Primo principio: $\delta u = \delta q + \delta l \qquad \delta s = \frac{\delta q}{T}$

Potenziali termodinamici

$$dU = TdS - PdV$$

$$dh = d(U + PV) = TdS + VdP$$

$$dF = d(U - TS) = -SdT - PdV \qquad + \sum_{i=1}^{c} \mu_i N_i$$

$$dG = d(H - TS) = -SdT + VdP$$

FASI

Componenti + 2 - Fasi = VAR. INTENSIVE INDIPENDENTI

Calori specifici

N. Atm.	c_p	c_v	$\gamma = c_p/c_v$
1	$5/2\,R$	$3/2\,R$	$5/3$
2	$7/2\,R$	$5/2\,R$	$7/5$
>2	$4\,R$	$3\,R$	$9/7$

con $R = 8314 \frac{J}{kmol\,k}$

$c_{p\,H_2O} = 4186\ J/kg\,k$

MM notevoli

$MM_{N_2} = 28\ kg/mol$

$MM_{O_2} = 32\ kg/mol$

$MM_{AIR} = 29\ kg/mol$

$R^* = \frac{R}{MM}$ notevoli

$$\boxed{R^*_{AIR} = 286{,}7 \frac{J}{kg\,k}}$$

Maxwell

$$\left(\frac{dT}{dV}\right)_S = -\left(\frac{dP}{dS}\right)_V$$

$$\left(\frac{dV}{dT}\right)_P = -\left(\frac{dS}{dP}\right)_T$$

$$\left(\frac{dP}{dT}\right)_V = \left(\frac{dS}{dV}\right)_T$$

$$\left(\frac{dT}{dP}\right)_S = \left(\frac{dV}{dS}\right)_P$$

ALTRO:

$$Pv = R^* T$$

$$K_p = \frac{1}{v}\frac{\partial v}{\partial T}\bigg|_P \qquad K_T = -\frac{1}{v}\frac{\partial v}{\partial P}\bigg|_T \qquad K_S = -\frac{1}{v}\frac{\partial v}{\partial P}\bigg|_S$$

$$P = \frac{R^* T}{v - b} - \frac{a}{v^2}$$

$$C_p - C_v = \frac{T v k_p^2}{k_T} \qquad \frac{K_S}{K_T} = \frac{C_v}{C_p} \qquad C_p = \frac{\delta q}{\partial T}\bigg|_p \qquad C_v = \frac{\delta q}{\delta T}\bigg|_v$$

	Solido/Liquido $dv = 0$	Aeriforme $k_P = 1/T$, $k_T = 1/P$
U (M_i, V, S)	$dU = C_v\,dT$	$dU = C_v\,dT$
S (M_i, V, U)	$ds = C_v \frac{dT}{T} = C_p \frac{dT}{T}$	$ds = C_p \frac{dT}{T} - R^* \frac{dP}{P}$ $ds = C_v \frac{dT}{T} + R^* \frac{dv}{v}$
H	$dh = C_p\,dT + v\,dP$	$dh = C_p\,dT$

Sistema fluente - Massa: $\frac{dM}{dt} = \sum \dot{M}_i - \sum \dot{M}_u$ $\quad \frac{d\rho}{\rho} + \frac{dw}{w} + \frac{dA}{A} = 0$

- Bilancio di energia

$$\frac{\partial}{\partial \tau}\left[M\left(u + gz + \frac{w^2}{2}\right)\right] = \sum_{i=1}^{m} \dot{M}_i \left(h_i + g z_i + \frac{w_i^2}{2}\right) + \dot{L}_{ut} + \dot{L}_{eq} + \dot{Q} - \sum_{u=1}^{m} \dot{M}_u \left(h_u + g z_u + \frac{w_u^2}{2}\right)$$

- Bilancio entropico

$$\left(\frac{\Delta S}{\Delta \tau}\right)_{SI} = \sum_{j=1}^{J} \int_i^f \frac{\delta \dot{Q}_{SISTEMA}}{T_{SISTEMA}} + \sum_{j=1}^{J} \int_i^f \frac{\delta \dot{Q}_{SORGENTE,j}}{T_{SORGENTE,j}} \qquad \left(\frac{\Delta S}{\Delta \tau}\right)_{SI} \geq 0$$

