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Fasori

Consuetudine ricorrente con segnali tempo varianti:

$E_j(t) = V_j \cos(\omega t + \varphi_j)$

$A_k(t) = I_k \cos(\omega t + \varphi_k)$

e un componente dinamico ricorrente

$\dot{x}(t) = \lambda x(t) + \sum_j E_j(t) \cdot g_j + \sum_k A_k(t) f_k$

$x(t) = x_{tr}(t) + x_e(t) = K e^{\lambda(t-t_0)} + x_p(t)$

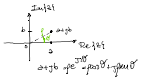
$x(t) \Big|_{\lim t \to \infty} \longrightarrow x_e(t)$ (*)

se y è una grandezza qualsiasi del circuito allora

(*) $y(t) = \sum_k y_k A_k(t) + \sum_j \varepsilon_j E_j(t) + \rho x(t)$

A regime $\lim t \to \infty$ $y(t) = Y \cos(\omega t + \vartheta)$

$z = a + jb \in \mathbb{C}$ $j^2 = -1$



$\mathrm{Re}\{z\} = \rho \cos\vartheta$

$\mathrm{Im}\{z\} = \rho \sin\vartheta$

$\angle z = \arctg(\frac{b}{a}) = \vartheta$

$|z| = \rho$

$a + jb = \rho e^{j\vartheta} = \rho \cos\vartheta + j \rho \sin\vartheta$

$\rho = \sqrt{a^2 + b^2}$ $\frac{b}{a} = \frac{\rho}{\rho} \frac{\sin\vartheta}{\cos\vartheta}$

$\vartheta = \arctan \frac{b}{a} (\pm \pi)$

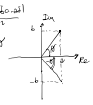
$(a+jb) + (c+jd) = a+c + j(b+d)$

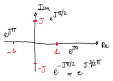
$(a+jb)(c+jd) = (ac - bd) + j(bc + ad)$

$\rho_1 e^{j\vartheta_1} \cdot \rho_2 e^{j\vartheta_2} = \rho_1 \rho_2 e^{j(\vartheta_1 + \vartheta_2)}$

$\frac{a+jb}{c+jd} = \frac{a+jb}{c+jd} \cdot \frac{c-jd}{c-jd} = \frac{ac+bd + j(bc-ad)}{c^2+d^2}$

$(a+jb)^* = a - jb$ $(\rho e^{j\vartheta})^* = \rho e^{-j\vartheta}$





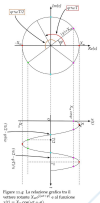


Figure 1.1.3 La relazione grafica tra il vettore rotante $X_m e^{j(\omega t + \varphi)}$ e il fasore $\bar{X} = X_m \cos\varphi + j X_m \sin\varphi$

$x(t) = X_m \cos(\omega t + \varphi)$ tempo

$\downarrow (\omega)$

$\bar{X} = X_m e^{j\varphi}$ fasore (C)

$x(t) = \mathrm{Re}\{\bar{X} e^{j\omega t}\} =$

$\mathrm{Re}\{|\bar{X}| e^{j\varphi} e^{j\omega t}\} =$

$= \mathrm{Re}\{|\bar{X}| e^{j(\omega t + \varphi)}\} =$

$= \mathrm{Re}\{|\bar{X}| (\cos(\omega t + \varphi) + j \sin(\omega t + \varphi))\} =$

$= |\bar{X}| \cos(\omega t + \varphi) = X_m \cos(\omega t + \varphi)$

- $\omega = 2 \cos\frac{1}{2}$ $\bar{X} = 2 + j$

$x(t) = \mathrm{Re}\{(2+j) e^{jt}\} =$

$= \mathrm{Re}\{\sqrt{5} e^{j\pi/4} e^{jt}\} =$

$= \sqrt{5} \cos(t + \pi/4)$

$\mathrm{Re}\{(2+j) e^{jt}\} = \mathrm{Re}\{(2+j)(\cos t + j \sin t)\} =$

$= \cos t - \sin t$

- $x(t) = 3 \sin(2t)$ $\bar{X} \neq 3$

$x(t) = 3 \cos(2t - \pi/2)$ $\bar{X} = 3 e^{-j\pi/2}$

$= -3j$

$x(t) = a x_1(t) + b x_2(t) = a \cdot \underbrace{X_1 \cos(\omega t + \varphi_1)}_{\bar{X}_1 = X_1 e^{j\varphi_1}} + b \underbrace{X_2 \cos(\omega t + \varphi_2)}_{\bar{X}_2 = X_2 e^{j\varphi_2}}$

$\bar{X} = a \bar{X}_1 + b \bar{X}_2$

- $\cos t - \sin t = x(t)$ $\cos t \to 1$, $-\sin t \to j$ $\bar{X} = 2 + j = \sqrt{5} e^{j\pi/4}$

