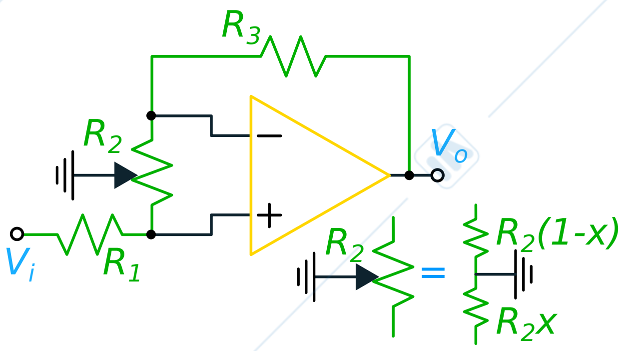
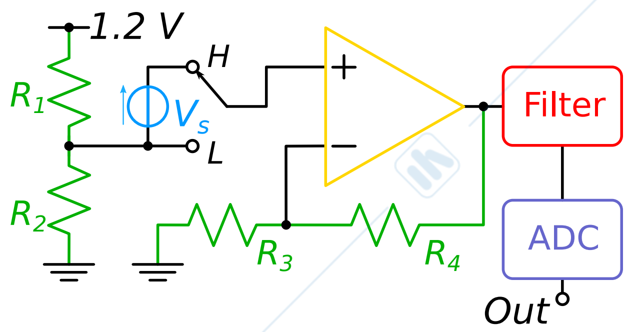
Electronics – January 15 2024 Prof. A. Spinelli

For a correct evaluation, please write your answers in a readable way; thank you





Solving six points correctly gives you 30/30

Problem 1

The scheme in the left figure is an audio gain stage that allows for a wide gain adjustement range and low noise, with a single potentiometer R_2. The OA has $A_0 = 120$ dB and poles at 60 Hz and 15 MHz. Parameters are $R_1 = 1$ kΩ, $R_3 = 8$ kΩ. In solving the scheme, replace the R_2 potentiometer with two resistors, as shown. x changes from 0 to 1 by rotating the wiper.

1. Compute the gain.
2. Compute the loop gain and find the maximum vale of R_2 that grants a phase margin larger than 45° for every x.
3. Compute the output rms noise voltage considering the equivalent noise source of the OA $\sqrt{S_V} = 3\ \text{nV}/\sqrt{\text{Hz}}$ plus resistor noise ($4k_BT \approx 1.646 \times 10^{-20}$ J). Consider $R_2 = 2$ kΩ and just evaluate the limit cases $x = 0$ and $x = 1$ for simplicity.
4. The proposed scheme only gives positive gains. Propose a simple modification that can give both positive and negative values when x goes from zero to one (hint: remember the inverting stage configuration and add just one resistor!).

Problem 2

A temperature sensor outputs a voltage $V_s = \alpha T$, with $\alpha = 40.7\ \mu\text{V}/°\text{C}$ and T expressed in °C. The circuit on the right is used to measure T in the range $[-100, +200]$°C. The ADC input range is $[0, 1.2]$ V. The OA has $V_{OS} = 10$ mV and input voltage noise $\sqrt{S_V} = 200\ \text{nV}/\sqrt{\text{Hz}}$. Filter BW is 30 Hz.

1. Neglect V_{OS} and find the parameter values that allows to measure T over the whole range.
2. Consider now V_{OS} and find the new parameter values. Remember that V_{OS} represents the absolute value of the offset.
3. To eliminate V_{OS}, its effect is first evaluated (switch L) and subtracted from the actual T measurement (switch H). Compute S/N for a minimum T difference of 1°C.
4. The OA has a flicker noise component with corner frequency $f_{nc} = 4$ kHz. Find the minimum reference frequency of a synchronous detection system that grants $S/N > 10$.

Allowed time: 2 hours 45 minutes – Do a good job!

Results will be posted by January 17th **Mark registration: Monday, January 22nd**

Solution

Problem 1

1.1
The voltage at the NI input of the OA is

$$V^+ = V_i \frac{R_2 x}{R_1 + R_2 x},$$

and the output becomes (NI stage gain)

$$V_o = V^+ \left(1 + \frac{R_3}{R_2(1-x)}\right) = V_i \frac{R_2 x}{R_1 + R_2 x} \frac{R_2(1-x) + R_3}{R_2(1-x)} = V_i \frac{x}{1+2x} \frac{10-2x}{1-x},$$

where the last expression holds for $R_2 = 2$ kΩ and is shown in Fig. 1 (left). Extreme gain values are zero and (ideally) infinite (or actually, A_0), as can be easily seen from the scheme.
Comment on the result: The advantage of this solution is that the gain is non-linear with x, with a shape that is (loosely) linear on a dB scale (if you don't get too close to the extremes), that is what you want in an audio equipment, and all this is achieved with a single potentiometer. The standard 20 kHz audio bandwidth can be simply set by a 1 nF capacitor in parallel to R_3.