- Turbomacchine

$$l_{ut} = h_u - h_i \longrightarrow \dot{L}_{ut,\,AREIF.} = \dot{M} C_p (T_2 - T_1) \qquad \eta^T_{isos} = \frac{l_{ut\ real}}{l_{ut\ isos}}$$

$$\dot{L}_{ut,\,LIQUIDO} = \dot{M} v (P_2 - P_1) \qquad \eta^C_{isos} = \frac{l_{isos}}{l_{real}}$$

- Scambiatori di calore

$$\dot{Q} = \dot{M}(h_u - h_i) \qquad Q_{H \to C} = -Q_{C \to H}$$

- Ugelli e diffusori

	$M<1$	$M>1$
Ugello → aumento della velocità	convergente	divergente
Diffusore → diminuzione della velocità	divergente	convergente

$$h_u - h_i + \frac{w_u^2 - w_i^2}{2} = 0 \Rightarrow \Delta h = \frac{w_i^2 - w_u^2}{2}$$

- Perdite di carico

$$\frac{\Delta P}{L} = -\frac{\varepsilon}{D}\frac{w^2 \rho}{2} \quad \text{con} \quad \varepsilon = \frac{64}{\rho w D}\cdot\mu = \frac{64}{wD}\nu = \frac{64}{Re}$$

ν = VISCOSITÀ CINEMATICA $= \frac{\mu}{\rho}$

$$Re = \frac{\rho w D}{\mu} = \frac{wD}{\nu}$$

Se D non è circolare uso $D_{eq} = 4\frac{\text{Area}}{\text{Perimetro}}$

$$\Delta P = -\beta\frac{w^2 \rho}{2}$$

- Laminazione

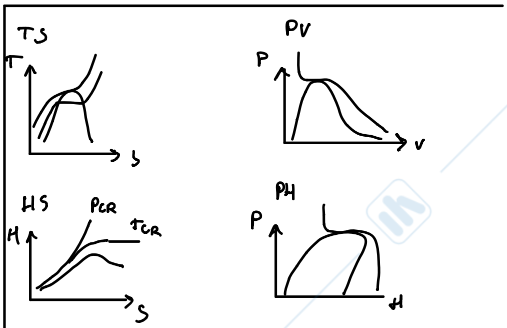
$$\Delta h = 0$$

Politropiche
- Gas perfetto
- c_x costante
- internamente reversibili

$$P v^k = \text{cost} \quad \text{con} \quad k = \frac{c_x - c_p}{c_x - c_v} \longrightarrow$$

ISOBARA	$c_x = c_p$	$k = 0$
ISOCORA	$c_x = c_v$	$k = \infty$
ISOTERMA	$c_x = \infty$	$k = 1$
ADIABATICA	$c_x = 0$	$k = c_p/c_v = \gamma$

$$c_x = \frac{k c_v - c_p}{k - 1}$$



$$\Delta s = c_x \ln\frac{T_f}{T_i}$$

$$\delta l_m = du - \delta q = c_v dT - c_x dT = (c_p - c_x)\, dT$$

$$T_{u\,IDEALE} = T_i\left(\frac{P_u}{P_i}\right)^{\frac{1-\gamma}{\gamma}}$$

$\longrightarrow$ Temperatura uscita turbomacchine reali

$$\eta^T_{isos} = \frac{l_{real}}{l_{isos}} = \frac{c_p\,(T_{u\,real} - T_i)}{c_p\,(T_{u\,ideale} - T_i)}$$

$$\Rightarrow T_{u\,real} = T_i + \eta^T_{isos}\,(T_{u\,ideal} - T_i) \quad \Rightarrow \Delta S = c_p \ln\frac{T_{u\,reale}}{T_i} - R^* \ln\frac{P_u}{P_i}$$

