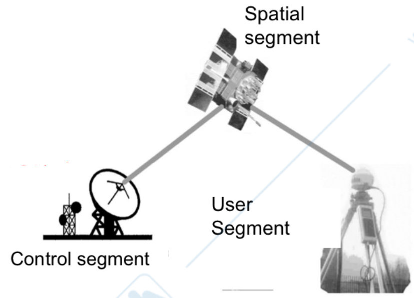
GNSS → GLOBAL NAVIGATION SATELLITE SYSTEM



- GPS → GLOBAL POSITIONING SYSTEM → USA
- GLONASS → GLOBALNAYA NAVIGATSIONNAYA SPUTNIKOVAYA SISTEMA → RUSSIA
- GALILEO → EUROPEAN

TO DETERMINE THE COORDINATE OF OUR ZONE WE NEED TO DETERMINE REFERENCE SYSTEM

SPATIAL SEGMENT → MADE UP BY SEVERAL SATELLITES ORBITING. MEO (MEDIUM EARTH ORBIT)
↳ ROLE → MANTAIN AN ACCURATE TIME REFERENCE AND TO BROADCAST SIGNAL FOR DEVICES POSITIONING.
↳ RECEIVE AND STORE INFORMATION FROM THE CONTROL SEGMENTS.
→ IT CAN NOT TAKE BACK SIGNALS

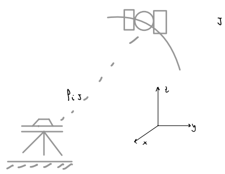
CONTROL SEGMENT → IT PERFORMS THE TRACKING OF THE SATELLITES AND CHECK THEIR CLOCKS (AND BIAS ESTIMATION)
↳ IT PERFORMS ORBITAL CORRECTIONS AND SEND USEFUL INFO
↳ UPLOADING NEW INFORMATIONS INTO SATELLITES

USER SEGMENTS → EACH PERSON WHO HAS A RECEIVER AND AN ANTENNA ABLE TO ACQUIRE THE SIGNAL

GPS STARTED ON THE 22/2/1978 (1° ONE, DEVELOPED IN USA)
↳ DEVELOPMENT OF CONTROL SEGMENT QUALITY → PRECISION OF BROADCAST EPHEMERIDS (COORDINATES) IS INCREASED FROM 3 IN 1994 TO 1 M IN 2005

→ THE CONCEPT OF SATELLITE POSITIONING → TRILATERATION ON SPACE WITH A SURVEY RANGE OF AT LEAST 4 SATELLITES. WE CAN WRITE A RANGE EQUATION (↓ PSEUDO RANGE) FOR EACH SATELLITE S

$$\rho_i^s(t) = \sqrt{(X^s(t)-X_i)^2+(Y^s(t)-Y_i)^2+(Z^s(t)-Z_i)^2}$$



$(XYZ)^s$: SATELLITE COORDINATES (EPHEMERIS)
$(XYZ)_i$: RECEIVER COORDINATES (UNKNOWN)
→ IN THE WGS 84 RS. RELATION WITH THE GEODETIC COORDINATES (φ, λ, h)

$$X = (N+h)\cos\varphi\cos\lambda$$
$$Y = (N+h)\cos\varphi\sin\lambda$$
$$Z = [N(1-e^2)+h]\sin\varphi$$

where $\begin{cases} N = a/W \\ W = \sqrt{1-e^2\sin^2\varphi} \end{cases}$

NB → SATELLITE COORDINATES ARE KNOWN
IN WGS 84 REFERENCE SYSTEM THERE ARE SOME RELATION BETWEEN USER DEVICE (ROVER) COORDINATES AND GEODETIC COORDINATES.
- CHANGING SATELLITES COULD RESULT IN A CHANGE OF COORDINATE AND WE MUST BE PRECISE WITH TRANSFORMATION.

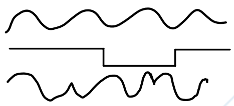
→ EVERYTHING DEPENDS ON THE CAPACITY TO COMMUNICATE BETWEEN THE SATELLITE AND THE ROVER.

SATELLITE SIGNALS

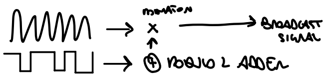
3 MAIN SIGNALS

- SINUSOIDAL COMPONENTS → CARRIER PHASE → WAVELENGHT AROUND 20 CM
  ↳ DEPENDING ON THE FREQUENCY

- IMPULSIVE COMPONENTS (CODE OR PSEUDORANGE) → TRANSITION OF +1 AND -1. THEY PRODUCE AN OFFSET OF π ON THE PHASE.



- BROADCAST → SIGNAL COMPOSED OF A CARRIER THAT IS MODULATED BY DIFFERENT CODES



FREQUENCIES

DEPENDING ON THE CONSTELLATION OF SATELLITES, THERE ARE DIFFERENT FREQUENCIES. EACH CONSTELLATION HAVE 3 DIFFERENT BANDS.
GALILEO HAS 4 IN ORDER TO GUARANTEE THE OVERLAPPING WITH OTHER CONSTELLATIONS. → WE NEED TO TIME - SYNCRONIZE ONE WITH THE OTHERS

HOW TO DETERMINE THE FREQUENCIES?

→ FOR GPS, WE HAVE AN ELEMENTAL / FUNDAMENTAL THAT WORKS AS FACTOR FOR ALL THE OTHERS.
→ LOWER FREQUENCIES L5 ARE MORE NOISE RESILIENT AND THEREFORE UTILIZED TO IMPROVE THE ESTIMATION OF IONOSPHERIC DELAY

GPS HAS CDMA: CODE DIVISION MULTIPLE ACCESS. THEIR MODULATION IS BASED ON CODE, HENCE RECEIVER ARE ABLE TO DISTINGUISH THE SATELLITE THAT IS TRANSMITTING BY USING THE CODE.

GLONASS HAS FDMA: FREQUENCY DIVISION MULTIPLE ACCESS. THIS MEANS THAT THE SATELLITES HAVE DIFFERENT FREQUENCY BASED ON A FUNDAMENTAL AND ON A FACTOR. EVERY SATELLITE HAS A PROPER FREQUENCY.