1.2
The calculation of the loop gain is straightforward and leads to

$$G_{loop} = -A(s) \frac{R_2(1-x)}{R_3 + R_2(1-x)},$$

from which it is clear that problems might arise at $x = 0$, where we have

$$G_{loop} = -A(s) \frac{R_2}{R_3 + R_2}.$$

To achieve a phase margin of 45°, the OA pole at 15 MHz must fall below the 0 dB. The gain of $A(s)$ at such frequency is

$$A_0 \times 60 = G \times 15 \times 10^6 \Rightarrow G = 4,$$

so that

$$G \frac{R_2}{R_3 + R_2} < 1 \Rightarrow R_2 < \frac{R_3}{3} = 2.67 \text{ k}\Omega.$$

Comment on the result: Please note that for $x \to 1$ we have $|G_{loop}| \to 0$ and the OA works in open loop, which is not great (see noise calculations later).

1.3
We follow the suggestion and consider first $x = 0$, i.e., grounded NI input. The scheme is a simple NI amplifier and output noise PSD becomes:

$$S_{V_o} = S_V \left(1 + \frac{R_3}{R_2}\right)^2 + \left(\frac{4k_BT}{R_3} + \frac{4k_BT}{R_2}\right) R_3^2 = 2.25 \times 10^{-16} + 6.58 \times 10^{-16} = 8.83 \times 10^{-16} \text{ V}^2/\text{Hz}.$$

With $R_2 = 2$ kΩ we have $G_{loop} = -A(s)/5$, and $f_{0dB} = 60/5 = 12$ MHz, so that

$$\overline{V_o^2} = S_{Vo} \frac{\pi}{2} f_{0dB} \approx (129\ \mu\text{V})^2 .$$

For $x = 1$ the OA is in open loop, and the gain is simply $A(s)$. The NI input noise PSD is

$$S_V^+ = S_V + \left(\frac{4k_BT}{R_1} + \frac{4k_BT}{R_2}\right)(R_1 \parallel R_2)^2 = 9 \times 10^{-18} + 1.1 \times 10^{-17} = 2 \times 10^{-17} \text{ V}^2/\text{Hz}$$

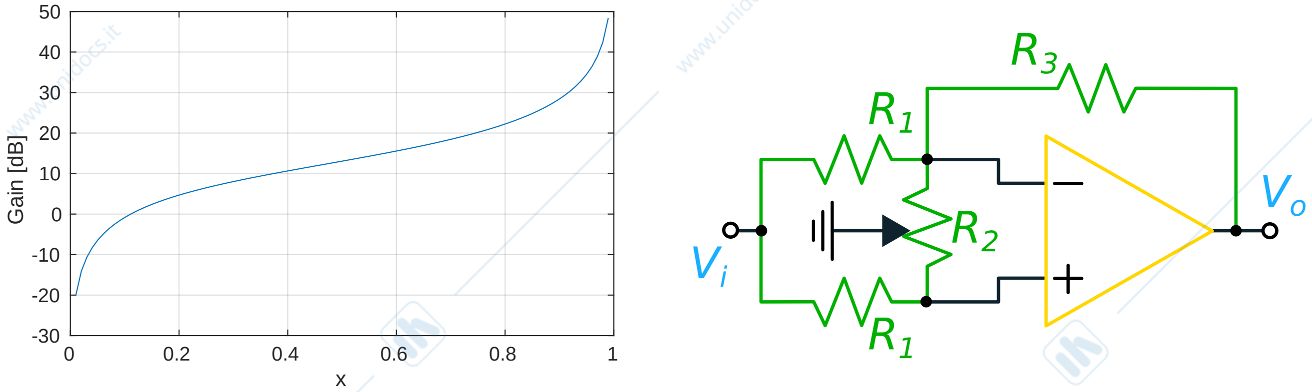


Figure 1: Left = Gain as a function of the pot position x. Right = modified scheme that provides negative gains.

and, considering $f_p = 60$ Hz:

$$S_{V_o} = S_V^+ |A(s)|^2 \Rightarrow \overline{V_o^2} = S_V^+ A_0^2 \frac{\pi}{2} f_p \approx (43.4 \text{ mV})^2 .$$

Comment on the result: This value is huge, but it should be kept in mind that comes at an unreasonable value of the gain. In reality, a small resistor placed in series to the potentiometer will limit the gain and the noise.

1.4

For $x = 1$ the ideal gain is infinite. If we want to add a negative gain, we need to work around $x = 0$, where the OA NI input is grounded. A negative gain can be obtained by adding a resistor R_1 from the input to the I input of the OA, as shown in Fig. 1, right. For $x = 0$ the gain becomes $-R_3/R_1$.

