Metal-semiconductor junction

1. Draw the qualitative band diagram for a junction between metal and n semiconductor (assume $q\Phi_{\mathrm{M}} > q\Phi_{\mathrm{S}}$), highlighting the most important energy levels and the energy barriers. Consider the three cases:
   (a) thermal equilibrium;
   (b) reverse bias;
   (c) forward bias with negligible voltage drop on the neutral regions
2. Derive the width of the semiconductor depleted region for a junction between metal and n semiconductor (assume $q\Phi_{\mathrm{M}} > q\Phi_{\mathrm{S}}$)
3. Assuming known the expression for the depleted region width, derive the depletion capacitance as a function of the applied bias voltage for a junction between metal and n semiconductor (assume $q\Phi_{\mathrm{M}} > q\Phi_{\mathrm{S}}$)
4. Derive the static characteristic of a junction between metal and n semiconductor (assume $q\Phi_{\mathrm{M}} > q\Phi_{\mathrm{S}}$)
5. Derive the expression of the differential conductance for a junction between metal and n semiconductor (assume $q\Phi_{\mathrm{M}} > q\Phi_{\mathrm{S}}$), and draw the small signal equivalent circuit

FET

1. Derive the charge control relation for a JFET or MESFET
2. Assuming known the charge control relation for an FET, derive the general expression for the device static characteristic
3. Discuss the differences between FET drain current saturation due to channel pinch off or due to carrier velocity saturation

Heterostructures

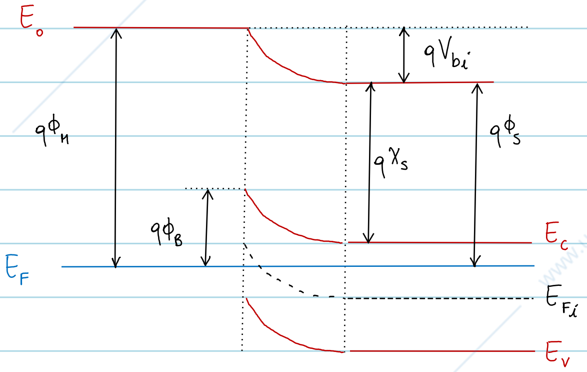
1. Derive the heterostructure affinity rule
2. Draw the qualitative equilibrium band diagram for an *nn* heterostructure, deriving the expression for the built in voltage (choose freely the order relationship between the material workfunctions)
3. Draw the qualitative equilibrium band diagram for a *pp* heterostructure, deriving the expression for the built in voltage (choose freely the order relationship between the material workfunctions)
4. Draw the qualitative equilibrium band diagram for an *pn* heterostructure, deriving the expression for the built in voltage (choose freely the order relationship between the material workfunctions)

MS JUNCTION

1 MS band diagram (n type) $q\phi_M > q\phi_S$

a) from Schottky theory as $q\phi_M > q\phi_S$ and the MS junction is characterized by a n-doped semicond., the junction is rectyfing

thermal equilibrium



$q\phi_S$: semicond workfunction as it is n-doped

$$q\phi_S = q\chi_s + K_B T \ln\left(\frac{N_c}{N_D}\right)$$

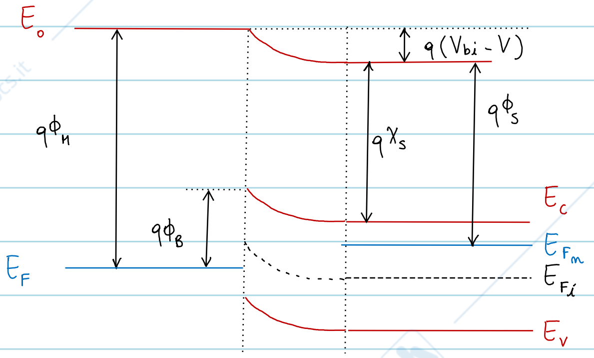
$q\phi_B$: Schottky barrier this is the energy barrier opposing to e^- flowing from metal to semicond

$$q\phi_B = q\phi_M - q\chi_s$$

qV_{bi} : voltage barrier across depleted region opposing to the e^- flowing from semicond to metal