$F_1 = 1602 + 0.5625\,K$ MHz
$F_2 = 1246 + 0.4375\,K$ MHz

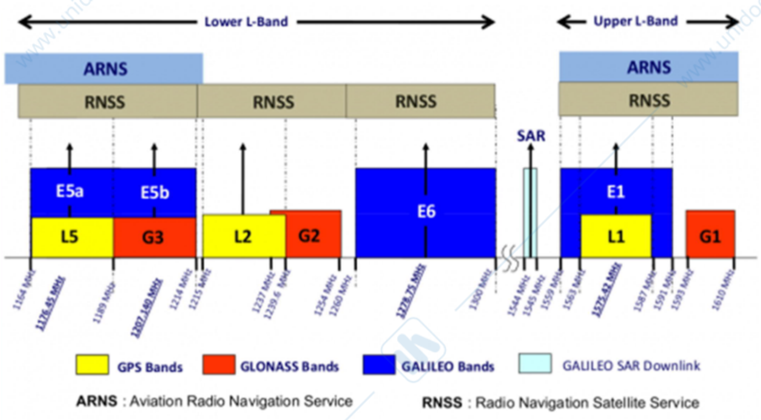
$\frac{f_1}{f_2} = \frac{9}{7}$

$-7 < K < 8$

- THE SATELLITE OF K BLOCK WILL BE THE THIRD FREQUENCY: $f_3 = \frac{3}{4} f_1$

GALILEO USES CDMA MODULATION. IT HAS ALSO MANY MORE SERVICES, OPEN, COMMERCIAL...

Frequencies



f0= 10.23MHz (154, 120, 115*f0)

GALILEO CONSTELLATION → REFERENCE TERRESTRIAL CONTROL SEGMENT → GTRF (GALILEO TERRESTRIAL REFERENCE FRAME)

BEIDOU CONSTELLATION 16 ACTIVE SATELLITES

TIME SYNCRONIZATION REFERENCE SYSTEM → GPS: 3(x,y,z) + 1 UNKNOWN → CLOCK + GLONASS: 1 MORE UNKNOWN, SYNCHRO BETWEEN GPS AND GLONASS TIME SCALE

SATELLITE POSITIONING
PRINCIPLES AND STRATEGIES

WE PREVIOUSLY SAW THE "STAND ALONE" APPROACH THAT TELLS US THE POSITION OF THE RECEIVE BASING THE CALCULATION ON PITAGORA'S THEOREM AND SATELLITE KNOWN POSITION.

→ HOW DO WE MEASURE SATELLITE DISTANCE? → WE USE TIME AS MEASURE

4 SATELLITES ARE NEEDED TO SOLVE THE SYSTEM (3 SPATIAL + 1 TEMPORAL)

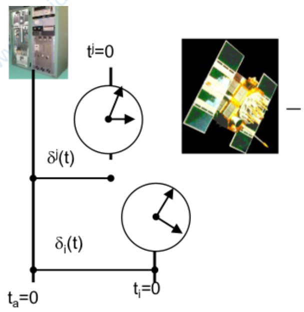
POINT POSITIONING

ASSUMING THAT THE SIGNAL COMING FROM THE SATELLITE IS CONTINUOUS, RECEIVER RECEIVE THE SIGNAL AND KNOWN WHAT SATELLITE IS THAT (CDMA OR FDMA).

ONCE THE RECEIVER IS ON, IT RECEIVES A SIGNAL FROM SATELLITES. THESE SIGNALS CONTAIN THE TIME-INFORMATION, IT TELLS THE EXACT TIME OF DEPARTURE OF THE SIGNAL FROM THE SATELLITE.

→ BY MEASURING THE TIME INTERVAL BETWEEN THIS INFORMATION AND THE PROPER INTERNAL TIME OF THE DEVICE, THE RECEIVER KNOWS THE Δt THAT THAT THE SIGNAL TOOK TO RUN AND, BY MULTIPLING FOR THE SPEED OF LIGHT, IT COMPUTES THE SATELLITE DISTANCE.

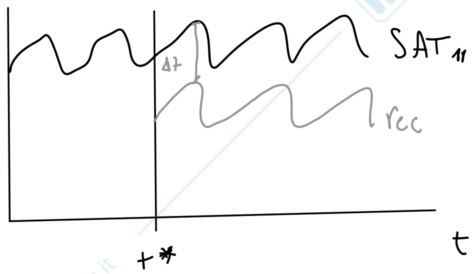
BIGGEST ERROR → CLOCK OFFSET *



$$R_i^s(t) = c(t^s - t_i) = c\Delta t$$

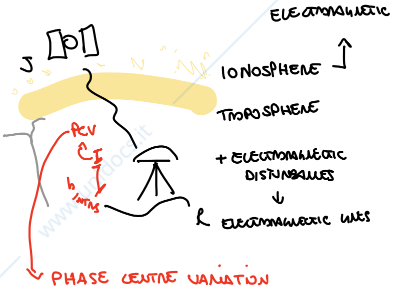
$$* \; R_i^s = c[\Delta t + \delta^s(t) - \delta_i(t)] = c[\Delta t + \underbrace{\delta_i^s(t)}_{\text{UNKNOWN}}]$$

$\delta_i(t)$ = OFFSET RECEIVER CLOCK ($\sim 10^{-3}\,s \cdot 3 \cdot 10^8\,m/s = 300\,km$)



CORRELATION BETWEEN SIGNALS

1) PSEUDORANGE OR CODE MEAS → SINUSOIDAL COMPONENTS
2) CARRIER PHASE MEAS.
3) BROADCAST → EPHEMERIDS



$$R = c\,\Delta t = \rho_R^s + c\,\Delta t$$

↳ ERROR → TIME OFFSET

$$= \rho_R^s + c(t^s - t_R)$$

$$= \rho_R^s + c\Delta t$$

$$= + c(t^s - t_R)$$

$$= T_R^s + I_R^s + E^s + PCV + \varepsilon$$

↳ SITE SPECIFIC ERRORS

· ERROR YOU REMOVE WHEN YOU KNOW THE SATELLITE

\+ UNCERTAINTY RELATED TO SATELLITE POSITION → EPHEMERIDS ERRORS

\+ INSTRUMENTAL HEIGHT

$$R = \sqrt{(x^s - x_R)^2 + (y^s - y_R)^2 + (z_s - z_R)^2} + c(t^s - t_R) + T_R^s + I_R^s + E_s + \varepsilon$$

