Analysis of circuits with negative feedback

Fondamenti di Elettronica

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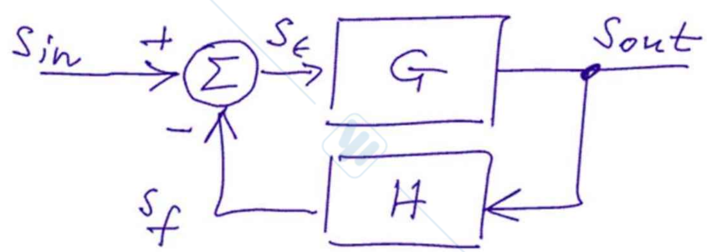
v1.6red

A mathematical introduction to feedback

- Feedback is an important concept in electronics. Feedback is covered also in courses on control theory but the complexity of circuit topologies require to develop a specific approach in order to fully analyze a real circuit within the framework of feedback theory. New concepts and terminology need to be developed (open and closed-loop parameters, loop gain, ideal gain, direct feed-through, etc.). It must be stressed that feedback is virtually present in all analog circuits, therefore it is important that students develop a good understanding of feedback concepts in circuits and learn a sound analysis method.

- In most circuit design books, feedback analysis is presented with reference to the classical block diagram (*) – consisting in a ***unilateral*** forward amplifier (forward gain, G) and a ***unilateral*** feedback network (feedback factor, H) - where s_{in} and s_{out} can each be either a current or a voltage.



- The closed-loop gain (s_{out}/s_{in}) is easily obtained from the following passages: $s_{out}=Gs_e=G(s_{in}-s_f)=G(s_{in}-Hs_{out})$ and, solving for (s_{out}/s_{in}), you get

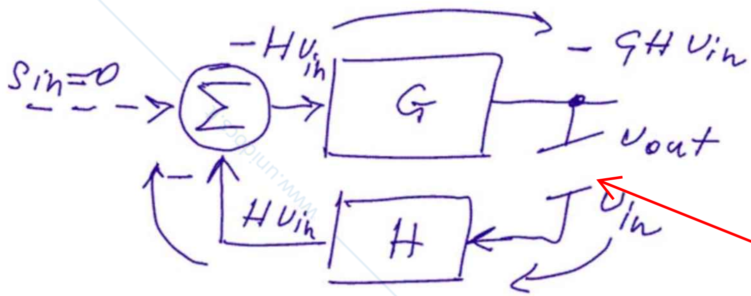
$$G_{closed-loop} = \frac{s_{out}}{s_{in}} = \frac{G}{1+GH}$$

(*) other possible block schemes could be employed, but all lead to the same general properties

Loop gain

- **Loop gain (G_{LOOP}):** it is the distinctive feature of the feedback system. It is the amplification of a test signal after "one lap around the feedback loop". It can be measured by inserting a test signal into the loop, with the input signal set to zero (it must be stressed that the insertion of the the test source must be such that it does not alter the loop gain). If G_{LOOP} is equal to zero there is no feedback in the circuit. The computation of the loop gain in the classic block diagram is shown below.

si spegne il segnale d'igresso e rimane solo l'anello



$$G_{LOOP} = \frac{u_{out}}{u_{in}} = -GH$$

si taglia poi l'anello inserendo un segnale test d'ingresso verso il blocco H

- The **"ideal" condition** for the feedback loop **is to have infinite G and hence infinite G_{LOOP}**.
- The **loop gain** can be either **positive** or **negative**. When the loop gain is negative, if you apply a positive step at the input, the loop returns a positive feedback signal s_f, which is subtracted from the input so that the error signal s_ε is smaller ($s_\varepsilon = s_i - s_f < s_i$)
- It will be clear after studying the stability of feedback systems (e.g. see Bode criterion for stability) that systems with positive feedback ($G_{LOOP} > 0$) tend to be unstable, i.e. with poles having positive real part, while systems with negative feedback ($G_{LOOP} < 0$) can be designed to be stable.
- Here we will restrict to the analysis of **negative feedback circuits**, which are commonly used to design analog amplifiers with superior performances.
- The inherent instability of **positive feedback circuits** is exploited in special circuits (comparators, oscillators, etc.).

