**Industrial Economics (02OJAPH)**

**Prof. Luigi Benfratello**

**Academic year 2015-16**

**Exam June, $27^{th}$ 2016**

**Instructions**: answer to all questions in 1 hour and 40 minutes

**Exercise 1 (7.5 points)** A market is characterized by the following inverse demand function: $p = 100 - q$, where $q$ is the quantity produced. The technology used to produce that good is described by the following cost function: $CT(q) = 2q + 2001$.

(1) Verify that the market is characterized by the presence of a natural monopoly;

(2) Calculate the solution in monopoly;

(3) Determine the socially optimal first and second best solutions;

(4) How would you modify the cost function to make the "natural monopoly status" of the market to depend on the size of the market?

**Solution**

(1) Natural monopoly is characterized (i.e. it is a sufficient and necessary condition) by cost sub-additivity. This entails that any amount of output is less costly produced by a single firm than more than one firm.
In the single product case, this leads to the following inequality:

$$TC^M(q) < TC^N(q)$$

i.e. the total cost of producing the quantity $q$ is lower for a monopolist ($TC^M$) than for $N$ firms ($TC^N$). In our case, this leads to:

$$\begin{aligned} 2q + 2001 &< N\left[2\left(\frac{q}{N}\right) + 2001\right] \\ 2q + 2001 &< 2q + N \times 2001 \\ 2001 &< N \times 2001 \end{aligned}$$

which is verified for any $N > 1$. It is also verified for any $q$, i.e. no matter the size of the market.
Alternatively, one could check the sufficient condition, which is the presence of scale economies. The average cost function is:

$$AC = \frac{CT(q)}{q} = \frac{2q + 2001}{q} = 2 + \frac{2001}{q}$$

As the average cost is clearly decreasing in $q$, there are economies of scale no matter the size of the market so that the sufficient condition for natural monopoly in a single product setting is verified.

(2) The monopoly solution is found by setting $MR = MC$, i.e. $100 - 2q = 2$, so that $q = 49$ and $p = 51$. The monopolist clearly enjoys a positive profit ($\pi = 400$).

(3) The first best solution is found by setting $p = MC$, i.e. $100 - q = 2$, so that $q = 98$ and $p = 2 = MC$. This solution clearly entails losses for the monopolist (equal to the fixed costs), so that it is not feasible. The second best solution is instead

found by setting $p = AC$, i.e. $100 - q = 2 + \frac{2001}{q}$. The second degree equation in $q$, $q^2 - 98q + 2001$ has two solution quantities, $q = 69$ and $q = 29$. Clearly, the first one (with $p = 31$) is the one implemented by a government which wants to maximise social welfare.

(4) If the marginal cost is constant, the presence of fixed cost leads to natural monopoly no matter the size of the market. It is therefore necessary to modify the cost function by making the marginal cost to be *increasing* in the quantity produced. In this way, the natural monopoly status will depend on the size of the market. An example would be $TC = 2q^2 + 2001$.

**Solution**

**Exercise 2 (7.5 points)** Consider an oligopolistic market where $N$ firms compete *à la Cournot*. The inverse demand is $p = 1 - Q$, where $Q = \sum_{j=1}^{N} q_j$, and total costs are $CT_i = A + q_i^2$.

(a) Find the values of $A$ such that at least one firm can stay in the market.

(b) Verify whether total surplus increases or decreases moving from a monopolistic to a duopolistic configuration. Explain.

(c) Derive the number of firms in a free entry equilibrium.

**Notice**: marginal costs are not constant

**Solution**

(a) The more general way to solve the problem is to write the profit function for a generic firm $i$ out of the $N$ firms:

$$\pi_i = q_i \left(1 - \sum_{j=1}^{N} q_j\right) - A - q_i^2$$

From the profit function the optimal quantity produced can be derived:

$$\frac{\partial \pi_i}{\partial q_i} = 1 - 2q_i - \sum_{j=1, j\neq i}^{N} q_j - 2q_i = 0$$

and by invoking the symmetry:

$$q^* = \frac{1}{N+3}$$
$$Q^* = \frac{N}{N+3}$$
$$p^* = \frac{3}{N+3}$$
$$CT = A + \frac{1}{(N+3)^2}$$
$$\pi_i = \frac{2}{(N+3)^2} - A$$

The highest value that $A$ can assume in order for a monopolist ($N = 1$) to have non-negative profits is $A \leq \frac{1}{8}$.

Alternatively, one can directly set up the monopolist's problem and find out the optimal quantity $\left(q_M = \frac{1}{4}\right)$. This leads to a profit function equal to

$$\pi_M = \frac{1}{8} - A$$

from which it is easily seen that the condition for at least on firm to operate is $A \leq \frac{1}{8}$.

(b) Total surplus $TS(N)$ equals aggregate profit of the $N$ firms plus consumer surplus $\left(\frac{(1-p^*)(Q^*)}{2}\right)$. Total surplus in our case becomes:

$$TS(N) = N\left[\frac{2}{(N+3)^2} - A\right] + \frac{N^2}{2(N+3)^2} = -NA + \frac{N(4+N)}{2(N+3)^2}$$

You can observe that for $A > 0$, $TS$ increases with $N$ up to a certain level $N^*$ and then decreases. This is because the duplication of fixed costs (detrimental to total welfare) ends up to more than compensate the reduction of firms' market power and hence price (beneficial to total welfare). Notice also that by increasing $A$, $TS$ decreases. For a subset of those values of A that allow firms to be in the market (i.e. $A \leq \frac{67}{800} \leq \frac{1}{8}$), when moving from $N = 1$ to $N = 2$, $TS$ increases.

Alternatively, one could have found the equilibrium for a Cournot duopoly, where each duopolist produces $q_C = \frac{1}{5}$. This leads to a total surplus of $\frac{4}{25} + \frac{4}{50} - 2A$ which compared with the one of monopoly, equal to $\frac{5}{32} - A$, leads to an increase in total surplus for values of $A \leq \frac{67}{800}$.

(c) Free entry equilibrium entail the entry until $\pi_i(N) = 0$. In this case, potential entrants will enter until $N < \sqrt{\frac{2}{A}} - 3$. The only way to find this value is to write the profit function for a generic firm $i$ as a function of $N$, as done in point (a) above.

**Exercise 3 (7.5 points)** Consider a linear city whose lenght is 1 with two firms, $A$ and $B$. Consumers are uniformly distributed along the interval $x \in [0,1]$. Firm $A$ is located in point $x_A = \frac{1}{4}$ and firm $B$ in $x_B = 1$. Every consumer buys one unit of the good. Transportation costs are equal to 4 times the distances between consumers and firms, i.e. $\tau = 4$.

(1) Suppose first that the good of firm $A$ is produced at a null marginal cost and its consumption gives to the consumer the gross surplus $v_A = 4$ whereas the good of firm $B$ gives the consumers the gross surplus $v_B = 6$ and its marginal cost is 1. Find the indifferent consumer and firms' equilibrium prices. Verify that the profits for the two firms are equal to 2.

(2) Suppose now that firm $B$ can save on marginal costs but only by reducing the gross surplus for the consumer. In particular, suppose that firm $B$ can produce the good at 0 marginal cost but providing to the consumer a gross surplus $v_B = 5$. Verify that the profits for the two firms are—as before—equal to 2 for both firms. Explain the counter intuitive result that the two firms have the same level of profit despite they have the same marginal cost but firm $B$ produces a "better"product.

**Solution**

(1) The indifferent consumer, located in $x$, will be indifferent between purchasing the good from firm $A$ or purchasing the good of firm $B$. This leads to the following equality:

$$4 - p_A - 4\left(x - \frac{1}{4}\right) = 6 - p_B - 4(1-x)$$

The indifferent consumer will then be located in:

$$x = \frac{3 + p_B - p_A}{8}$$

so that the demands for firms $A$ and $B$ will be, respectively, $\frac{3+p_B-p_A}{8}$ and $1 - \frac{3+p_B-p_A}{8}$.
This leads to the following profit functions:

$$\pi_A = p_A\left(\frac{3 + p_B - p_A}{8}\right)$$
$$\pi_B = (p_B - 1)\left(1 - \frac{3 + p_B - p_A}{8}\right)$$

The FOCs are:

$$\frac{\partial \pi_A}{\partial p_A} = \frac{3 + p_B - p_A}{8} - \frac{p_A}{8} = 0 \Rightarrow p_A = \frac{3 + p_B}{2}$$
$$\frac{\partial \pi_B}{\partial p_B} = 1 - \frac{3 + p_B - p_A}{8} - \frac{p_B - 1}{8} = 0 \Rightarrow p_B = \frac{6 + p_A}{2}$$

Solving the system of two FOCs we obtain:

$$p_A = 4$$
$$p_B = 5$$
$$x = \frac{1}{2}$$
$$\pi_A = 2$$
$$\pi_B = 2$$

(2) The indifferent consumer, located in $x$, will be indifferent between purchasing the good from firm $A$ or purchasing the good from firm $B$. This leads now to the following equality:

$$4 - p_A - 4\left(x - \frac{1}{4}\right) = 5 - p_B - 4\left(1 - x\right)$$

The indifferent consumer will then be located in:

$$x = \frac{4 + p_B - p_A}{8}$$

so that the demands for firms $A$ and $B$ will be, respectively, $\frac{4+p_B-p_A}{8}$ and $1 - \frac{4+p_B-p_A}{8}$.
This leads to the following profit functions:

$$\pi_A = p_A\left(\frac{4 + p_B - p_A}{8}\right)$$
$$\pi_B = p_B\left(1 - \frac{4 + p_B - p_A}{8}\right)$$

The FOCs are:

$$\frac{\partial \pi_A}{\partial p_A} = \frac{4 + p_B - p_A}{8} - \frac{p_A}{8} = 0 \Rightarrow p_A = \frac{4 + p_B}{2}$$
$$\frac{\partial \pi_B}{\partial p_B} = 1 - \frac{4 + p_B - p_A}{8} - \frac{p_B}{8} = 0 \Rightarrow p_B = \frac{4 + p_A}{2}$$

Solving the system of two FOCs we obtain:

$$p_A = 4$$
$$p_B = 4$$
$$x = \frac{1}{2}$$
$$\pi_A = 2$$
$$\pi_B = 2$$

The counter intuitive result that the two firms have the same level of profit despite they have the same marginal cost but firm $B$ produces a "better" product can be explained by the better position of firm $A$, which enjoys a sort of "monopoly" over the segment $\left[0, \frac{1}{4}\right]$

**Exercise 4 (7.5 points)** Suppose a researcher wants—by using cross-sectional data at the industry level—to model profitability ($\Pi$, measured as net profits/Equity) as a function of: concentration in the market (measured by a concentration index, $CR4$, ranging from 0 to 100) and the extent of entry barriers due to product differentiation, measured by *Advertisement expenses/Sales* ($ASR$), and to the absolute cost advantage, measured by the *Absolute Capital Requirement* ($ACR$) (the amount of money necessary to build an optimally sized plant, in million US \$). $ACR$ and $CR4$ enter the model non-linearly: $ACR$ enters in log whereas for the concentration variable the dummy $CR4 > 70$ (taking the value 1 if $CR4 > 70$, 0 otherwise) is also included in the model alongside with the variable $CR4$ itself.

The model is therefore:

$$\Pi_i = \beta_0 + \beta_1 ASR_i + \beta_2 log(ACR)_i + \beta_3 CR4_i + \beta_4 \left(CR4 > 70\right)_i + u_i \qquad (1)$$

and the OLS estimate are (standard errors in round brackets below the corresponding coefficient):

$$\begin{aligned}\widehat{\Pi_i} = &\underset{(0.01)}{0.04} + \underset{(0.15)}{0.25} ASR_i + \underset{(0.003)}{0.006} log(ACR)_i + \underset{(0.001)}{0.001} CR4_i \\ &+ \underset{(0.01)}{0.03} \left(CR4 > 70\right)_i \quad R^2 = 0.57\end{aligned} \qquad (2)$$

(1) Carefully interpret the coefficients

(2) Provide an economic explaination for the sign of the estimated coefficients

(3) Test at the $\alpha = 5\%$ level, that the coefficient of $log(ACR)$ is equal to 0 against a bilateral alternative, assuming the 4 OLS assumptions hold

(4) How would you test that the concentration ratio has no effect at all on profitability?

(5) Compute the expected profitability for an industry with $ASR = 0.01$, $log(ACR) = 2$, $CR4 = 72$.

**Solution**

(1) As for the coefficients, the intercept $\beta_0$ has the usual interpretation of the expected value of the dependent variable when all the regressors are equal to 0, i.e. it is the expected profitability of an industry with $ASR = 0$, $CR4 = 0$, and $log(ACR) = 0$, i.e. $ACR = 1$. It has clearly only a geometrical interpretation. The coefficients

of $ASR$ and $CR4$ have the usual interpretation, being the expected effect of a unit change in the regressor on the dependent variable. In terms of estimated coefficients, by increasing the $ASR$ ratio by 1, the expected profitability will increase of 0.25 and by increasing the $CR4$ by 1 will lead to an increase in expected profitability of the order of 0.001. The coefficient of $log(ACR)$, $\beta_2$ divided by 100 gives the expected impact on profitability of a 1% increase in $ACR$. The coefficient of the dummy $CR4 > 70$ imply that for those industries for which the dummy is equal to 1, the intercept has a shift of 0.03, i.e. they experience a higher profitability, holding the other regressors fixed.

(2) The sign we obtain by estimating the model by OLS are those we expect according to theory. The effect of entry barriers is positive on profitability, as it is the effect of concentration. This latter effect can be explained either in the light of the increased ease of collusion or in the light of the Cournotian paradigm, which predicts higher profitability the lower the number of firms in the market (and hence the higher the concentration). The dummy for the high level of concentration ($CR4 > 70$) is positive as we would expect that those industries with high concentration have a higher profitability which goes bejond the linear effect captured by the regressor $CR4$.

(3) To test that the coefficient of $log(ACR)$ is equal to 0 against a bilateral alternative it is sufficient to construct the corresponding *t test*, which is equal to 2. This value is close (actually, slightly above) to the critical value for the $\alpha = 5\%$ level (1.96, so that the corresponding pvalue is slightly below 5%. In turn, we reject at 10% and—marginally—at 5% but we do not reject at 1%.

(4) To test that the concentration ratio has no effect at all on profitability we have to test the joint hypothesis $H_0 : \beta_3 = \beta_4 = 0$. This has to be done with an usual *F test*.

(5) Given the values of the estimated coefficients and of the regressors, the expected profitability will be:

$$\widehat{\Pi}_i = 0.04 + 0.25 \times 0.01 + 0.006 \times 2 + 0.001 \times 72 + 0.03 \times 1 = 0.1565$$

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**Exam July, $13^{th}$ 2016**

**Instructions**: answer to all questions in 1 hour and 40 minutes

**Exercise 1 (7.5 points)** Consider three firms, producing an homogeneous product. Marginal costs are constant and equal to 60. The inverse demand function is $p = 100 - Q$, where $Q$ is the total quantity. The timing of the game is the following:

(a) in stage 1, firm 1 and firm 2 decide whether to merge or not;
(b) in stage 2, the firms in the market compete simultaneously on quantities.