UNDER CERTAIN CONDITIONS } POST-PROC 2-5 mm; REAL-TIME 2-5 cm

REAL TIME	POST PROCESSES
	1) RELATIVE POSITION: SINGLE, DOUBLE, TRIPLE } DIFFERENCES
	2) MODEL USING DATA
	3) COMBINATIONS

IF I USE ANOTHER SATELLITE AND I DO THE DIFFERENCE, I TAKE AWAY 1 BIAS — SINGLE DIFFERENCE

$$R_R^J = \rho_R^J + c\,(t^J - \cancel{t_R}) + T_R^J + I_R^J + E_R^J + \varepsilon \quad (-)$$

$$R_R^K = \rho_R^K + c\,(t^K - \cancel{t_R}) + T_R^K + I_R^K + E_R^K + \varepsilon$$

$$R_R^{JK} = \rho_R^{K} + c\,t^{JK} + T_R^{JK} + I_R^{JK} + E_R^{JK} + \varepsilon^{JK}$$

IF I PUT ANOTHER RECEIVER →  M (x_M, y_M, z_M) KNOWN

R_R^{JK} . . .

$$R_M^{JK} = \rho_M^{K} + c\,t^{JK} + T_M^{JK} + I_M^{JK} + E_M^{JK} + \varepsilon^{JK}$$

→ WE CAN TAKE OF 2 BIAS

· IF THE DISTANCE BETWEEN THE 2 RECEIVER IS LESS THAN 20 KM I^{JK} AND T^{JK} CAN BE CONSIDERED NEGLIGIBLE

$$R_{MR}^{JK} = \rho_{MR}^{JK} + \Delta T_{MR}^{JK} + \Delta I_{MR}^{JK}$$

DOUBLE DIFFERENCE

ALSO E_{MR}^{JK} CAN BE REMOVED, BETWEEN THE SATELLITES. THEY ARE THE SAME

PSEUDO RANGE → LESS ACCURATE WE HAVE → COMBINATION BETWEEN SIMULATION AND METEOROLOGICAL ONE

→ POINT POSITIONING WITH CODE MEASUREMENTS

POSITIONING IS REALISED EPOCH BY EPOCH.

IF I CONSIDER 5 SATELLITES (APEX), THE LINEARIZED MODEL IS

$$\begin{bmatrix} \frac{X_i^{(0)}-X^{(1)}}{\rho_i^{1(0)}} & \frac{Y_i^{(0)}-Y^{(1)}}{\rho_i^{1(0)}} & \frac{Z_i^{(0)}-Z^{(1)}}{\rho_i^{1(0)}} & -1 \\ \frac{X_i^{(0)}-X^{(2)}}{\rho_i^{2(0)}} & \frac{Y_i^{(0)}-Y^{(2)}}{\rho_i^{2(0)}} & \frac{Z_i^{(0)}-Z^{(2)}}{\rho_i^{2(0)}} & -1 \\ \frac{X_i^{(0)}-X^{(3)}}{\rho_i^{3(0)}} & \frac{Y_i^{(0)}-Y^{(3)}}{\rho_i^{3(0)}} & \frac{Z_i^{(0)}-Z^{(3)}}{\rho_i^{3(0)}} & -1 \\ \frac{X_i^{(0)}-X^{(4)}}{\rho_i^{4(0)}} & \frac{Y_i^{(0)}-Y^{(4)}}{\rho_i^{4(0)}} & \frac{Z_i^{(0)}-Z^{(4)}}{\rho_i^{4(0)}} & -1 \\ \frac{X_i^{(0)}-X^{(5)}}{\rho_i^{5(0)}} & \frac{Y_i^{(0)}-Y^{(5)}}{\rho_i^{5(0)}} & \frac{Z_i^{(0)}-Z^{(5)}}{\rho_i^{5(0)}} & -1 \end{bmatrix} \begin{pmatrix} x_i \\ y_i \\ z_i \\ c\delta_i \end{pmatrix} - \begin{pmatrix} R_i^{(1)}+c\delta^{(1)}-\rho_i^{1(0)} \\ R_i^{(2)}+c\delta^{(2)}-\rho_i^{2(0)} \\ R_i^{(3)}+c\delta^{(3)}-\rho_i^{3(0)} \\ R_i^{(4)}+c\delta^{(4)}-\rho_i^{4(0)} \\ R_i^{(5)}+c\delta^{(5)}-\rho_i^{5(0)} \end{pmatrix} = \begin{pmatrix} \nu^{(1)} \\ \nu^{(2)} \\ \nu^{(3)} \\ \nu^{(4)} \\ \nu^{(5)} \end{pmatrix}$$

· IT IS BETTER TO REALISE THE RECEIVER CLOCK OFFSET IN METER (MULTIPLIED BY SPEED TIME) IN ORDER TO AVOID AN ILL-CONDITIONED MATRIX.