quando il guadagno ad anello è positivo il circuito risulta di solito instabile

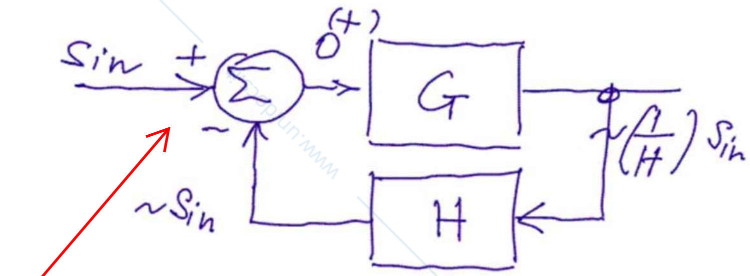
General properties of ***negative*** feedback systems

- The main variables (s_{out} , s_f , s_ε) of a negative feedback system, in response to the input signal (s_{in}), are summarized below. The asymptotic behavior for G→∞ is also shown.

$$\frac{s_{out}}{s_{in}} = \frac{G}{1+GH} \xrightarrow{G\to\infty} \left(\frac{1}{H}\right)$$

$$\frac{s_f}{s_{in}} = \frac{GH}{1+GH} \xrightarrow{G\to\infty} 1$$

$$\frac{s_\varepsilon}{s_{in}} = \frac{1}{1+GH} \xrightarrow{G\to\infty} 0^+$$



il segnale di feedback tende ad uguagliare il segnale di ingresso

- It is interesting to note that **for G→∞ the closed-loop gain does not depend on G any more but it depends only on the feedback factor H**. In this limiting condition, the feedback signal (s_f) is forced to be exactly equal to the input signal, so that the error signal (s_ε) tends to zero.
- The **asymptotic (or *ideal*) closed-loop gain, G_{ID}, equals (1/H).** Therefore in order to have closed-loop gain with magnitude >1 the feedback factor H is an attenuator.
- It is normally easier to make a ***precise*** attenuator (e.g. the attenuation of a resistive divider is related to the ***ratio*** of resistances, which can be made very precisely), than to make an amplifying stage with precise gain because the parameters of any active device depend on several variables (time, environmental conditions, aging, etc.).
- This means that using the concept of feedback **we can design an amplifier with very precise gain (more precise than a single amplifying stage) by using a precise attenuator and an (intrinsically less precise) amplifying stage, provided it has very large gain**.

Application of feedback to real circuits

- The **classical feedback diagram** is very useful as it can explain **all the general properties of feedback.** However it is an idealized picture that **does not fit all real circuits** (see Notes below).
- Alternatively we can re-express the closed-loop gain in terms of two other relevant quantities: the **Loop Gain (G_{LOOP})** and the **Ideal Gain (G_{ID})**:

$$G_{closed-loop} = \frac{s_{out}}{s_{in}} = \frac{G}{1+GH} = \left(\frac{1}{H}\right)\frac{GH}{1+GH} = G_{ID}\frac{-G_{LOOP}}{1-G_{LOOP}}$$

$$G_{LOOP} = \frac{u_{out}}{u_{in}} = -GH$$

$$G_{ID} = \frac{s_{out}}{s_{in}} = \frac{G}{1+GH} \xrightarrow{G\to\infty} \left(\frac{1}{H}\right)$$

- G_{ID} corresponds to the asymptotic gain of the system, which is typically the designed gain and it is a meaningful quantity by itself. By computing the actual value of G_{LOOP} one obtains the exact closed-loop gain.
- The main advantage of this formulation is that **both quantities can be derived directly from the circuit**, with no previous knowledge of G and H and of the block diagram.

