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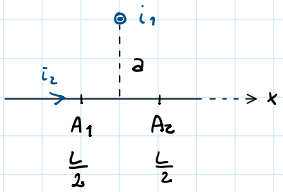
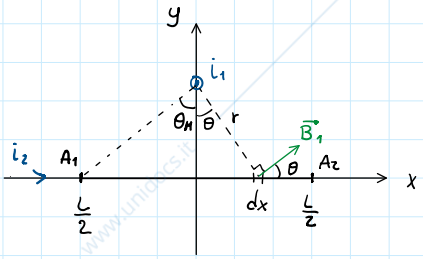
Esercitazione 10

giovedì 11 dicembre 2025 10:09

Interazione tra conduttori

1) 

Determinare il momento delle forze agenti sul tratto A_1A_2 del conduttore percorso da i_2 orientato lungo x in seguito all'azione del conduttore percorso da i_1 orientato lungo z.



$$\vec{B}_1 = B_1 \hat{u} = B_1 \cos\theta \, \hat{u}_x + B_1 \sin\theta \, \hat{u}_y$$

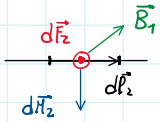
$$d\vec{l}_2 = dx \, \hat{u}_x \quad ; \quad B_1 = \frac{\mu_0 i_1}{2\pi r}$$

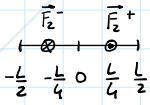
$$d\vec{F}_2 = i_2 \, d\vec{l}_2 \times \vec{B}_1 = i_2 \, dx \, B_1 \, \hat{u}_x \times (\cos\theta \, \hat{u}_x + \sin\theta \, \hat{u}_y) =$$

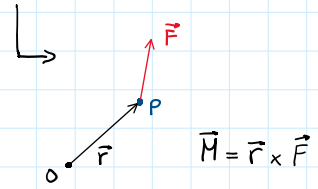
$$= i_2 B_1 \sin\theta \, dx \, \hat{u}_x \times \hat{u}_y = i_2 B_1 \sin\theta \, dx \, \hat{u}_z = \frac{\mu_0 i_1 i_2 \sin\theta}{2\pi r} dx \, \hat{u}_z$$

$d\vec{M}$ rispetto all'asse y

$$d\vec{M}_2 = (x \, \hat{u}_x) \times d\vec{F}_2 = \frac{\mu_0 i_1 i_2 \sin\theta \, x}{2\pi r} \hat{u}_x \times \hat{u}_z = -\frac{\mu_0 i_1 i_2 \sin\theta \, x}{2\pi r} dx \, \hat{u}_y$$







$\vec{M} = \vec{r} \times \vec{F}$

rotazione attorno y in senso orario:

$$\begin{cases} x = a \cdot tg\theta \\ dx = \dfrac{a}{\cos^2\theta} d\theta \\ x = r\sin\theta \implies r = \dfrac{x}{\sin\theta} \\ \theta_M = \arctg \dfrac{L}{2a} \end{cases}$$

$$d\vec{M}_2 = \frac{\mu_0 i_1 i_2 a}{2\pi} tg^2\theta \, d\theta \, \hat{u}_y$$

$$\vec{M}_2 = -\hat{u}_y \frac{\mu_0 i_1 i_2 a}{2\pi} \int_{-\theta_M}^{\theta_M} tg^2\theta \, d\theta = -\hat{u}_y \frac{\mu_0 i_1 i_2 a}{\pi} \left[\frac{L}{2a} - \arctg \frac{L}{2a} \right]$$

$$\left(\int tg^2\theta \, d\theta = tg\theta - \theta \right)$$

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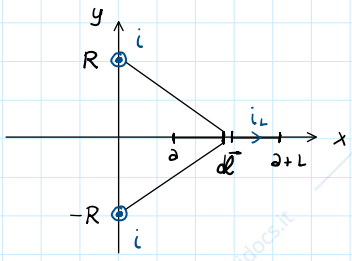
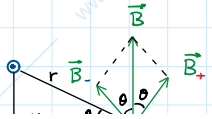
2) 

Calcolare la $\vec{F}$ sul conduttore L, i_L immerso nel campo $\vec{B}$ creato da due correnti $\parallel$ ∞ i, i.



$$d\vec{l} = dx \, \hat{u}_x$$

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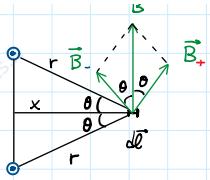


$$d\vec{l} = dx\,\hat{u}_x$$

$$|\vec{B}_-| = |\vec{B}_+| = \frac{\mu_0 i}{2\pi r} \qquad r = \sqrt{x^2+R^2} \quad ; \quad \cos\theta = \frac{x}{\sqrt{x^2+R^2}}$$

$$|\vec{B}| = 2|\vec{B}_-|\cos\theta = \frac{\mu_0 i\, x}{\pi(x^2+R^2)} \quad ; \quad \vec{B} = \frac{\mu_0 i\, x}{\pi(x^2+R^2)}\,\hat{u}_y$$

