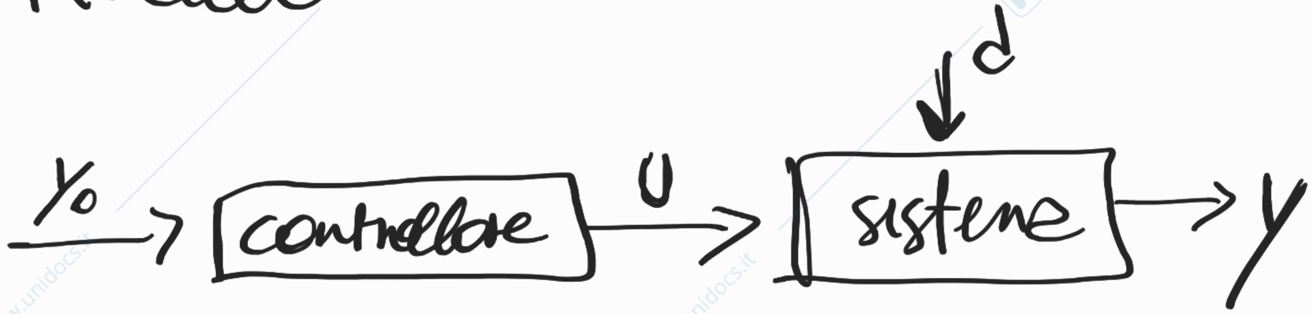
Lez : SISTEMI

Modello



sistema : $y = f(u)$

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

$$\frac{dx(t)}{dt} = \dot{x}(t)$$

Classificazione:

- STATICO / DINAMICO → CI
- t. VARIANTE / INVARIANTE → coeffic
- SISO / MIMO : $u(t)$, $y(t)$
- PROPRIO / strett proprio
  $D \neq 0$ $\quad$ $D = 0$
- LINEARE / NON LINEARE

\+ ORDINE → $x(t)$

Linearizzazione:

LOCALE

retta tg

numero

$$y = f(u) \simeq f(\bar{u}) + \left.\frac{\partial f}{\partial u}\right|_{\bar{u}} (u - \bar{u})$$

sistemi linearizzazione LOCALE nel punto di equilibrio

$$y = g(x, u)$$
$$\dot{x} = f(x, u)$$

$$u(t) = \bar{u} \longrightarrow x(t) = \bar{x}$$
$$y(t) = \bar{y}$$

$$0 = f(\bar{x}, \bar{u}) \longrightarrow \bar{x}$$

$$\bar{y} = g(\bar{x}, \bar{u}) \longrightarrow \bar{y}$$

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

$$u(t) = \bar{u}$$

$$0 = A\bar{x} + B\bar{u} \longrightarrow \bar{x} = A^{-1}B\bar{u}$$

$\exists!$ sse $\det A \neq 0$

$$\bar{y} = C\bar{x} + D\bar{u} = $$
$$\stackrel{!}{=} \underbrace{[CA^{-1}B + D]}\bar{u}$$

$$\begin{cases} \delta \dot{x}(t) = A\,\delta x(t) + B\,\delta u(t) \\ \delta y(t) = C\,\delta x(t) + D\,\delta u(t) \end{cases}$$

$$\dot{x} - \dot{x}_0 = \delta \dot{x}$$

$$x - \bar{x} = \delta x$$

$$A = \frac{\partial f}{\partial x}\Big|_{\bar{x},\bar{u}} \qquad B = \frac{\partial f}{\partial u}\Big|_{\bar{x},\bar{u}}$$

$$C = \frac{\partial g}{\partial x}\Big|_{\bar{x},\bar{u}} \qquad D = \frac{\partial g}{\partial u}\Big|_{\bar{x},\bar{u}}$$

Momento LIBERO: $u(t) = 0$

$$\dot{x}(t) = Ax(t) \longrightarrow x_L = x_0 e^{At}$$

Momento FORZATO: $u(t) \neq 0$

$$\dot{x}(t) = Ax(t) + Bu(t)$$

Formule di Lagrange:
$$\begin{cases} x(t) = x_0 e^{At} + \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau \\ y(t) = Cx(t) + Du(t) \end{cases}$$

$$e^{At}$$

$$A = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{\lambda_1 t} & & & 0 \\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ 0 & & & e^{\lambda_n t} \end{bmatrix}$$