Notes:

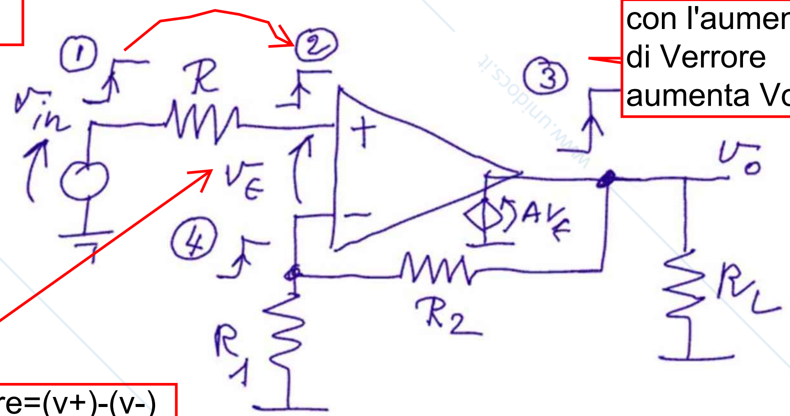
- *The classical feedback diagram assumes unilateral blocks (while it is intuitive that even simple resistive networks have bilateral transfer), moreover in electronic circuits the analysis is complicated by the interaction of the feedback network with the forward amplifier (i.e. the gain of a stage depends on the ratio of its output resistance to the load resistance, the well-known "load effect").*
- *In order to apply it to a real feedback circuit, we have to manipulate the circuit to fit the idealized block diagram, i.e. we have to define the blocks G, H in terms of unilateral two-port networks. Then one can use the known expressions to compute the variables (s_{out}, s_f, s_ε). This procedure is tedious and not always feasible due to the above mentioned assumptions.*

Computation of the ideal gain in real circuits

- Let us analyze the following feedback circuit in a qualitative way:

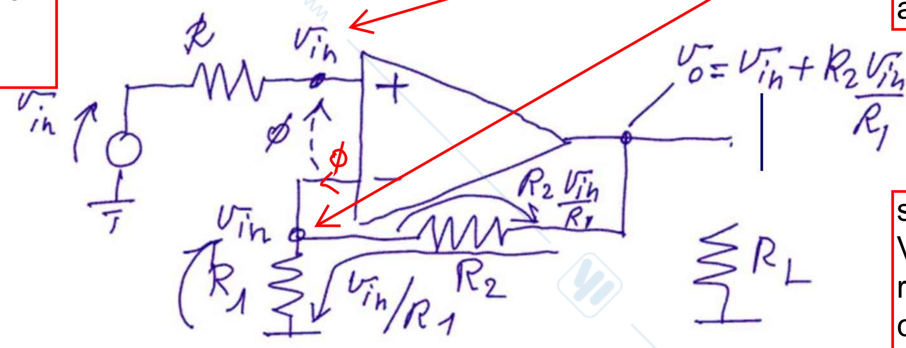
- The input voltage v_{in} (1) is applied to the positive input of the opamp (2) (there is no current on R) and produces a positive step at the output of Vout (3) and therefore a positive step vf is fed back to the negative opamp input (4). Therefore , due to the KVL at the v_{in}-v_ε-v_f loop, the differential voltage $v_\varepsilon = v^+ - v^-$ which drives the op-amp is reduced by the action of negative feedback.

do in ingresso un segnale a scalino



Verrore=(v+)-(v-)

con l'aumentare di Verrore aumenta Vo



il morsetto negativo tenderà a chiudere il contatto uguagliando Vin al morsetto pos.

se conoscono Vin, riesco a ricavare la corrente che passa per R1, applicando LKT ai rami di R1 e R2 (deve scorrere stessa corrente perchè dal ramo del morsetto negativo non passa corrente) trovo le tensioni sulle R e quindi trovo Vo

- If the loop gain is high, Vε tends to zero (i.e. V- tends to be closer to V+) and, asymptotically for A→∞, you have V-=Vin. The current on R1 (=Vin/R1) flows in R2 so that the output voltage Vout=Vin+Vin(R2/R1)=Vin(1+R2/R1). This represents the “ideal” gain of the circuit.