$(\omega = 1)$

$\frac{d}{dt} x(t) = \frac{d}{dt} X_m \cos(\omega t + \varphi) = -\omega X_m \sin(\omega t + \varphi) =$

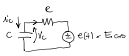
$\bar{X} = X_m e^{j\varphi}$ $= -\omega X_m \cos(\omega t + \varphi - \pi/2) = y(t)$

$\bar{Y} = -\omega X_m e^{j(\varphi - \pi/2)} = -\omega X_m e^{-j\pi/2} e^{j\varphi} =$

$= j\omega \underbrace{X_m e^{j\varphi}}_{\bar{X}} = j\omega \bar{X}$

$\frac{d^2}{dt^2} x(t) = \frac{d}{dt}\left(\frac{d}{dt} x(t)\right) \longrightarrow (j\omega)^2 \bar{X} = -\omega^2 \bar{X}$

$\frac{d^n}{dt^n} x(t) \longleftrightarrow (j\omega)^n \bar{X}$



$e(t) = E \cos(\omega t + \varphi)$

V_C di REGIME ?

$C \frac{dV_C}{dt} = -\frac{V_C - e(t)}{R} \Rightarrow$

$\frac{dV_C}{dt} = -\frac{1}{RC} V_C + \frac{e(t)}{RC}$

$V_{C_{rp}}(t) = V \cos(\omega t + \varphi)$ $\bar{V}_{C_{rp}} = V e^{j\varphi}$

$\frac{d}{dt} v_{C_{rp}}(t) \longrightarrow j\omega V e^{j\varphi}$

$j\omega \bar{V}_C = -\frac{1}{RC} \bar{V}_C + \frac{1}{RC} E e^{j\varphi}$

$\bar{V}_C \left(j\omega + \frac{1}{RC}\right) = \frac{1}{RC} e^{j\varphi} E$

$\bar{V}_C \frac{1 + j\omega RC}{RC} = \frac{1}{RC} e^{j\varphi} E$

$\bar{V}_C = \frac{e^{j\varphi} E}{1 + j\omega RC}$ $V_{C_{rp}}(t) = \mathrm{Re}\{\bar{V}_C e^{j\omega t}\}$

$V_{C_{rp}}(t) = \mathrm{Re}\left\{\frac{(1 - j\omega RC) e^{j\varphi}}{1 + (\omega RC)^2} e^{j\omega t}\right\} = \frac{E}{1 + (\omega RC)^2} \mathrm{Re}\{(1 - j\omega RC) \cdot$

$= \frac{E}{1 + (\omega RC)^2} \{\cos(\omega t + \varphi) + RC\omega \sin(\omega t + \varphi)\}$ $(\cos(\omega t + \varphi) + j \sin(\omega t + \varphi))\}$

quando un regime raggiungibile (con AC)

$V(t)$ di regime ?

$e(t) = E \cos \omega t$

$\frac{V}{R} = \frac{v_L - e(t)}{R}$ $i_L + \frac{L \frac{di_L}{dt} - e(t)}{R} = \alpha\left(L \frac{di_L}{dt} - e(t)\right)$

$L\left(\frac{1}{R} - \alpha\right) \frac{d}{dt} i_L = -i_L + e(t)\left(\frac{1}{R} - \alpha\right)$

$\frac{d}{dt} i_L = -\frac{R}{L(1 - \alpha R)} i_L + e(t) \frac{1 - \alpha R}{R} \frac{R}{L(1 - \alpha R)}$

$\frac{d}{dt} i_L = -\frac{R}{L(1 - \alpha R)} i_L + \frac{E}{L} \cos \omega t$

$j\omega \bar{i}_L = -\frac{R}{L(1 - \alpha R)} \bar{i}_L + \frac{E}{L}$

$\bar{i}_L \left(j\omega + \frac{R}{L(1 - \alpha R)}\right) = \frac{E}{L}$

$1 - \alpha R > 0$

Questa condizione è ulteriore rispetto alla condizione di stabilità

$\bar{i}_L = \frac{(1 - \alpha R) E}{R + j\omega L(1 - \alpha R)}$

$i_L(t) = \mathrm{Re}\{\bar{i}_L e^{j\omega t}\} = \mathrm{Re}\left\{\frac{(1 - \alpha R) E}{R + j\omega L(1 - \alpha R)} e^{j\omega t}\right\} =$

$\frac{(1 - \alpha R) E}{R^2 + \omega^2 L^2 (1 - \alpha R)^2} \mathrm{Re}\{(R - j\omega L(1 - \alpha R))(\cos \omega t + j \sin \omega t)\} =$

$= \gamma \left(R \cos \omega t + \omega L (1 - \alpha R) \sin \omega t\right)$

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