· 3 DIFFERENT TIME REFERENCE SCALE → 3 DIFFERENT RECEIVER OFFSET

G = GPS

R = GLONASS

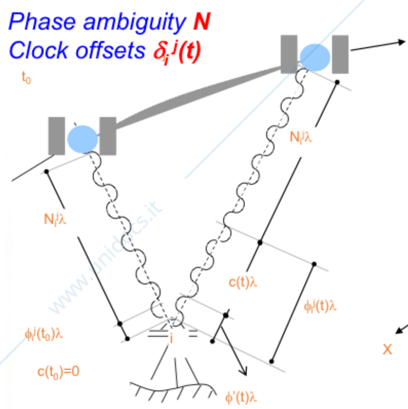
E = GALILEO

$$\begin{bmatrix} \left(\frac{X_r-X^{1G}}{\rho_r^{1G}}\right)_{(0)} & \left(\frac{Y_r-Y^{1G}}{\rho_r^{1G}}\right)_{(0)} & \left(\frac{Z_r-Z^{1G}}{\rho_r^{1G}}\right)_{(0)} & -1 & 0 & 0 \\ \left(\frac{X_r-X^{2G}}{\rho_r^{2G}}\right)_{(0)} & \left(\frac{Y_r-Y^{2G}}{\rho_r^{2G}}\right)_{(0)} & \left(\frac{Z_r-Z^{2G}}{\rho_r^{2G}}\right)_{(0)} & -1 & 0 & 0 \\ \left(\frac{X_r-X^{3G}}{\rho_r^{3G}}\right)_{(0)} & \left(\frac{Y_r-Y^{3G}}{\rho_r^{3G}}\right)_{(0)} & \left(\frac{Z_r-Z^{3G}}{\rho_r^{3G}}\right)_{(0)} & -1 & 0 & 0 \\ \left(\frac{X_r-X^{1R}}{\rho_r^{1G}}\right)_{(0)} & \left(\frac{Y_r-Y^{1R}}{\rho_r^{1G}}\right)_{(0)} & \left(\frac{Z_r-Z^{1R}}{\rho_r^{1G}}\right)_{(0)} & 0 & -1 & 0 \\ \left(\frac{X_r-X^{2R}}{\rho_r^{2G}}\right)_{(0)} & \left(\frac{Y_r-Y^{2R}}{\rho_r^{2G}}\right)_{(0)} & \left(\frac{Z_r-Z^{2R}}{\rho_r^{2G}}\right)_{(0)} & 0 & -1 & 0 \\ \left(\frac{X_r-X^{1E}}{\rho_r^{1G}}\right)_{(0)} & \left(\frac{Y_r-Y^{1E}}{\rho_r^{1G}}\right)_{(0)} & \left(\frac{Z_r-Z^{1E}}{\rho_r^{1G}}\right)_{(0)} & 0 & 0 & -1 \\ \left(\frac{X_r-X^{2E}}{\rho_r^{1G}}\right)_{(0)} & \left(\frac{Y_r-Y^{2E}}{\rho_r^{1G}}\right)_{(0)} & \left(\frac{Z_r-Z^{2E}}{\rho_r^{1G}}\right)_{(0)} & 0 & 0 & -1 \end{bmatrix} \begin{pmatrix} \Delta X_r \\ \Delta Y_r \\ \Delta Z_r \\ c\delta_r^G(t) \\ c\delta_r^R(t) \\ c\delta_r^E(t) \end{pmatrix} - \begin{pmatrix} R_r^{1G}+c\delta^{1G}-\rho_r^{1G(0)} \\ R_r^{2G}+c\delta^{2G}-\rho_r^{2G(0)} \\ R_r^{3G}+c\delta^{3G}-\rho_r^{3G(0)} \\ R_r^{1R}+c\delta^{1R}-\rho_r^{1R(0)} \\ R_r^{2R}+c\delta^{2R}-\rho_r^{2R(0)} \\ R_r^{1E}+c\delta^{1E}-\rho_r^{1E(0)} \\ R_r^{2E}+c\delta^{2E}-\rho_r^{2E(0)} \end{pmatrix} = \nu$$

· POINT POSITIONING WITH CARRIER PHASE MEASUREMENTS

L1, L2, L5 ($\lambda \sim 20$ cm)

RANGE = Σ ϕ (PHASE DISPLACEMENTS) + n° CYCLES FROM t_0



$$\phi_i^j(t) = \frac{1}{\lambda}\rho_i^j(t) + N_i^j + f^j \Delta\delta_i^j(t)$$

→ N IS A NEW UNKNOWN FOR EACH SATELLITE

→ N IS CONSTANT IF CONTINUOUS SATELLITE TRACKING (NO CYCLE SLIP)

→ POSITIONING FEASIBLE ONLY WITH N ESTIMATION

HERE, PHASE AMBIGUITIES ARE INCLUDED.

$$\begin{bmatrix} \frac{X_i^{(0)}-X^{(1)}}{\rho_i^{1(0)}} & \frac{Y_i^{(0)}-Y^{(1)}}{\rho_i^{1(0)}} & \frac{Z_i^{(0)}-Z^{(1)}}{\rho_i^{1(0)}} & \lambda_i^{(1)} & 0 & 0 & 0 & 0 & -1 \\ \frac{X_i^{(0)}-X^{(2)}}{\rho_i^{2(0)}} & \frac{Y_i^{(0)}-Y^{(2)}}{\rho_i^{2(0)}} & \frac{Z_i^{(0)}-Z^{(2)}}{\rho_i^{2(0)}} & 0 & \lambda_i^{(2)} & 0 & 0 & 0 & -1 \\ \frac{X_i^{(0)}-X^{(3)}}{\rho_i^{3(0)}} & \frac{Y_i^{(0)}-Y^{(3)}}{\rho_i^{3(0)}} & \frac{Z_i^{(0)}-Z^{(3)}}{\rho_i^{3(0)}} & 0 & 0 & \lambda_i^{(3)} & 0 & 0 & -1 \\ \frac{X_i^{(0)}-X^{(4)}}{\rho_i^{4(0)}} & \frac{Y_i^{(0)}-Y^{(4)}}{\rho_i^{4(0)}} & \frac{Z_i^{(0)}-Z^{(4)}}{\rho_i^{4(0)}} & 0 & 0 & 0 & \lambda_i^{(4)} & 0 & -1 \\ \frac{X_i^{(0)}-X^{(5)}}{\rho_i^{5(0)}} & \frac{Y_i^{(0)}-Y^{(5)}}{\rho_i^{5(0)}} & \frac{Z_i^{(0)}-Z^{(5)}}{\rho_i^{5(0)}} & 0 & 0 & 0 & 0 & \lambda_i^{(5)} & -1 \end{bmatrix} \begin{pmatrix} x_i \\ y_i \\ z_i \\ N_i^{(1)} \\ N_i^{(2)} \\ N_i^{(3)} \\ N_i^{(4)} \\ N_i^{(5)} \\ c\delta_i \end{pmatrix} - \begin{pmatrix} \phi_i^{(1)}+c\delta^{(1)}-\rho_i^{1(0)} \\ \phi_i^{(2)}+c\delta^{(2)}-\rho_i^{2(0)} \\ \phi_i^{(3)}+c\delta^{(3)}-\rho_i^{3(0)} \\ \phi_i^{(4)}+c\delta^{(4)}-\rho_i^{4(0)} \\ \phi_i^{(5)}+c\delta^{(5)}-\rho_i^{5(0)} \end{pmatrix} = \begin{pmatrix} \nu^{(1)} \\ \nu^{(2)} \\ \nu^{(3)} \\ \nu^{(4)} \\ \nu^{(5)} \end{pmatrix}$$

→ IT IS NECESSARY TO COLLECT MORE EPOCHS TO ESTIMATE THE AMBIGUITIES.