λ = autovalori di A

$e^{\lambda t}$ = modi di A

$$\det(\lambda I - A) = 0$$

diagonalizzazione : λ regolari

$mg = ma$

$\lambda \longrightarrow Av = \lambda v \longrightarrow T_D^{-1}$

$\longrightarrow T_D$

$$e^{At} = T_D^{-1} e^{A_D t} T_D$$

$$A_D = T_D A T_D^{-1}$$

1. Classificazione e forma di stato

Sistema:
$$\begin{cases} \dot{x}_1(t) = 2x_1(t) + 3x_2(t)(1+\alpha x_2(t)) + u(t), \quad \alpha \in \mathbb{R} \\ \dot{x}_2(t) = -x_1(t) + x_2(t) \\ y(t) = x_1(t) + 3u(t) \end{cases}$$

1) Classificare sistema:

- DINAMICO
- 2° ORDINE $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$
- SISO: $y(t), u(t)$ scalari
- $D = 3$ proprio
- t. INVARIANTE

$\alpha = 0$
$$\begin{cases} \dot{x}_1(t) = 2x_1(t) + 3x_2(t) + u(t) \\ \end{cases}$$

lineare

$\alpha \neq 0$
$$\dot{x}_1(t) = 2x_1(t) + 3x_2(t) + 3\alpha x_2^2(t) + u(t)$$

non lineare

2) Sistema forma di stato

$$\begin{cases} \dot{x}_1(t) = 2x_1(t) + 3x_2(t) + u(t) \quad \left(1 + \alpha x_2 \right., \ \alpha \in \mathbb{R} \\ \dot{x}_2(t) = -x_1(t) + x_2(t) \\ y(t) = x_1(t) + 3u(t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

$$A = \begin{array}{c|cc} & x_1 & x_2 \\ \hline \dot{x}_1 & 2 & 3 \\ \dot{x}_2 & -1 & 1 \end{array} \qquad B = \begin{array}{c|c} & u \\ \hline \dot{x}_1 & 1 \\ \dot{x}_2 & 0 \end{array}$$

$$C = \begin{array}{c|cc} & x_1 & x_2 \\ \hline y & 1 & 0 \end{array} \qquad D = \begin{array}{c|c} & u \\ \hline y & 3 \end{array}$$

2. Circuito RC

$$I_R(t) = I_c(t)$$

1) Derivare e classificare modello in forma di stato

$$v(t) - I_R(t)R - v_c(t) = 0$$

$$v_c(t) \longrightarrow I_c(t) = C\frac{dv_c(t)}{dt}$$

$$v(t) - I_c(t)R - v_c(t) = 0$$

$$v(t) - RC\frac{dv_c(t)}{dt} - v_c(t) = 0$$

$$u(t) = v(t)$$

$$x(t) = v_c(t) \longrightarrow \frac{dv_c(t)}{dt} = \dot{x}(t)$$

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$\frac{dv_c(t)}{dt} = -\frac{1}{RC}v_c(t) + \frac{1}{RC}v(t)$$

$$\begin{cases} \dot{x}(t) = -\frac{1}{RC}x(t) + \frac{1}{RC}u(t) \\ y(t) = x(t) \end{cases}$$

$$D = 0$$

$$C = 1$$

$$A = \left[-\frac{1}{RC}\right] = a \qquad B = b = \frac{1}{RC}$$

Classificatore:

- DINAMICO
- 1° ORDINE $x(t)$
- t. INVARIANTE
- LINEARE
- SISO
- $D = 0 \Rightarrow$ strettam proprio

2) Determinare MOVIMENTO FORZATO STATO e USCITA per $u(t) = 1$, $\forall t \geq 0$

