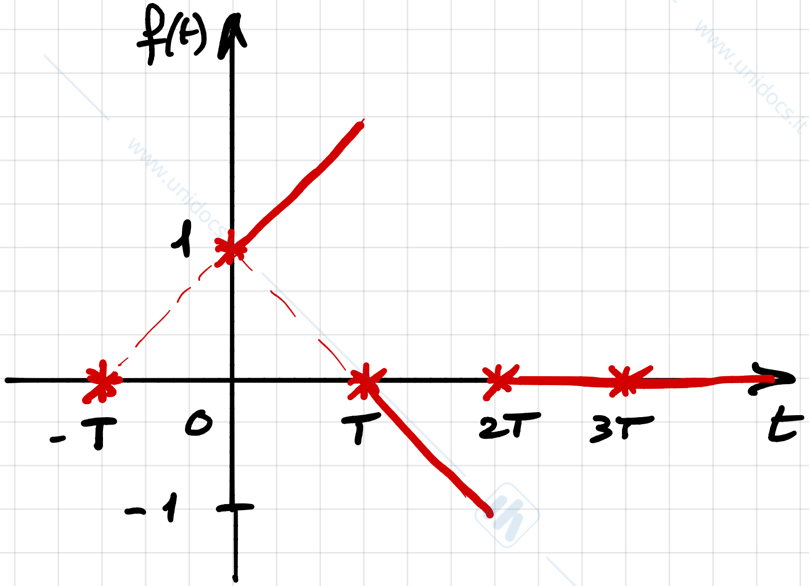
$$\Downarrow$$

STIMA APPROX.!

$$\mu_{MAX} = \frac{\pi}{3T}$$

(CON ALTRI METODI SI VEDRA' CHE $\mu_{MAX} = \frac{1}{T}$)

ESERCIZIO 4 ANALISI IN FREQUENZA DEL FOH (FIRST ORDER HOLD)



$$f^*(k) = imp^*(k)$$

$$f(t) = \underbrace{sca(t) + \frac{1}{T} ram(t)}_{g(t)} - 2\left(sca(t-T) + \frac{1}{T} ram(t-T)\right) + sca(t-2T) + \frac{1}{T} ram(t-2T) =$$

$$= g(t) - 2g(t-T) + g(t-2T) \triangleq h_1(t)$$

$$G(s) = \mathcal{L}[g(t)] = \frac{1}{s} + \frac{1}{Ts^2} = \frac{1+sT}{Ts^2}$$

$$F(s) = \mathcal{L}[f(t)] = \frac{1+sT}{Ts^2}\left(1 - 2e^{-sT} + e^{-2sT}\right) = \frac{(1+sT)(1-e^{-sT})^2}{Ts^2} \triangleq H_1(s)$$

_ Si noti che:

$$H_1(s) = \frac{1+sT}{T} H_0^2(s)$$

$$H_1(0) = T, \quad \left|H_1\left(j\frac{\pi}{T}\right)\right| = \frac{4T}{\pi^2}\sqrt{1+\pi^2} \simeq 1.3T, \quad H_1\left(j\frac{2\pi}{T}\right) = 0$$

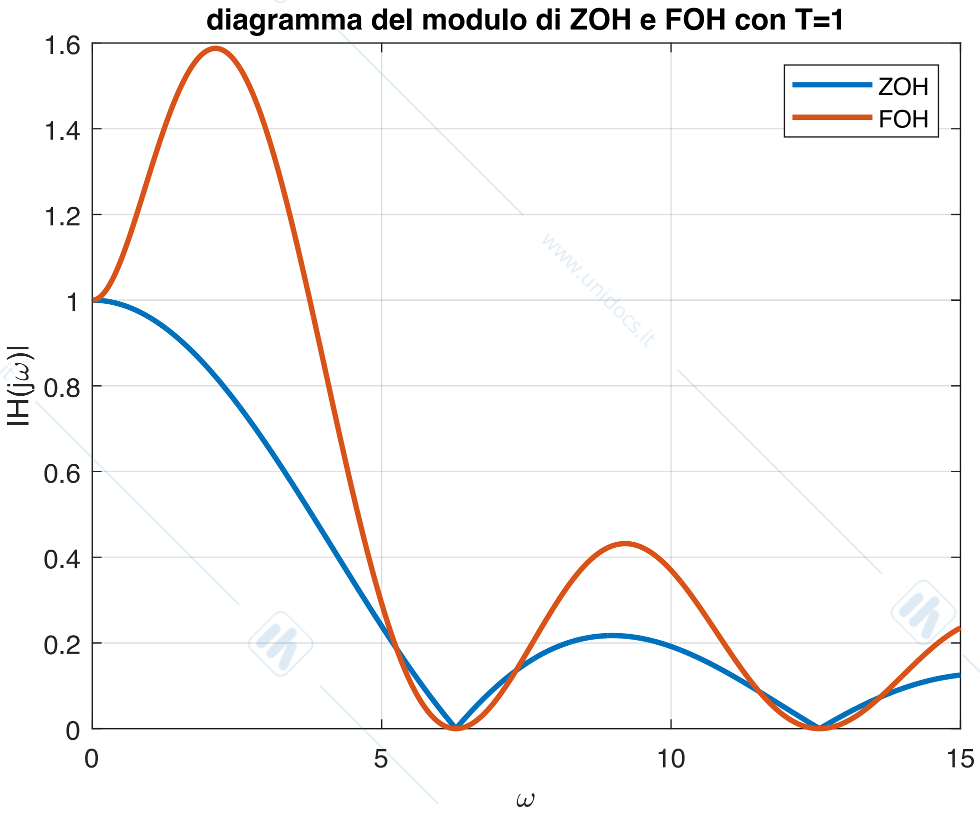


diagramma del modulo di ZOH e FOH con T=1
ZOH
FOH
|H(jω)|
ω
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15