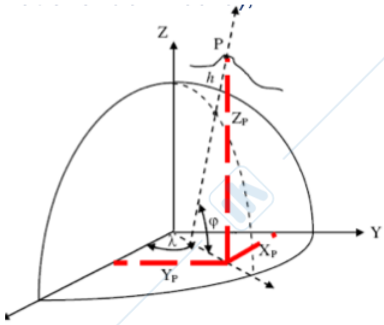
→ WHEN N IS ESTIMATED, THEY ARE REMOVED BY UNKNOWNS.

↳ INITIALISATION

REFERENCE SYSTEM OF GPS POSITIONING

WGS84 (WORLD GEODETIC SYSTEM 1984) IS A GEOCENTRIC SYSTEM OF ECEF (EARTH CENTRED EARTH FIXED) DEFINED BY:

→ ORIGIN IN THE MASS CENTRE OF THE EARTH
→ Z ORIENTED ALONG WITH THE CONVENTIONAL POLE SUGGESTED BY IERS
→ X ORIENTED ALONG THE INTERSECTION POINT BETWEEN THE EQUATORIAL PLANE AND THE "ZERO" MEDIAN, DEFINED BY (BUREAU INTERNATIONAL DE L'HEURE)
→ Y COMPLETES THE CLOCKWISE TERN



WGS84 WAS IDENTICAL TO DATUMNAD83 (NORTH AMERICA DATUM, 1983) WHICH USES THE ELLIPSOID GRS80

TIME REFERENCE SCALE

→ TIME IS ONE OF THE MAIN OBSERVATION IN THE GNSS POSITIONING, AND EACH SYSTEM HAS ITS OWN TIME REFERENCE SYSTEM. MORE IMPORTANT STANDARD:

• TAI (INTERNATIONAL ATOMIC TIME) → DEFINED BY THE CAESIUM ATOM'S RADIATION AND MANTAINED BY 200 ATOMIC CLOCKS LOCATED IN MORE THAN 70 LABORATORIES.
↳ VERY ACCURATE. INSTANT 0 IS JANUARY 1ST 1958, IN A DAY TAI → 86400 SECONDS.

• GMT (GREENWICH MEAN TIME MEDIUM) → BASED ON CELESTIAL OBSERVATIONS RELATED TO EARTH'S ROTATION TIME. CHANGES IN THE ROTATIONAL SPEED OF THE DELAY WITH RESPECT TO THE GMT TIME OF ATOMIC CLOCKS

• UTC (COORDINATES UNIVERSAL TIME) → DERIVED FROM GMT, IT IS NOT BASED ON CELESTIAL PHENOMENA BUT IS MANTEINED BY ATOMIC CLOCKS. → MAXIMUM DEVIATION 0.9 S, EVERY 1.S MEANS BY APPLYING CORRECTION OF 1 OR LEAP SECOND, SINCE 1972.

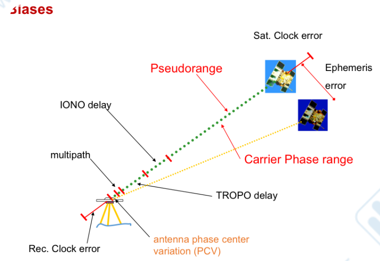
THERE IS A 37 SECOND TIME DIFFERENCE BETWEEN TAI AND UTC

GLONASS TIME → KEPT BY THE CLOCK OF THE CENTRAL SYNCHRONIZER MASER (CS) TIME AND IS PERIODICALLY CORRECTED BY AN INTEGER NUMBER OF SECONDS (LEAP SECONDS) TO ALIGN IT WITH THE UTC TIME · SOVIET UTC (SU) = UTC + 3 HOURS (DIFFERENCE OF 1 ms)

GALILEO TIME → GST (GALILEO SYSTEM TIME): STARTS AT 00:00 ON 22 AUGUST 1999. NAVIGATION MESSAGE CONTAINS THE COEFFICIENTS USED TO CALCULATE THE OFFSET BETWEEN GST AND UTC SCALE

BEIDOU TIME SCALE → BDT (BEIDOU NAVIGATION SATELLITE SYSTEM TIME). TIME STARTS ON 00:00:00 ON JANUARY 1ST 2006, UTC TIME.

BIASES AND CORRELATIONS



→ ATMOSPHERE INTERACTION WITH THE EM SIGNAL BROADCASTED BY THE SATELLITE.

1) IONOSPHERE AND TROPOSPHERE INTERACT AND DISTURB THE SIGNAL INTRODUCING BIAS
2) SATELLITE POSITION ERRORS AND ANTENNA RELATED ERRORS.
3) ERRORS RELATED TO BOTH SATELLITE AND RECEIVER CLOCK.

BIASES CAN BE SPATIAL OR TIME-CORRELATED (SPATIAL CORRELATION IS USUALLY STRONGER) DEPENDING OR NOT ON SIGNAL FREQUENCY (DIFFERENT ON L1 AND L2)

Nature of bias	*Bias*	Spatial correlation
Not dispersive (frequency independent)	Ephemeris	High on big distance (>100 km)
	Troposphere	Regional (about 10 km)
Not dispersive (but frequency dep.)	Receiver clock	Identical on the same station
	Satellite clock	Identical on the same satellite
Dispersive (depending on f and they are different on L1 o L2)	Ionosphere	Regional (about 10 km)
	Multipath	No correlation. Site depending
	center phase antenna variation	No correlation. Site depending

DISPERSIVE AND NOT DISPERSIVE BIAS. NOT DISPERSIVE BIAS DO NOT DEPEND ON FREQUENCY.

