giovedì 19 marzo 2020 13:03

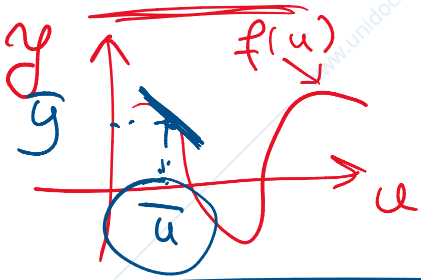
LINEARIZZAZIONE PER SISTEMI DINAMICI LTI

DINAMICO
↳ N.L e T.I

Pto di EQUIL. → APPROX LINEARE
$u = \bar{u}$ cost
↳ $\bar{x}, \bar{y}$

STATICA

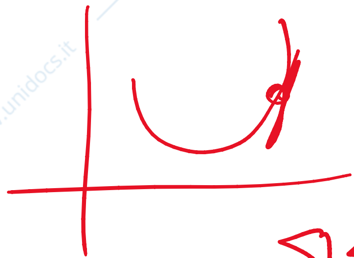
PUNTO di LINEAR.



APP. LIN

SISTEMI DINAMICI LTI

NL e TI → $u = \bar{u}$ COSTANTE → APPROX. LINEARE ATTORNO AD UN PUNTO DI EQ
$\bar{x}, \bar{y}$



SISTEMA LINEARIZZATO

SISTEMA DINAMICO LTI TANGENTE A QUELLO N.L. NEL PUNTO di EQ

PROCEDIMENTO DI LINEARIZZAZIONE

(1) SISTEMA DINAMICO NL e TI

① SISTEMA DINAMICO NL e T I

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = g(x(t), u(t)) \end{cases}$$

CONSIDERO UN INGRESSO $u(t) = \bar{u}$ COSTANTE $\forall t \geq 0$

Assumo che, ASSOCIATI a $u(t) = \bar{u}$ ci siano uno STATO e UNA USCITA di EQ $\bar{x}$ e $\bar{y}$ $\Rightarrow$ $\boxed{\begin{matrix} f(\bar{x}, \bar{u}) = 0 \\ \bar{y} = g(\bar{x}, \bar{u}) \end{matrix}}$

② CALCOLO del SISTEMA LINEARIZZATO

SERIE di TAYLOR di 1° ORD

$$(\square)\quad f(x,u) \simeq \cancel{f(\bar{x},\bar{u})}^{\,0} + \left.\frac{\partial f(x,u)}{\partial x}\right|_{EQUIL.} \underbrace{(x-\bar{x})}_{\delta x} + \left.\frac{\partial f(x,u)}{\partial u}\right|_{EQ} \underbrace{(u-\bar{u})}_{\delta u}$$

È UN NUMERO

$$(\circ)\quad g(x,u) \simeq \underbrace{g(\bar{x},\bar{u})}_{\bar{y}} + \left.\frac{\partial g(x,u)}{\partial x}\right|_{EQ} (x-\bar{x}) + \left.\frac{\partial g(x,u)}{\partial u}\right|_{EQ} (u-\bar{u})$$

Definiamo $\left.\begin{matrix} \delta x = x - \bar{x} \\ \delta y = y - \bar{y} \end{matrix}\right\}$ VAR. di STATO, USCITA del sistema

$$(*)\ \delta y = y - g(\bar{x},\bar{u}) \qquad \boxed{\delta y = y - \bar{y}} \quad \text{USCITA} \qquad \text{del sistema}$$

$$\delta u = u - \bar{u} \quad \text{INGRESSO} \qquad \text{linearizz}$$

EQUAZIONI DEL SISTEMA LINEARIZZATO (LTI)

$$\delta \dot{x} = \dot{x} \overset{(\square)}{\simeq} \boxed{\left.\frac{\partial f}{\partial x}\right|_{EQ}}^{A} \delta x + \boxed{\left.\frac{\partial f}{\partial u}\right|_{eq}}^{B} \delta u$$

$$\delta y \overset{(0),(*)}{=} \boxed{\left.\frac{\partial g}{\partial x}\right|_{EQ}}^{C} \delta x + \boxed{\left.\frac{\partial g}{\partial u}\right|_{EQ}}^{D} \delta u$$

$$f(x,u) \simeq \underset{0}{\cancel{f(\bar{x},\bar{u})}} + \frac{\partial f}{\partial x} \underbrace{(x-\bar{x})}_{\delta x} + \left.\frac{\partial f}{\partial u}\right|_{EQ} \underbrace{(u-\bar{u})}_{\delta u}$$

ALL'EQ

$$0 = f(\bar{x},\bar{u})$$

$$\delta x = x - \bar{x} \quad \text{COSTANTE}$$

$$\dot{x} = f(x,u)$$

$$\delta \dot{x} = \dot{x} \simeq \left.\frac{\partial f}{\partial x}\right|_{ep} \delta x + \left.\frac{\partial f}{\partial u}\right|_{ep} \delta u$$

$$g(x,u) \simeq \boxed{g(\bar{x},\bar{u})} + \left.\frac{\partial g}{\partial x}\right|_{ep} (x-\bar{x}) + \left.\frac{\partial g}{\partial u}\right|_{ep} (u-\bar{u})$$