$\Rightarrow$ Formule di Lagrange

$$\begin{cases} x(t) = \cancel{x_0 e^{At}} + \int_0^t e^{A(t-\tau)} B u(\tau) d\tau \\ y(t) = C x(t) + D u(t) \end{cases}$$

$$x_0 = 0$$

$$x_F(t) = \int_0^t e^{-\frac{1}{RC}(t-\tau)} \frac{1}{RC} \cancel{1} \, d\tau =$$

$$= e^{-\frac{1}{RC}t} \int_0^t e^{\frac{1}{RC}\tau} \frac{1}{RC} d\tau =$$

$$= e^{-\frac{1}{RC}t} \left[e^{\frac{1}{RC}\tau} \right]_0^t =$$

$$= \cancel{1} - e^{-\frac{1}{RC}t}$$

$$y_F(t) = x_F(t) = 1 - e^{-\frac{1}{RC}t}$$

3) $x(0) \neq 0$ e $u(t) = 1 \ \forall t$?

$$x_L(t) = x_0 e^{At} = x_0 e^{-\frac{1}{RC}t}$$

$$x(t) = x_L(t) + x_F(t) = x_0 e^{-\frac{1}{RC}t} + (1 - e^{-\frac{1}{RC}t})$$

4) $x(0) = 1$ e $u(t) = 1 \ \forall t$?

$$x(t) = \cancel{e^{-\frac{1}{RC}t}} + (1 - \cancel{e^{-\frac{1}{RC}t}}) = 1$$

$$u(t) = 1 \longrightarrow x(t) = 1$$

equilibrio

3. Sistema:

$$\dot{x}(t) = |x(t)|^3 - 3x^2(t) + 3|x(t)| - u(t)$$

1) punti equilibrio associati a $u(t) = \bar{u} = 1$, $t \geq 0$

$$0 = |\bar{x}|^3 - 3\bar{x}^2 + 3|\bar{x}| - \bar{u}$$

$$\begin{cases} a) \ \bar{x}^3 - 3\bar{x}^2 + 3\bar{x} - \bar{u} = 0 & x \geq 0 \\ b) \ -\bar{x}^3 - 3\bar{x}^2 - 3\bar{x} - \bar{u} = 0 & x < 0 \end{cases}$$