INTER CHANNEL BIAS RELATED TO THE RECEIVER COMPONENT.

INTER-CHANNEL BIAS

THE SIGNAL HAS MADE THE FOLLOWING PATHS:

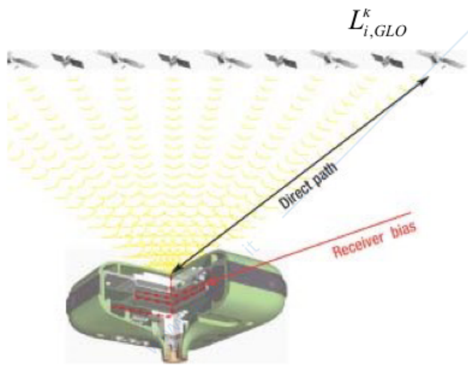
- SATELLITE-ANTENNA (IT IS UNIQUE FOR ALL SATELLITES)
- FROM ANTENNA TO ELECTRONICAL RECEIVER COMPONENTS

→ SEVERAL BIASES FOR EACH SATELLITE K

IT IS DIVIDED IN:

- CONSTANT PART $L_{i,GLO}$
- INTER-CHANNEL BIASES δ

$L_{i,GLO}^{k} = L_{i,GLO} + \delta_{i,GLO}^{k}$



$$R_{i,GLO}^{k}(t) + c\delta_{i,GLO}^{k}(t) = \rho_{i,GLO}^{k}(t) - c\delta_{i,GLO}(t) + cL_{i,GLO}^{k} =$$
$$= \rho_{i,GLO}^{k}(t) - c\left(\delta_{i,GLO}(t) - L_{i,GLO}\right) + c\,\delta_{i,IC}^{k}$$

→ INTER-CHANNEL BIAS ESTIMATION IS MANDATORY WITH CARRIER PHASE POSITIONING

ERRORS AND MEASUREMENTS

VALUE' = VALUE + BIASES + AE + GE
where

$$P_r^j = \rho_r^j - cdT_r + cdt^j + I_r^i + T_r^j + E_r^j + \mu_r^j + \varepsilon$$

$$\boxed{\phi_r^j = \rho_r^j - cdT_r + cdt^j - I_r^i + T_r^j + E_r^j + \mu_r^{\phi j} + \varepsilon^{\phi} - \lambda N_r^j}$$

VALUE = real values
BIASES = systematic errors
AE = accidental errors → measurements noise → 1% λ
GE = gross errors (outliers) → errors due to the human factor

BIASES SUMMARY

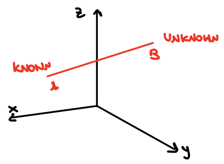
THE PRESENCE OF BIASES DECREASES POSITION ACCURACY. FOR HIGH POSITIONING ACCURACY, WE CAN FOLLOW 3 WAYS:

1) RELATIVE OR DIFFERENTIAL POSITIONING. → TO REDUCE BIAS BY DIFFERENCING THE POSITION OF 2 RECEIVERS
- • RELATIVE POSITIONING → POST-PROCESSING
- •• DIFFERENTIAL POSITIONING → REAL-TIME

2) TO COMBINE OBSERVATIONS AND FREQUENCIES
- • IONO-FREE: TO REMOVE THE IONOSPHERIC DELAY, MAKING A COMBINATION WITH L_1 AND L_2
- •• GEOMETRY-FREE: TO REMOVE THE TROPO+EPH COMPONENTS

3) TO MODEL BIASES USING MATHEMATICAL MODELS
- • KLOBUCHAR MODEL FOR IONOSPHERIC DELAY
- •• SAASTAMOINEN OR HOPFIELD FOR TROPOSPHERIC DELAY
- ••• POLYNOMIALS FOR CLOCK ERRORS

RELATIVE POSITIONING



THE GOAL IS TO ESTIMATE BIASES BY USING A KNOWN POINT. THE UNKNOWN POINT POSITION IS THEN EVALUATED. THE "BASELINE" VECTOR CONNECTS THE TWO. THE BASELINE CAN BE ESTIMATED PERFORMING SEVERAL MEASURE ON A.

• RELATIVE POSITIONING REQUIRES SIMULTANEOUS OBSERVATIONS. LINEAR COMBINATION CAN BE PERFORMED

$$X_B = X_A + b_{AB}$$

• RELATIVE POSITIONING CAN BE PERFORMED WITH CODE OR CARRIER PHASE →

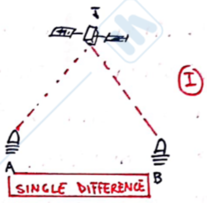
$$b_{ab} = \begin{bmatrix} X_b - X_a \\ Y_b - Y_a \\ Z_b - Z_a \end{bmatrix} = \begin{bmatrix} \Delta X_{AB} \\ \Delta Y_{AB} \\ \Delta Z_{AB} \end{bmatrix}$$

SINGLE DIFFERENCE

→ TWO POINTS (A,B) AND ONE SATELLITE (J) ARE INVOLVED. TWO CARRIER PHASE EQUATIONS CAN BE WRITTEN, ONE FOR RECEIVER A AND ONE FOR RECEIVER B, BY REFERRING TO THE SAME SATELLITE. (I)

BY SUBTRACTING THESE, WE GET A CARRIER-PHASE DIFFERENCE BETWEEN THE TWO AT THE SAME TIME
→ WE ALSO REDUCE THE SATELLITE CLOCK BIAS ERROR → THE TROPOSPHERIC AND IONOSPHERIC ERRORS ARE SO LITTLE IN COMPARISON WITH OTHERS (SATELLITE ALTITUDE > 20000 km, BASELINE SHORTER THAN 20-50 km) THAT THEY CAN BE NEGLECTED.