$$y = g(x,u)$$

$$\bar{y} = g(\bar{x},\bar{u})$$

$$\delta y = y - \bar{y}$$

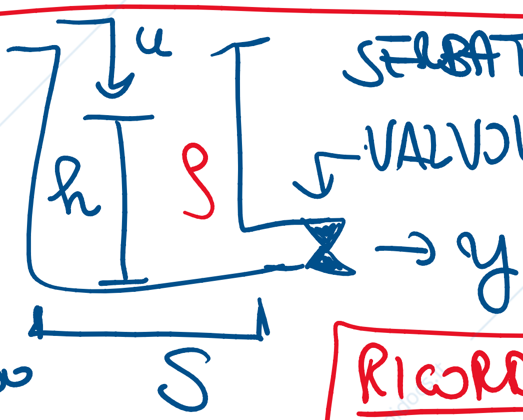
$$\delta y = \left.\frac{\partial g}{\partial x}\right| \delta x + \left.\frac{\partial g}{\partial u}\right|_{ep} \delta u$$

$$\delta y = \frac{\partial g}{\partial x}\Big|_{eq} \delta x + \frac{\partial g}{\partial u}\Big|_{eq} \delta u$$

$$\underset{m \times m}{A_{LIN}} = \left[\frac{\partial f}{\partial x}\Big|_{eq} \right] \qquad \underset{m \times m}{B_{LIN}} = \left[\frac{\partial f}{\partial u}\Big|_{eq} \right]$$

$$\underset{p \times m}{C_{LIN}} = \left[\frac{\partial g}{\partial x}\Big|_{eq} \right] \qquad \underset{p \times m}{D_{LIN}} = \left[\frac{\partial g}{\partial u}\Big|_{eq} \right]$$

ESEMPIO



SERBATOIO

VALVOLA DI USCITA

u = PORTATA ENTRANTE

$x = h$: LIVELLO LIQUIDO

S: AREA SERBATOIO

y = PORTATA IN USCITA

VALVOLA PORTATA IN USCITA $\sim \alpha\sqrt{h}$

RICORDA

$$\dot{h} = \frac{1}{\rho S}\left(W_i - W_o\right)$$

($\dot h \to x$, PORTATA IN $W_i \to u$, PORTATA OUT W_o)

$\boxed{\text{EQ. STATO} \quad (NL, TI)}$

$$\begin{cases} \dot{x} = \dfrac{1}{\rho S}\left(u - \alpha\sqrt{x}\right) \\ y = \alpha\sqrt{x} \end{cases}$$

① $\dot{x}(t) = \frac{1}{...}\left(-\alpha\sqrt{x(t)} + u(t) \right)$

(1)

$$\boxed{\begin{aligned}\dot{x}(t) &= \frac{1}{\rho S}\left(-\alpha\sqrt{x(t)} + u(t)\right)\\ y(t) &= \alpha\sqrt{x}\end{aligned}}$$

NON LIN
T. INV

$S = 10m^2$
$\rho = 1,\ \alpha = 1$

$$\begin{cases}\dot{x}(t) = -\frac{1}{10}\sqrt{x} + \frac{1}{10}u \\ y = \sqrt{x}\end{cases}$$

INGRESSO

$u(t) = \bar{u} = 1,\ t \geq 0$

CALCOLO $\bar{x}, \bar{y}$

$$0 = -\frac{1}{\cancel{10}}\sqrt{\bar{x}} + \frac{1}{\cancel{10}}\bar{u}$$

$$\boxed{\begin{aligned}\dot{x} &= \frac{1}{10}\sqrt{x} + \frac{1}{10}u \\ y &= \sqrt{x}\end{aligned}}$$

$\bar{u} = \sqrt{\bar{x}} \iff \bar{x} = 1$

$$\boxed{\bar{y} = \sqrt{\bar{x}} = 1}$$

CONDIZ EQ

$$\boxed{\begin{aligned}\bar{x} &= 1\\ \bar{y} &= 1\\ \bar{u} &= 1\end{aligned}}$$

CALCOLO SIST. LIN

$$\delta\dot{x} = \left.\frac{\partial f}{\partial x}\right|_{ep}\delta x + \left.\frac{\partial f}{\partial u}\right|_{ep}\delta u$$

$$= \left.\left(-\frac{1}{10}\left(\frac{1}{2}x^{-1/2}\right)\right)\right|_{eq}\delta x + \frac{1}{10}\delta u$$

$$\delta\dot{x} = -\frac{1}{20}\delta x + \frac{1}{10}\delta u$$

$$\boxed{\delta\dot{x} = -\frac{1}{20}\delta x + \frac{1}{10}\delta u}$$

$$y = \underbrace{\sqrt{x}}_{g(x,u)}$$

$$\delta y = y - \bar{y} = y - g(\bar{x}, \bar{u})$$

$$= \left.\frac{\partial g}{\partial x}\right|_{EQ} \delta x + \left.\frac{\partial g}{\partial u}\right|_{eq} \delta u$$

$$= \left.\frac{1}{2}x^{-1/2}\right|_{eq} \delta x + 0\, \delta u$$

$$\boxed{\delta y = \frac{1}{2}\delta x}$$

~~EQ.~~ SISTEMA LIN

$$\boxed{\begin{aligned}\delta\dot{x} &= -\frac{1}{20}\delta x + \frac{1}{10}\delta u \\ \delta y &= \frac{1}{2}\delta x\end{aligned}}$$

$A_{LIN} = -\frac{1}{20}$

$B_{LIN} = \frac{1}{10}$

$C_{LIN} = \frac{1}{2}$

$D_{LIN} = 0$