The merger reduces the marginal costs of the merged firm to $60 - A$. The profits obtained by the merged firm are shared equally by firm 1 and firm 2.

(1) Compute the profits of all firms if firms 1 and 2 do not merge;
(2) Would firms 1 and 2 merge if $A = 25$, i.e. after the merge marginal cost is equal to 35 for merged firms?
(3) Find the lowest value of $A$ which leads firm 3 to exit the market (i.e. to produce a quantity equal to 0) in stage 2 after firms 1 and 2 merge;
(4) Modify the answers to the previous points if firms compete in prices and not in quantities.

## Solution

(1) If firms 1 and 2 do not merge, the setting is the one of a 3 firm symmetric Cournot model with constant marginal costs. In this setting, we know that

$$q_i = \frac{A - c}{(N + 1)b} \; for \; i = 1, 2, 3$$

This leads to an equilibrium quantity of $q_i = \frac{100-60}{4\times 1} = 10$. Total quantity $Q = 3 \times 10 = 30$ so that $p = 70$. In turn, profits are: $\pi_1 = \pi_2 = \pi_3 = (70 - 60) \times 10 = 100$.

(2) To answer whether firms 1 and 2 would merge if the merger brings a reduction of marginal cost $A = 25$, i.e. if after the merge marginal cost is equal to 35 for merged firms, we have to set our problem considering that after the merge the new entity (call it $M$) will produce a quantity $q_M$ with a marginal cost equal to

$c_M = 60 - A$, equal to 35 in the specific case $A = 25$.

The profit for the new firm will be:

$$\pi_M = (100 - q_M - q_3) \times q_M - c_M \times q_M \tag{1}$$

The FOC is:

$$\frac{\partial \pi_M}{\partial q_M} = 100 - 2q_M - q_3 - c_M = 0$$

which leads to the best response function:

$$q_M = 50 - \frac{q_3 + c_M}{2}$$

The profit for the non merging firm, i.e. firm 3, will be:

$$\pi_3 = (100 - q_M - q_3) \times q_3 - 60 \times q_3 \tag{2}$$

The FOC is:

$$\frac{\partial \pi_3}{\partial q_3} = 100 - 2q_3 - q_M - 60 = 0$$

which leads to the best response function:

$$q_3 = 20 - \frac{q_M}{2}$$

By solving the two equation system, we have:

$$q_M = \frac{160}{3} - \frac{2}{3}c_M$$

$$q_3 = 20 - \frac{160}{6} + \frac{1}{3}c_M$$

So if $c_M = 35$, $q_M = 30$ and $q_3 = 5$ and $\pi_M = 900$. As the profit of the new firm is more than the aggregate profit in the case firms do not merge (200), firms will merge.

(3) The lowest value of $A$ which leads firm 3 to exit the market (i.e. to produce a quantity equal to 0) in stage 2 after firms 1 and 2 merge, can be easily found by setting:

$$q_3 = 20 - \frac{160}{6} + \frac{1}{3}c_M = 0$$

This leads to $c_M$=20 so that $A = 40$.

(4) If firms compete in prices and not in quantities, i.e. Bertrand competition, then if they do not merge and all three firms have the same marginal cost, then $p_i = 60\ for\ i = 1, 2, 3$ and $\pi_i = 0\ for\ i = 1, 2, 3$. Then it is convenient for the merging firms to merge for any $A > 0$, as in this case they will induce firm 3 exit the market by setting $p_M = 60 - \varepsilon$ ($\varepsilon < A$) and having all market with a positive profit.

**Exercise 2 (7.5 points)** Firms 1 and 2 compete in prices in a non-address differentiated product market. Production costs are normalised to 0, the game is static and the market demand for products 1 and 2 are, respectively:

$$q_1 = 1 - bp_1 + dp_2$$
$$q_2 = 1 - bp_2 + dp_1$$

with $b, d > 0$ and $b > d$.

(a) Find prices, quantities and profits at equilibrium.

(b) Clearly explain the role of the parameter $d$ in the solution formulas found in the previous point.

**Solution**

(a) To find prices (and then quantities and profits at equilibrium), it is sufficient to write the profit function for the two firms and find the first order conditions:

$$\pi_1(p_1, p_2) = p_1 \times q_1 = p_1 \times (1 - bp_1 + dp_2)$$
$$\pi_2(p_1, p_2) = p_2 \times q_2 = p_2 \times (1 - bp_2 + dp_1)$$

$$\frac{\partial \pi_1}{\partial p_1} = 1 - 2bp_1 + dp_2 = 0 \Leftrightarrow p_1 = \frac{1 + dp_2}{2b}$$
$$\frac{\partial \pi_2}{\partial p_2} = 1 - 2bp_2 + dp_1 = 0 \Leftrightarrow p_2 = \frac{1 + dp_1}{2b}$$

Notice the best response functions are sloping upward, as prices are strategic complements variables.
Solving the system of best response functions we get: $p_1 = p_2 = \frac{1}{2b-d}$. By substituting these values in the demand functions we get $q_1 = q_2 = \frac{b}{2b-d}$ and hence $\pi_1 = \pi_2 = \frac{b}{(2b-d)^2}$.

(b) The parameter $d$ is a parameter which measures the sensitivity of demand of each product with respect to the price of the other product. As $d > 0$, the products are substitutes. Notice however that we cannot perform comparative statics by changing $d$ alone. Remember that $d$ is a transformation of the original parameter of product differentiation entering consumers' utility function. As also the parameter $b$ (and the intercept, in this case normalised to 1) are also a transformation of the original parameter of product differentiation entering consumers' utility function, we cannot change $d$ alone. This explains the paradox that by increasing $d$, prices, quantities and profit at equilibrium increase with $d$ (and do not decrease, as we would expect if $d$ was a parameter of product differentiation).

**Exercise 3 (7.5 points)** A firm produces three products, the demands for which are independent. Consumers (or group of consumers) have the following reservation prices:

| *Consumer* | *Good* 1 | *Good* 2 | *Good* 3 |
|---|---|---|---|
| $A$ | 25 | 95 | 60 |
| $B$ | 40 | 80 | 60 |
| $C$ | 80 | 40 | 60 |
| $D$ | 95 | 25 | 60 |

(a) Suppose first that all three products are produced at 0 marginal cost. Consider three alternative pricing strategies:

- selling the three goods separately;
- pure bundling (i.e. selling only the three goods together);
- mixed bundling (i.e. selling the three goods together but also the goods separately or in bundle of two goods).

For each strategy, determine the optimal prices to be charged and the resulting profits. Which strategy would be best?

(b) Now suppose that the production of each good entails a marginal cost of 30$. How this information changes your answer to point (a)? Carefully explain why the optimal strategy is now different.

(c) Suppose now that the production of each good entails a marginal cost of 10$. How this information changes your answer to point (b)? Carefully explain.

**Solution**

(a) Notice first that the sum of the three reservation prices, for all four consumers, is constant (and equal to 180). This means that the three reservation prices lie on a plane. This is equivalent to the two good case when the reservation prices lie on a straight line. Given the assumption that all three products are produced at 0 marginal cost, we have the two conditions which make pure bundling the preferred choice.

To verify this, consider first the strategy of selling the three goods separately. The optimal price for each of the good must be set according to the standard rule $MC = MR$. This leads to the optimal prices $p_1 = 80$, $p_2 = 80$, and $p_3 = 60$. This leads to a revenue of 560 (equal to the profit, as marginal costs are 0). The pure bundling strategy has as optimal price for each bundle $p_B = 180$, the sum of the reservation prices of the goods for each of the four consumers. This leads to a revenue (profit) of 720. As for mixed bundling, no matter which good is bundled and which is unbundled, the profit will be less than the one of pure bundling. Just as an example, consider the following strategy (which will prove to be the optimal one under point (b) below): to sell the bundle of 3 goods at $p_B = 180$ and to sell the bundle of goods 1 and 3 at $p_{13} = 155 - \varepsilon$ and the bundle of goods 2 and 3 at $p_{23} = 155 - \varepsilon$. Consumers $B$ and $C$ will buy the bundle of three goods, consumer $A$ will buy the bundle composed of goods 2 and 3 whereas consumer

$D$ will buy the bundle composed of goods 1 and 3. The profit (revenue) of this strategy is $670 - 2\varepsilon$, which is less than the profit of pure bundling.

(b) If the production of each good entails a marginal cost of 30\$, there is room—at least in principle—for mixed bundling to be the best strategy.

To see whether this is the case, consider first the strategy of selling the three goods separately. The optimal price for each of the good are equal than before: $p_1 = 80$, $p_2 = 80$, and $p_3 = 60$. This leads to a revenue of 560. Now the cost is equal to $30 \times 8 = 240$, so the profit is 320. The pure bundling strategy—as before—has as optimal price for each bundle $p_B = 180$ and a revenue of 720. Now the cost is equal to $30 \times 12 = 360$, so the profit is 360. As for mixed bundling, the optimal strategy entails to induce consumer $A$ not to buy good 1 and consumer $D$ not to buy good 2, as the marginal willingness to pay the two consumers have for those goods is less than the marginal cost of production. Therefore, the optimal strategy is to sell the bundle of 3 goods at $p_B = 180$ and to sell the bundle of goods 1 and 3 at $p_{13} = 155 - \varepsilon$ and the bundle of goods 2 and 3 at $p_{23} = 155 - \varepsilon$. Consumers $B$ and $C$ will buy the bundle of three goods, consumer $A$ will buy the bundle composed of goods 2 and 3 whereas consumer $D$ will buy the bundle composed of goods 1 and 3. The profit of this strategy is $670 - 2\varepsilon - 10 \times 30 = 370 - 2\varepsilon$, which is now more than the profit of pure bundling. The mixed bundling is now the best strategy as it allows the firm not to produce some units of the goods which are evaluated by some consumers less than the production cost. In fact, the difference in profit of the two alternative (pure and mixed bundling) is equal (not considering the $2\varepsilon$) to the loss pure bundle entails by selling products 1 and 2 to consumers $A$ and $D$: $2 \times (30 - 25) = 10$.

(c) If the production of each good entails a marginal cost of 10\$, instead of 30\$, the reservation price of each good for each consumer is higher than the marginal cost of production. In turn, pure bundling is again the best strategy. Put differently, when the reservation prices are perfectly correlated, mixed bundling emerges as the best strategy if the marginal cost are not only strictly positive but also higher than the willingness to pay (reservation price) of some consumers.

**Exercise 4 (7.5 points)** Suppose a researcher wants—by using a cross-section of firm level data within a given industry—to assess the relative performance, in terms of productivity, of private and public (i.e. State) owned firms. To this end, he estimates the following Cobb-Douglas production function augmented with a dummy variable—*State Owned*—for public ownership (i.e. the dummy *State Owned* equals 1 if the firm is under public ownership and 0 otherwise):

$$log(Y)_i = \beta_0 + \beta_K log(K)_i + \beta_L log(L)_i + \beta_3(State\ Owned)_i + u_i \tag{3}$$

(1) Carefully interpret the coefficients

(2) Suggest the likely sign of the coefficients, providing the economic explaination for your suggestions

(3) How would you address the same question by using the cost function instead of the production function?

(4) Suppose now that the researcher wants to use the production function approach but he also wants to impose the constant return to scale restriction, i.e. $\beta_K + \beta_L = 1$. How he has to modify the previous model to take into account this restriction?

**Solution**

(1) The coefficients for $log(K)$ and $log(L)$ have to be interpreted as elasticities of the dependent variable with respect to each of the input, Capital and Labour. So, a 1% increase in Labour (resp., Capital) will lead to a $\beta_L$% (resp., $\beta_K$%) in output $Y$. The intercept, $\beta_0$ is the expected level of the dependent variable when the regressors are equal to 0, i.e. for a firm with $K = L = 1$ under private ownership (i.e. $State\ Owned = 0$). $exp\,(\beta_0)$ is therefore the average productivity for firms under private ownership whereas $exp\,(\beta_0 + \beta_3)$ is the average productivity for those firms under public ownership.

(2) As for the likely sign of the coefficients, $\beta_L$ and $\beta_K$ will be positive, as the marginal productivity of factors is positive. For the coefficient $\beta_0$ we cannot predict the sign. The coefficient $\beta_3$ is likely to be negative as State Owned firms are usually (in developed countries) badly managed for political interference problems, so that they are less productive than privately owned firms.

(3) Differential productivity analyses can be run also by estimating a cost function instead of the production function. In this case, the rationale is that a firm will be more (resp., less) productive than another firm if it has a lower (resp., higher) cost of production, holding the output and the price of factor fixed. So the corresponding equation would be:

$$log(C)_i = \beta_0 + \beta_K log(price_K)_i + \beta_L log(price_L)_i + \beta_Y log(Y)_i + \beta_3(State\ Owned)_i + u_i \tag{4}$$

In this case, a positive value of the coefficient $\beta_3$ has to be expected. In fact, a positive value of the coefficient entail higher costs and hence lower productivity.

(4) Equation (3) allows unrestricted values of the two elasticities $\beta_K$ and $\beta_L$. If the researcher wants to impose the constant return to scale restriction, i.e. $\beta_K + \beta_L =$

1, the previous model would be modified as follows:

$$\begin{aligned}
log(Y)_i &= \beta_0 + \beta_K log(K)_i + \beta_L log(L)_i + \beta_3 (State\ Owned)_i + u_i \\
log(Y)_i &= \beta_0 + \beta_K log(K)_i + (1-\beta_K) log(L)_i + \beta_3 (State\ Owned)_i + u_i \\
log(Y)_i - log(L)_i &= \beta_0 + \beta_K \left[log(K)_i - log(L)_i\right] + \beta_3 (State\ Owned)_i + u_i \\
log\left(\frac{Y}{L}\right)_i &= \beta_0 + \beta_K log\left(\frac{K}{L}\right)_i + \beta_3 (State\ Owned)_i + u_i \qquad (5)
\end{aligned}$$

The last expression (eq. 5) is the one the researcher would estimate if he wants to impose the constant return restriction. Notice that this equation contains one parameter less than the original one (eq. 3) and that a new dependent variable $log\left(\frac{Y}{L}\right)$ and a new regressor $log\left(\frac{K}{L}\right)$ have been used in place of the original ones.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2015-16**
**Exam September, $16^{th}$ 2016**

**Instructions**: answer to all questions in 1 hour and 40 minutes

**Exercise 1 (7.5 points)** Consider two firms, producing an homogeneous product. The inverse demand function is $p = 3 - Q$, where $Q$ is the total quantity. Marginal costs are different for the two firms: total cost function for firm 1 is $TC_1(q_1) = \frac{q_1}{2}$ (i.e. constant marginal costs) whereas total cost function for firm 2 is $TC_2(q_2) = q_2^2$.