$$qV_{bi} = q\phi_M - q\phi_S$$

c) forward bias $(V>0)$



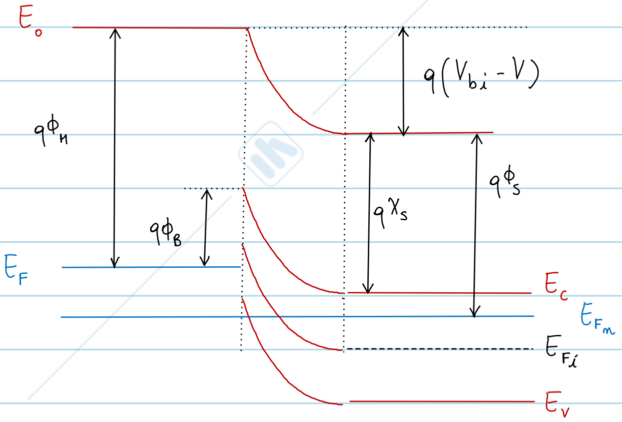
the potential energy on the depleted region is decreased $qV_{bi} \to q(V_{bi}-V)$, $V>0$

as the energy barrier $q\phi_B$ is indipendent to the applied bias, the current contribution made of e^- leaving the metal remains constant (small contribution due to thermoionic emission)

As the energy barrier on the depleted region decreases, the main current contribution is characterized by e^- leaving the semiconductor. The presence of this energy barrier (depleted region) gives the junction the diode like behaviour characterized by the static characteristic

$$I = I_S \exp\left(\frac{V}{V_T} - 1\right)$$

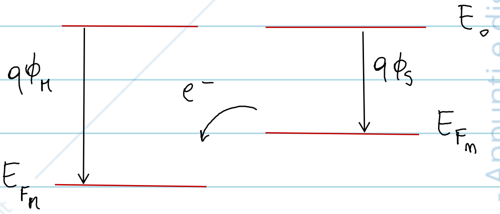
b) reverse bias ($V<0$)

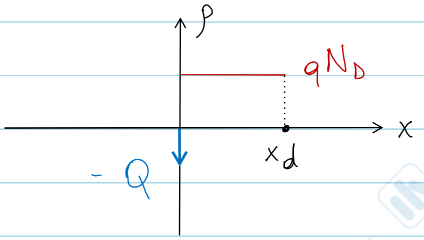


as in reverse bias condition the energy barrier at the depleted region increases and $q\phi_B$ remains constant the main current contribution is due to e^- leaving metal due to thermoionic emission. From Richardson law

$$J_{th} = A_0 T^2 \exp\left(-\frac{q\phi_B}{K_B T}\right)$$

2 as $q\phi_M > q\phi_S$ and n-doped semicond the junct. is rectifying
as $q\phi_M > q\phi_S$ during the transient to equilibr. after joining the two materials, e^- will flow from the semicond. to the metal The following charge distribution is the result





$$\rho(x) = -Q\delta(x) + \begin{cases} 0, & x<0 \\ qN_D, & 0<x<x_d \\ 0, & x>x_d \end{cases}$$

imposing the global neutrality condition $\int_{-\infty}^{+\infty} \rho(x)\,dx = 0$

$$-Q + qN_D x_d = 0 \iff -Q = -qN_D x_d$$

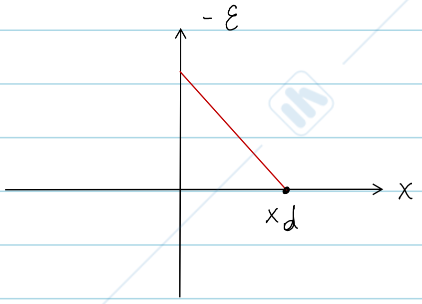
from Gauss law $\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{\epsilon} \iff \mathcal{E}(x) = \mathcal{E}(x\to-\infty) + \int_{-\infty}^{x} \rho(y)\,dy$

ideal metal and neutral regions ⇒ $\mathcal{E}(x\to\pm\infty) = 0$

$$\mathcal{E}(x) = \begin{cases} 0, & x<x_d \\ -\frac{qN_D}{\epsilon}x_d + \frac{qN_D}{\epsilon}x = \frac{qN_D}{\epsilon}(x-x_d), & 0<x<x_d \\ 0, & x>x_d \end{cases}$$

as $\frac{d\varphi}{dx} = -\mathcal{E}(x)$ <=> $\varphi(x) = \cancel{\varphi(x \to -\infty)} + \int_{-\infty}^{x} [-\mathcal{E}(y)]\,dy$

