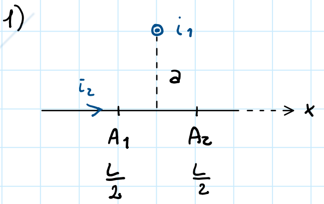
Esercitazione 10

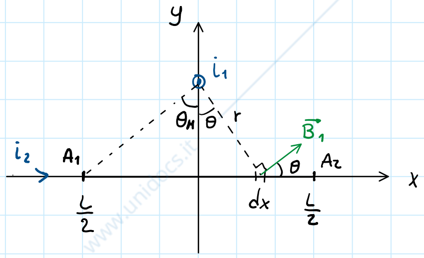
giovedì 11 dicembre 2025 10:09

Interazione tra conduttori

1)



Determinare il momento delle forze agenti sul tratto $A_1 A_2$ del conduttore percorso da i_2 orientato lungo x in seguito all'azione del conduttore percorso da i_1 orientato lungo z.



$$\vec{B}_1 = B_1 \hat{u} = B_1 \cos\theta \, \hat{u}_x + B_1 \sin\theta \, \hat{u}_y$$

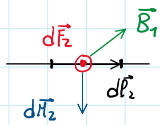
$$d\vec{l}_2 = dx \, \hat{u}_x \qquad ; \qquad B_1 = \frac{\mu_0 i_1}{2\pi r}$$

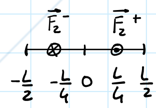
$$d\vec{F}_2 = i_2 \, d\vec{l}_2 \times \vec{B}_1 = i_2 \, dx \, B_1 \, \hat{u}_x \times (\cos\theta \, \hat{u}_x + \sin\theta \, \hat{u}_y) =$$

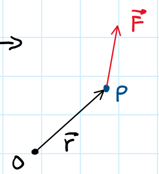
$$= i_2 B_1 \sin\theta \, dx \, \hat{u}_x \times \hat{u}_y = i_2 B_1 \sin\theta \, dx \, \hat{u}_z = \frac{\mu_0 i_1 i_2 \sin\theta}{2\pi r} dx \, \hat{u}_z$$

$d\vec{M}$ rispetto all'asse y

$$d\vec{M}_2 = (x \, \hat{u}_x) \times d\vec{F}_2 = \frac{\mu_0 i_1 i_2 \sin\theta \, x}{2\pi r} \hat{u}_x \times \hat{u}_z = - \frac{\mu_0 i_1 i_2 \sin\theta \, x \, dx}{2\pi r} \hat{u}_y$$







$$\vec{M} = \vec{r} \times \vec{F}$$

rotazione attorno y in senso orario:

$$\begin{cases} x = a \cdot tg\theta \\ dx = \frac{a}{\cos^2\theta} d\theta \\ x = r\sin\theta \implies r = \frac{x}{\sin\theta} \\ \theta_M = arctg \frac{L}{2a} \end{cases}$$

$$d\vec{M}_2 = \frac{\mu_0 i_1 i_2 a}{2\pi} tg^2\theta \, d\theta \, \hat{u}_y$$

$$\vec{M}_2 = - \hat{u}_y \frac{\mu_0 i_1 i_2 a}{2\pi} \int_{-\theta_M}^{\theta_M} tg^2\theta \, d\theta = - \hat{u}_y \frac{\mu_0 i_1 i_2 a}{\pi} \left[\frac{L}{2a} - arctg \frac{L}{2a} \right]$$

$$\left(\int tg^2\theta \, d\theta = tg\theta - \theta \right)$$

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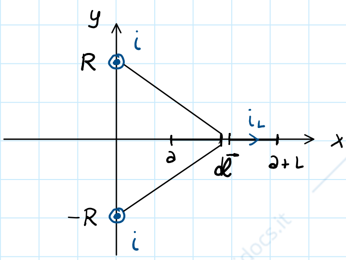
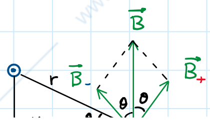
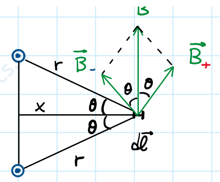
2)



Calcolare la $\vec{F}$ sul conduttore L, i_L immerso nel campo $\vec{B}$ creato da due correnti // ∞ i, i.



$$d\vec{l} = dx \, \hat{u}_x$$



$$d\vec{l} = dx\, \hat{u}_x$$

$$|\vec{B}_-| = |\vec{B}_+| = \frac{\mu_0 i}{2\pi r} \qquad r = \sqrt{x^2+R^2} \quad ; \quad \cos\theta = \frac{x}{\sqrt{x^2+R^2}}$$

$$|\vec{B}| = 2|\vec{B}_-|\cos\theta = \frac{\mu_0 i\, x}{\pi(x^2+R^2)} \quad ; \quad \vec{B} = \frac{\mu_0 i\, x}{\pi(x^2+R^2)}\,\hat{u}_y$$