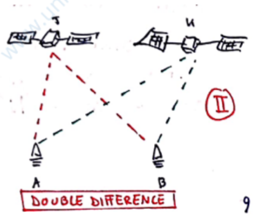
→ WE ARE NOW LEFT WITH THE RECEIVER CLOCK ERROR, THAT IS REMOVABLE BY LOOKING AT A SECOND SATELLITE K. (II)



$$\Phi_A^{j}(t) - f^{j}\delta^{j}(t) = \frac{1}{\lambda}\rho_A^{j}(t) + N_A^{j} - f^{j}\delta_A(t)$$
$$\Phi_B^{j}(t) - f^{j}\delta^{j}(t) = \frac{1}{\lambda}\rho_B^{j}(t) + N_B^{j} - f^{j}\delta_B(t)$$
$$\Phi_{AB}^{j}(t) = \frac{1}{\lambda}\rho_{AB}^{j}(t) + N_{AB}^{j} - f^{j}\delta_{AB}(t)$$

• WE'LL HAVE 2 DIFFERENCES, SUBTRACTING THEM, RECEIVER CLOCK ERROR WILL BE DELETED. THIS CAN BE DONE BECAUSE J AND K CLOCKS ARE PERFECTLY ESTIMATED, SO ARE PERFECTLY SYNCRONISED AND WE CAN MIX THE MEASUREMENTS PERFORMED AT THE SAME TIME.

WE ARE LEFT NOW WITH GEOMETRIC UNKNOWN (3) AND PHASE AMBIGUITY (1).



$$\Phi_{AB}^{j}(t) = \frac{1}{\lambda}\rho_{AB}^{j}(t) + N_{AB}^{j} - f^{j}\delta_{AB}(t)$$
$$\Phi_{AB}^{k}(t) = \frac{1}{\lambda}\rho_{AB}^{k}(t) + N_{AB}^{k} - f^{k}\delta_{AB}(t)$$
$$\Phi_{AB}^{jk}(t) = \frac{1}{\lambda}\rho_{AB}^{jk}(t) + N_{AB}^{jk}$$

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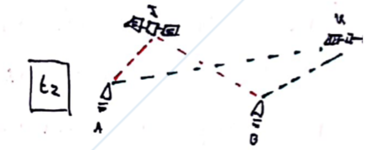
TRIPLE DIFFERENCES

TO ELIMINATE THE TIME-INDIPENDENT AMBIGUITY, TWO EPOCHS HAVE BEEN CONSIDERED. → WE CAN CANCELL EFFECTS OF THE AMBIGUITIES → THEY ARE TIME-INDIPENDENT

• DISADVANTAGE → THEY ARE STRONGLY CORRELATED AND LESS ACCURATE, TWO DIFFERENCES ARE REQUIRED FOR ONE TRIPLE → REAL TIME IS NOT POSSIBLE!



$$\Phi_{AB}^{jk}(t_1) = \frac{1}{\lambda}\rho_{AB}^{jk}(t_1) + N_{AB}^{jk}$$
$$\Phi_{AB}^{jk}(t_2) = \frac{1}{\lambda}\rho_{AB}^{jk}(t_2) + N_{AB}^{jk}$$
$$\Phi_{AB}^{jk}(t_{12}) = \frac{1}{\lambda}\rho_{AB}^{jk}(t_{12})$$

NB → RELATIVE POSITIONING IS FOR POST PROCESSING

$$N_{AB}^{Sk} = \phi_{AB}^{kS}(t) - \frac{1}{\lambda}\rho_{AB}^{Sk}(t)$$

$$\rho_{AB}^{Sk}(t) = \lambda\left[\phi_{AB}^{Sk}(t) - N_{AB}^{Sk}\right]$$

BY DOING A TRIPLE DIFFERENCE WE ESTIMATE AT LEAST A ROUGH BASELINE.
FINAL ESTIMATION IS PERFORMED INTO THE GNSS SOFTWARE AS FOLLOW:

1) ESTIMATE AN APPROXIMATE SOLUTION OF POSITIONING (FEW METERS ACCURACY)
2) SINGLE DIFFERENCES BETWEEN 2 RECEIVERS AND 1 SATELLITE, FOR EACH SATELLITE AVAILABLE
3) TRIPLE DIFFERENCES, WE GET Δx, Δy, Δt WITH FEW cm ACCURACY.
4) PHASE-AMBIGUITY IS USED INSIDE THE DOUBLE DIFFERENCE EQUATION IN ORDER TO ESTIMATE N.

THE RATIO TEST

↳ STRONG INDICATOR FOR CORRECT AMBIGUITY PHASE FIXING.

"N" IS REPRESENTING PHASES → SHOULD ALWAYS BE AN INTEGER (IT CAN BE A FLOAT)

SINCE WE ESTIMATE SEVERAL FLOATERS Ns, THEY CAN BE GIVEN TOGETHER WITH A SQUARE ERROR σ, OR VARIANCE.
BY PERFORMING THE RATIO BETWEEN THE FIRST AND SECOND SMALLER VALUES WE GET RATIO

$$\text{RATIO} = \frac{\sigma_0^2(II)}{\sigma_0^2(I)}$$

IF RATIO ≥ THRESHOLD, $\sigma_0^2(I)$ IS REFERRED TO THE RIGHT N ESTIMATION AS INTEGER

ACCEPTABLE VALUE FOR RATIO

RATIO ≥ 3 (FOR 5 km BASELINE)

UNTIL RATIO IS SMALLER THAN 3 WE SHOULD KEEP COMPUTING N TILL WE FIND THE MOST ACCURATE

EXAMPLE

$N_1 = 21.34$
$N_2 = 19 \pm 0.03$
$N_3 = 20 \pm 0.06$
$N_4 = 22 \pm 0.03$
$N_5 = 23 \pm 0.04$
$N_6 = 21 \pm 0.01$

→ $\text{RATIO} = \frac{(0{,}03)^2}{(0{,}01)^2} = 9 > 3$

N = 21 IS OK

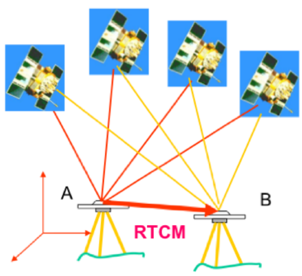
IF RATIO < 3 → $N \in \mathbb{R}$, FLOAT

DIFFERENTIAL POSITIONING

DGPS → FEW METERS ACCURACY (PERFORMED IN REAL TIME)