0 as it is the choosen reference



$$\varphi(x) = \begin{cases} 0, & x < x_d \\ \int_0^x -\frac{qN_D}{\epsilon}(y - x_d)\,dy = \left[-\frac{qN_D}{2\epsilon}(y - x_d)^2\right]_0^x = -\frac{qN_D}{2\epsilon}(x - x_d)^2 + \frac{qN_D}{2\epsilon}x_d^2 \\ \frac{qN_D}{2\epsilon}x_d^2, & x > x_d \end{cases}$$

as $\varphi(x_d) - \varphi(0) = V_{bi}$ => $V_{bi} = \frac{qN_D}{2\epsilon}x_d^2$

$$\boxed{x_d = \sqrt{\frac{2\epsilon}{qN_D}V_{bi}}}$$

3 the depleted region width for a rectyf. MS junction

$$x_d = \sqrt{\frac{2\epsilon}{qN_D}V_{bi}}$$

assuming low injection, the applied voltage bias drops entirely on the depleted region which width and charge:

$$x_d = \sqrt{\frac{2\epsilon}{qN_D}[V_{bi} - v(t)]}$$

considering the charge in the depleted region

$$Q = qN_D x_d = qN_D x_d[v(t)]A = A\sqrt{2qN_D\epsilon(V_{bi} - v)}$$

$$C_d = \left|\frac{dQ}{dv}\right| = A\sqrt{2qN_D\epsilon}\,\frac{1}{2\sqrt{(V_{bi} - v)}} = \boxed{A\sqrt{\frac{q\epsilon N_D}{2(V_{bi} - v)}}}$$

4 static characteristic MS junction

assuming low injection, in neutral regions free carriers may be assumed in thermal equilibrium

defining:

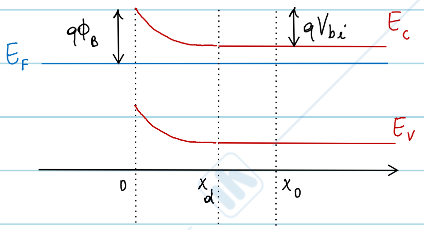
F_{MS} as the flux of e from metal to n-side per u. time, area as a result of thermoionic emission

F_{SM} as the flux of e from n-side to metal //

so the current contributions

$F_{MS} \Rightarrow I_{MS} < 0$, $F_{SM} \Rightarrow I_{SM} > 0$ ⇨ total current $I = I_{SM} - |I_{MS}|$

I_{SM} is related to the free e density in $x=0$ as e in $x=0$ have sufficient energy to overcome the energy barrier qV_{bi} considering th. equilibrium from Boltzmann



$$n(0) = N_c \exp\left[-\frac{E_c(0) - E_F}{kT}\right] = N_c \exp\left[-\frac{q\phi_B}{kT}\right] \quad [1]$$

considering $x = x_0$ in the neutral region of the semiconductor

$$n_0 = N_D = N_c \exp\left[-\frac{E_c(x_0) - E_F}{kT}\right] \iff N_c = N_D \exp\left[\frac{E_c(x_0) - E_F}{kT}\right]$$

substituting in [1]

$$n(0) = N_D \exp\left[\frac{E_c(x_0) - E_F - q\phi_B}{kT}\right]$$

as $q\phi_B - [E_c(x_0) - E_F] = qV_{bi}$

$$n(0) = N_D \exp\left[-\frac{qV_{bi}}{kT}\right]$$

at th. equilibrium $I = 0$ $\Rightarrow$ $|I_{MS}| = I_{SM}$ & $n(0) = N_D \exp\left[-\frac{qV_{bi}}{kT}\right]$

considering an applied voltage bias and assuming low injection, it is still possible to describe majority carriers with Boltzmann relations

$$I_{SM} \propto n(0) = N_D \exp\left[-\frac{qV_{bi} - qV}{kT}\right]$$

as $q\phi_B$ is indipendent of the applied bias, $|I_{MS}|$ remains constant so out of equilibrium

substituting in the total current expression $\quad |I_{MS}| \propto N_D \exp\left[-\dfrac{qV_{bi}}{kT}\right]$

$$I = I_{SM} - |I_{MS}| \propto N_D \exp\left[-\frac{qV_{bi} - qV}{kT}\right] - N_D \exp\left[-\frac{qV_{bi}}{kT}\right]$$

$$= N_D \exp\left[-\frac{qV_{bi}}{kT}\right]\left\{\exp\left[\frac{qV}{kT}\right] - 1\right\}$$