(1) Find price, quantities and profits at the Cournot equilibrium and compute the market shares of the two firms;

(2) Suppose that firms 1 and 2 merge into a single firm, which is therefore a monopolist with two plants. Find the monopolistic solution (quantity, price, and profits);

(3) Suppose now that the demand function is $p = 3d - Q$, with $d$ a constant $> 1$. How do you expect the market shares computed in point (1) would change? Provide an economic intuition.

**Solution**

(1) This is a standard asymmetric Cournot model. The two reaction functions are:

$$q_1 = \frac{5 - 2q_2}{4}$$

$$q_2 = \frac{3 - q_1}{4}$$

Solving the system, we get: $q_1^* = 1$; $q_2^* = \frac{1}{2}$; $p^* = \frac{3}{2}$; $\pi_1^* = 1$; $\pi_2^* = \frac{1}{2}$. The two market shares are: $ms_1^* = \frac{2}{3}$; $ms_2^* = \frac{1}{3}$

(2) If the firm merges, the merged firm is a monopolist with two plants. The problem of the firm is to maximise, choosing **both** $q_1$ and $q_2$, the profit which is given by:

$$\pi_M(q_1, q_2) = (3 - q_1 - q_2)(q_1 + q_2) - \frac{q_1}{2} - q_2^2$$

Finding the two FOCs and solving the system, we get: $q_1^* = 1$; $q_2^* = \frac{1}{4}$; $p^* = \frac{7}{4}$; $\pi_M^* = \frac{13}{8}$. As we can see, the total quantity has decreased and the price increased with respect to the duopoly solution. As for profit, monopoly profits are higher than the sum of the profits of the two duopolists.

(3) The constant $d$ multiplies the intercept, which measures the size of the market. When the size of the market increases, both firms benefit from an increased demand, but the firm with increasing marginal cost will be at disadvantage with respect to the firm with constant marginal cost. Therefore, the market share of firm 1 will be higher than $\frac{2}{3}$ if $d > 1$

**Exercise 2 (7.5 points)** Suppose a market in which a firm is monopolist (incumbent) and another firm is considering whether to enter or not in the market (potential entrant). Suppose that the incumbent—in case the potential entrant enters the market—can either trigger a price war or it can accommodate entry. The payoff for the firms are: $\Pi^m = 8$ (monopoly profit for the incumbent if the potential entrant does not enter); $\Pi^d = 3$ (duopoly profit for both firms if the potential entrant enters and the incumbent accommodates entry); $\Pi^w = -1$ (price war profit for both firms if the potential entrant enters and the incumbent triggers the price war); 0 (profit of the potential entrant if it does not enter the market).

(a) Model the game in normal form and find the Nash Equilibria;

(b) Model the game in extensive form and find the Subgame Perfect Nash Equilibria. Clearly explain why one of the Nash Equilibria found in point (a) is not a SPNE;

(c) Suppose the incumbent can—before the potential entrant decides whether to enter or not—make an irreversible investment $c$ (such as capacity or advertising) which prepares the firm in case of price war. Include this additional stage in the game and find the values of $c$ that can lead the potential entrant not to enter the market. Discuss the importance of the irreversibility of $c$ in this equilibrium.

**Solution**

(a) The normal form can be derived by lectures notes. There are two Nash Equilibria: (Accommodate, Enter) and (price war, Not Enter).

(b) The extensive form can be derived by lectures notes. By using backward induction, the only Subgame Perfect Nash Equilibria is (Accommodate, Enter). The other Nash Equilibria (price war, Not Enter) is not SPNE as it is a Nash Equilibrium in the normal form due to the non credible threat that the incumbent will trigger a price war.

(c) The structure of the game if there is a pre entry additional stage in which the incumbent can make an irreversible investment preparing for the price war can be derived from the lectures notes. The value of $c$ which can make the incumbent strategy credible is $c \in (4, 5)$. The investment must be irreversible as it has to alter payoffs of the incumbent. In other terms, the payoff of the incumbent if it makes the investment, the potential entrant enters and the incumbent accommodates will be $\pi^d - c$ if the investment is irreversible but $\pi^d$ if it is not irreversible as the same investment can be used for other purposes.

**Exercise 3 (7.5 points)** A night club faces two consumers of whisky, one with high demand and the other with low demand. Their demand is:

$$q_i^h = 30 - p$$
$$q_i^l = 10 - 0.5p$$

Marginal cost are supposed, for sake of simplicity, equal to 0.

(a) Suppose first that the night club does not want to price discriminate so that it charges a single price for both consumers. Find the optimal price, and the ensuing quantities demanded, profits, and consumer surplus;

(b) Now suppose that the firms can recognize the two consumers and implement a third degree price discrimination. Find the optimal prices, and the ensuing quantities demanded, profits, and consumer surpluses. Compare, providing an economic rationale, with the results obtained in point (a) above;

(c) Suppose now that instead of performing a third degree price discrimination the firm implement a two-part tariff, equal for both consumers, composed of an entrance fee ($T$) and a price $p$ for each consumption of whisky. Find the optimal entrance fee and the optimal price for each consumption. Compute profits and consumer surpluses and compare, providing the economic rationale, with the results obtained in points (a) and (b) above.

**Solution**

(a) Without price discrimination, the night club faces an aggregate demand, given by the sum of the two demands. This is $Q = 40 - \frac{3}{2}p$. Inverting the demand, finding the marginal revenue and setting equal to the marginal cost leads to: $Q = 20$ and $p = \frac{40}{3}$. The profit is $\pi = \frac{800}{3}$. Consumers' surpluses are: $CS^h = \frac{1250}{9}$ and $CS^l = \frac{100}{9}$, so that total consumers' surplus is 150.

(b) If the night club can discriminate, it will charge $p^h = 15$ and $p^l = 10$. The quantities are $q^h = 15$ and $q^l = 5$. The profit is now $\pi = 275$, higher than before. Given total quantity is unchanged, total consumer surplus must have decreased. This is the case as it is now equal to $112.5 + 25 = 137.5$. As we can see, the low demand consumer are better off whereas the high demand consumer is worse off. This is unsurprising as the consumer who is penalised by third degree discrimination is the one with more rigid demand.

(c) If the night club can impose a two-part tariff, it will set the entrance fee price equal to the surplus of the low demand consumer. This will depend on price so

$$T(p) = \frac{1}{2} \times \left(10 - \frac{1}{2}p\right)(20 - p) \tag{1}$$

Revenues for drinks are also function of the price

$$Revenues(p) = p \times (30 - p) + p \times (10 - \frac{1}{2}p) \tag{2}$$

Total profit will be the sum of the entrance fee (one for each consumer) and of revenues from drink:

$$\pi(p) = 2 \times \frac{1}{2} \times \left(10 - \frac{1}{2}p\right)(20 - p) + p \times (30 - p) + p \times (10 - \frac{1}{2}p) \qquad (3)$$

Solving with respect to $p$ we get: $p^* = 10$ and $T^* = 25$. Profit will be now equal to 300, higher than in case (a) and (b) above as the night club is able to fully capture the surplus from the low demand consumer, although it is not able to do the same with the high demand one. Consumer surpluses are $CS^h = 175$ and $CS^l = 0$.

**Exercise 4 (7.5 points)** Suppose a researcher wants—by using cross-sectional data on 284 industries—to model Price Cost Margin ($PCM$, measured as Sales minus Total Variable Costs over Sales) as a function of: concentration in the market (measured by a concentration index, $CR4$, ranging from 0 to 100) and the extent of entry barriers due to product differentiation, measured by *R&D Expenditures over Sales* ($R\&D$), and to the absolute cost advantage, measured by the *Absolute Capital Requirement* ($ACR$) (the amount of money necessary to build an optimally sized plant, in million US \$). $ACR$ and $CR4$ enter the model non-linearly: $ACR$ enters in log whereas $CR4$ enters with a second degree polynomial.
The model is therefore:

$$PCM_i = \beta_0 + \beta_1 R\&D_i + \beta_2 log(ACR)_i + \beta_3 CR4_i + \beta_4 (CR4_i)^2 + u_i \qquad (4)$$

and the OLS estimate are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{PCM}_i = \underset{(0.05)}{0.03} + \underset{(0.08)}{0.20} R\&D_i + \underset{(0.02)}{0.04} log(ACR)_i + \underset{(0.02)}{0.01} CR4_i + \underset{(0.001)}{0.0018} (CR4_i)^2 \quad R^2 = 0.42$$

(1) Carefully interpret the coefficients;
(2) Provide an economic explanation for the sign of the estimated coefficients;
(3) Test at the $\alpha = 5\%$ level, that the coefficients of $CR4$ and $CR4^2$ are individually equal to 0 against a bilateral alternative, assuming the 4 OLS assumptions hold;
(4) Knowing that $\hat{\rho}_{t_1,t_2} = 0.83$, test at the $\alpha = 5\%$ level that the coefficients of $CR4$ and $CR4^2$ are jointly equal to 0 against a bilateral alternative, assuming the 4 OLS assumptions hold. Provide the intuition for the different result obtained with respect to point (3) (Hint: the critical value at the 5% level is 3.0);
(5) How would you expect the $R^2$ to change if we estimate model (1) by dropping the $(CR4_i)^2$ variable? Provide the sign and an approximate magnitude of the change.

**Solution**

(1) The intercept ($\beta_0$) is the expected level of the dependent variable when the regressors are equal to 0, i.e. for an industry with $R\&D = 0$, $log(ACR) = 1$ and $CR4 = 0$. It has, in this case, only a geometrical interpretation. The coefficient $\beta_1$ has to be interpreted as the expected change in the dependent variable for a unit increase in $R\&D$. The coefficient $\frac{\beta_2}{100}$ is the expected change in the dependent variable for a 1% increase in $ACR$. The coefficients $\beta_3$ and $\beta_4$ cannot be interpreted in isolation as $CR4$ enters the equation in a quadratic form. They compose the marginal impact (the expected change in the dependent variable for an very small change in $CR4$):

$$\frac{\partial PCM}{\partial CR4} = \beta_3 + 2 \times \beta_4 CR4 \tag{5}$$

(2) Barriers to entry lead to higher market power, so the sign of both $\beta_1$ and $\beta_2$ is expected to be positive. Furthermore, we expect the effect of an increase in market concentration to be positive and increasing in market concentration itself. So we expect both $\beta_3$ and $\beta_4$ to be positive.

(3) By performing individual $t$ tests, we get $t_1 = \frac{0.01}{0.02} = 0.5$ and $t_2 = \frac{0.0018}{0.001} = 1.8$. Both are smaller than the critical value 1.96 so we cannot reject the null hypothesis.

(4) To permorm a joint test, we have to use the $F$ statistics whose formula is:

$$F = \frac{1}{2} \frac{t_1^2 + t_2^2 - 2 \times \hat{\rho}_{t_1,t_2} t_1 \times t_2}{1 - \hat{\rho}_{t_1,t_2}^2} \tag{6}$$

By replacing the values in the text, we get a value of 3.21 which exceeds the critical value of 3.0. So we reject the null hypothesis of equality to zero of both coefficients. The different result we get in the joint test with respect tf the individual ones is due to the collinearity between the two variables which inflates the standard errors.

(5) By dropping a regressor, the $R^2$ cannot increase. Given the high collinearity between $CR4$ and $CR4^2$, the information contained in the latter is to a great extent also present in the former. So by dropping $CR4^2$ from the regression, we expect the $R^2$ to decrease of a small amount, say to the value of 0.39.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2015-16**
**Exam January, $26^{th}$ 2017**

**Instructions**: answer to all questions in 1 hour and 45 minutes

**Exercise 1 (7.5 points)** Consider an industry where the technology is such that the total cost for producing the good is equal to $TC = cq^2 + 100$, where $c$ is a positive constants and $q$ is the quantity produced by each firm in the market.

(a) Find the values of $q$ for which the market can be a natural monopoly.

(b) Provide an economic explanation why the size of the interval of $q$ for which we can have a natural monopoly depends negatively on the positive constant $c$.

(c) Find the values of $q$ for which the market can be a natural duopoly.

(d) Find the condition for the market to be a natural monopoly (check)

**Exercise 2 (7.5 points)** Suppose an duopolistic market whose demand is $P(Q) = a - Q$ and firms have symmetric marginal costs $c$ and no fixed costs (*i.e.* $TC_i = cq_i$).

(a) Find the Cournot and the collusive (i.e. joint profit maximization) output ($q^C$, $\frac{q^M}{2}$) and the corresponding profits ($\Pi^C$, $\frac{\Pi^M}{2}$)

(b) Find the best response of the each firm when the other is producing the collusive output ($q^D$) and the corresponding profits ($\Pi^D$, $\frac{\Pi^{MD}}{2}$)

(c) Suppose the game is repeated an infinite number of times and that firms have the following grim punishment strategy: "Produce $\frac{q^M}{2}$ in the first stage. In the following stages, produce $\frac{q^M}{2}$ iff all the players always produced $\frac{q^M}{2}$ before, otherwise produce $q^C$". Find the discount factor $\delta$ which makes this strategy a SPNE.

**Solution**

(a) The Cournot equilibrium is given by:

$$q^C = \frac{a-c}{3}; \quad p^C = \frac{a+2c}{3}; \quad \Pi^C = \frac{(a-c)^2}{9}$$

whereas the collusive equilibrium is:

$$\frac{q^M}{2} = \frac{a-c}{4}; \quad p^M = \frac{a+c}{2}; \quad \frac{\Pi^M}{2} = \frac{(a-c)^2}{8}$$

Clearly,

$$q^C = \frac{a-c}{3} > \frac{q^M}{2} = \frac{a-c}{4}$$
$$p^C = \frac{a+2c}{3} < p^M = \frac{a+c}{2}$$
$$\Pi^C = \frac{(a-c)^2}{9} < \frac{\Pi^M}{2} = \frac{(a-c)^2}{8}$$

(b) The best response of the each firm when the other is producing the collusive output is:

$$q^D = \frac{3}{8}(a-c)$$

This leads to

$$p\left(q^D, q^{MD}\right) = \frac{3a+5c}{8}$$

and

$$\Pi^D = \frac{9}{64}(a-c)^2 \quad \frac{\Pi^{MD}}{2} = \frac{6}{64}(a-c)^2$$

Clearly,

$$q^D = \frac{3}{8}(a-c) > q^C = \frac{a-c}{3} > \frac{q^M}{2} = \frac{a-c}{4}$$
$$\Pi^D = \frac{9}{64}(a-c)^2 > \frac{\Pi^M}{2} = \frac{(a-c)^2}{8} > \frac{\Pi^{MD}}{2} = \frac{6}{64}(a-c)^2$$

(c) The following grim punishment strategy: "Produce $\frac{q^M}{2}$ in the first stage. In the following stages, produce $\frac{q^M}{2}$ iff all the players always produced $\frac{q^M}{2}$ before, otherwise produce $q^C$" is a SPNE if the following inequality hold:

$$\Pi^D + \sum_{t=2}^{\infty} \Pi^C \delta^{t-1} < \sum_{t=1}^{\infty} \frac{\Pi^M}{2} \delta^{t-1}$$

i.e.

$$\Pi^D + \Pi^C \frac{\delta}{1-\delta} < \frac{\Pi^M}{2} \frac{1}{1-\delta}$$

This inequality in our case becomes

$$\frac{9}{64}(a-c)^2 + \frac{(a-c)^2}{9} \frac{\delta}{1-\delta} < \frac{(a-c)^2}{8} \frac{1}{1-\delta}$$

which is satisfied if $\delta > \frac{9}{17}$.