↳ ONE RECEIVER (MASTER) IS LOCATED ON THE REFERENCE SITE WITH KNOWN COORDINATES, AND THE REMOTE IS USUALLY MOVING

→ THE REFERENCE STATION CALCULATES THE PSEUDORANGE CORRECTION PRC AND THE RANGE RATE CORRECTION RRC WHICH ARE TRANSMITTED IN REAL TIME



$$PRC^j(t_0) = -R_A^j(t_0) + \rho_A^j(t_0)$$

→ THE ROVER APPLIES IN RT THE CORRECTION TO THE MEASURE OF PSEUDORANGES AND PERFORMS POINT POSITIONING WITH THE CORRECTED PSEUDORANGES

- PSEUDORANGE AT BASE STATION A

$$R_A^S(t_0) = \rho_A^S(t_0) + c\delta^S(t_0) - c\delta_A(t_0) + \Delta\rho_A^S(t)_{EPH} + \Delta\rho_A^S(t_0)_{IO} + \Delta\rho_A^S(t_0)_{TROPHO}$$

- PSEUDORANGE CORRECTION FOR SATELLITE S

$$PRC^S(t_0) = -c\delta^S(t_0) + c\delta_A(t_0) - \Delta\rho_A^S(t)_{EPH} - \Delta\rho_A^S(t_0)_{IO} - \Delta\rho_A^S(t_0)_{TROPHO}$$

PRC IN AN EPOCH → $PRC^S(t) = PRC^S(t_0) + RRC(t - t_0) = \rho_{AB}^S(t_0) - c\,\Delta\delta_{AB}(t)$

NB → DIFFERENTIAL POSITIONING IS FOR REAL TIME

RTK POSITIONING → FEW CM ACCURACY

↳ SAME APPROACH OF DGPS, FOR THE CARRIER PHASE

THIS APPROACH IS LIKE DOUBLE DIFFERENCING WITH ZERO LATENCY

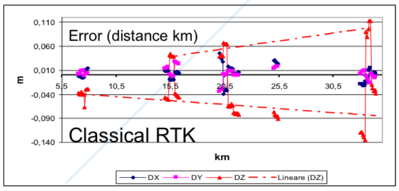
$$\lambda \phi_A^s(t_0) = \rho_A^s(t_0) + c\delta^s(t_0) - c\delta_A(t_0) + \lambda N_A^s + \Delta\rho_A^s(t)_{EPH} - \Delta\rho_A^s(t_0)_{IO} - \Delta\rho_A^s(t_0)_{TROPO}$$

BROADCASTING RTK CORRECTION HAVE TO BE PERFORMED ACCORDING TO PAPER TECHNICS.

RADIO, GSM (LONGER DISTANCES), INTERNET IP ARE THE 3 MAIN WAYS

$$\lambda \phi_B^s(t)_{corr} = \rho_B^s(t) + c\cancel{\delta_{AB}}(t) + \lambda N^s_{AB}$$

RTK positioning accuracy



N.B.: the correction degrades as the base-rover distance increases

➔ Distance ~ 15-30 km from the base station (if greater, all biases don't correctly allow the ambiguity estimation)

Idea: to model the biases within a network of master stations and create a virtual reference station (VRS) close to the rover receiver.

NETWORK REAL TIME KINEMATIC POSITIONING → FEW CENTIMETERS ACCURACY IN RT, MILLIMETERS IN POST PROCESS

A CONTROL CENTRE RECEIVES DATA FROM ALL PERMANENT STATIONS, AND A MODEL OF CORRECTION IS MADE.

↳ VALID ONLY FOR SPATIAL BIAS

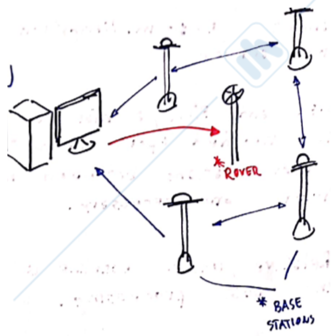
CONTROL CENTRE IS INDIPENDENT OF A SINGLE STATION

COMMUNICATION

- CONTROL CENTRE AND SINGLE PERMANENT STATION
- CONTROL CENTRE

NETWORK RTK IMPLEMENTATION

1) VIRTUAL REFERENCE STATION
2) MULTI REFERENCE STATION
3) NETWORK RTK WITH CORRECTIONS TRANSMITTING BY CELL → MASTER AUXILIARY



RECEIVERS

THERE ARE 3 MAIN CLASSES FOR DEFINING RECEIVERS

- QUALITY OF DATA, POSITIONING PERFORMANCE

— MASS MARKET RECEIVER: SINGLE SOLUTION, STAND ALONE ACCURACY, 5÷10 m. NOT PRECISE.

— SINGLE FREQUENCY: USE L1, REAL TIME OR STATIC POSITIONING, ACCURACY TILL mm-cm

— DUAL FREQUENCY: MULTIPLE CONSTELLATION, MULTIPLE FREQUENCIES, VERY HIGH ACCURACY.

TO RECAP ABOUT POSITIONING

- STAND ALONE POSITIONING LEAD ONLY TO PSEUDORANGE SOLUTIONS WITH SINGLE POSITION SOLUTION. PRECISION ABOUT 5-10 m. NAVIGATION BUT NOT SURVEY.

- DGPS AND RTK: A MASTER STATION IS NEEDED APART FROM THE RECEIVER. COMMUNICATION BETWEEN MASTER AND ROVER ARE NECESSARY.

- RELATIVE, WHERE BASELINE IS ESTIMATED IN POST PROCESSING.

Methods	Session	Baseline	Precision	Rate [s]	Note
Static	> ½ h 1 h 3÷4 h > 4 h	10 km 20÷30 km >100 km	10^{-6}÷10^{-8}	15÷60	Dual Freq. if baseline > 20 km
Rapid/static	20÷30 min (L1) 6÷8 min (L1+L2)	<10÷15km	10^{-6}	5÷15	Require a good satellite const.
Kinematic-stop and go	< 1 min (at least 2 epochs)	Some km	cm	1÷5	Good satellites visibility ➢ Up tp 30 min ➔ L1 ➢ 5÷6 min ➔ L1+L2 ➢ OTF ➔ L1+L2
Kinematic continuous	continuous	Some km	cm	1÷5 20 Hz	As stop and go