so the static characteristics

$$\boxed{I = I_S\left\{\exp\left[\frac{V}{V_T} - 1\right]\right\}}$$

, I_S reverse saturation current of the MS junction

5 in small signal conditions

$v(t) = V_0 + v_{ss}(t)$, $i(t) = I_0 + i_{ss}(t)$ $\quad$ where $I_0 = i_{dc}[V_0]$ as expressed by the static characteristics

$$i_{dc}[v(t)] = i_{dc}[V_0 + v_{ss}(t)]$$

$$Q_f[v(t)] = Q_f[V_0 + v_{ss}(t)] \qquad i_{dc}[v(t)] = I_S\left\{\exp\left[\frac{v(t)}{V_T}\right] - 1\right\}$$

as $|v_{ss}(t)| \ll |V_0|$, using a first order approximation

$$i_{dc}[v(t)] \approx i_{dc}[V_0] + \left.\frac{\partial i_{dc}}{\partial v}\right|_{v=V_0} \cdot v_{ss}(t)$$

$$Q_f[v(t)] \approx Q_f[V_0] + \left.\frac{\partial Q_f}{\partial v}\right|_{v=V_0} \cdot v_{ss}(t)$$

defining

$$\boxed{g_{d_0} = \left.\frac{\partial i_{dc}}{\partial v}\right|_{v=V_0}} = \frac{I_S}{V_T}\exp\left[\frac{V_0}{V_T}\right] = \boxed{\frac{I_S + I_0}{V_T}}$$

differential conductance

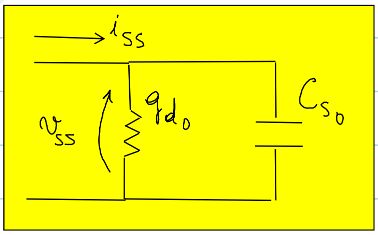
$$C_{S_0} = \left.\frac{\partial Q_f}{\partial v}\right|_{v=V_0} = A\sqrt{\frac{q\epsilon N_D}{2(V_{bi} - V_0)}}$$

depletion (differential) capacitance

the current relation, substituting the approx

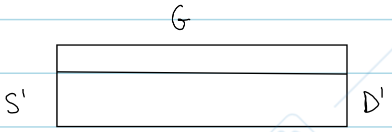
$$i(t) = I_0 + i_{ss}(t) \approx i_{dc}[v(t)] + \frac{d}{dt}[-Q_f(t)] \simeq$$

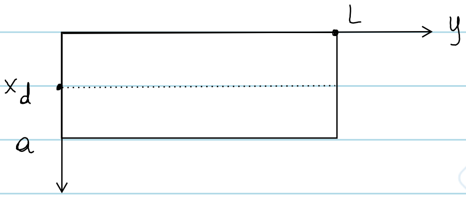
$$i_{dc}(V_0) + g_{d_0} v_{ss}(t) + \frac{d}{dt}\left\{ -Q_f\Big|_{V_0} + C_{s_0} v_{ss}(t) \right\} = \boxed{I_0 + g_{d_0} v_{ss}(t) + C_{s_0}\frac{dv_{ss}}{dt}}$$



FET

1 considering the region under the gate





S', D' intrinsic source, drain

from MS junction, depl. width $x_d = \sqrt{\frac{2\epsilon}{qN_D}(V_{bi} - V_{GS})}$

defining pinch off voltage $V_{po} = (V_{bi} - V_{GS})$ so that $x_d = a$

$$a^2 = \frac{2\epsilon}{qN_D} V_{po} \iff V_{po} = \frac{qN_D}{2\epsilon} a^2$$

applying a positive voltage V_{DS}
at the source MS j. reverse bias V_{GS}
at the drain MS j. reverse bias $V_{GS} - V_{DS} < V_{GS}$
$\Rightarrow$ depleted region not uniform

properties of the channel varies with y
distributed model of the channel by dividing the channel in Δy inf. slices (within Δy uniform properties)

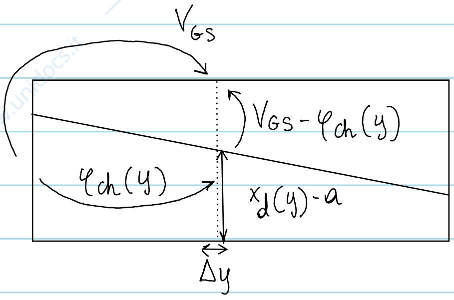
charge per unit area of free carriers in channel

$$Q_n(y) = -q \int_{x_d(y)}^{a} n(x,y)\, dx \simeq -q \int_{x_d(y)}^{a} N_D\, dx = -qN_D [a - x_d(y)]$$

by applying $V_{DS} > 0$ also channel potential not uniform

channel potential $\varphi_{ch}(y) = \frac{1}{a - x_d} \int_{x_d(y)}^{a} \varphi(x,y)\, dx$