Dunque applicando la condizione che Ve sia circa 0, quindi che la tensione ai morsetti tende a essere uguale, troviamo la Vo e il guadagno in condizioni asintotiche che ci serviranno per ricostruire il gudagno nel caso realistico

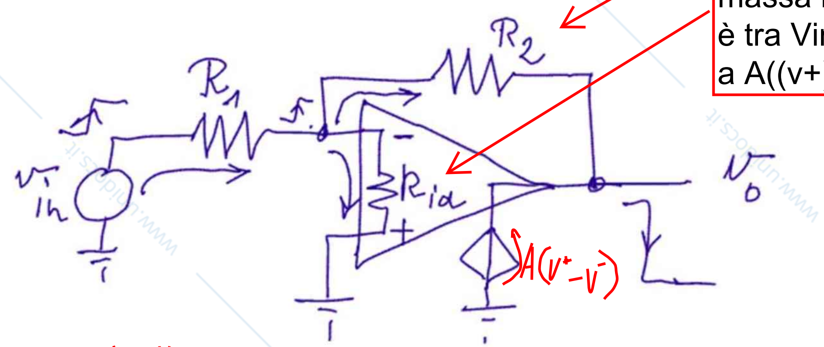
Computation of the ideal gain in real circuits (2)

- Let's see a second example.

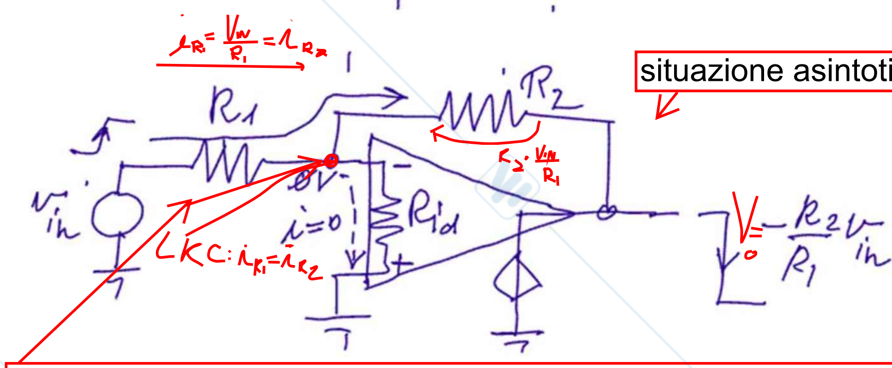
- The positive step Vin, applied to the negative branch of the opamp, forces a current in R1 and a fraction of it will go into Rid. This fraction produces a positive step on V- and, being V+ at ground, a negative differential voltage Vε=(V+-V-) which drives the opamp output. The negative output voltage increases the voltage drop on R2 and therefore the current fraction flowing into R2 increases, thus reducing the current fraction going into Rid.

- If the loop gain is high, the current in Rid and V- tends to be closer to 0 and, asymptotically for A→∞, you have V-=0. The current on R1 (=Vin/R1) flows in R2 (as Rid can not take current) so that the output voltage is given by the drop on R2: Vout=-(Vin/R1)*R2=Vin(-R2/R1), which shows the ideal gain of this circuit.

NON c'è un parallelo tra R2 e Rid poichè il ramo di Rid va a massa mentre R2 è tra Vin e il nodo a A((v+)-(v-))



situazione asintotica



mi basta conoscere il pot a un nodo per risolvere il circuito in un caso ideale

la grandezza ((v+)-(v-))che pilota il blocco finito tende a 0, in questo caso (v+) è a massa quindi (v-) tende a diventare circa 0

In conclusion, to compute the ideal gain directly on the circuit:

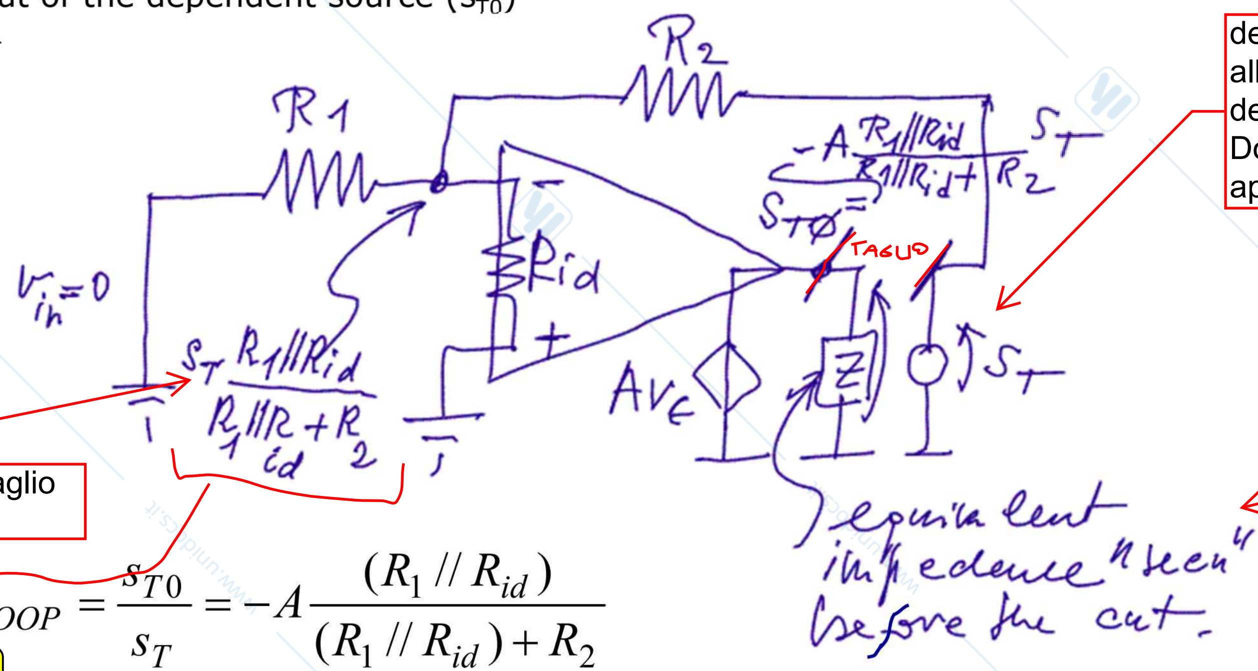
- **set the controlling quantity (in the example $v_\varepsilon=v^+-v^-$) of the dependent source to zero** (v_ε is like the error signal in the classical block diagram)
- **Find the output variable with constraint $v_\varepsilon=0$.**
- $G_{ID} = (V_{out}/V_{in})_{v_\varepsilon=0}$

Computation of the loop gain in real circuits

- We need to break the loop without altering the signal transfer along the loop. To this end **we identify the dependent source which has generally the role to provide gain in the feedback loop**. This is straightforward as a dependent source is present in the equivalent model of any active device (op-amp or transistor).

- Then:
- Set all ***independent*** sources to zero
- Break the connection between the ***dependent*** source and the rest of the circuit and restore the original impedance (Z).
- Drive the circuit at the break point with a test source (s_T) of same type of the dependent source to probe the loop gain
- Find the output of the dependent source (s_{T0})
- $G_{LOOP} = s_{T0}/s_T$

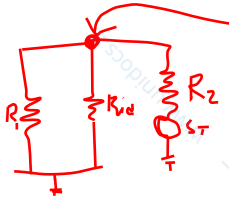
si spengono innanzitutto gli ingressi



devo entrare all'interno dell'nello chiuso. Dovrò tagliare, aprire l'anello.

metto un'impedenza equivalente(a tutta l'impedenza vista dal gen.pilotato prima del taglio) sul ramo del gen.pil.