Problem 2

2.1

We label $V_{ref} = 1.2 \text{ V } R_2/(R_1 + R_2)$ the voltage after the $R_1 - R_2$ divider, and compute the minimum and maximum OA output voltages:

$$V_{min} = (V_{ref} + \alpha T_{min})G$$
$$V_{max} = (V_{ref} + \alpha T_{max})G,$$

where $G = 1 + R_4/R_3$ is the NI stage gain. We now have

$$V_{min} > 0 \Rightarrow V_{ref} + \alpha T_{min} > 0 \Rightarrow V_{ref} > -\alpha T_{min} = 4.1 \text{ mV}$$

and, setting $V_{min} = 0$,

$$V_{max} < 1.2 \text{ V} \Rightarrow (V_{ref} + \alpha T_{max})G = (V_{min} + \alpha(T_{max} - T_{min}))G < 1.2 \text{ V} \Rightarrow$$
$$\alpha(T_{max} - T_{min})G < 1.2 \text{ V} \Rightarrow G < 98.3.$$

Possible (ideal) resistor values are $R_1 = 292$ kΩ, $R_2 = 1$ kΩ, $R_3 = 1$ kΩ, $R_4 = 97.3$ kΩ. Actual values will be a bit different because of commercial availability.

2.2

The worst case happens when V_{OS} lowers V_{min} and increases V_{max}. The new requirements become then

$$(V_{ref} + \alpha T_{min} - V_{OS})G > 0 \Rightarrow V_{ref} > 14.1 \text{ mV}$$
$$(V_{ref} + \alpha T_{max} + V_{OS})G < 1.2 \text{ V} \Rightarrow G < 37.3.$$

Note the large change in values, consequence of the large V_{OS}: its value is equivalent to a temperature difference of $V_{OS}/\alpha \approx 246$°C, i.e., almost equal to the entire signal range! New ideal values are then $R_1 = 84$ kΩ, $R_2 = 1$ kΩ, $R_3 = 1$ kΩ, $R_4 = 36.3$ kΩ.

2.3

The difference operation removes the LF offset, meaning that we can switch back to the resistor values computed in #2.1. We have then for the OA output noise

$$S_{V_o} = S_V G^2 + 4k_BT(R_1 \parallel R_2)G^2 + 4k_BTR_3(G-1)^2 + 4k_BTR_4 \approx G^2 S_V = 3.9 \times 10^{-10}\ \text{V}^2/\text{Hz},$$

and S/N becomes ($\Delta T = 1°\text{C}$, $BW_n = \frac{\pi}{2} \times 30$ Hz):

$$\left(\frac{S}{N}\right) = \frac{\alpha \Delta T G}{\sqrt{2S_{V_o} BW_n}} \approx \frac{\alpha \Delta T}{\sqrt{2S_V BW_n}} \approx 21,$$

where the factor 2 comes from the difference operation.

2.4

If the switch is driven by a square wave at frequency f_r, the OA output is also a square wave whose peak-to-peak amplitude $A = \alpha G \Delta T$ represents the signal. If we pick a sinusoidal demodulation, this will extract the fundamental harmonic of the square wave, with amplitude $2A/\pi$. We then get

$$\left(\frac{S}{N}\right) = \frac{2A/\pi}{\sqrt{4S_x(f_r)G^2 BW_n}}.$$

If we demodulate below f_{nc} we have $S_x = K/f_r$ and

$$\frac{\alpha \Delta T}{\pi}\sqrt{\frac{f_r}{KBW_n}} > 10 \Rightarrow f_r > \frac{100\pi^2 KBW_n}{(\alpha \Delta T)^2} \approx 2.86 \text{ kHz}.$$

Note that this value is close to f_{nc}, so we should also account for the WN component. A more exact calculation is obtained with $S_x \approx G^2(K/f_r + S_V)$ and

$$\left(\frac{S}{N}\right) = \frac{A/\pi}{\sqrt{S_x(f_r)BW_n}} > 10 \Rightarrow f_r > \frac{100\pi^2 BW_n K}{(\alpha \Delta T)^2 - 100\pi^2 BW_n S_V} = 10 \text{ kHz}.$$

A final, probably unsolicited, comment is that the system in #2.3 already rejects some LF noise! Let'do some math: The sampling times difference is limited by the $f_p = 30$ Hz filter, as its output must reach the steady-state before sampling. Considering 5 time constant, we have

$$t_s \geq \frac{5}{2\pi 30} = 26.5 \text{ ms},$$

meaning that the output FN contribution becomes (see lesson slide on correlated double sampling for derivation of the WF):

$$\overline{n_{FN}^2} = KG^2 \int_0^{x_p} \frac{1-\cos x}{x} dx \approx 2.38KG^2 = 3.8\ G^2 \times 10^{-10}\ \text{V}^2/\text{Hz},$$

where $x = 2\pi f t_s$ and $x_p = 2\pi f_p t_s = 5$. We have then

$$\left(\frac{S}{N}\right) = \frac{\alpha \Delta T G}{\sqrt{2S_{V_o} BW + n_{FN}^2}} \approx \frac{\alpha \Delta T}{\sqrt{2S_V BW + 2.38K}} \approx 2.$$

close to the requirement, but no cigar!