$\varphi_{ch}(0) = 0$
$\varphi_{ch}(y = L) = V_{DS}$
φ_{ch} non decreasing funct. of y



substituting in the relation for the depl. width

$$x_d(y) = \sqrt{\frac{2\epsilon}{qN_D}[V_{bi} - (V_{GS} - \varphi_{ch}(y))]} = a\sqrt{\frac{[V_{bi} - (V_{GS} - \varphi_{ch}(y))]}{V_{p0}}}$$

charge control law $Q_m = Q_m(\varphi_{ch})$

$$Q_m = -qN_D[a - x_d(y)] = \boxed{-qN_D a \left[1 - \sqrt{\frac{[V_{bi} - (V_{GS} - \varphi_{ch}(y))}{V_{p0}}}\right]}$$

2

MESFET static output characteristics $I_D = I_D(V_{DS}, V_{GS})$
under the hypotheses:
I_D only due to majority carriers in the channel
gradual channel approx (yields if $a \gg L$) only force causing the drifting of the free carriers is the y component of the electric field $\mathcal{E}_y = -\frac{d\varphi_{ch}}{dy}$

the drift current density in the channel $J_n = q n \mu_n \mathcal{E}_y$
integrating

$$I_D = -\oint_{xz} J_n \, d\sigma = -\oint_{xz} q n \mu_n \mathcal{E}_y \, d\sigma = \int_0^W \int_{x_d}^a q n \mu_n \frac{d\varphi_{ch}}{dy} \, dx \, dz =$$

current from drain to source

$$= q W \mu_n \frac{d\varphi_{ch}}{dy} \int_{x_d}^a n(x,y) \, dx = -W \mu_n Q_n \frac{d\varphi_{ch}}{dy}$$

integration along z (invariance)

considering $V_{DS} < V_{DSS} \Rightarrow Q_n \neq 0 \ \forall y \in [0, L]$

$$\int_0^L I_D \, dy = -\int_0^L W \mu_n Q_n \frac{d\varphi_{ch}}{dy} \, dy = -W \mu_n \int_0^L Q_n \frac{d\varphi_{ch}}{dy} \, dy =$$

considering $\alpha = \varphi_{ch}(y)$, $d\alpha = \frac{d\varphi_{ch}}{dy} dy$

$$= -W \mu_n \int_{\alpha(0) = \varphi_{ch}(0) = 0}^{\alpha(L) = \varphi_{ch}(L) = V_{DS}} Q_n(\alpha) \, d\alpha$$

$$L I_D = -W \mu_n \int_0^{V_{DS}} Q_n(\varphi_{ch}) \, d\varphi_{ch} \Leftrightarrow I_D = \frac{W}{L} \mu_n \int_0^{V_{DS}} \left[-Q_n(\varphi_{ch}) \right] d\varphi_{ch}$$

substituting the charge control law and solving the integral

$$Q_n = -q N_D a \left(1 - \sqrt{\frac{V_{bi} - (V_{GS} - \varphi_{ch}(y))}{V_{p0}}} \right)$$

$$I_D = q \mu_n \frac{W}{L} N_D a \left\{ V_{DS} - \frac{2}{3} V_{p0} \left[\left(\frac{V_{bi} - V_{GS} + V_{DS}}{V_{p0}} \right)^{3/2} - \left(\frac{V_{bi} - V_{GS}}{V_{p0}} \right)^{3/2} \right] \right\}$$

where $G_M = q \mu_n \frac{W}{L} N_D a$ is the maximimum theoretical value of G_{ch}

considering $V_{DS} \geq V_{DSS}$ where V_{DSS} is the saturation drain bias the channel is pinched off before the drain $\Rightarrow Q_n = 0$, $y \in [L', L]$

assuming that $L - L' \ll L$ as a first order approximation it can be written

$$\Rightarrow I_D \simeq \frac{W}{L}\mu_n \int_0^{V_{DSS}} [-Q_n(\varphi_{ch})]\, d\varphi_{ch}$$

which is indipendent of V_{DS}

so assuming $V_{GS} \geq V_{th_0}$

$$I_D = \begin{cases} G_n \left\{ V_{DS} - \frac{2}{3} V_{po} \left[\left(\frac{V_{bi} - V_{GS} + V_{DS}}{V_{po}} \right)^{3/2} - \left(\frac{V_{bi} - V_{GS}}{V_{po}} \right)^{3/2} \right] \right\}, & V_{DS} \leq V_{DSS} \\ G_n V_{po} \left\{ \frac{V_{GS} - V_{th_0}}{V_{po}} - \frac{2}{3} \left[1 - \left(1 - \frac{V_{GS} - V_{th_0}}{V_{po}} \right)^{3/2} \right] \right\} & V_{DS} > V_{DSS} \end{cases}$$