a) $\bar{x}^3 - 3\bar{x}^2 - 3\bar{x} - 1 = 0$

$$(\bar{x} - 1)^3 = 0 \implies \bar{x} = 1$$

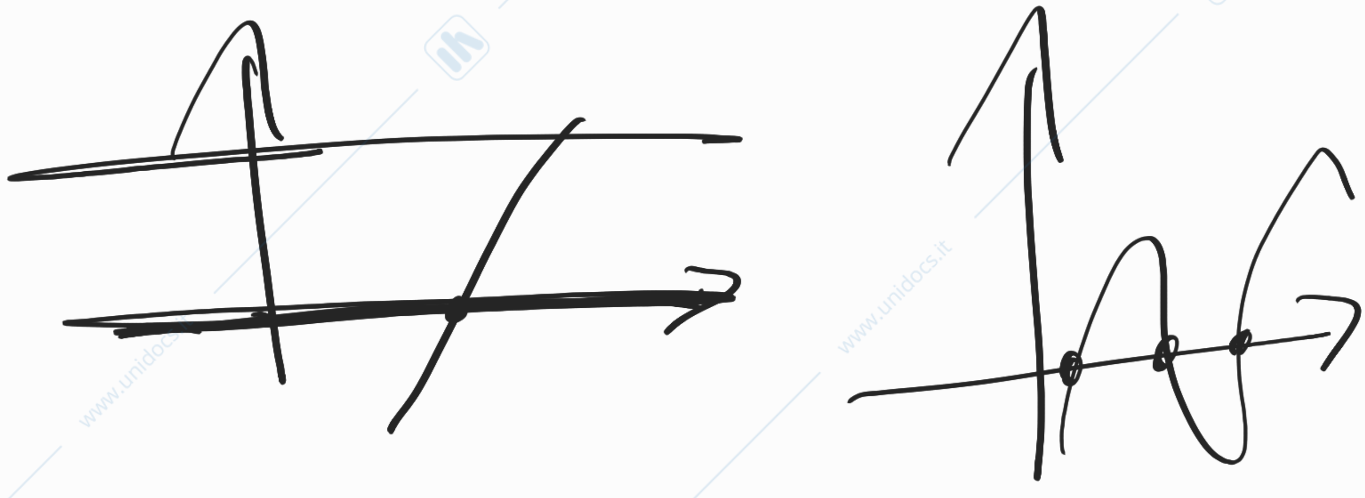
$$(\bar{x}, \bar{u}) = (1, 1)$$

b) $-\bar{x}^3 - 3\bar{x}^2 - 3\bar{x} - 1 = 0$

$-(\bar{x}+1)^3 = 0 \quad \Rightarrow \bar{x} = -1$

$(\bar{x}, \bar{u}) = (-1, 1)$

2 punti equilibrio sse sistema NON LINEARE



2) Linearizzazione intorno del punto di equilibrio

$$\dot{x}(t) = \underline{|x(t)|^3 - 3x^2(t) + 3|x(t)|} \underbrace{-u(t)}$$

$B = -1$

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$A = \left.\frac{\partial f}{\partial x}\right|_{(\bar{x},\bar{u})}$$

$$f(x,u) = \begin{cases} x^3(t) - 3x^2(t) + 3x(t) - u(t) & x \geq 0 \\ -x^3(t) - 3x^2(t) - 3x(t) - u(t) & x < 0 \end{cases}$$

1. eq $(\bar{x},\bar{u}) = (1,1)$

$$\left.\frac{\partial f}{\partial x}\right|_{\bar{x},\bar{u}} = \left. 3x^2(t) - 6x(t) + 3 \right|_{\bar{x},\bar{u}} = 0$$

2. eq $(\bar{x},\bar{u}) = (-1,1)$

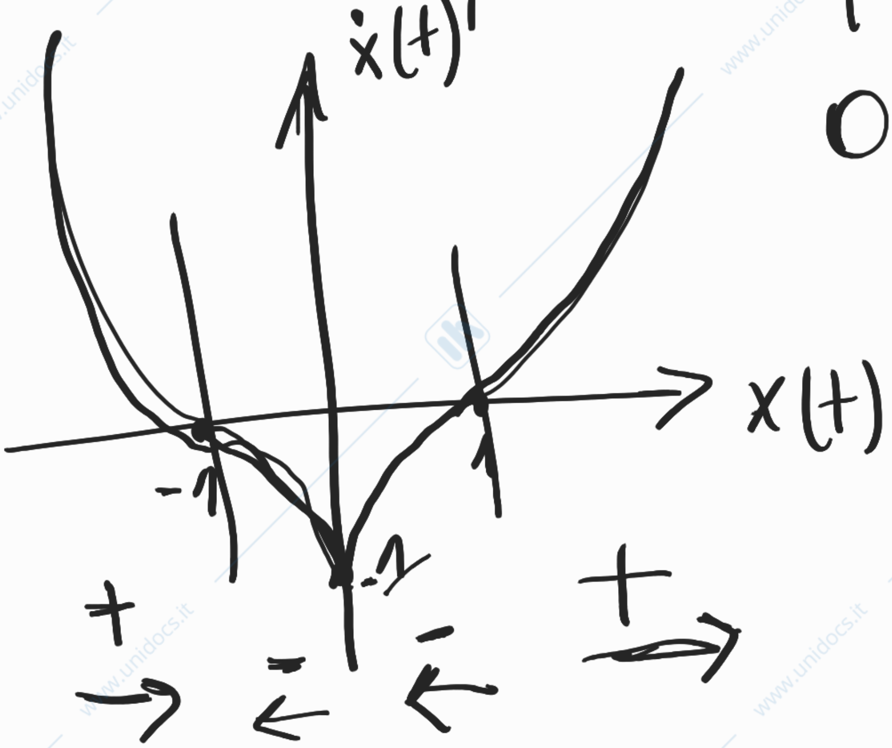
$$\frac{\partial f}{\partial x}\bigg|_{\bar{x},\bar{u}} = -3x^2(t) - 6x(t) - 3\bigg|_{\bar{x}} = 0$$

$$\delta \dot{x}(t) = \cancel{A\,\delta x(t)} + B\,\delta u(t) =$$

$$= -\delta u(t)$$

3) stabilità punti equilibrio

$$0 = f(\bar{x},\bar{u})$$



(attrattivo)