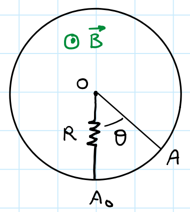
$$d\vec{F} = i_L\, d\vec{l}\times\vec{B} = i_L\, dx\,\hat{u}_x \times \frac{\mu_0 i\, x}{\pi(x^2+R^2)}\,\hat{u}_y = \hat{u}_z\,\frac{\mu_0 i i_L\, x\, dx}{\pi(x^2+R^2)}$$

$$\rightarrow \vec{F} = \hat{u}_z\,\frac{\mu_0 i i_L}{\pi}\int_a^{a+L}\frac{x\,dx}{x^2+R^2} = \hat{u}_z\,\frac{\mu_0 i i_L}{\pi}\,\frac{1}{2}\ln(x^2+R^2)\Big|_a^{a+L} = \hat{u}_z\,\frac{\mu_0 i i_L}{2\pi}\ln\left(\frac{R^2+(a+L)^2}{R^2+a^2}\right)$$

se $a = -\frac{L}{2} \Rightarrow \vec{F} = 0$

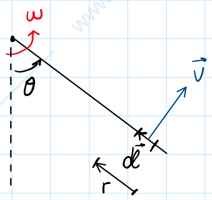
Legge di Faraday

3)

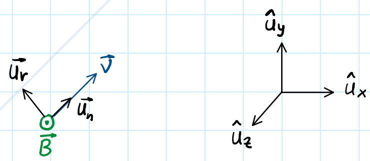


In presenza di un campo $\vec{B}$, un'asta OA è vincolata a ruotare attorno alla sua estremità O con velocità angolare ω. L'estremità A si trova in contatto elettrico con l'anello di raggio $OA = l$ e centro O. Determinare la corrente i che circola nel circuito di resistenza R.

$\vec{B} = B\,\hat{u}_z \quad ; \quad \vec{\omega} = \omega\,\hat{u}_z$

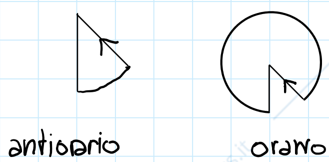


; centro di $d\vec{l}$



$$d\vec{l} = dr\,\hat{u}_r \qquad \vec{v} = v\,\vec{u}_n \qquad v = \omega(l-r)$$

Circuito $AOA_0A_+ \equiv AOA_0A_-$



I metodo)

Forza di Lorentz : $\vec{F} = q\vec{v}\times\vec{B} = qv\,\hat{u}_n\, B\,\hat{u}_z = qvB\,\hat{u}_n\times\hat{u}_z = qvB(-\hat{u}_r) =$

I metodo)

Forza di Lorentz : $\vec{F} = q\vec{v} \times \vec{B} = qv\,\hat{u}_n B\,\hat{u}_z = qvB\,\hat{u}_n \times \hat{u}_z = qvB\,(-\hat{u}_r) =$

$= -qvB\,\hat{u}_r$

elettroni $q = -e$ (e modulo della carica elettr.) $\longrightarrow$ $\vec{F} = evB\,\hat{u}_r$

$\Rightarrow$ gli elettroni si spostano da A verso O

$\boxed{\vec{E} = \vec{v} \times \vec{B}}$ $\quad (\vec{F} = q\vec{v} \times \vec{B}) \quad \longrightarrow \quad \vec{E} = \frac{\vec{F}}{q} = \frac{-qvB\,\hat{u}_r}{q} = -vB\,\hat{u}_r$

$$V_{AO} = \mathcal{E} = \int_{AO} \vec{E}\,d\vec{\ell} = -B\,\hat{u}_r \int_{AO} v\,d\vec{\ell} = -B\,\hat{u}_r \int_{AO} v\,dr\,\hat{u}_r = -B \int_{AO} v\,dr = -B \int_{AO} \omega(\ell - r)\,dr =$$

$$= B\omega \int_{AO} (r-\ell)\,d(r-\ell) \rightarrow w = r - \ell \;;\; \text{in A}: w = -\ell \;;\; \text{in O}: w = 0 \longrightarrow$$

$$\longrightarrow B\omega \int_{-\ell}^{0} w\,dw = B\omega \left[\tfrac{1}{2} w^2\right]_{-\ell}^{0} = B\omega\left[0 - \tfrac{1}{2}\ell^2\right] = -\tfrac{1}{2} B\omega\ell^2$$

II metodo)

Faraday :

ϕ  $$\mathcal{E} = -\frac{\partial \phi}{\partial t} = -\frac{\partial}{\partial t}\Big(B \cdot \underbrace{\tfrac{1}{2}\ell^2\theta}_{\text{area settore}}\Big) = -\frac{1}{2} B\ell^2 \frac{\partial \theta}{\partial t} = -\frac{1}{2} B\omega\ell^2$$

ϕ_-  $$\mathcal{E}_- = -\frac{\partial \phi_-}{\partial t} = -\frac{\partial}{\partial t} B\left(\pi\ell^2 - \tfrac{1}{2}\ell^2\theta\right) = \frac{1}{2} B\ell^2 \frac{\partial \theta}{\partial t} = \frac{1}{2} B\omega\ell^2$$

$$\Longrightarrow \quad i = \frac{|\mathcal{E}|}{R} = \frac{B\omega\ell^2}{2R}$$