3 In low field condition the e mobility is constant. Increasing V_{DS} results to an increase in magnitude of the electric field in the channel. It's known that for a large enough value of the electric field, the e drift velocity saturates and remains constant as $\mathcal{E}$ continues to increase.

High freq. FET often have very short channel so the drain current could saturate for a value of V_{DS} lower than V_{DSS}, as a consequence of the phenomena previously described.

$$I_D = -WQ_n(\varphi_{ch})\mu_n \frac{d\varphi_{ch}}{dy} = WQ_n(\varphi_{ch})\mu_n \mathcal{E}_y = W \quad \text{as } v_n = -\mu_n \mathcal{E}$$

$$= WQ_n(\varphi_{ch})[-v_n]$$

defining $\mathcal{E}_c$ as the value for the electric field such that for $\mathcal{E} > \mathcal{E}_c \Rightarrow v_n = v_{n,sat}$

by applying $V_{DS} \geq V_{DSS,v} \Rightarrow \mathcal{E} > \mathcal{E}_c \qquad I_D = -WQ_n v_{n,sat}$

Another mechanism for which the drain current become indipendent from the applied V_{DS} is the channel pinch-off. This phenomena occours as the charge per unit area of free carriers in the channel is a decreasing function of the channel potential. At the drain $\varphi_{ch} = V_{DS}$

$$Q_n = -qN_D a\left(1-\sqrt{\frac{[V_{bi}-(V_{GS}-V_{DS})}{V_{p0}}}\right)$$

for $V_{DS} = V_{DSS} = V_{GS} - (V_{bi} - V_{p0}) \Rightarrow Q_n(y=L) = 0$

As $V_{DS} \geq V_{DSS}$ the channel is pinched off and Q_n becomes null for $L' < L$. Assuming $L \simeq L'$ it can be written (neglecting channel length modulation)

$$I_D \simeq \frac{W}{L}\mu_n \int_0^{V_{DSS}} [-Q_n(\varphi_{ch})]\,d\varphi_{ch} \qquad \text{for } V_{DS} > V_{DSS}$$

To understand which mechanism is causing the I_D saturation

$V_{DSS} < V_{DSS,v} \Rightarrow$ pinch-off

$V_{DSS,v} < V_{DSS} \Rightarrow$ velocity saturation

HETEROSTRUCTURES

1 The behaviour of heterostructures strongly depends of the band structure difference between the two materials used to build the structure.

Considering an ideal heterostructure (negligible mismatch between the lattice constants of the two materials) considering an heterostructure made up of material A and material B it can be written

$E_{C_B} = U_0 - q\chi_B$, $E_{C_A} = U_0 - q\chi_A$

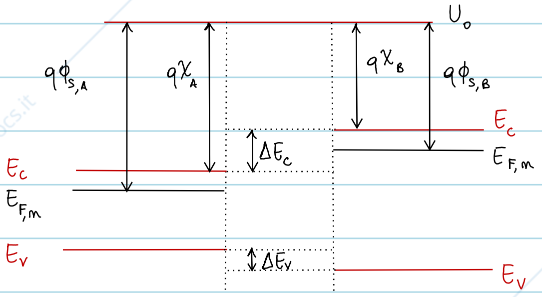
$$\Delta E_C = E_{C_B} - E_{C_A} = U_0 - q\chi_B - U_0 + q\chi_A = -q\Delta\chi$$

$$\Delta E_V = E_{V_B} - E_{V_A} = E_{C_B} - E_{g_B} - (E_{C_A} - E_{g_A}) = \Delta E_C - \Delta E_g = -q\Delta\chi - \Delta E_g$$

so the affinity rule permits to compute the conduction and valence band offset starting from the affinity difference $\Delta\chi$. It's also worth noting that $|\Delta E_c| + |\Delta E_v| = |\Delta E_g|$. ΔE_c, ΔE_v are conserved when joining the two materials.