**Exercise 3 (7.5 points)** Consider a market where a single firm operates and the inverse demand function is $P(q) = 2 - q$, where $q$ is the quantity produced. Be

$VC(q,x) = (1-x)q$ firm's variable cost function which is function of the amount invested in innovation, noted as $x$. The investment in innovation has an additional cost equal to $I(x) = \frac{x^2}{2}$. The monopolist chooses at the first stage the amount invested in innovation, $x$, and in the second stage sets the price, $p$.

(1) Find the optimal choice of the firm in prices, quantity and in the amount invested in innovation, $x$

(2) Suppose the market is regulated and the regulator fixes the price of the good or service. Find the optimal price $p^r$ which maximizes total welfare under the firm budget constraint. Given the price $p^r$, find the amount of the investment $x^r$ that the firm is incentivated to do. What would happen if the regulator fixes a price of first best?

**Solution**

(1) In the second stage, by setting $MR = MC$ the firm chooses:

$$q(x) = \frac{x+1}{2}$$

so that

$$p(x) = \frac{3-x}{2} \quad \Pi(x) = \frac{1}{4}(-x^2 + 2x + 1)$$

In the first stage, by maximizing profits as a function of $x$ the firm chooses $x^* = 1$ so that $q^* = 1, \quad p^* = 1, \quad \Pi^* = \frac{1}{2}$.

(2) If the regulator chooses the optimal price $p^r$ which maximizes welfare under the firm budget constraint, then $p^r = AC$. Although profits are 0, there is no reason why the optimal investment in innovation has to be 0, as the cost in innovation is somehow compensated by a higher price.
Instead, if the regulator fixes a price of first best, i.e. the one such that $p^r = MC$ we have:

$$p(x) = 1-x \quad q(x) = x+1 \quad \Pi(x) = (1+x)(1-x) - (1-x)(1+x) - \frac{x^2}{2}$$

By setting the first order conditions we see that $x^* = 0$. The rationale is that the investment in innovation decreases marginal cost but also decreases price set by regulator, so the monopolist has no incentive to carry out any investment.

**Exercise 4 (7.5 points)** Suppose a researcher wants—by using data on different personal computers sold in a given town—to model the price of computers as a function of some regressors.

(1) Provide a model in which:
- There are at least three regressors;
- There is at least one non linear function of the variables (i.e. logs, polynomial, or interactions);

(2) Provide reasonable estimates of the parameters and provide the economic explanation for the sign of the estimated coefficients;

(3) Provide estimated standard errors for the estimated coefficients and test at the $\alpha = 5\%$ level, that one of the coefficient is equal to 0 against a bilateral alternative, assuming the 4 OLS assumptions hold;

(4) Provide a value for $\hat{\rho}_{t_1,t_2}$, and test at the $\alpha = 5\%$ level that two coefficients are jointly equal to 0 against a bilateral alternative, assuming the 4 OLS assumptions hold;

(5) Provide consistent values for the $R^2$, the RMSE and the (unconditional) standard deviation of the dependent variable.

**Solution**

(1) One reasonable specification (among many others) could be:

$$log(P)_i = \beta_0 + \beta_1(RAM)_i + \beta_2(Hard\_Disk\_Capacity)_i + \beta_3(brand)_i + u_i$$

where *brand* is a dummy for being the PC branded (Samsung, Apple, Acer, Lenovo) or not. Notice that it is reasonable to use a log-lin model as it implies that further increases in characteristics have more than proportional effects on price (i.e. the increase from 12 to 16 GB of RAM has a higher effect on price than the increase from 4 to 8). Instead, a lin-log model would be inappropriate as it would imply the opposite effect.

(5) Suppose $sd(log(price)) = 2$. Then if $R^2 = 0.5$ (medium fit), we expect the RMSE to be around 1.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2016-17**
**Exam June, $30^{th}$ 2017**

**Instructions**: answer to all questions in 1 hour and 40 minutes

### Exercise 1 (7.5 points)

Consider a market in which two firms, 1 and 2, produce a homogenous good under constant returns to scale. Marginal costs are $c_1 = 0$ and $c_2 = c$, respectively, where $0 < c \leq \frac{1}{2}$. The two firms face the demand function $Q(p) = 1 - p$. Suppose first that firms interact only once. Find the equilibrium (quantities, price, profits) and comment on the role of $c$ in the following cases:

(a) Cournot game;

(b) Bertrand game;

(c) Stackelberg game where firm 1 acts as leader;

(d) Joint profit maximization;

Suppose now that the two firms play an infinite number of times

(e) Do you think that a collusive equilibrium in prices is sustainable in the game played an infinite number of times? Explain

### Solution

(a) It is a standard Cournot with asymmetric cost. The reaction functions are:

$$q_1(q_2) = \frac{1 - q_2}{2} \quad (1)$$

$$q_2(q_1) = \frac{1 - q_1 - c}{2} \quad (2)$$

so that

$$q_1^* = \frac{1}{3} + \frac{c}{3} \quad (3)$$

$$q_2^* = \frac{1}{3} - \frac{2c}{3} \quad (4)$$

and

$$Q^* = \frac{2}{3} - \frac{c}{3} \quad (5)$$

$$p^* = \frac{1}{3} + \frac{c}{3} \tag{6}$$

$$\pi_1 = \left(\frac{1}{3} + \frac{c}{3}\right) \times \left(\frac{1}{3} + \frac{c}{3}\right) = \frac{1}{9}\left(1 + c^2 + 2c\right) \tag{7}$$

$$\pi_2 = \left(\frac{1}{3} - \frac{2c}{3}\right) \times \left(\frac{1}{3} + \frac{c}{3}\right) = \frac{1}{9}\left(1 - 2c^2 - c\right) \tag{8}$$

Quantity and hence profits for firms 1 are positively related to the marginal cost of firm 2, $c$. If $c = 0$ all the previous results revert to those of a standard Cournot model with symmetric, null marginal cost and $b = 1$

(b) This is a standard Bertrand model with asymmetric costs. Firm 1 will set the $p_1^* = c - \epsilon$ and will serve the whole demand whereas firm 2 will set $p_2^* = c$ and will produce a null amount. The quantity for firm 1 is $Q = 1 - p = 1 - c + \epsilon$ so that profit for firm 1 is $\pi_1 = (1 - c + \epsilon) \times (c - \epsilon) = c - c^2 + 2\epsilon c - \epsilon - \epsilon^2 > 0 =$.
Notice that the parameter $c$ affects equilibrium price and hence quantity and profits. In particular, neglecting in the profit expression the $\epsilon$ term, we have

$$\frac{\partial \pi_1}{\partial c} = 1 - 2c$$

which is positive for $c < \frac{1}{2}$

(c) By inserting the reaction function of firm 2 into the profit function of firm 1 we get:

$$\pi_1 = \left(\frac{1}{2} - \frac{q_1}{2} + \frac{c}{2}\right) q_1$$

so that the Stackelberg equilibrium quantities

$$q_1^* = \frac{1}{2} + \frac{c}{2}$$

$$q_2^* = \frac{1}{4} - \frac{3c}{4}$$

and

$$Q^* = \frac{3 - c}{4} \tag{9}$$

$$p^* = \frac{1}{4} + c \tag{10}$$

$$\pi_1 = \frac{1}{2}\left(c^2 + \frac{5}{4}c + \frac{1}{4}\right) \tag{11}$$

$$\pi_2 = \frac{1}{4}\left(-3c^2 + \frac{1}{4}c + \frac{1}{4}\right) \tag{12}$$

Here the parameter $c$ reinforces the first mover advantage for firm 1, increasing its quantity and profits. Notice also that if $c > \frac{1}{3}$, the follower will produce a 0 amount.

(d) In the joint profit maximization problem, the two firms will act as a unique monopolist and jointly produce the monopoly quantity. Notice that firm 1 will produce the whole monopoly quantity whereas firm 2 will produce 0. This is because the marginal cost of firm 1 is lower than the one of firm 2 for any level of production.

(e) A collusive equilibrium in prices can be sustainable if firms play a Bertrand game an infinite number of times. In fact, it can be convenient for firm 1 not to play the Nash equilibrium in all the stages of the game ($p = c - \epsilon$) but to set a price equal to the monopoly price ($p = \frac{1}{2}$) and collude with the other firm which will also set the same price. With equal prices, the two firms will evenly share the demand. The rationale here is that firm 1 can enjoy a higher price, with respect to $p = c - \epsilon$, which can more than compensate firm 1 from the loss of sharing the demand with firm 2. It can be proved that the ensuing grim strategy can be an equilibrium in the infinitely repeated game for discount factors sufficiently high: "At time 0 firm $i$, $i = 1, 2$, sets a price $p = 1/2$, and both firms serve an equal fraction of the market demand. Firm $i$, $i = 1, 2$, keeps setting $p = 1/2$ if firm $j \neq i$ has always set $p = 1/2$ in the past. If at any point in time firm $j$ sets a price different from $p = 1/2$, then firm $i$ reverts to the once-shot Nash equilibrium strategy"

**Exercise 2 (7.5 points)**

Consider the following demands for two substitute products:

$$D_1(p_1, p_2) = 1 - p_1 + bp_2$$

$$D_2(p_1, p_2) = 1 - p_2 + bp_1$$

where $p_1$ and $p_2$ are the prices of goods 1 and 2 and $1 > b > 0$ is the product differentiation parameter. The marginal cost of production of each product is constant and equal to $c$.

(a) Assume that the two products are produced by a monopolist who makes a mistake by fixing the prices of the two goods considering them as independent, i.e. assuming that $b = 0$. Find the optimal prices and quantities for the two goods;

(b) Repeat the previous point considering that the monopolist chooses the prices of the two products correctly, i.e. considering $b \neq 0$ and comment why the results are different from those obtained in the previous point;

(c) Consider now that each of the two goods is offered by one of two different firms competing in prices. Find the equilibrium prices and quantities and compare with the results obtained in the previous points.

**Solution**

(a) The monopolist maximizes its profits:

$$\pi = (p_1 - c)(1 - p_1) + (p_2 - c)(1 - p_2)$$

Setting the two first order conditions:

$$\frac{\partial \pi}{\partial p_1} = 1 - 2p_1 + c = 0$$

$$\frac{\partial \pi}{\partial p_2} = 1 - 2p_2 + c = 0$$

By using symmetry $p_2^* = p_1^*$ we find

$$p_2^* = p_1^* = \frac{1+c}{2}$$

and

$$q_2^* = q_1^* = 1 - (1+b)\,\frac{1+c}{2}$$

(b) The monopolist maximizes its profits:

$$\pi = (p_1 - c)(1 - p_1 + bp_2) + (p_2 - c)(1 - p_2 + bp_1)$$

Setting the two first order conditions:

$$\frac{\partial \pi}{\partial p_1} = 1 - 2p_1 + 2bp_2 + c - bc = 0$$

$$\frac{\partial \pi}{\partial p_2} = 1 - 2p_2 + 2bp_1 + c - bc = 0$$

By using symmetry $p_2^* = p_1^*$ we find

$$p_2^* = p_1^* = \frac{1 + c - bc}{2(1-b)}$$

and

$$q_2^* = q_1^* = 1 - (1+b)\,\frac{1 + c - bc}{2\,(1-b)}$$

The correct price, $\frac{1+c-bc}{2(1-b)}$, is higher than the wrong one $\frac{1+c}{2}$. In turn, the monopolist produces too much of the two goods when he uncorrectly consider $b = 0$. This result can be explained in the light of the negative externality that each of the good exerts on the demand of the other good.

(c) The two oligopolists maximise their profits:

$$\pi_1 = (p_1 - c)\,(1 - p_1 + bp_2)$$

$$\pi_2 = (p_2 - c)\,(1 - p_2 + bp_1)$$

Setting the two first order conditions:

$$\frac{\partial \pi_1}{\partial p_1} = 1 - 2p_1 + bp_2 + c = 0$$

$$\frac{\partial \pi_2}{\partial p_2} = 1 - 2p_2 + bp_1 + c = 0$$

and invoking symmetry $p_2^* = p_1^*$ we get:

$$p_2^* = p_1^* = \frac{1 + c}{2 - b}$$

and

$$q_2^* = q_1^* = 1 - (1+b)\,\frac{1 + c}{2 - b}$$

We can see that the price set by each of the duopolists is between the two prices set by the monopolist. This is because the duopolist only partially internalise the effect of his choice on the choice of the other duopolist.

**Exercise 3 (7.5 points)**

Consider an island where there is a natural gas plant owned by the local government which produces 4.6 million $M^3/year$ at a variable cost of 1 euro/$M^3$, plus a fix cost of 1.25 Million euros per year to pay capital expenses. Consumption is split between residential consumers (2.4 million $M^3$/year) and an industrial plant which uses gas to produce methanol (2.2 million $M^3$/year). The price is initially set at 0.8 euro/$M^3$.

The government is short of funds and hires you to propose a new tariff level to allow the plant to cover all its cost. They give you historical data which indicates that if you increase tariffs in 0.1 euro/$M^3$, residential consumption will decline by 0.2 million $M^3$/year (since firewood and kerosene are available) and industrial consumption will decline by 0.1 million $M^3$/year (since there are no substitutes for gas). Therefore, the demand curves for each segment are (in millions):

$$q_1 = 4 - 2p \text{ (residential)} \tag{13}$$

$$q_2 = 3 - p \text{ (industrial)} \tag{14}$$

(1) Illustrate graphically the initial economic situation of the gas plant. Is it efficient for the local economy? What are the losses of the plant?

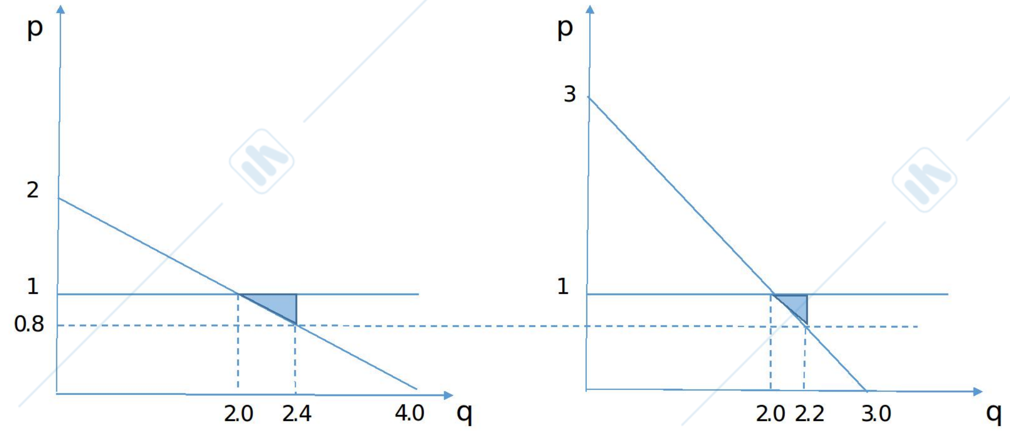
(2) Following the criterion to charge the same price to all consumers, propose a price level which allows the plant to cover all its cost. What will the production and consumption levels be? Is it economically efficient?