ASINTOTICAM STABILE

INSTABILE

4. Movimento forzato e stabilità

Sistema:
$$\begin{cases} \dot{x}_1(t) = -2x_1(t) + 2x_2(t) + u(t) \\ \dot{x}_2(t) = -4x_2(t) \end{cases}$$

1) Classificazione:

- 2° ordine
- t. INVARIANTE
- D e uscita nulla

- single input
- DIAGONALE

2) Determinare A e B + autovalori e modi A

$$A = \begin{bmatrix} -2 & 2 \\ 0 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\det(\lambda I - A) = 0$$

$$\det \begin{bmatrix} \lambda+2 & -2 \\ 0 & \lambda+4 \end{bmatrix} = 0$$

$$(\lambda+2)(\lambda+4) = 0 \rightarrow \lambda_1 = -2, \quad \lambda_2 = -4$$

stabili λ con $\mathrm{Re} < 0$.

$\Longrightarrow$ sisteme AS. STABILE

MODI sisteme : $e^{\lambda_1 t}$ $e^{\lambda_2 t}$

e^{-2t} e e^{-4t} $\xrightarrow[t \to +\infty]{} 0$

3) Determenare movimento stato per $x(0) = [0,0]^T$, $u(t) = e^t$

$$\dot{x}(t) = \underbrace{\begin{bmatrix} -2 & 2 \\ 0 & 4 \end{bmatrix}}_{\text{triangolare}} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}$$

$$\dot{x}_2(t) = 4x_2(t) \quad + \cancel{Bu(t)}$$

metodo a cascata

$$x_2(t) = x_{20} e^{At} \qquad A = 4$$

$$x_2(t) = x_{20} e^{4t} = 0$$

$$\dot{x}_1(t) = -2x_1(t) + \underbrace{2x_2(t) + u(t)}_{:w(t)}$$

$$A = -2 \qquad B = 1$$

$$\dot{x}_1(t) = -2x_1(t) + 1 \cdot \underline{w(t)}$$

$$x_1(t) = x_{10} e^{4t} + \int_0^t e^{A(t-\tau)} B\, w(\tau)\, d\tau$$

$$= \cancel{0 e^{-2t}} + \int_0^t e^{-2(t-\tau)} \cdot \cancel{1} \left[\cancel{2x_2(\tau)} + u(\tau)\right] d\tau$$

$$x_1(t) = e^{-2t}\int_0^t e^{2\tau}\cdot e^{\tau}\,d\tau =$$

$$= e^{-2t}\cdot\left[\frac{e^{3\tau}}{3}\right]_0^t =$$

$$= \frac{e^{t} - e^{-2t}}{3}$$

movimento stato combinaz lineare INGRESSO e MODI del sistema

4) Calcolare movimento sistema per $x(0) = [0\ 0]^T$ e $u(t) = 1$

$$x_2(t) = x_{20} e^{At} = 0 \cdot e^{4t} = 0$$

$$x_1(t) = \int_0^t e^{-2(t-\tau)} \, 1 \, d\tau$$

$$w(t) = \underbrace{u(t)}_{1} + \cancel{2x_2(t)} \qquad B = 1$$

$$x_1(t) = e^{-2t} \int_0^t e^{2\tau} \, d\tau = e^{-2t} \left[\frac{e^{2\tau}}{2} \right]_0^t = \frac{1 - e^{-2t}}{2}$$

