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- $a^x \Rightarrow C.E: a > 0$
- $\log_a(x) \Rightarrow C.E: x > 0$
  $a > 1$ o $0 < a < 1$

Proprietà log ed esponenziali:

1) $a^{x_1} \cdot a^{x_2} = a^{x_1 + x_2}$

2) $\log_a(x_1 \cdot x_2) = \log_a(x_1) + \log_a(x_2)$

3) $a^{\log_a(x_1 \cdot x_2)} = x_1 \cdot x_2$

4) $a^{\log_a(x_1) + \log_a(x_2)} = x_1 \cdot x_2$

5) $\log_a(x^b) = b \cdot \log_a(x)$

6) $\log_b(x) = \dfrac{\log_a(x)}{\log_a(b)}$

# Funzioni trigonometriche

- $\frac{1}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = 1 + \tan^2 x$

- $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
- $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

- $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
- $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

- $\sin(2x) = 2\cos(x)\sin(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$

- $\cos(x) = \cos\left(\frac{x}{2}+\frac{x}{2}\right) = 1 - \sin^2\left(\frac{x}{2}\right)$
- $2\sin^2\left(\frac{x}{2}\right) = 1 - \cos(x)$

- $\sin^2\left(\frac{x}{2}\right) = \frac{1-\cos(x)}{2}$
- $\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos(x)}{2}$

# Limiti notevoli

- $\lim a^n = \begin{cases} +\infty & \text{se } a>1 \\ 1 & \text{se } a=1 \\ 0 & \text{se } -1<a<1 \\ \nexists & \text{se } a \leq -1 \end{cases}$
- $\lim n^b = \begin{cases} +\infty & \text{se } b>0 \\ 1 & \text{se } b=0 \\ 0 & \text{se } b<0 \end{cases}$
- $\lim \sqrt[n]{a} = 1 \quad \forall a \geq 0$
- $\log_a(n) \lneq n^b \lneq a^n$ → gerarchia degli infiniti
- $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$
- $\lim_{x\to +\infty} a^x = \begin{cases} +\infty & a>1 \\ 0 & 0<a<1 \end{cases}$
- $\lim_{x\to -\infty} a^x = \begin{cases} 0 & a>1 \\ +\infty & 0<a<1 \end{cases}$
- $\lim_{x\to +\infty} \frac{\log_e(x)}{x^b} = 0$ con $b>0$
- $\lim_{x\to +\infty} x^b = \begin{cases} +\infty & b>0 \\ 0 & b<0 \end{cases}$
- $\lim_{x\to 0^+} \log_e(x) = -\infty$
- $\lim_{x\to \pm\infty} \left(1+\frac{1}{x}\right)^x = e$
- $\lim_{x\to \pm\infty} x \cdot \log_e\left(1+\frac{1}{x}\right) = 1$
- $\lim_{x\to 0} \frac{\log_e(1+x)}{x} = 1$
- $\lim_{x\to 0} \frac{e^x-1}{x} \Rightarrow \lim_{y\to 0} \frac{y}{\ln(y+1)} = 1$
- $\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log_e a} = \frac{1}{\ln a}$
- $\lim_{x\to 0} \frac{a^x-1}{x} = \log_e a$
- $\lim_{x\to 0} \frac{\tan x}{x} = 1$
- $\lim_{x\to +\infty} \frac{\sin(x)}{x} = 0$
- $\lim_{x\to 0} \frac{1-\cos(x)}{x^2} = \frac{1}{2}$
- $\lim_{x\to +\infty} \log_e(x) = +\infty$
- $\lim_{x\to 0^+} x^b \log(x) = 0$
- $\lim_{x\to \pm\infty} \left(1+\frac{T}{x}\right)^x = e^T \quad \forall T \in \mathbb{R}$
- $\lim_{x\to +\infty} \frac{x^b}{a^x} = 0$ con $b>0$ e $a>1$
- $\lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$
- $\lim_{x\to 0^\pm} (1+Tx)^{1/x} = e^T$

# Derivate

$y = K \rightarrow y' = 0$

$y = x^n \rightarrow y' = n x^{n-1}$

$y = \sin x \rightarrow y' = \cos x$

$y = \cos x \rightarrow y' = -\sin x$

$y = tg\, x \rightarrow y' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

$y = cotg\, x \rightarrow y' = \frac{-1}{\sin^2 x}$

$y = \arcsin x \rightarrow y' = \frac{1}{\sqrt{1 - x^2}}$

$y = x \rightarrow y' = 0$

$y = \log_a x \rightarrow y' = \frac{1}{x \ln a}$

$y = a^x \rightarrow y' = a \ln a$

$y = \sqrt{x} \rightarrow y' = \frac{1}{2\sqrt{x}}$

$y = \ln x \rightarrow y' = \frac{1}{x}$

$y = e^x \rightarrow y' = e^x$

$y = \arccos x \rightarrow y' = -\frac{1}{\sqrt{1 - x^2}}$

$y = \arctan x \rightarrow y' = \frac{1}{1 + x^2}$

$y = arccot\, x \rightarrow y' = -\frac{1}{1 +}$

$D[f(x) + g(x)] = f'(x) + g'(x)$

$D[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$D\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

$D\left[\frac{1}{f(x)}\right] = -\frac{f'(x)}{(f(x))^2}$

$D[f(g(x))] = f'(g(x)) \cdot g'(x)$

$y = \{f(x)\}^{g(x)} \longrightarrow y' = \{f(x)\}^{g(x)} \cdot \left\{ g'(x) \ln f(x) + \frac{g(x)}{f(x)} \cdot f'(x) \right\}$



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