Aria umida

umidità assoluta $x = \dfrac{M_{VAP}}{M_{AS}} \Rightarrow M_{AU} = (1+x)\,M_{AS}$ $\quad x_{max} = \dfrac{MM_v}{MM_{AS}}\,\dfrac{P_{vs}}{(P - P_{vs})}$

$$x = \frac{MM_v}{MM_{AS}}\,\frac{\varphi P_{vs}}{P - \varphi P_{vs}}$$

umidità relativa $\varphi = \dfrac{P_v}{P_{vs}(T_{BS})}$

grado di saturazione $\psi = \dfrac{x}{x_{max}}$

Da $T_{aria\,umida} \rightarrow P_{vs}$

$\varphi = \dfrac{P_v}{P_{vs}} \Rightarrow P_v = \varphi P_{vs}$

$P_v \rightarrow$ TROVO $T_{RUGIADA}$

SE $T_p < T_{RUGIADA}$ HO CONDENSA

Trasmissione del calore

Irraggiamento

$$\dot{E}_{1,netta} = -\frac{\sigma (T_2 - T_1)^4}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{1\to 2}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

con

$A_1 = A_2 = A$

$F_{1\to 2} = F_{2\to 1} = 1$

$$\dot{E}_{1,2} = -\frac{A\sigma\,(T_2^4 - T_1)^4}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

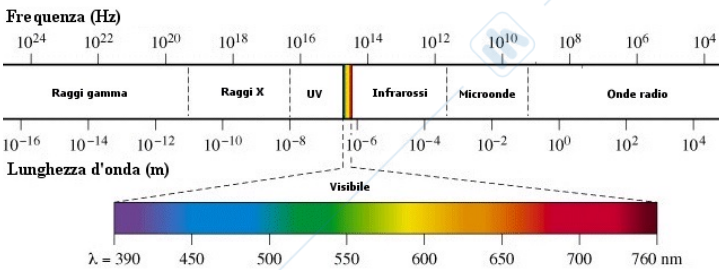
con

$A_1 << A_2$

$F_{1\to 2} = 1$

$F_{2\to 1} = \dfrac{A_1}{A_2} \approx 0$

$$\dot{E}_{1,2} = -A_1 \varepsilon_1 \sigma\,(T_2^4 - T_1^4)$$



Wien:

$$\lambda_{del\,max}\,T = 2897{,}6\ \mu m K$$

Conduzione

Legge di Fourier : $\dot{q}'' = -\Lambda_{\bar{p},\bar{d},T} \nabla T \;\rightarrow\; \rho c \frac{dT}{d\tau} = \nabla(\Lambda_{\bar{p},d,T} \nabla T) + \dot{U}'''$

Equazione di Poisson $\nabla^2 T = -\frac{\dot{U}'''}{\lambda}$

Equazione di Laplace $\nabla^2 T = 0$

Generazione interna di potenza

→ integrazione indefinita

coordinate cartesiane	coordinate cilindriche	coordinate sferiche
$\frac{\partial^2 T}{\partial x^2} = -\frac{\dot{U}'''}{\lambda}$	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{\dot{U}'''}{\lambda}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = -\frac{\dot{U}'''}{\lambda}$
$\frac{\partial T}{\partial x} = -\frac{\dot{U}''' x}{\lambda} + C_1$	$\frac{\partial T}{\partial r} = -\frac{\dot{U}''' r}{2\lambda} + \frac{C_1}{r}$	$\frac{\partial T}{\partial r} = -\frac{\dot{U}''' r}{3\lambda} + \frac{C_1}{r^2}$
$T = -\frac{\dot{U}''' x^2}{2\lambda} + C_1 x + C_2$	$T = -\frac{\dot{U}''' r^2}{4\lambda} + C_1 \ln(r) + C_2$	$T = -\frac{\dot{U}''' r^2}{6\lambda} - \frac{C_1}{r} + C_2$
$\dot{Q}'' = \dot{U}''' x - C_1\lambda$	$\dot{Q}'' = \frac{\dot{U}''' r}{2} - \frac{C_1\lambda}{r}$	$\dot{Q}'' = \frac{\dot{U}''' r}{3} - \frac{C_1\lambda}{r^2}$