$$x_{11}(t) = \frac{e^{t} - e^{-2t}}{3} \xrightarrow{t \to +\infty} +\infty$$

$$x_{12}(t) = \frac{1 - e^{-2t}}{2} \longrightarrow \frac{1}{2}$$

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31/08/2020

1) sist dinamico NON LINEARE

$\dot{x}_1 = -x_1 x_2^3 + 1$

$\dot{x}_2 = -x_1 + u$

$y = \dfrac{x_1}{x_2}$

2) stati equilibrio per $u(t) = \bar{u}$

- $0 = -\bar{x}_1 \bar{x}_2^3 + 1$

$0 = -\bar{x}_1 + \bar{u} \rightarrow \bar{x}_1 = \bar{u}$

$\bar{y} = \dfrac{\bar{x}_1}{\bar{x}_2}$

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$$0 = -\bar{U}\bar{X}_2^3 + 1$$

$$\bar{X}_2^3 = \frac{1}{\bar{U}} \longrightarrow \bar{X}_2 = \sqrt[3]{\frac{1}{\bar{U}}} = \frac{1}{\sqrt[3]{\bar{U}}}$$

$$\bar{Y} = \frac{\bar{U}}{\sqrt[3]{\frac{1}{\bar{U}}}} = \bar{U} \cdot \sqrt[3]{\bar{U}}$$

b) sistema linearizzato punti eq

$$\begin{cases} \delta\dot{x}(t) = A\,\delta x(t) + B\,\delta u(t) \\ \delta y(t) = C\,\delta x(t) + D\,\delta u(t) \end{cases}$$

$$\delta\dot{x}_2 = -\delta x_1 + \delta u$$

$$\dot{x}_1 = -x_1 x_2^3 + 1$$

$$\delta \dot{x}_1(t) = H\,\delta x_1(t) + I\,\delta x_2(t) + K\,\delta u$$

$$A = \begin{array}{c} \begin{matrix} \qquad H \qquad & \qquad I \qquad \end{matrix} \\ \begin{bmatrix} \left.\frac{\partial f}{\partial x_1}\right|_{eq} & \left.\frac{\partial f}{\partial x_2}\right|_{eq} \\ \left.\frac{\partial g}{\partial x_1}\right|_{eq} & \left.\frac{\partial g}{\partial x_2}\right|_{eq} \end{bmatrix} \end{array}$$

$$\left.\frac{\partial f}{\partial x_1}\right|_{eq} = -\bar{x}_2^3 = -\frac{1}{u}$$

$$\left.\frac{\partial f}{\partial x_2}\right|_{eq} = -3\bar{x}_1\bar{x}_3^2 = \frac{-3u}{\sqrt[3]{(\bar{u})^2}}$$

$$\delta \dot{x}_1(t) = -\frac{1}{U}\delta x_1(t) - \frac{3U}{\sqrt[3]{U^2}}\delta x_2(t)$$

$$y = \frac{x_1}{x_2} \qquad D = 0$$

$$C = \left[\left.\frac{\partial h}{\partial x_1}\right|_{eq} \quad \left.\frac{\partial h}{\partial x_2}\right|_{eq}\right]$$

$$\left.\frac{\partial h}{\partial x_1}\right|_{eq} = \frac{1}{\bar{x}_2} = \sqrt[3]{U}$$

$$\left.\frac{\partial h}{\partial x_2}\right|_{eq} = \frac{-x_1}{(x_2)^2} = -U\cdot\sqrt[3]{U^2}$$

$$\delta y(t) = \sqrt[3]{U}\,\delta x_1(t) - U\sqrt[3]{U^2}\,\delta x_2(t)$$

c) stabilità equilibri

$$A = \begin{bmatrix} -\frac{1}{U} & -\frac{3U}{\sqrt[3]{U^2}} \\ -1 & 0 \end{bmatrix}$$

$$\det(\lambda I - A) = 0$$

$$\det \begin{bmatrix} \lambda + \frac{1}{U} & \frac{3U}{\sqrt[3]{U^2}} \\ 1 & \lambda \end{bmatrix} = 0$$

$$\lambda^2 + \frac{1}{U}\lambda - \frac{3U}{\sqrt[3]{U^2}} = 0$$

eq 2° grado C.N ASINT

STABILE coeff tutti concordi

$\Longrightarrow$ coeff discordi $\Longrightarrow$ sistema linearizzato INSTABILE

$\Longrightarrow$ sist partenza INSTABILE