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Equazioni a coefficienti costanti

$\dot{x}(t) = ax(t) + b(t)$ omogenea

$\dot{x}(t) = ax(t)$

$x(t) = x_0 + m \cdot x(t)$

$m = \frac{dx}{dt} \cdot (x + y)$

$m \cdot y = (x + x \cdot x(t))$

$\cdot\ x_0^{\text{omogenea}}$

$\frac{d}{dt} x(t) = a x(t)$

$x(t) = e^{at} \quad \frac{d}{dt} e^{at} = a e^{at}$

$x(t) = K e^{at} \quad \frac{d}{dt} K e^{at} = a K e^{at} = a x(t)$

$x(t) = K e^{a(t-t_0)} \quad \frac{d}{dt} K e^{a(t-t_0)} = a K e^{a(t-t_0)}$

$x_0(t) = K e^{a(t-t_0)}$

$x(t_0) = K = x_0$

$t_0 = 0$

$x_p(t)$ integrale particolare | $\frac{d}{dt} x_p(t) = a x_p(t) + m(t)$, $\forall t$

$x_0(t)$ soluzione di: $\frac{d}{dt} x_0(t) = a x_0(t) + m(t)$

$x_0(t) = x_0^{\text{omogenea}} + x_p(t) = K e^{a(t-t_0)} + x_p(t)$

$x_0(t) = x_0 \cdot (t, x_0)$

$\begin{cases} \dot{x}(t) = a x(t) + m(t) \\ x(t_0) = x_0 \end{cases}$ $\quad x(t) \in C_1(\mathbb{R})$

$x_0(t) = K e^{a(t-t_0)} + x_p(t)$

$x_0(t_0) = x_0 = K e^{a(t_0-t_0)} + x_p(t_0)$

$x_0 = K + x_p(t_0)$

$K = x_0 - x_p(t_0)$

$x_0(t) = (x_0 - x_p(t_0)) e^{a(t-t_0)} + x_p(t)$

$m(t) \in C_1(\mathbb{R})$

$\begin{cases} \dot{x} = x + t \\ x(t_0) = 2 \end{cases}$ $\quad x(t) = x_0(t) + x_p(t, t_0, x_0)$

$t_0 = 2$

$x_0 = -x_{00} \quad x_{00} = K e^{a(t-t_0)}$

$x_0(t) = x_0(t) + x_p(t)$

$x(t) = t^2 \qquad x(t) = t$

$\frac{d}{dt} t^2 = 2t + 2 \qquad \frac{d}{dt} t = 1 + 2$

$2t = 2t + 2 \qquad 0 \neq 1 + 2 \neq 1$

$x(t) = t$

$\frac{d}{dt} t = 1 + 2$

$1 = 1 + 2 \quad \boxed{t = 2}$

$x_0(t) = K e^{(t-t_0)} + 2$

$x_0(t) = x_0 = K e^{(t_0-t_0)} + 2$

$x_0 = 1 + 2$

$x_0(t) = -K e^{(t-t_0)} + 2$

$\frac{d}{dt}\left(e^{(t-t_0)}\right) = -e^{(t-t_0)}$

$= e^{(t-t_0)} \qquad = -e^{(t-t_0)}$

$x_0 \qquad x_0 = -K e^{(t_0-t_0)} + 2$

$0 = 0$

$x_0(t) = (x_0 - x_p(t_0)) e^{a(t-t_0)} + x_p(t)$

transiento

$x_0(t) \xrightarrow[t \to +\infty]{} x_p(t)$

$t - t_0 = \Delta t$ se $\frac{dx}{dt}$ con $a < 0$

$\lim_{t \to +\infty} e^{-\Delta t \frac{dx}{dt}} = e^{-\infty}$

$\dot{x} = ax + m(t)$

$\dot{x} = \lambda x + m(t)$

Equazione lineare

$[\lambda t] = \frac{1}{s}$

$\tau = |\lambda|^{-1} \qquad (\tau) = s$

$x_0 = K e^{\lambda(t-t_0)} \qquad x_0 = K e^{\frac{\operatorname{sgn}(\lambda)}{\tau}(t-t_0)}$

$\begin{cases} \dot{x} = \lambda x + m(t) \\ x(t_0) = x_0 \end{cases}$ $\quad x_0(t) = \underbrace{(x_0 - x_p(t_0)) e^{\lambda(t-t_0)}}_{\text{transiento } (\lambda < 0)} + \underbrace{x_p(t)}_{\text{regime } (\lambda < 0)}$

Vista di asintoticamente stabile

$\dot{x} = \lambda x + m(t)$

$x(t_0) = x_0 \qquad x(t_0) = x_0$

$x_0^1(t) = (x_0 - x_p(t_0)) e^{\lambda(t-t_0)} + x_p(t)$

$x_0^2(t) = (x_0 - x_p(t_0)) e^{\lambda(t-t_0)} + x_p(t)$

$|x_0^1(t) - x_0^2(t)| = |x_0 - x_0| e^{\lambda(t-t_0)} \xrightarrow[\lambda < 0]{t \to \infty} 0$