Generazione interna di potenza

→ integrazione definita

parete piana	cilindro cavo	sfera cava
$\begin{cases} T = -\frac{\dot{U}'''}{2\lambda}x^2 + C_1 x + C_2 \\ T(x=0) = T_1 \\ T(x=s) = T_2 \end{cases}$	$\begin{cases} T = -\frac{\dot{U}'''}{4\lambda}r^2 + C_1 \ln(r) + C_2 \\ T(r=R_i) = T_1 \\ T(r=R_e) = T_2 \end{cases}$	$\begin{cases} T = -\frac{\dot{U}'''}{6\lambda}r^2 - \frac{C_1}{r} + C_2 \\ T(r=R_i) = T_1 \\ T(r=R_e) = T_2 \end{cases}$
$C_1 = \frac{\dot{U}''' s}{2\lambda} + \frac{T_2 - T_1}{s}$	$C_1 = \frac{\dot{U}'''}{4\lambda}\frac{R_e^2 - R_i^2}{\ln\left(\frac{R_e}{R_i}\right)} + \frac{T_2 - T_1}{\ln\left(\frac{R_e}{R_i}\right)}$	$C_1 = \frac{\dot{U}'''}{6\lambda}\frac{R_e^2 - R_i^2}{\frac{1}{R_i} - \frac{1}{R_e}} + \frac{T_2 - T_1}{\frac{1}{R_i} - \frac{1}{R_e}}$
$C_2 = T_1$	$C_2 = T_1 + \frac{\dot{U}'''}{4\lambda}R_i^2 - C_1 \ln(R_i)$	$C_2 = T_1 + \frac{\dot{U}'''}{6\lambda}R_i^2 + \frac{C_1}{R_i}$
$T(x) = \frac{\dot{U}'''}{2\lambda}(s-x)\,x + \frac{T_2 - T_1}{s}x + T_1$		
$\dot{Q}''(x) = \dot{U}'''\left(x - \frac{s}{2}\right) - \lambda\frac{T_2 - T_1}{s}$		

No generazione di potenza

parete piana	cilindro cavo	sfera cava
$\begin{cases} T = C_1 x + C_2 \\ T(x=0) = T_1 \\ T(x=s) = T_2 \end{cases}$	$\begin{cases} T = C_1 \ln(r) + C_2 \\ T(r=R_i) = T_1 \\ T(r=R_e) = T_2 \end{cases}$	$\begin{cases} T = -\frac{C_1}{r} + C_2 \\ T(r=R_i) = T_1 \\ T(r=R_e) = T_2 \end{cases}$
$T = \frac{T_2 - T_1}{s}x + T_1$	$T = \frac{\ln(r/R_i)}{\ln(R_e/R_i)}(T_2 - T_1) + T_1$	$T = \frac{R_i}{r}\frac{R_e - r}{R_e - R_i}T_1 + \frac{R_e}{r}\frac{r - R_i}{R_e - R_i}T_2$
$\dot{Q}'' = -\frac{T_2 - T_1}{\frac{s}{\lambda}}$	$\dot{Q} = -\frac{T_2 - T_1}{\frac{\ln(R_e/R_i)}{2\pi\lambda L}}$	$\dot{Q} = -\frac{T_2 - T_1}{\frac{R_e - R_i}{4\pi\lambda R_i R_e}}$

Condizioni al contorno per trovare le due incognite

- Temperature note
- Temperature tra strati $\rightarrow$ continuità
- Flusso termico $= -\lambda \dfrac{\partial T}{\partial x} = \dot{Q}''$

Convezione

Newton $\rightarrow$ $\dot{Q}'' = -h\,(T_p - T_f)$

Calcolo di h $\rightarrow$ $Nu = \dfrac{h\,D_{car}}{\lambda_f}$ scambio convettivo in conduttivo fluido $\quad Bi = \dfrac{h\,D_{car}}{\lambda_s}$