St=Stest col taglio



$$G_{LOOP} = \frac{s_{T0}}{s_T} = -A \frac{(R_1 // R_{id})}{(R_1 // R_{id}) + R_2}$$

proviamo a tagliare in un altro punto

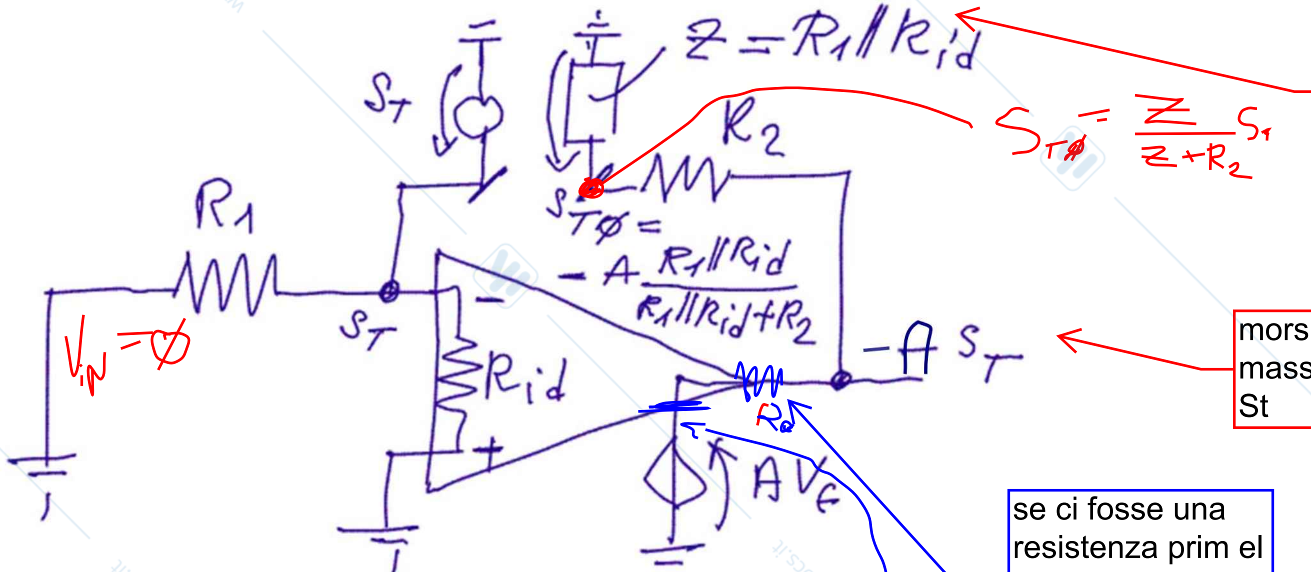
Computation of the loop gain in real circuits (2)

- Breaking the loop may alter the impedance level at the break point and therefore it may lead to a wrong value of G_{LOOP}, i.e. different from the loop gain in closed-loop condition. Breaking at the dependent source is a smart way to avoid the problem, as the output of the dependent source does not depend on the value of the load impedance (Z) which need not be computed.

- Breaking at a different point along the loop is also possible but - as the break point is no more generally driven by an ideal source – in order to obtain the correct loop gain one must compute the original load impedance Z at the break as it now affects the result. The computation of the loop gain is shown in the following scheme. The result is of course the same as before.

in generale possiamo tagliare dove ci pare però dobbiamo calcolarci l'impedenza eq. nel caso in cui Z compare nell'espress. finale

ci conviene comunque tagliare in punto in cui non dobiamo calcolare l'impedenza eq. prima del taglio



mors.pos. a massa e neg. a St

se ci fosse una resistenza prim el gen. pil. dobbiamo tagliare qui, osssia la testa del gen.

questo guadagno non è infinito come quello ideale

$$G_{LOOP} = \frac{s_{T0}}{s_T} = -A\frac{(R_1 // R_{id})}{(R_1 // R_{id}) + R_2}$$