(3) Assuming you are now allowed to differentiate prices charged to consumers, would you change your recommendation and how? Can you improve the efficiency of the economy? (hint: you dont need to provide exact numeric answers)

(4) Suppose now you are requested to provide advice not only regarding tariffs, but also about income taxes and subsidies in the island. What would your recommendation be in this case? What would be the production and consumption levels in this case?

**Solution**

(1) The equilibrium is not efficient, since there are dead weight losses (DWL): the blue area. This area corresponds to the difference between the cost of production (marginal cost) and the value of consumption (demand curve).



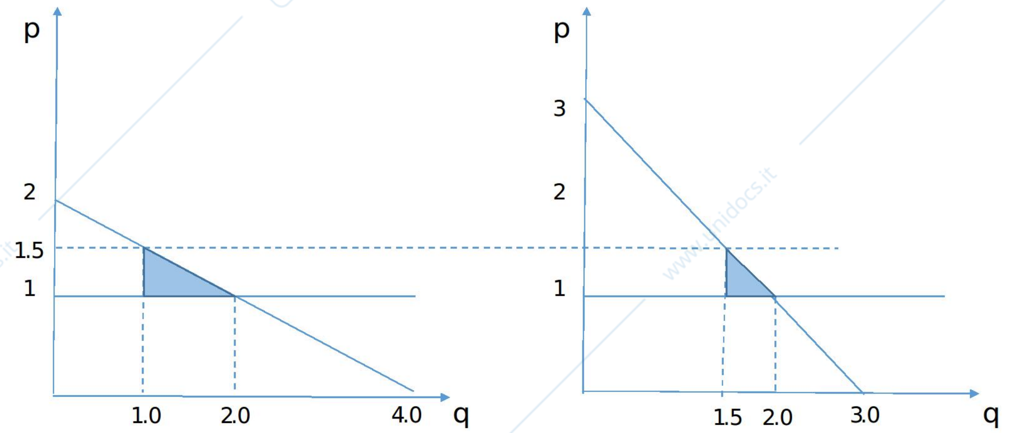

The losses of the plant are 2.17 Million euros per year:

$$4.6 \times 0.2 + 1.25 = 0.92 + 1.25 = 2.17$$

(2) We need to set a unique price that allows the plant to cover both variable and fixed costs, i.e.:

$$(p-1)(q_1+q_2) = 1.25$$

or $p = AC$. Graphically it can be seen that the condition is met at $p = 1.5$; $q_1 = 1.0$; $q_2 = 1.5$; $production : 2.5$




The same solution can be found by computing the aggregate demand ($q = 7 - 3p$) and solving the following equation:

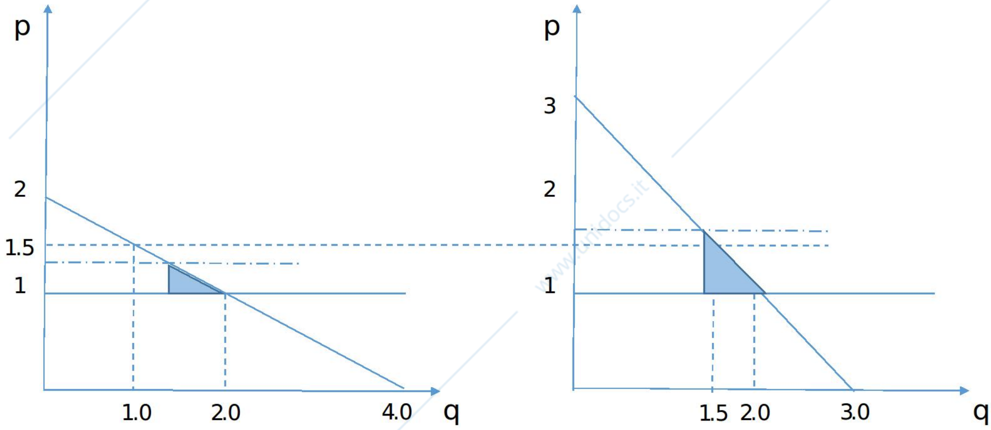
$$(p-1)(7-3p) = 1.25$$

The optimal price is the lowest of the two solutions to this equation.
Such a price is not economically efficient, since there are DWL.

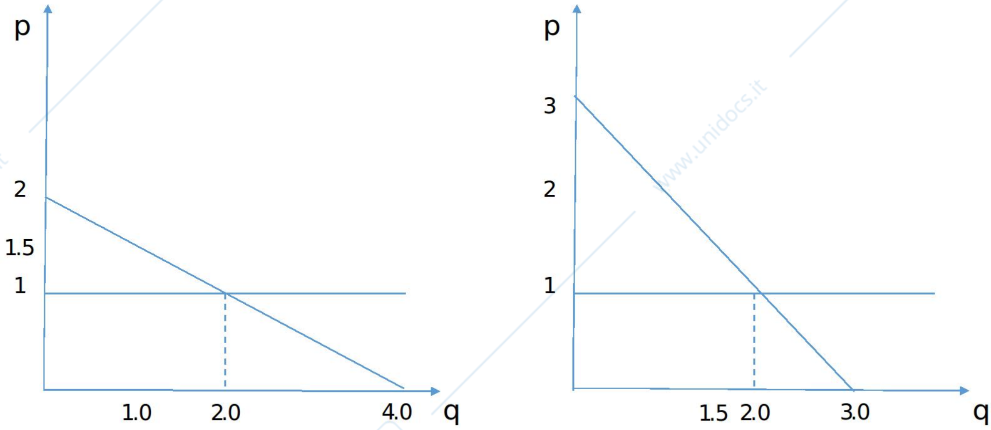
(3) The second best solution is obtained by applying Ramsey pricing rules, according to which price-cost margins are inversely related to price elasticities. It follows that:

- $p_1 < 1.5$ (but higher than 1.0)
- $p_2 > 1.5$
- $q_1 > 1.0$ (but smaller than 2.0)
- $q_2 < 1.5$

This solution improves the efficiency of the economy, since it allows full recovery of costs while minimizing dead weight losses (s.t. revenue constraint). Graphically:




(4) The optimal economic solution is to set prices equal to marginal cost ($p_1 = p_2 = 1$) and to provide a lump sum subsidy of euro 1.25 million to the company, collected from income taxes (rather than consumption taxes). Consumption levels would be 1 million $M^3/year$ for residential and industrial consumer each. Production would be 2 million $M^3/year$. There are no DWL in this case.



**Exercise 4 (7.5 points)** Suppose that firms operating in a regulated industry differ according to the ownership (some are publicly owned, others are privately owned) and their regulatory regime (some are regulated with a price cap mechanism, others with a Rate of Return mechanism).

A researcher wants first to model the productivity differential between public and private firms. To this end, he estimates the following model:

$$log(Y)_i = \beta_0 + \beta_K log(K)_i + \beta_L log(L)_i + \beta_3 (State\ Owned)_i + u_i \quad (15)$$

where $Y$ is output, $L$ is labour, $K$ is capital and *State Owned* is a dummy taking the value of 1 if the firm is publicly owned. The OLS estimate are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{log(Y)}_i = \underset{(0.01)}{0.2} + \underset{(0.10)}{0.32} log(K)_i + \underset{(0.15)}{0.65} log(L)_i - \underset{(0.02)}{0.07} (State\ Owned)_i \qquad R^2 = 0.85 \quad (16)$$

(1) Carefully interpret the coefficients in (15);

(2) Test at the $\alpha = 5\%$ level, that the coefficients of $log(K)$ is equal to 0 against a bilateral alternative, assuming the 4 OLS assumptions hold;

(3) Test at the $\alpha = 5\%$ level, that the coefficients of $log(K)$ and $log(L)$ are jointly equal to 0, assuming the 4 OLS assumptions hold and knowing that $\hat{\rho}_{t_1,t_2} = -0.3$

(4) Suppose that the researcher estimates the following simpler version of equation (15):

$$log(Y)_i = \beta_0 + \beta_K log(K)_i + \beta_3 (State\ Owned)_i + u_i \quad (17)$$

Provide reasonable estimates for equation (17) (i.e. write the analogous of equation (16)) and provide an explaination for your choices;

Suppose now that the researcher wants to take into account the different regulatory regimes:

(5) How would you modify equation (15) to model a different effect of ownership according to the different regulatory regimes? What is the expected sign of the new parameters?

(6) How would you test the hypothesys that the regulatory regime has no effect on firms' productivity?

## Solution

(1) The coefficients for $log(K)$ and $log(L)$ have to be interpreted as elasticities of the dependent variable with respect to each of the input, Capital and Labour. So, a 1% increase in Labour (resp., Capital) will lead to a $\beta_L\%$ (resp., $\beta_K\%$) in output

$Y$. The intercept, $\beta_0$, is the expected level of the dependent variable when the regressors are equal to 0, i.e. for a firm with $K = L = 1$ under private ownership (i.e. $State\ Owned = 0$). $exp\,(\beta_0)$ is therefore the average productivity for firms under private ownership whereas $exp\,(\beta_0 + \beta_3)$ is the average productivity for those under public ownership.

(2) By performing an individual $t$ test on the coefficient of $log(K)$, $\beta_K$, we get $t = \frac{0.32-0}{0.1} = 3.2$. As this value is outside the acceptance interval $(-1.96, +1.96)$ we reject the null hypothesis.

(3) To perform a joint test of two restrictions, we have to use the $F$ statistics whose formula is:

$$F = \frac{1}{2} \times \frac{t_1^2 + t_2^2 - 2 \times \hat{\rho}_{t_1,t_2} \times t_1 \times t_2}{1 - \hat{\rho}^2_{t_1,t_2}} \tag{18}$$

In our case the null is $H_0: \quad \beta_K = \beta_L = 0$, the formula becomes:

$$F = \frac{1}{2} \times \frac{4.3^2 + 3.2^2 - 2 \times -0.3 \times 4.3 \times 3.2}{1 - (-0.3)^2} \approx 20.5 \tag{19}$$

As the value of the statistics is greater than the critical value at $\alpha = 5\%$ of the $\frac{\chi^2}{2}$ distribution (equal to 3), we reject the null hypothesis.

(4) If we omit the log of labour from the equation, the fit of the model will decrease substantially and the coefficient of capital will be biased upward as the two factors of production are likely to be positively correlated (at least in a cross section) and the coefficient of labour is positive. No a priori sign could be provided for the coefficient of the ownership dummy. Reasonable estimates would be:

$$\widehat{log(Y)}_i = \underset{(0.01)}{0.2} + \underset{(0.20)}{0.97} log(K)_i - \underset{(0.02)}{0.07} (State\ Owned)_i \quad R^2 = 0.45 \tag{20}$$

(5) To take into account the effect of the regulatory regime, we should construct an additional dummy, *Price Cap*, which takes value of 1 if the firm is regulated with a Price Cap regime and 0 if the firm is regulated with a RoR regime. This extended model would be:

$$log(Y)_i = \beta_0 + \beta_K log(K)_i + \beta_L log(L)_i + \beta_3 (State\ Owned)_i + \beta_4 (Price\ Cap)_i + u_i \tag{21}$$

In this model $exp(\beta_4)$ represents the productivity differential between firms regulated with a price cap regime with respect to those regulated under a RoR regime. We expect this coefficient to be positive as price cap regime incentivates firms to use efficiently factors of production and hence leads to higher productivity. Notice that in this model the effect of regulatory regime *does not* depend on ownerhsip. To answer the question, we have to make the model even more flexible by interacting the ownership and the regulatory regime variables:

$$\begin{aligned} log(Y)_i = &\beta_0 + \beta_K log(K)_i + \beta_L log(L)_i + \beta_3 (State\ Owned)_i + \beta_4 (Price\ Cap)_i \\ &+ \beta_5 (State\ Owned \times Price\ Cap)_i + u_i \end{aligned} \tag{22}$$

In this model $exp(\beta_3)$ represents the productivity differential between public and private firms when the firm is regulated with a RoR regime (dummy $Price\ Cap = 0$) whereas $exp(\beta_4)$ represents the productivity differential between firms regulated with a price cap regime with respect to those regulated under a RoR regime when they are private (dummy $State\ Owned = 0$). As before, we expect the former to be negative and the latter to be positive. As for $exp(\beta_5)$, it is the increment of the differentials when the other dummy $= 1$. We expect it to be negative as the incentivating effect of the price cap regulation should be stronger for private than for public firms.

(6) To test the hypothesys that the regulatory regime has no effect on firms' productivity, we should perform a joint test of the null $H_0 : \beta_4 = 0$ *and* $\beta_5 = 0$ in model (22). If one limits to model (21), a t-test of the null hypothesis $H_0 : \beta_4 = 0$ should be performed.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2016-17**
**Exam July, $14^{th}$ 2017**

**Instructions**: answer to all questions in 1 hour and 40 minutes

## Exercise 1 (7.5 points)

Suppose a competitive market with 100 firms. The total (internal) cost funtion for each of these firms is $TC(q) = 160q + q^2$ so that the marginal (internal) cost function is$MC(q) = 160 + 2q$. Alongside with the previous internal total cost, production also entails *external* costs represented by the following function $EC(q) = 40q + q^2$ so that the marginal *external* cost is $MEC(q) = 40 + 2q$ where $q$ is the quantity produced by each firm (in kilos) and all monetary variables are in \$. Market demand is given by $Q = 100,000 - 10p$ where $Q$ is total (market) quantity and price is in \$.

Find:

(a) The competitive equilibrium (price and quantity);

(b) The efficient quantity (i.e. the quantity produced if producers take into account the externality);

(c) The welfare loss due to the externality;

(d) The optimal Pigouvian tax which corrects for the externality.

Suppose now that the market is not competitive but is monopolised by a single firm. Discuss, on *qualitative* grounds, whether in presence of externalities:

(e) the superiority of competition with respect to monopoly in terms of social welfare is always true if the monopolist does not discriminate;

(f) the superiority of competition with respect to monopoly in terms of social welfare is always true if the monopolist performs a first degree (or perfect) discrimination.

### Solution

(a) To find the competitive equilibrium (price and quantity) we have to find the intersection of the market demand and the market supply. The latter is the sum of the individual firm supply functions. The individual firm supply function is found by setting price equal marginal cost and solving for the quantity:

$$p = 160 + 2q \Rightarrow q = \frac{p}{2} - 80 \quad (1)$$

The market supply is the sum of the quantities given price:

$$Q = 100 \times \left(\frac{p}{2} - 80\right) = 50p - 8,000 \tag{2}$$

By setting market supply equal to marked demand we find the equilibrium values for price and quantity: $p^* = 1,800\$$ and $Q^* = 82,000$.