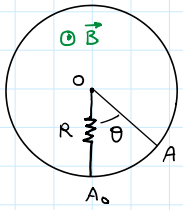
$$d\vec{F} = i_L\, d\vec{l}\times\vec{B} = i_L\, dx\,\hat{u}_x \times \frac{\mu_0 i\, x}{\pi(x^2+R^2)}\,\hat{u}_y = \hat{u}_z\,\frac{\mu_0 i i_L\, x\, dx}{\pi(x^2+R^2)}$$

$$\rightarrow \vec{F} = \hat{u}_z\,\frac{\mu_0 i i_L}{\pi}\int_{\partial}^{\partial+L}\frac{x\,dx}{x^2+R^2} = \hat{u}_z\,\frac{\mu_0 i i_L}{\pi}\,\frac{1}{2}\ln(x^2+R^2)\Big|_{\partial}^{\partial+L} = \hat{u}_z\,\frac{\mu_0 i i_L}{2\pi}\ln\left(\frac{R^2+(\partial+L)^2}{R^2+\partial^2}\right)$$

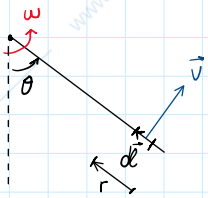
se $\partial = -\frac{L}{2} \Rightarrow \vec{F} = 0$

Legge di Faraday

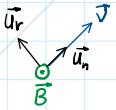
3) In presenza di un campo $\vec{B}$, un'asta OA è vincolata a ruotare attorno alla sua estremità O con velocità angolare ω. L'estremità A si trova in contatto elettrico con l'anello di raggio $OA = \ell$ e centro O. Determinare la corrente i che circola nel circuito di resistenza R.

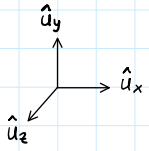


$\vec{B} = B\,\hat{u}_z$; $\vec{\omega} = \omega\,\hat{u}_z$



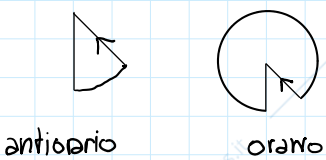
; centro di $d\vec{l}$





$d\vec{l} = dr\,\hat{u}_r \qquad \vec{v} = v\,\vec{u}_n \qquad v = \omega(\ell - r)$

Circuito $AOA_0A_+ \equiv AOA_0A_-$



I metodo)

Forza di Lorentz: $\vec{F} = q\vec{v}\times\vec{B} = q v\,\hat{u}_n\, B\,\hat{u}_z = q v B\,\hat{u}_n\times\hat{u}_z = q v B(-\hat{u}_r) =$

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I metodo)

Forza di Lorentz: $\vec{F} = q\vec{v} \times \vec{B} = qv\,\hat{u}_n\, B\,\hat{u}_z = qvB\,\hat{u}_n \times \hat{u}_z = qvB(-\hat{u}_r) =$

$= -qvB\,\hat{u}_r$

elettroni $q = -e$ (e modulo della carica elettr.) $\longrightarrow$ $\vec{F} = evB\,\hat{u}_r$

$\Rightarrow$ gli elettroni si spostano da A verso O

$\boxed{\vec{E} = \vec{v} \times \vec{B}}$ $\left(\vec{F} = q\vec{v} \times \vec{B}\right)$ $\longrightarrow$ $\vec{E} = \frac{\vec{F}}{q} = \frac{-qvB\,\hat{u}_r}{q} = -vB\,\hat{u}_r$

$$V_{AO} = \mathcal{E} = \int_{AO} \vec{E}\,d\vec{\ell} = -B\,\hat{u}_r \int_{AO} v\,d\vec{\ell} = -B\,\hat{u}_r \int_{AO} v\,dr\,\hat{u}_r = -B\int_{AO} v\,dr = -B\int_{AO} \omega(\ell - r)\,dr =$$

$$= B\omega \int_{AO} (r-\ell)\,d(r-\ell) \longrightarrow \omega = r - \ell \;;\; \text{in A: } \omega = -\ell \;;\; \text{in O: } \omega = 0 \longrightarrow$$

$$\longrightarrow B\omega \int_{-\ell}^{0} \omega\,d\omega = B\omega\left[\tfrac{1}{2}\omega^2\right]_{-\ell}^{0} = B\omega\left[0 - \tfrac{1}{2}\ell^2\right] = -\frac{\ell}{2}B\omega\ell^2$$

II metodo)

Faraday:

ϕ

$$\mathcal{E} = -\frac{\partial\phi}{\partial t} = -\frac{\partial}{\partial t}\left(B \cdot \underbrace{\tfrac{1}{2}\ell^2\theta}_{\text{area settore}}\right) = -\frac{1}{2}B\ell^2\frac{\partial\theta}{\partial t} = -\frac{1}{2}B\omega\ell^2$$

ϕ_-

$$\mathcal{E}_- = -\frac{\partial\phi_-}{\partial t} = -\frac{\partial}{\partial t}B\left(\pi\ell^2 - \tfrac{1}{2}\ell^2\theta\right) = \frac{1}{2}B\ell^2\frac{\partial\theta}{\partial t} = \frac{1}{2}B\omega\ell^2$$

$$\Rightarrow \quad i = \frac{|\mathcal{E}|}{R} = \frac{B\omega\ell^2}{2R}$$

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