Altri gruppi adimensionali

$Re = \dfrac{\rho_f\, w\, D_{car}}{\mu_f} = \dfrac{w\, D_{car}}{\nu}$ forze d'inerzia in forze viscose

$Pr = \dfrac{c_f\, \mu_f}{\lambda_f}$ proprietà di trasporto quantità di moto in calore

MET. FUSO, H, O, AIR, H_2O, OIL CRESCENTE

$Gr = \dfrac{g\, \rho_f^2\, \kappa_{pf}\,(T_f - T_p)\, D_{car}^3}{\mu_f^2}$ galleggiamento in viscosità

$Ra = Gr\, Pr$

$T_{f\,film} = \dfrac{T_p + T_\infty}{2}$

$Ri = \dfrac{Gr}{Re^2}$ $\begin{cases} \gg 1 & \text{convezione naturale} \\ = 1 & \text{convezione mista} \\ \ll 1 & \text{convezione forzata} \end{cases}$

Trasmissione mista

- Senza generazione interna di potenza

$$\dot{Q}'' = -\frac{(T_{\infty DX} - T_{\infty SX})}{\frac{1}{h_{SX}} + \frac{1}{h_{DX}} + \frac{S_i}{\lambda_i}} \qquad \dot{Q} = \dot{Q}'' \cdot A$$

- Elementi in parallelo

$$\dot{Q} = -\frac{(T_{\infty DX} - T_{\infty SX})}{\frac{1}{A_1 h_1} + \frac{S_A}{A_A \lambda_A} + \frac{1}{\frac{A_B \lambda_B}{S_B} + \frac{A_C \lambda_C}{S_C}} + \frac{S_D}{A_D \lambda_D} + \frac{1}{A_D h_D}}$$

Parte in parallelo considerata isoterma ortogonalmente al flusso

OPPURE

$$\dot{Q} = \dot{Q}_{ABD} + \dot{Q}_{ACD}$$

- Misto cilindrico $\longrightarrow$ conduzione integrata in coordinate cilindriche
convezione su AREA ESTERNA

Transitori - Parametri concentrati

- APPLICABILE SE

$$Bi = \frac{h\, D_{car}}{\lambda_S} < 0{,}1$$

- Solo scambio convettivo

$$\frac{\partial U}{\partial \tau} = \dot{Q} = -hA(T - T_\infty)$$

$$\rho V c \frac{\partial T}{\partial \tau} = -hA(T - T_\infty)$$

$$\frac{\partial T}{\partial \tau} = -\frac{hA}{\rho V c}(T - T_\infty)$$

Pongo $\vartheta = (T - T_\infty)$ $\quad d\vartheta = dT$

$$\frac{d\vartheta}{\vartheta} = -\frac{hA}{\rho V c}\, d\tau \qquad \vartheta = \vartheta_0\, e^{-\frac{hA}{\rho V c}\Delta\tau}$$

- Se lungo $\dot{Q}$

$$T_f = T_i + \frac{\dot{Q}}{\rho c V}\Delta\tau$$

$$\Rightarrow T = T_\infty + (T_0 - T_\infty)\, e^{-\frac{hA}{\rho V c}\Delta\tau}$$

- Convezione + generazione interna di potenza

$$\rho V c \frac{\partial T}{\partial \tau} = -h A (T - T_\infty) + \dot{U}''' V$$

$$\Rightarrow T = T_\infty + \left(T_0 - T_\infty - \frac{\dot{U}''' V}{hA}\right) e^{-\frac{hA}{\rho V c}\Delta\tau} + \frac{\dot{U}''' V}{hA}$$

$$\overset{\tau \to \infty}{\Rightarrow} T = T_\infty + \frac{\dot{U}''' V}{hA}$$

Scambiatori di calore

$$\dot{Q} = A \frac{\Delta T_{ML}}{\frac{1}{h_H} + R_{fouling} + R_{fouling} + \frac{1}{h_C}}$$

Scambio di calore con vap. in transizione

$$\dot{Q} = \dot{M}\, DX\, \lambda$$

$$\Delta T_{ML} = \frac{\Delta T_L - \Delta T_0}{\ln \frac{\Delta T_L}{\Delta T_0}}$$

Altezze e HTU [...]