(b) To find the efficient quantity (i.e. the quantity produced if producers take into account the externality) we have to replicate the previous steps but using the marginal social cost and not the marginal internal cost to derive firms' supply function. The marginal social cost is the sum of the marginal internal cost and of the marginal external cost.

$$p = (160 + 2q) + (40 + 2q) = 200 + 4q \Rightarrow q = \frac{p}{4} - 50 \tag{3}$$

The market supply is the sum of the quantities given price:

$$Q = 100 \times \left(\frac{p}{4} - 50\right) = 25p - 5,000 \tag{4}$$

By setting market supply equal to marked demand we find the equilibrium values for price and quantity: $p^e = 3,200\$$ and $Q^e = 70,000$.

(c) To find the welfare loss due to the externality, it is sufficient to notice it correspond to the area of a triangle with height the difference between the optimal and the competitive quantities and as base the marginal external cost evaluated at the competitive quantity produced by each firm.
Given that at the competitive quantity each firm is producing $\frac{82,000}{100} = 820$ kilos, the marginal external cost is equal to $MEC(820) = 40 + 2 \times 820 = 1,680$. The difference between competitive and efficient production is $82,000 - 70,000 = 12,000$ so that the loss is:

$$Loss = \frac{1}{2} \times 12,000 \times 1,680 = 1,008,000\$ \tag{5}$$

per year.

(d) To find the optimal Pigouvian tax which corrects for the externality it is sufficient to equate the market demand depending on price to the market supply depending to the price minus the tax:

$$Q^D = 100,000 - 10P \tag{6}$$

$$Q^S = 50(P - T) - 8000 \tag{7}$$

from which

$$100,000 - 10P = 50(P - T) - 8,000 \Rightarrow 50T = 60P - 108,000 \tag{8}$$

It is a single equation in two unknowns. It can be solved by setting the price equal to $p = 3,000$ (the efficient price found before) so that the quantity exchanged is the efficient one previously found, $70,000$. The optimal tax amounts to $1,440\$$.

(e) In presence of negative externalities, the competitive output is larger than the efficient one. It can happen, then, that a non discriminating monopolist, by producing less than the competitive output, is more efficient than perfect competition in maximising social welfare.

(f) A perfectly discriminating monopolist produces the same amount than in perfect competition. Therefore, the two cases are perfectly equivalent in terms of social welfare, no matter the presence of externality.

**Exercise 2 (7.5 points)**

Consider an oligopolistic market where firms produce a homogenous good. Costs are symmetric among all firms: marginal costs are equal for all firms ($c_i = c \, \forall i = 1, \ldots, n$) and all firms incur in a fixed cost $f$. The (inverse) demand function is linear: $p(Q) = a - bQ$.

(a) Consider a simplified version of the demand function: $p(Q) = 1 - Q$ (i.e. $a = b = 1$), and suppose that $c = 0$ (null marginal costs), that $n = 2$ (duopoly) and that firms compete by setting simultaneously the quantities (Cournot model). Find the equilibrium quantities, price, and profits and comment on the role of $f$;

(b) For the same demand, marginal cost, and number of firms of point (a), find the equilibrium quantities, price, and profits and comment on the role of $f$ in the case one firm sets the quantity before the other (Stackelberg model);

(c) For the two cases above, compute the Herfindahl-Hirschman concentration index and comment about the differences between point (a) and (b) (if any);

(d) For the two cases above, compute the $CR4$ concentration index (the sum of the market shares of the four largest firms) and comment about the differences between point (a) and (b) (if any) and with respect with the results in point (c);

(e) For a generic demand function $p(Q) = a - bQ$ and a generic marginal cost $c \neq 0$ find the number of firms in a free entry Cournot equilibrium. Comment on the role of $f$ (notably when $f = 0$) and $a$, and provide the economic intuition for your results.

**Solution**

(a) From the generic results for a Cournot model with symmetric, constant marginal cost and linear demand:

$$q_i^* = \frac{a-c}{(n+1)b} \; i = 1, 2 \tag{9}$$

$$p_i^* = \frac{a+nc}{n+1} \tag{10}$$

$$\pi_i^* = \frac{\left(\frac{a-c}{(n+1)}\right)^2}{b} - f \tag{11}$$

Setting $a = b = 1$ and $c = 0$:

$$q_i^* = \frac{1}{3} \; i = 1, 2 \tag{12}$$

$$p_i^* = \frac{1}{3} \tag{13}$$

$$\pi_i^* = \left(\frac{1}{3}\right)^2 - f = \frac{1}{9} - f \tag{14}$$

In this model the role of $f$ is simply to decrease firms' profits. It has no role in the choice of the equilibrium quantities.

(b) In a Stackelberg model, with $a = b = 1$ and $c = 0$, the optimal quantities are:

$$q_1^* = \frac{1}{2} \tag{15}$$

$$q_2^* = \frac{1}{4} \tag{16}$$

so that

$$p_i^* = \frac{1}{4} \tag{17}$$

$$\pi_1^* = \frac{1}{8} - f \tag{18}$$

$$\pi_2^* = \frac{1}{16} - f \tag{19}$$

The role of $f$ here is more subtle. Not only it decreases the profit, but it also might induce the leader to have entry deterrence behaviour and to produce the limit output instead of the Stackelberg quantity

(c) The two market shares in point (a) are: $ms_1 = ms_2 = \frac{1}{2}$ and in point (b) $ms_1 = \frac{2}{3}$ and $ms_2 = \frac{1}{3}$. Therefore:

$$HHI = \sum_{i=1}^{n} ms_i^2 = 0.5$$

in the case of Cournot and

$$HHI = \sum_{i=1}^{n} ms_i^2 = \frac{5}{9}$$

in the Stackelberg case. As $\frac{5}{9} > 0.5$, the Herfindahl index correctly indicates that the presence of a leader with higher market share than the other duopolist makes the market more concentrated

(d) The two market share in point (a) are: $ms_1 = ms_2 = \frac{1}{2}$ and in point (b) $ms_1 = \frac{2}{3}$ and $ms_2 = \frac{1}{3}$. Therefore:

$$CR4 = \sum_{i=4}^{n} ms_i = 1$$

in the case of Cournot and

$$CR4 = \sum_{i=4}^{n} ms_i = 1$$

in the Stackelberg case. The $CR4$ is not able to indicate that the second case is more concentrated. This shows that the HHI is a better concentration index than the CR4.

(e) The free entry equilibrium can be found by setting to 0 the formula for the profits in a Cournot with $n$ firms and fixed cost $f$:

$$\pi(n) = \left(\frac{a-c}{n+1}\right)^2 \frac{1}{b} - f$$

Solving for $n$ we get:

$$n^* = \frac{a-c}{\sqrt{fb}} - 1$$

Decreasing $f$ there is room, everything else equal, for more firms in the market as the quantity sold (which does not depend on $f$) covers more easily the fixed costs. If $f$ goes to 0 then the number of firms goes to infinity as there is room for any additional firm as the firms already in the market will accomodate the additional entry reducing the quantity they produce.
The parameter $a$ represents in this kind of model the dimension of the market. Unsurprisingly, if we increase the dimension of the market, everything else equal, the number of firms in the market will increase.

**Exercise 3 (7.5 points)**

Telecom Italy provides voice, data, ICT and digital content services. It also owns 70% of Persidera, which provides access to digital terrestrial networks to TV channels.
Vivendi is a French media group that controls several companies, including Havas (which provides advertising in Italy) and owns 29% of Mediaset, which also provides access to digital terrestrial networks to TV channels and together with Persidera, have a dominant position in this market in Italy. Vivendi also has a 24% stake in Telecom Italy and recently was able to name 2/3 of the board of this Company.
The European Commission (EC) has analyzed the plan of Vivendi to acquire shares and gain de facto control of Telecom Italy, reaching a final decision in May 30th, 2017.

(a) What has the EC to do with this matter?
(b) In what market(s) could competition be affected (restricted or enhanced) by the deal?
(c) What kind of restraints on competition could arise from this deal?
(d) What do you think (or know) was ther decision taken by te EC and for what reasons?

**Solution**

(a) Because the EC is the enforcement agency (prosecutor and judge) for Competition Policy in the EU. It overseas competition in European markets and approves mergers.

This case involves a possible merger between Vivendi and Telecom Italy.

In fact, Merger Regulations in the EU prohibits concentrations that impede effective competition, as a result of the creation or strengthening of a dominant position

(b) The deal could affect competition in the market of services provided by digital terrestrial networks for TV channels in Italy, restricting competition or allowing more efficient operation and exploitation of economies of scale.

Potentially it could also restrict competition in the market for advertising in Italy, since Havas and Telecom Italy are important players in the advertising market

(c) This deal could facilitate an horizontal restraint: a collusion between Persidera and Mediaset in order to increase the price of services of digital terrestrial networks.

Potentially it could also introduce exclusionary behavior or restraints in advertising: an agreement between Havas and Telecom Italy which discriminates against producers or consumers of advertising

(d) Three alternatives are possible:

(a) Allow the deal

(b) Reject the deal

(c) Approve the deal subject to conditions: the sale of Mediaset by Vivendi

Explanation is essential. Best answers should be (c), since it solves the potential collusion in the market for network services (that was the decision taken by the EC). (a) and (b) could be partially right if reasonable explanation is given

In fact, the EC approved the acquisition and take of control of Telecom Italy by Vivendi, conditional on the divestment of Telecom Italy's stake in Persidera.

Such decision was based on the following conclusions:

- The deal would create incentives to raise prices charged to TV stations for access to the terrestrial network (due to Vivendis control of Persidera and Mediaset)
- The deal would not restrict competition in the advertising and media market

**Exercise 4 (7.5 points)** A researcher, by using a sample of 1,254 transactions of residential houses in the Torino area, has estimated the following hedonic model:

$$price_i = \beta_0+\beta_1\, area_i+\beta_2\, area_i^2+\beta_3\, high\ floor_i+\beta_4\, elevator_i+\beta_5\,(high\ floor\times elevator)_i+u_i \quad (20)$$

where *price* is the price of the house in euro per square meter, *area* is the area of the house in square meters, $area^2$ is the area of the house squared, *high floor* is a dummy taking the value of 1 if the house is at the third or higher floor of the building, *elevator* is a dummy taking the value of 1 if the building has an elevator, and $high\ floor_i \times elevator$

is the interaction of the two previous dummies.

The OLS estimates are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{price}_i = \underset{(596.0)}{2,719} + \underset{(1.199)}{3.965}\ area_i - \underset{(0.003)}{0.006}\ area_i^2 - \underset{(78.0)}{238.1}\ high\ floor_i + \underset{(51.7)}{263.1}\ elevator_i + \underset{(80.4)}{288.2}\ (high\ floor \times elevator)_i \quad R^2 = 0.53 \tag{21}$$

(1) Carefully interpret the coefficients in (20);

(2) Test at the $\alpha = 5\%$ level, that the coefficient of *elevator* is equal to 200 against a bilateral alternative, assuming the 4 OLS assumptions hold;

(3) Suppose that the researcher estimates a simpler version of equation (20) where the regressor $area^2$ is omitted. Carefully explain what would happen to the estimates you obtained in equation (21);

(4) Suppose you are offered a house of 100 square meters, located at the second floor of a building with elevator at a price of 3,000 euros per square meter. Based only on the estimated model (21) would you buy that house? Explain your choice.

**Solution**

(1) The intercept $\beta_0$ is the expected price when all regressors are equal to 0. As there is no flat with a 0 area, it has just a geometric, and not an economic, interpretation. $\beta_1$ and $\beta_2$ cannot be interpreted in isolation. They are the coefficients of the polynomial linking the price of the flat to its area. They constitute the marginal effect, i.e. the marginal change in the price of the flat for an infinitesimal change in the area: $\frac{\partial E(price)}{\partial area} = \beta 1 + 2 \times \beta_2 \times area$. The coefficient of the dummy *high floor*, $\beta_3$ is the increment in price when the flat is at a high floor but the building has no elevator (unsurprisingly, its estimate is negative), the coefficient of the dummy *elevator*, $\beta_4$, is the increment in price when the flat is in a building with elevator and is in a low floor (unsurprisingly, its estimate is positive as the elevator benefits all the houses in the building), and the coefficient of the interaction ($high\ floor \times elevator$), $\beta_5$ is the increment in the effect of the two dummies when the other is equal to 1 (unsurprisingly, it is positive as the effect of elevator is greater when the flat is located in a high floor of the building).

(2) By performing an individual $t$ test on the coefficient of *elevator*, $\beta_4$, we get $t = \frac{263.1-200}{51.7} \approx 1.2$. As this value is inside the acceptance interval $(-1.96, +1.96)$ we do not reject the null hypothesis.

(3) If we omit $area^2$ from the equation, the fit of the model will decrease but not substantially as the two variables *area* and $area^2$ are correlated. Furthermore, the coefficient of *area* will be biased downward as *area* and $area^2$ are *positively* correlated and the coefficient of $area^2$ is negative. No a priori sign could be provided for the coefficients

of the other variables. By estimating the model with the same data but without the regressor $area^2$ we get an estimated value of $\beta_1 = 2.331$ and a $R^2 = 0.52$.

(4) The expected value of such a house would be: $2,719 + 3.965 \times 100 - 0.006 \times 100^2 - 238.1 \times 0 + 263.1 \times 1 + 288.2 \times 0 = 3,318.6$. Being the expected value higher than the proposed price, you should buy the house.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2016-17**
**Exam September, $15^{th}$ 2017**

**Instructions**: answer to all questions in 1 hour and 40 minutes

### Exercise 1 (7.5 points)

Consider a market in which two firms, 1 and 2, produce a homogenous good under constant returns to scale. The two firms face the demand function $Q(p) = a - bp$. Marginal costs are $c_1 = 0$ and $c_2 = c$, respectively, where $0 < c \leq \frac{a}{2}$.

Find the equilibrium (quantities, price, profits) in the following cases:

(a) Cournot game;

(b) Bertrand game without capacity constraints;

(c) Stackelberg game where firm 2 acts as leader;

Suppose now that the firms are symmetric in costs ($c_1 = c_2 = 0$) but they face different capacity constraints: capacity constraint of firm 1 is $k_1 = 1$ and capacity constraint of firm 2 is $k_2 = 2$. Suppose also that $a = 7$ and that $b = 1$.

(d) Find the Bertrand equilibrium (quantities, price, profits)

### Solution

(a) It is a standard Cournot with asymmetric cost. The inverse demand function is $p(Q) = \frac{a}{b} - \frac{Q}{b}$. Setting $a^* = \frac{a}{b}$ and $b^* = \frac{1}{b}$ we can use the standard formulas.