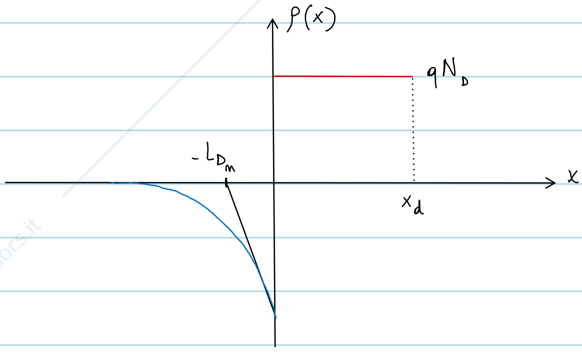
2 nn heterojunction

separate material band diagram at th. eq.

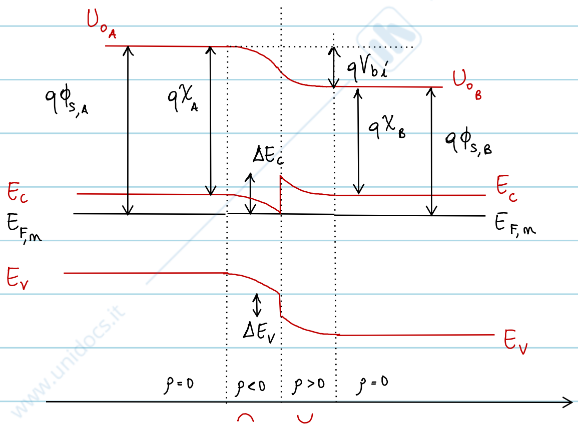


as $E_{F,m_B} > E_{F,m_A}$ when joining the two materials, during the transient towards equilibr. there will be a net flow of electrons from B to A (flow of holes from A to B)

as a consequence the following graph for charge density can be constructed



so a qualitative band diagram of the heterojunct. at th. eq. can be constructed



defining $-q\varphi = U_0$

$$-qV_{bi} = U_{0_B} - U_{0_A} =$$

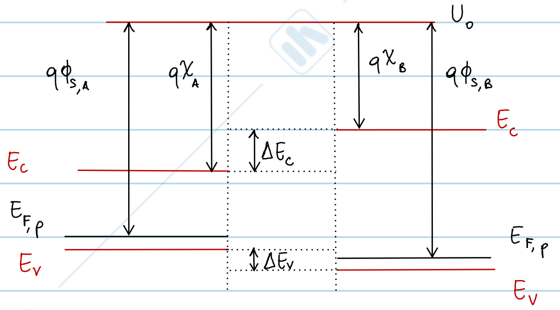
$$= q\phi_{s_B} - q\phi_{s_A} =$$

$$= q\chi_B + KT\ln\frac{N_{C_B}}{N_{D_B}} - q\chi_A - KT\ln\frac{N_{C_A}}{N_{D_A}} =$$

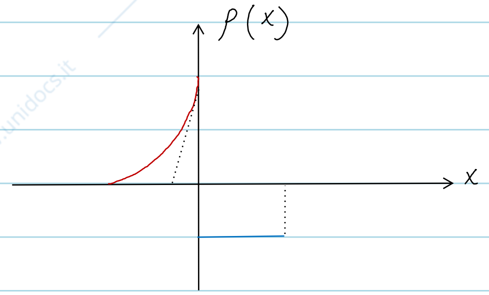
$$= \Delta q + KT\ln\frac{N_{C_B}N_{D_A}}{N_{D_B}N_{C_A}}$$

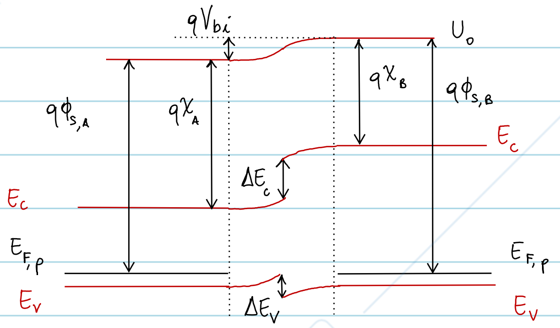
3 pp heterostructure

separate materials band diagram



as $E_{F_{p_B}} > E_{F_{p_A}}$ when joining the two materials during the transient towards th. equilibr. there will be a flow of holes from B to A