Cicli termodinamici

$$\begin{cases} \Sigma L + \Sigma Q_H + \Sigma Q_C = 0 \\ S_p = -\Sigma \dfrac{Q_H}{T_H} - \Sigma \dfrac{Q_C}{T_C} \end{cases}$$

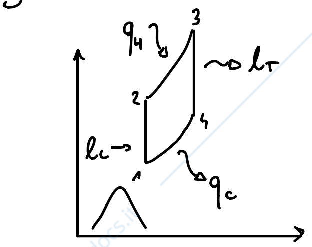
SE S_H e S_C T VARIANO

$$S_p = M_C C_p \ln \frac{T_{2C}}{T_{1C}} + M_H C_p \ln \frac{T_{2H}}{T_{1H}}$$

Parametri prestazionali

	DIRETTO	INVERSO
I° Principio	$\eta_I = -\dfrac{\Sigma L}{\Sigma Q_H}$	$\varepsilon = \dfrac{\Sigma Q_C}{\Sigma L}$ $\quad COP = -\dfrac{\Sigma Q_C}{\Sigma L}$
II° Principio confronto con Carnot	$\eta_{II} = \dfrac{\eta_I}{1 - \frac{T_C}{T_H}} = \dfrac{Q_{H\,IDEAL}}{Q_{H\,REAL}}$	$\eta_{II\,\varepsilon} = \dfrac{\varepsilon}{\frac{T_C}{T_H - T_C}} = \dfrac{L_{IDEALE}}{L_{REALE}}$ $\quad \eta_{II\,COP} = \dfrac{COP}{\frac{T_H}{T_H - T_C}} = \dfrac{L_{IDEALE}}{L_{REALE}}$

Joule



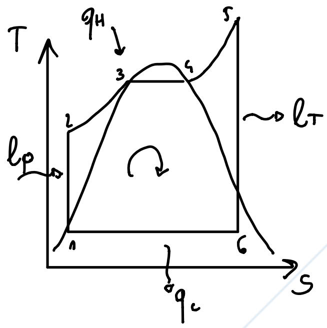
$$\begin{cases} q_H = h_3 - h_2 \\ l_T = h_4 - h_3 \\ l_c = h_2 - h_1 \\ q_c = h_1 - h_4 \end{cases}$$

$$\eta_I = -\frac{l_c + l_T}{q_H} = 1 + \frac{q_c}{q_H} = 1 - \frac{T_1}{T_2}$$

$$= 1 - \frac{T_4}{T_3} = 1 - \left(\frac{P_{23}}{P_{41}}\right)^{\frac{1-\gamma}{\gamma}} \qquad \gamma = \frac{c_p}{c_v}$$

$$\eta_{RIG} = \frac{T_y - T_2}{T_4 - T_x}$$

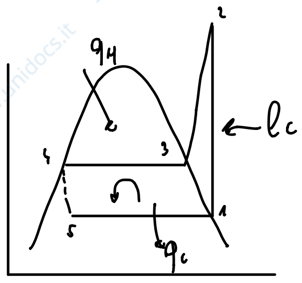
Rankine **NO GAS PERFETTO**



$$\begin{cases} l_p = h_2 - h_1 \\ q_H = h_5 - h_2 \\ l_T = h_6 - h_5 \\ q_C = h_1 - h_6 \end{cases}$$

$$\eta_I = -\frac{l_T + l_P}{q_H} = \frac{q_H + q_C}{q_H}$$

inverso



$$\begin{cases} l_c = h_2 - h_1 \\ q_H = h_4 - h_2 \\ q_c = h_1 - h_5 \end{cases}$$

$$\varepsilon = \frac{h_1 - h_5}{h_2 - h_1}$$

$$COP = -\frac{h_4 - h_2}{h_2 - h_1}$$