The reaction functions are:

$$q_1(q_2) = \frac{a^*}{2b^*} - \frac{q_2}{2} = \frac{a - q_2}{2} \tag{1}$$

$$q_2(q_1) = \frac{a^* - c}{2b^*} - \frac{q_1}{2} = \frac{a - cb - q_1}{2} \tag{2}$$

so that

$$q_1^* = \frac{a^* + c}{3b^*} = \frac{a + bc}{3} \tag{3}$$

$$q_2^* = \frac{a^* - 2c}{3b^*} = \frac{a - 2bc}{3} \tag{4}$$

and

$$Q^* = \frac{2a^* - c}{3b^*} = \frac{2a - bc}{3} \tag{5}$$

$$p^* = \frac{a^* + c}{3} = \frac{a + bc}{3b} \tag{6}$$

$$\pi_1 = \frac{a^* + c}{3b^*} \times \frac{a^* + c}{3} = \frac{(a^* + c)^2}{9b^*} = \frac{b\left(\frac{a}{b} + c\right)^2}{9} \tag{7}$$

$$\pi_2 = \frac{a^* - 2c}{3b^*} \times \left(\frac{a^* + c}{3} - c\right) = \frac{(a^* - 2c)^2}{9b^*} = \frac{b\left(\frac{a}{b} - 2c\right)^2}{9} \tag{8}$$

Quantity and hence profits for firms 1 are positively related to the marginal cost of firm 2, $c$. If $c = 0$ all the previous results revert to those of a standard Cournot model with symmetric, null marginal cost.

(b) This is a standard Bertrand model with asymmetric costs. Firm 1 will set the $p_1^* = c - \epsilon$ and will serve the whole demand whereas firm 2 will set $p_2^* = c$ and will produce a null amount. The quantity for firm 1 is $Q = a - bp = a - b(c - \epsilon)$ so that profit for firm 1 is $\pi_1 = a - b(c - \epsilon) \times (c - \epsilon) > 0$.

(c) By inserting the reaction function of firm 1 into the profit function of firm 2 we get:

$$\pi_2 = \left(\frac{a^*}{2} - \frac{b^* q_2}{2} - c\right) q_2$$

so that the Stackelberg equilibrium quantities

$$q_2^* = \frac{a^*}{2b^*} - \frac{c}{b^*} = \frac{a}{2} - bc$$

$$q_1^* = \frac{a^*}{4b^*} + \frac{c}{2b^*}\frac{a}{4} + \frac{bc}{2}$$

and

$$Q^* = \frac{3a^*}{4b^*} - \frac{c}{2b^*} = \frac{3a}{4} - \frac{cb}{2} \tag{9}$$

$$p^* = \frac{1}{4}a^* + \frac{c}{2} = \frac{a}{4b} + \frac{c}{2} \tag{10}$$

$$\pi_1 = \frac{(a^*)^2 + 2a^*c + 2c + 4c^2}{16b^*} = \frac{b\left[(\frac{a}{b})^2 + 2\frac{a}{b}c + 2c + 4c^2\right]}{16} \tag{11}$$

$$\pi_2 = \frac{2(a^*)^2 - 8a^*c + 8c^2}{16b^*} = \frac{b\left[2(\frac{a}{b})^2 - 8\frac{a}{b}c + 8c^2\right]}{16} \tag{12}$$

You can verify that, in terms of quantities, if the cost disadvantage is low $(c < \frac{a}{6})$ the first mover advantage compensates the cost disadvantage as firm 2 produces a higher quantity than firm 1.

(d) If $c_1 = c_2 = 0$, $k_1 = 1$, $k_2 = 2$, $a = a^* = 7$ and $b = b^* = 1$ we have:

$$q_1(q_2) = \frac{7}{2} - \frac{q_2}{2} = \frac{7 - q_2}{2} \tag{13}$$

$$q_2(q_1) = \frac{7}{2} - \frac{q_1}{2} = \frac{7 - q_2}{2} \tag{14}$$

If firm 2 produces at its capacity ($q_2 = k_2 = 2$):

$$q_1(2) = \frac{7-2}{2} = \frac{5}{2} \tag{15}$$

so that $k_1 = 1 < q_1^*(k_2)$

If firm 1 produces at its capacity ($q_1 = k_1 = 1$):

$$q_2(1) = \frac{7-1}{2} = 3 \tag{16}$$

so that $k_2 = 2 < q_2^*(k_1)$

Therefore, we are in the case in which each firm would like to expand its production but it cannot due to the capacity constraints. In this case, the equilibrium price will be $p(k_1 + k_2) = 7 - 1(3) = 4$. Quantities produced will be the constraints and the profits will be equal to the revenue: $\pi_1 = 1 \times 4 = 4$ and $\pi_2 = 2 \times 4 = 8$.

**Exercise 2 (7.5 points)**

Consider a standard linear city *à la Hotelling* (i.e unitary length with uniform distribution of the consumers) with linear transportation costs $\theta$ and three firms $(a,b,c)$ localized in $x_a = 0$, $x_c = 1$, and $x_b = x$, with $0 < x < 1$. The three firms compete in prices and their production costs are null.

(a) Verify that at equilibrium the profit of firm $b$ does not depend on its position $x$. Provide the economic rationale of this result.

(b) How the equilibrium found in point (a) would change if firm $b$ has a constant production cost $c > 0$ whereas firms $a$ and $c$ still have null production costs?

Assume an equilibrium exists.

**Solution**

(a) If an equilibrium $[p_a^*, p_b^*, p_c^*]$ exists, then two indifferent consumers are localized in $x_{ab}$, between $a$ and $b$, and in $x_{bc}$, between $b$ and $c$.

Consider the consumer who is indifferent between firm $a$ and firm $b$. For this consumer it must hold the following equality:

$$\theta x_{ab} + p_a = \theta (x - x_{ab}) + p_b \tag{17}$$

whereas for the consumer who is indifferent between firm $b$ and firm $c$ the following equality must hold:

$$\theta (x_{bc} - x) + p_b = \theta (1 - x_{bc}) + p_c \tag{18}$$

From (17):

$$2\theta x_{ab} = \theta x + p_b - p_a \tag{19}$$

and from (18):

$$2\theta x_{bc} = \theta (1 + x) + (p_c - p_b) \tag{20}$$

The coordinates of the position of the indifferent consumers are therefore:

$$x_{ab} = \frac{x}{2} + \frac{p_b - p_a}{2\theta} \tag{21}$$

$$x_{bc} = \frac{1 + x}{2} + \frac{p_c - p_b}{2\theta} \tag{22}$$

The payoffs of the firms are:

$$\pi_a = p_a x_{ab} = p_a \times \left(\frac{x}{2} + \frac{p_b - p_a}{2\theta}\right) \tag{23}$$

$$\begin{aligned}\pi_b &= p_b\left(x_{bc} - x_{ab}\right) = p_b\left[\left(\frac{1+x}{2} + \frac{p_c - p_b}{2\theta}\right) - \left(\frac{x}{2} + \frac{p_b - p_a}{2\theta}\right)\right] \\ &= p_b \times \left(\frac{1}{2} + \frac{p_a + p_c}{2\theta} - \frac{p_b}{\theta}\right)\end{aligned} \tag{24}$$

$$\pi_c = p_c(1 - x_{bc}) = p_c \times \left[1 - \left(\frac{1+x}{2} + \frac{p_c - p_b}{2\theta}\right)\right] = p_c \times \left(\frac{1}{2} - \frac{x}{2} - \frac{p_c - p_b}{2\theta}\right) \tag{25}$$

To obtain the Nash equilibrium in prices we have to derive the profit functions, set equal to 0 and solve the system of the reaction functions.

$$\frac{\partial \pi_a}{p_a} = \frac{x}{2} + \frac{p_b}{2\theta} - \frac{p_a}{\theta} = 0 \tag{26}$$

$$\frac{\partial \pi_b}{p_b} = \left(\frac{1}{2} + \frac{p_a + p_c}{2\theta} - \frac{p_b}{\theta}\right) - \frac{p_b}{\theta} = 0 \tag{27}$$

$$\frac{\partial \pi_c}{p_c} = \left(\frac{1}{2} - \frac{x}{2} - \frac{p_c - p_b}{2\theta}\right) - \frac{p_c}{2\theta} = \frac{1}{2} - \frac{x}{2} - \frac{p_c}{\theta} + \frac{p_b}{2\theta} = 0 \tag{28}$$

From (26) and (28):

$$p_a = \frac{\theta x}{2} + \frac{p_b}{2} \tag{29}$$

$$p_c = \theta\left(\frac{1-x}{2}\right) + \frac{p_b}{2} \tag{30}$$

If we substitute in (27) and then back in (26) and (28) we get:

$$p_a^* = \theta\left(\frac{1+2x}{4}\right) \tag{31}$$

$$p_b^* = \frac{\theta}{2} \tag{32}$$

$$p_c^* = \theta\left(\frac{3-2x}{4}\right) \tag{33}$$

As it can be seen, at the equilibrium both $(x_{bc} - x_{ab})$ and $p_b$ do not depend on $x$. In particular, $(x_{bc} - x_{ab}) = \frac{1}{2}$, and then $\pi_b = \frac{\theta}{4}$. This result can be explained as—at the equilibrium—a variation of $x$ determines a decrease of the demand in the direction of the variation, and, at the same time, an *equal* increase in the opposite direction. This explains why prices and profits of $b$ are independent on $x$.

(b) If firm $b$ incurs in a positive marginal cost $c$, its payoff (eq. (24)) is modified into

$$(p_b - c) \times \left(\frac{1}{2} + \frac{p_a + p_c}{2\theta} - \frac{p_b}{\theta}\right) \tag{34}$$

In turn, the derivative of payoff (eq. (27)) becomes

$$\frac{\partial \pi_b}{p_b} = \left(\frac{1}{2} + \frac{p_a + p_c}{2\theta} - \frac{p_b}{\theta}\right) - \frac{p_b - c}{\theta} = \frac{1}{2} + \frac{p_a + p_c}{2\theta} - \frac{2p_b}{\theta} + \frac{c}{\theta} = 0 \tag{35}$$

Formulas (31) and (32) are unchanged. By inserting them into equation (35) we get:

$$p_a^* = \theta\left(\frac{1+2x}{4}\right) + \frac{c}{3} \tag{36}$$

$$p_b^* = \frac{\theta}{2} + \frac{2c}{3} \tag{37}$$

$$p_c^* = \theta\left(\frac{3-2x}{4}\right) + \frac{c}{3} \tag{38}$$

As we can see, equilibrium prices are higher than before for all firms. Profits are higher now for firms $a$ and $c$ but lower for firm $b$. Indifferent consumer $x_{ab}$ will move to the right and indifferent consumer $x_{bc}$ will move to the left.

**Exercise 3 (7.5 points)** A owner of a movie theater faces two groups of consumers: adults (with high demand) and students (with low demand). Their demand is:

$$q^A = 1600 - 100p \tag{39}$$
$$q^S = 800 - 100p \tag{40}$$

Marginal cost are constant and equal to 2. Suppose no fixed cost.

(a) Suppose first that the movie theater owner does not want to price discriminate so that it charges a single price for both groups of consumers. Find the optimal price, and the ensuing quantities demanded, profits, and consumer surplus;

(b) Now suppose that the movie theater owner can recognize the two groups of consumers (via ID cards) and implements a third degree price discrimination. Find the optimal prices, and the ensuing quantities demanded, profits, and consumer surpluses. Compare, providing an economic rationale, with the results obtained in point (a) above;

**Solution**

(a) Without price discrimination, the movie theater faces an aggregate demand, given by the sum of the two demands. This is $Q = 2,400 - 200p$. Inverting the demand we find $p = 12 - 0.005Q$. Marginal revunue is $MR = 12 - 0.01Q$ and setting equal to the marginal cost leads to: $Q^* = 1,000$ and $p^* = 7$. The profit is $\pi = 7,000 - 2,000 = 5,000$. Inverse demands for each category are: $p^A = 16 - 0.01q^A$ and $p^S = 8 - 0.01q^S$. Consumers' surpluses are therefore: $CS^A = \frac{(16-7)900}{2} = 4,050$ and $CS^S = \frac{(8-7)100}{2} = 50$, so that total consumers' surplus is $4,100$.

(b) If the movie theater can discriminate, it will charge $p^A = 9$ and $p^S = 5$. The quantities are $q^A = 700$ and $q^S = 300$. The profit is now $\pi = 700 \times 9 + 300 \times 5 - 1,000 \times 2 = 5,800$, higher than before. Given total quantity is unchanged, total consumer surplus must have decreased. This is the case as $CS^A = \frac{(16-9)700}{2} = 2,450$ and $CS^S = \frac{(8-5)300}{2} = 450$, so that total consumers' surplus is $2,900$. As we can see, the low demand consumer are better off whereas the high demand consumer is worse off. This is unsurprising as the consumer who is penalised by third degree discrimination is the one with more rigid demand. In fact, the elasticities of the two demands are $-100 \times \frac{7}{100} = -7$ for students and $-100 \times \frac{7}{900} = -0.78$ for adults.

**Exercise 4 (7.5 points)** A researcher, by using a sample of 603 Barolo and Barbaresco wine bottles for the 1995-98 vintages, has estimated the following hedonic model:

$$log(price)_i = \beta_0 + \beta_1\ type_i + \beta_2\ an97 + \beta_3\ (type \times\ an97)_i + \beta_4\ alc_i + \beta_5\ fit_i + u_i \quad (41)$$

where *price* is the price of the bottle in euro; *type* is a dummy variable taking a value of 1 if the bottle is of Barolo and 0 if it is of Barbaresco; $an97$ is a dummy taking a value of 1 if the wine vintage is 1997 and 0 if the vintage is 1995, 1996 or 1998; $type \times\ an97$ is the interaction between the two previous dummies; *alc* is the alcholic content of the wine (in %) and *fit* is a measure of how famous the wine producer of the specif bottle is, i.e. the number of awards the producer received from a well known wine guide.

The OLS estimates are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{log(price)}_i = \underset{(0.52)}{1.99} + \underset{(0.034)}{0.038}\ type_i + \underset{(0.052)}{0.051}\ an97_i + \underset{(0.060)}{0.078}\ (type \times\ an97)_i + \underset{(0.038)}{0.076}\ alc_i$$
$$+ \underset{(0.003)}{0.045}\ fit_i \qquad R^2 = 0.35 \quad (42)$$

(1) Carefully interpret the coefficients in (41);

(2) Test at the $\alpha = 5\%$ level, the null hypothesis that the coefficient of *alc* is equal to 0 ($H_0 : \beta_4 = 0$) against a bilateral alternative ($H_1 : \beta_4 \neq 0$), assuming the 4 OLS assumptions hold;

(3) Test at the $\alpha = 5\%$ level, the null hypothesis that the coefficient of *alc* is equal to 0 ($H_0 : \beta_4 = 0$) against the unilateral alternative that the coefficient of $alc > 0$ ($H_1 : \beta_4 > 0$), assuming the 4 OLS assumptions hold;

(4) Explain how to test that the coefficients of *type* and $type \times\ an97$ are jointly equal to 0, assuming the 4 OLS assumptions hold and provide the economic interpretation of this test;

(5) Suppose that you can collect additional observations so that your sample size is increased. Suppose specifically that it is increased by a factor of 4, i.e. that instead of estimating model (41) with 603 observations you estimate it on 2,412 observations. How would you expect the estimated results in equation (42) and the result ot the tests in points (2) and (3) above would change?