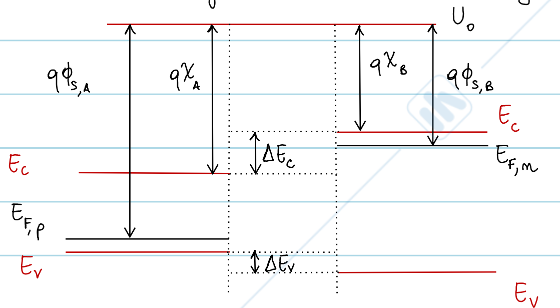


defining $-q\varphi = U_0$

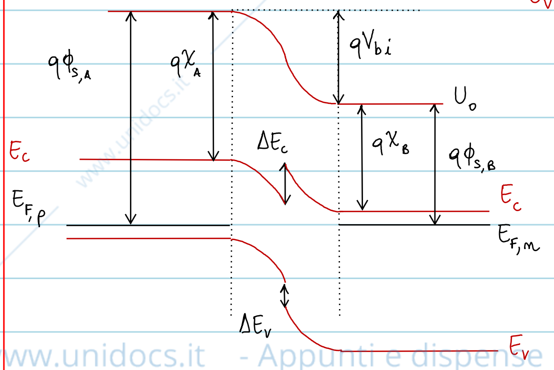
$$-qV_{bi} = q\phi_{s_B} - q\phi_{s_A} = q\chi_B + E_{g_B} - K_BT\ln\frac{N_{V_B}}{N_{A_B}} - q\chi_A - E_{g_A} + K_BT\ln\frac{N_{V_A}}{N_{A_A}} =$$

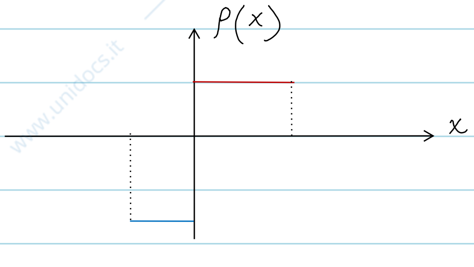
$$= \Delta\chi + \Delta E_g + K_BT\ln\left[\frac{N_{V_A}\,N_{A_B}}{N_{A_A}\,N_{V_B}}\right]$$

4 considering the band diagram for a pn heterost. at th. equilibr.



as $E_{F,n_B} > E_{F,p_A}$ when joining the two materials during the transient towards th. equilibr. electrons will flow from B to A





$$-q\varphi = U_0$$

$$-qV_{bi} = q\phi_{S_B} - q\phi_{S_A} = q\chi_B + K_B T \ln \frac{N_{C_B}}{N_{D_B}} - q\chi_A - E_{g_A} + K_B T \ln \frac{N_{V_A}}{N_{A_A}} =$$

$$= \Delta\chi - E_{g_A} + K_B T \ln \frac{N_{C_B} N_{V_A}}{N_{D_B} N_{A_A}}$$

5 main heterostructures features exploited in electronic and optoelectronic applications

heterostructure's band discontinuities are exploited for the making of bipolar transistors HBT. In fact exploiting an heterostructure to make the EB junct. of a bipolar transistor allows to solve the performance limitations of these devices. As ΔE_V acts as an energy barrier for holes in the base, it is possible to obtain large values of β_F without requiring large doping levels. The band discontinuity ΔE_C allows only high energy e^- to be injected into the base causing the transit time $\tau_t \downarrow \Rightarrow$ better freq perf. $b \rightarrow 1$

Free carriers confinement is an important characteristic of heterostructures. For electronic applications, quantum well (one-dimensional spatial confinement) is exploited in the realization of HEMT. In particular the quantum well, in which free e^- are able to move in two direction (forming a two-dimensional gas), is exploited as the conductive channel of these transistors. Also quantum wires (two dimensional spatial confim.) are exploited in research level FETs to realize the conductive channels of these transistors.

Regarding optical applications the quantum well structure is exploited to realize heterostructure LED. An LED is basically a pn junction: in forward bias as carriers recombination prevails in the depleted region the materials have to be choosen in such a way that optical generation is the prevalent phenomena.

In order to increase the efficiency of the conversion of free carriers into photons an heterostructure is used. By spatially confining the free carriers injected into the depleted region with a quantum well placed in the depleted region. As the free carriers in quantum well are spatially alligned, the ratio between emitted optical power and the injected current is increased.

The same principle is also exploited to realize LASERS where a double heterostructure is used in order to confine vertically the field in a similar way to the horizontal LED. The difference from the LEDs is that a resonating cavity is realized in order to increase stimulated emission to obtain better spectral purity.

In conclusion heterostructure's quantum well is also exploited to realize photodetectors.