$\lambda = 0$ asintoticamente stabile

$|x_0 - x_0| e^{\lambda(t-t_0)} \Big|_{\lambda = 0} = |x_0 - x_0|$

$x_0(t) = (x_0 - x_p(t_0)) e^{\lambda(t-t_0)} + x_p(t)$

$x_0 e^{\lambda(t-t_0)}$ primo termine | $(x_p(t) - x_p(t_0)) e^{\lambda(t-t_0)}$ secondo termine

se $x_0 = x_p(t_0) \longrightarrow$ resta $x_0 e^{\lambda(t-t_0)}$ primo termine

$\cdot\ m(t) = \sum_{k=0}^{n} a_k t^k = a_n t^n + \dots + a_1 t + a_0$

$x_{p0}(t) = \sum_{k=0}^{n} b_k t^k = b_n t^n + \dots + b_1 t + b_0$

$\begin{cases} \dot{x}(t) = -x(t) + t^2 + 1 \\ x(0) = 0 \end{cases}$ $\quad \lambda = -1$ asintoticamente stabile, $\lambda = 0$ asintoticamente stabile, $\lambda = 1$ instabile

$x_0(t) = b_2 t^2 + b_1 t + b_0$

$x_0(t) = K e^{-(t-t_0)} = K e^{-t} \qquad \tau = \frac{1}{|-1|} = \frac{1}{1}$

$\frac{d}{dt}(b_2 t^2 + b_1 t + b_0) = -(b_2 t^2 + b_1 t + b_0) + t^2 + 1$

$2b_2 t + b_1 = -b_2 t^2 - b_1 t - b_0 + t^2 + 1$

$(-b_2 - 1) t^2 + (2b_2 + b_1) t + b_1 + b_0 - 1 = 0$

$\begin{cases} -b_2 - 1 = 0 \\ 2b_2 + b_1 = 0 \\ b_1 + b_0 - 1 = 0 \end{cases} \qquad \begin{aligned} b_2 &= -1 \\ b_1 &= -2 \\ b_0 &= 1 - b_1 = 1 + 2 = 3 \end{aligned}$

$x_{p0}(t) = -t^2 - 2t + 3$

$x_0(t) = K e^{-(t-t_0)} + 3 - 2t - t^2$

$x_0(0) = x_0 = 0 \Rightarrow K + 3 = 0 \qquad K = -3$

$x_0(t) = -3 e^{-(t-t_0)} - t^2 - 2t + 3$

$\cdot\ m(t) = x_{0n} \cos(\omega t + \varphi) = x_{0n} \cos\varphi \cdot \cos\omega t - x_{0n} \sin\varphi \sin\omega t$

$= A \cos\omega t + B \sin\omega t$

$x_p(t) = x_{p0} \cos(\omega t + \varphi_0) = A_0 \cos\omega t + B_0 \sin\omega t$

$A \cos\omega t + B \sin\omega t \qquad x_{0n} \cos\varphi = A$

$x_{0n} \sin\varphi = B$

$\sqrt{A^2 + B^2} = x_{0n}$

$\frac{B}{A} = -\frac{x_{0n} \sin\varphi}{x_{0n} \cos\varphi} \qquad \varphi = \operatorname{atan}\left(-\frac{B}{A}\right)$

$\dot{x} = -x + \cos t = -x + \cos(\omega t + 0) \; ; \; x(0) = 0$

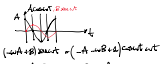
$x_{p0} = X \cos(\omega t + \varphi) = A \cos\omega t + B \sin\omega t$

$\dot{x} = -x \qquad x_{00}(t) = K e^{-(t-t_0)} = K e^{-(t-0)}$

$x_0(t) = K e^{-(t-0)} + A \cos\omega t + B \sin\omega t$

$\frac{d}{dt}(A \cos\omega t + B \sin\omega t) = -(A \cos\omega t + B \sin\omega t) + \cos\omega t$

$-\omega A \sin\omega t + \omega B \cos\omega t = -A \cos\omega t - B \sin\omega t + \cos$



$(-\omega A + B) \sin\omega t = (-A - \omega B + 1) \cos\omega t$

$\begin{cases} -\omega A + B = 0 \\ -A - \omega B + 1 = 0 \end{cases} \qquad B = \omega A \qquad -A - \omega^2 A = -1$

$A = \frac{1}{1 + \omega^2} \qquad B = \frac{\omega}{1 + \omega^2}$

$x_0(t) = K e^{-(t - \pi/2)} + \frac{1}{1 + \omega^2} \cos\omega t + \frac{\omega}{1 + \omega^2} \sin\omega t$

$x_0(\pi/2) = 0 = K + \frac{\omega}{1 + \omega^2} \qquad K = -\frac{\omega}{1 + \omega^2}$

$\dot{x} = -x + t + 1 + \cos\omega t$

$x_{p0}(t) = b t + b_0 + A \cos\omega t + B \sin\omega t$

$x_{00}(t) = K e^{-(t - t_0)}$

$\frac{d}{dt}(b t + b_0 + A \cos\omega t + B \sin\omega t) = -b t - b_0 - A \cos\omega t - B \sin\omega t + t + 1 + \cos\omega t$

$b - A\omega \sin\omega t + B\omega \cos\omega t = -b t - b_0 - A \cos\omega t - B \sin\omega t + t + 1 + \cos\omega t$

$(b - 1) t + (b + b_0 - 1) + (-A\omega + B) \sin\omega t + (B\omega + A - 1) \cos\omega t = 0$

$\begin{cases} b - 1 = 0 \\ b + b_0 - 1 = 0 \\ -A\omega + B = 0 \\ B\omega + A - 1 = 0 \end{cases} \qquad \begin{aligned} b &= 1 \\ b_0 &= 1 - b = 0 \end{aligned}$

$x_0(t) = x_{00}(t) + x_p^1(t) + x_p^2(t)$

$x_0(t_0) = x_0 = K + x_p^1(t_0) + x_p^2(t_0)$

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