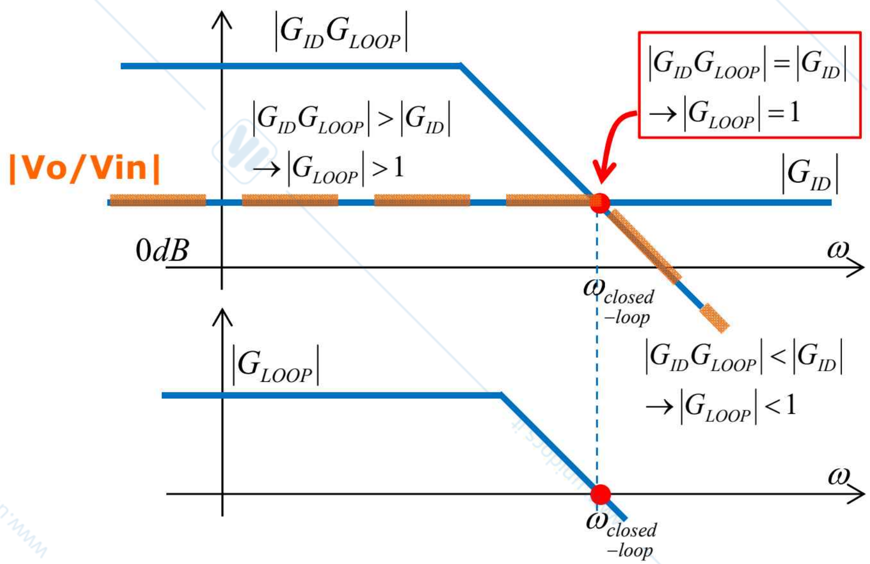
Frequency response of the closed loop gain: graphical method (1)

- When either G_{ID} or G_{LOOP} depend on frequency (s=jω), we can obtain the frequency response of the closed loop gain Vo/Vin(s) analytically, i.e. by direct substitution of $G_{ID}(s)$ or $G_{LOOP}(s)$ in the following equation.

$$G_{closed-loop} = \frac{v_{out}}{v_{in}} = G_{ID}\frac{-G_{LOOP}}{[1-G_{LOOP}]}$$

- However, in order to know the poles of Vout/Vin(s) (and be able to draw its Bode plot) we have to compute the roots of the expression at the denominator [1-$\boldsymbol{G_{LOOP}(s)}$]. In the case of 1st order systems (i.e. $\boldsymbol{G_{LOOP}(s)}$ with 1 pole), the analytical procedure is trivial but it quickly becomes cumbersome for 2nd order or unpractical for higher order.
- A useful method to draw an approximated Bode plot of |Vo/Vin(jω)| is based on the 2 asymptotic curves of Vout/Vin(s), obtained in the limit of |$\boldsymbol{G_{LOOP}}$| much larger or much smaller than 1:

$$\frac{v_{out}}{v_{in}}(j\omega) = G_{ID}(j\omega)\frac{-G_{LOOP}(j\omega)}{1-G_{LOOP}(j\omega)} \rightarrow \begin{cases} -G_{ID}(j\omega)\cdot G_{LOOP}(j\omega) & \text{for } |G_{LOOP}| << 1 \\ G_{ID}(j\omega) & \text{for } |G_{LOOP}| >> 1 \end{cases}$$

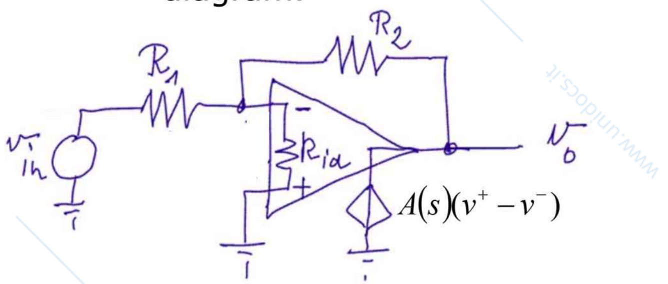


- The Bode diagram of the 2 asymptotic curves is straightforward (zeros and poles are known).
- The Bode diagram of the closed-lop gain |Vo/Vin| can be obtained by joining the sections of the 2 asymptotic curves in their respective regions of validity (dashed orange).
- It is useful to verify that when the curve $|G_{ID}\ G_{LOOP}|$ is above $|G_{ID}|$, this corresponds to the case $|G_{LOOP}|>1$ and viceversa (see also the associated plot of $|G_{LOOP}|$ below). This allows an easy selection of the valid asymptotic curve at any frequency.
- The point of intersection of the 2 curves (red point) is the breakeven point (i.e. $|G_{LOOP}|=1$), where |Vo/Vin| leaves the ideal behaviour $|G_{ID}|$. That frequency typically corresponds to the dominant pole of your circuit.
- ***Note: this graphical method is rigorous only for first-order systems. However, due to its simplicity, it is often used in circuit design for approximate estimation of the dominant pole of a feedback circuit.***

Frequency response of the closed loop gain: graphical method (2)

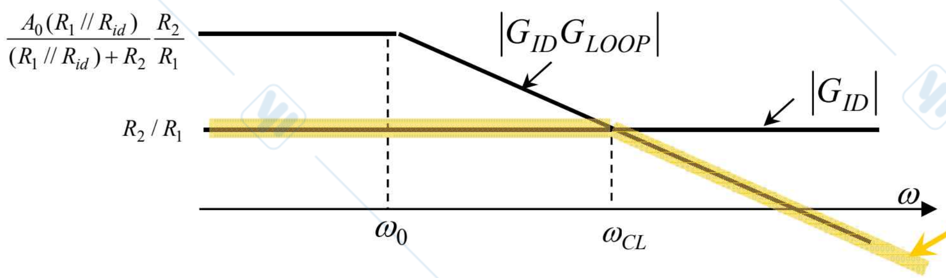
Example:

- The frequency response of the closed-loop gain of the previous circuit example (slides 7-8-9) is shown below (yellow line). It follows the asymptotic curve $|G_{ID}|$ when $|G_{LOOP}|>>1$ while is follows the curve $|G_{ID}\ G_{LOOP}|$ when $|G_{LOOP}|<<1$. The frequency at which $|G_{LOOP}|=1$ (i.e. the crossing point between the two asymptotic curves) corresponds to the closed-loop pole (ω_{CL}).
- Once the Bode plot is drawn, the analytical expression of Vo/Vin(s) can be easily deduced from the diagram.



$$G_{ID} = -\frac{R_2}{R_1},\quad G_{LOOP} = -A(s)\frac{(R_1 // R_{id})}{(R_1 // R_{id}) + R_2} = -\frac{A_0}{1+s/\omega_0}\frac{(R_1 // R_{id})}{(R_1 // R_{id}) + R_2}$$

$$\Rightarrow G_{ID}G_{LOOP} = -\frac{A_0}{1+s/\omega_0}\frac{(R_1 // R_{id})}{(R_1 // R_{id}) + R_2}\frac{R_2}{R_1}$$



$$\left|\frac{v_{out}}{v_{in}}\right| = \left|\frac{-R_2/R_1}{1+s/\omega_{CL}}\right|_{s=j\omega}$$

computation of ω_{CL} :

$$\frac{R_2}{R_1}\omega_{CL} = \frac{A_0(R_1 // R_{id})}{(R_1 // R_{id}) + R_2}\frac{R_2}{R_1}\omega_0$$

$$\Rightarrow \omega_{CL} = \omega_0 \frac{A_0(R_1 // R_{id})}{(R_1 // R_{id}) + R_2} =$$

$$= 2\pi \cdot GBWP \frac{A_0(R_1 // R_{id})}{(R_1 // R_{id}) + R_2}$$

$$(GBWP = A_0 f_0 = gain \times bw\ of\ O.A.)$$

- ***Note: it is useful to remind that this is the same level of approximation of the usual straight-line approximation of the Bode plot of transfer functions with real poles or zeros: we neglect 1 when $\omega\tau$ <<1 or we neglect $\omega\tau$ when $\omega\tau$ <<1.***