**Solution**

(1)
- The intercept $\beta_0$ is the expected log price when all regressors are equal to 0, i.e. the wine is a Barbaresco, the vintage is different from the year 1997, the alcholic content is 0 and the number of awards the producer received from a well known wine guide is also 0. Given our interest in price, and not in log price, it is more interesting to refer to price: $exp\,(\beta_0)$ is the expected price when all regressors are equal to 0. In any case, as there is no wine with a 0 alcolic rate, it has just a geometric, and not an economic, interpretation.

- $(exp\,(\beta_1) - 1) \times 100$ (resp. $\beta_1$) is the expected percentage change in price (resp. expected chance in log price) when the wine is Barolo with respect to a Barbaresco one, holding the other regressors fixed, and the vintage is different from the year 1997.
- $(exp\,(\beta_2) - 1) \times 100$ (resp. $\beta_2$) is the expected percentage change in price (resp. expected chance in log price) when the vintage is year 1997 with respect to the other vintages, holding the other regressors fixed, and the wine is a Barbaresco one.
- $(exp\,(\beta_3) - 1) \times 100$ (resp. $\beta_3$) is the increment in the expected percentage change in price (resp. expected chance in log price) when the vintage is year 1997 with respect to the other vintages and the wine is a Barolo one. It is also the increment in the expected percentage change in the price (resp. expected chance in log price) when the wine is a Barolo one and the vintage is year 1997
- $\beta_4 \times 100$ (resp. $\beta_4$) is the expected percentage change in the price (resp. expected chance in log price) when the alcolic content increases of one unit (i.e. of 1%) and the other regressors are held fixed
- $\beta_5 \times 100$ (resp. $\beta_5$) is the expected percentage change in price (resp. expected chance in log price) when the variable $fit$ increases of one unit and the other regressors are held fixed

(2) By performing an individual $t$ test on the coefficient of $alc$, $\beta_4$, we get $t = \frac{0.076-0}{0.038} = 2$. As this value is outside the acceptance interval $(-1.96, +1.96)$ we do not reject the null hypothesis.

(3) A unilateral test would be performed again by an individual $t$ test on the coefficient of $alc$, $\beta_4$, getting $t = \frac{0.076-0}{0.038} = 2$. What changes with respect to the previous case is the acceptance region, which will be $(-\infty, +1.64)$ as low values of the statistics (below $-1.96$) cannot be interpreted against the null. Also in this case we reject the null.

(4) The test that the coefficients of $type$ and $type \times\ an97$ are jointly equal to 0 must be performed through an $F$ test, whose formula is

$$F = \frac{1}{2} \times \frac{t_1^2 + t_2^2 - 2 \times \hat{\rho}_{t_1,t_2} \times t_1 \times t_2}{1 - \hat{\rho}^2_{t_1,t_2}} \tag{43}$$

The economic interpretation of this test is that the type of wine (Barolo or Barbaresco) has no effect on price. Notice that this information is not provided by a single test that $\beta_1 = 0$, as the latter simply tests that the effect of type is null if the vintage is different from 1997

(5) If we increase the number of observation in the sample the estimates will become more precise. This will affect the standard errors of the OLS estimates and, in turn, the $t$ tests in points (2) and (3). The $R^2$ will not be affected as the increase in the sample size does not induce a better fit in the model.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2016-17**
**Exam January, $22^{nd}$ 2018**

**Instructions**: answer to all questions in 1 hour and 40 minutes

### Exercise 1 (7.5 points)

Consider a market in which three firms (1, 2 and 3), produce a homogenous good. The three firms face the inverse demand function $p = 3-Q$ where $Q$ is total quantity. Marginal costs equal for three firms $c_1 = c_2 = c_3 = 1$ and fixed costs are null.

(a) Find the equilibrium (quantities, price, profits) supposing that firms play a Cournot game and compute the HHI index;

(b) Suppose that two firms (say, firms 2 and 3) merge to form a new entity, firm M. Supposing the same marginal costs as before and that firms still play a Cournot game, compute the profits of the two firms (firm 1 and firm M) and assess whether it is profitable for firms 2 and 3 to merge. Interpret the result. Compute the HHI index.

(c) Replicate points (a) and (b) above by supposing Bertrand and not Cournot competition (suppose no capacity constraints);

(d) Based on previous results, comment whether a Competition authority should be more concerned about the merger in a Cournot or in a Bertrand setting. Comment about the use of the HHI index to assess the desirability of the merger.

### Solution

(a) It is a standard Cournot model with symmetric and constant marginal costs and linear demand. Hence:

$$q_i = \frac{a-c}{(n+1)b} \qquad i = 1,2,3 \tag{1}$$

$$p_i = \frac{a+nc}{n+1} \qquad i = 1,2,3 \tag{2}$$

$$\pi_i = \frac{\left(\frac{a-c}{n+1}\right)^2}{b} \qquad i = 1,2,3 \tag{3}$$

Setting $a = 3$, $b = 1$, $c = 1$ and $n = 3$:

$$q_i = \frac{3-1}{4} = \frac{1}{2} \qquad i = 1,2,3 \tag{4}$$

$$p_i = \frac{6}{4} = \frac{3}{2} \qquad i = 1,2,3 \tag{5}$$

$$\pi_i = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad i = 1, 2, 3 \tag{6}$$

$$Q = \frac{3}{2} \tag{7}$$

The market share of each firm is equal to $ms_i = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$ so that the $HHI = \sum_{i=i}^{n} ms_i^2 = 3 \times \left(\frac{1}{3}\right)^2 = \frac{1}{3}$ (Notice that in many cases, the HHI is computed by using market share in percentages, so that the HHI would be multiplied by 10,000, in our case would be equal to 3,333)

(b) If the two firms merge and there is no gain in costs, the model becomes a standard Cournot duopoly model with symmetric and constant marginal costs and linear demand. The two firms in the market are firm 1 and firm $M$. Hence

$$q_i = \frac{3-1}{3} = \frac{2}{3} \quad i = 1, M \tag{8}$$

$$p_i = \frac{5}{3} \quad i = 1, M \tag{9}$$

$$\pi_i = \left(\frac{2}{3}\right)^2 = \frac{4}{9} \tag{10}$$

$$Q = \frac{4}{3} \tag{11}$$

The profit of the merged firm, $\frac{4}{9}$ is less than the sum of the profits of the two entities $(2 \times \frac{1}{4} = \frac{1}{2})$. This is because the increase in price is not able to compensate the decrease in the quantity produced. So the merger is not profitable.
The market share of each firm is equal to $ms_i = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2}$ so that the $HHI = \sum_{i=1}^{M} ms_i^2 = 2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{2}$. Clearly, the market has become more concentrated and the HHI has increased.

(c) If the three firms compete in price (Bertrand competition) each will set $p_i = MC = 1$ and the only equilibrium in case of equal marginal costs is that the three firms equally share the market:

$$q_i = \frac{Q}{3} \quad i = 1, 2, 3 \tag{12}$$

where $Q$ is given by $1 = 3 - Q$ so that $Q = 2$ and $q_i = \frac{2}{3}$, $i = 1, 2, 3$. Of course $\pi_i = 0$, $i = 1, 2, 3$ and $HHI = \sum_{i=i}^{n} = ms_i^2 = 3 \times \frac{1}{3}^2 = \frac{1}{3}$ as in the Cournot case.

If the two firms merge and there is no gain in costs, the model becomes a standard Bertrand duopoly model with symmetric and constant marginal costs and linear demand between firm 1 and firm $M$. Hence $p_i = MC = 1$ and the only equilibrium in case of equal marginal costs is that the two firms equally share the market $q_i = \frac{Q}{2}$, $i = 1, M$ where $Q$ is given as before by $1 = 3 - Q$ so that $Q = 2$ and $q_i = \frac{2}{2} = 1$, $i = 1, M$. Of course $\pi_i = 0$, $i = 1, M$. The profit of the merged firm is null and equal to the sum of the profits of the two entities which are all null as well. Firms are indifferent

with respect to the decision to merge or not.
$HHI = \sum_{i=1}^{M} ms_i^2 = 2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{2}$ as in the Cournot case.

(d) Competition authorities should be more concerned with the merger in the Cournot than in Bertrand setting. In the former case, the merger leads to a price increase and therefore to a higher market power for the firms in the market. As for HHI, it equally increases after the merger no matter the kind of competition. This shows that the usual practice to assess the potential danger of a merger by considering the increase in HHI it might lead to is not satisfactory as it emphasizes the change in market structure without considering the kind of rivalry in place in the market.

**Exercise 2 (7.5 points)**

Consider a linear city whose lenght is 1 with two firms, $A$ and $B$. Consumers are uniformly distributed along the interval $x \in [0, 1]$. Firm $A$ is located in point $x_A = 0$ and firm $B$ in $x_B = \frac{3}{4}$. Every consumer buys one unit of the good. Transportation costs are equal to 4 times the distances between consumers and firms, i.e. $\tau = 4$.

(a) Suppose first that the good of firm $A$ is produced with a marginal cost of 1 and its consumption gives to the consumer the gross surplus $v_A = 6$ whereas the good of firm $B$ gives the consumers the gross surplus $v_B = 4$ and it is produced at null marginal cost. Find the indifferent consumer and firms' equilibrium prices. Verify that the profits for the two firms are equal to 2.

(b) Suppose now that firm $A$ can save on marginal costs but only by reducing the gross surplus for the consumer. In particular, suppose that firm $A$ can produce the good at 0 marginal cost but providing to the consumer a gross surplus $v_A = 5$. Verify that the profits for the two firms are—as before—equal to 2 for both firms. Explain the counter intuitive result that the two firms have the same level of profit despite they have the same marginal cost but firm $A$ produces a "better" product.

(c) How would you expect that the result of equality of profits of the two firms would be affected if we suppose transportation costs are equal to 6 (and not to 4) times the distances, i.e. $\tau = 6$?

**Solution**

(a) The indifferent consumer, located in $x$, will be indifferent between purchasing the good from firm $A$ or purchasing the good of firm $B$. This leads to the following equality:

$$6 - p_A - 4x = 4 - p_B - 4\left(\frac{3}{4} - x\right)$$

The indifferent consumer will then be located in:

$$x = \frac{5 + p_B - p_A}{8}$$

so that the demands for firms $A$ and $B$ will be, respectively, $\frac{5+p_B-p_A}{8}$ and $1-\frac{5+p_B-p_A}{8}$. This leads to the following profit functions:

$$\pi_A = (p_A - 1)\left(\frac{5+p_B-p_A}{8}\right)$$
$$\pi_B = p_B\left(1-\frac{5+p_B-p_A}{8}\right)$$

The FOCs are:

$$\frac{\partial \pi_A}{\partial p_A} = \frac{5+p_B-p_A}{8} - \frac{p_A-1}{8} = 0 \Rightarrow p_A = 3 + \frac{p_B}{2}$$
$$\frac{\partial \pi_B}{\partial p_B} = \frac{3-p_B+p_A}{8} - \frac{p_B}{8} = 0 \Rightarrow p_B = \frac{3+p_A}{2}$$

Solving the system of two FOCs we obtain:

$$p_A = 5$$
$$p_B = 4$$
$$x = \frac{1}{2}$$
$$\pi_A = 2$$
$$\pi_B = 2$$

(b) The indifferent consumer, located in $x$, will be indifferent between purchasing the good from firm $A$ or purchasing the good from firm $B$. This leads now to the following equality:

$$5 - p_A - 4x = 4 - p_B - 4\left(\frac{3}{4} - x\right)$$

The indifferent consumer will then be located in:

$$x = \frac{4+p_B-p_A}{8}$$

so that the demands for firms $A$ and $B$ will be, respectively, $\frac{4+p_B-p_A}{8}$ and $1-\frac{4+p_B-p_A}{8}$. This leads to the following profit functions:

$$\pi_A = p_A\left(\frac{4+p_B-p_A}{8}\right)$$
$$\pi_B = p_B\left(1-\frac{4+p_B-p_A}{8}\right)$$

The FOCs are:

$$\frac{\partial \pi_A}{\partial p_A} = \frac{4+p_B-p_A}{8} - \frac{p_A}{8} = 0 \Rightarrow p_A = \frac{4+p_B}{2}$$
$$\frac{\partial \pi_B}{\partial p_B} = 1 - \frac{4+p_B-p_A}{8} - \frac{p_B}{8} = 0 \Rightarrow p_B = \frac{4+p_A}{2}$$

Solving the system of two FOCs we obtain:

$$p_A = 4$$
$$p_B = 4$$
$$x = \frac{1}{2}$$
$$\pi_A = 2$$
$$\pi_B = 2$$

The counter intuitive result that the two firms have the same level of profit despite they have the same marginal cost but firm $A$ produces a "better" product can be explained by the better position of firm $B$, which enjoys a sort of "monopoly" over the segment $\left[\frac{3}{4}, 1\right]$

(c) The increase in $\tau$ from 4 to 6 (i.e. higher transportation costs) entails higher advantage of localization. In turn, the firm better localised (firm $B$) will now enjoy higher profits than firm $A$.
This will be true for both point (a) and (b) above. Let's verify for point (a). The new indifferent consumer will be located in point

$$x = \frac{\frac{13}{2} + p_B - p_A}{12}$$

so that if $p_B = p_A$ the indifferent consumer will be located in $\frac{13}{24}$. In case of $p_B = p_A$ with $\tau = 4$, the indifferent consumer will be located in $\frac{5}{8} = \frac{15}{24}$. As $\frac{13}{24} < \frac{15}{24}$, firm B will have higher demand (and hence higher profits) in the case $\tau = 6$ with respect to the $\tau = 4$ case.

**Exercise 3 (7.5 points)** A monopolist serves two types of consumers, $l$ and $h$, with demands given by $q_l = 1 - p_l$ and $q_h = 2 - p_h$ respectively; l-type consumers account for $\frac{1}{3}$ of the entire population, while h-type consumers for $\frac{2}{3}$. Assume production costs equal to 0 for simplicity.

(a) Compute the equilibrium price and quantities in case price discrimination is not allowed;

(b) Derive the consumer surplus and the monopolist profits in case a unique (i.e. for both type of users) two part-tariff $T + pq$ is offered, where $p$ is the per-unit price of the good and $T$ the fixed fee;

(c) From the point of view of the monopolist, is it better equilibrium 1) or 2)? Why? And what about welfare? Explain your results.

**Solution**

(a) Without price discrimination, the monopolist serves an aggregate demand given by the weighted average of the two demands:

$$Q = \frac{1}{3}(1 - p_l) + \frac{2}{3}(2 - p_h) \tag{13}$$

As the monopolist applies a single price to the two markets, the profit function is (production costs are 0 so it is equal to the revenue function):

$$\pi = \left[\frac{1}{3}(1 - p_l) + \frac{2}{3}(2 - p_h)\right] p \tag{14}$$

The FOC of the monopolist are then:

$$\frac{5}{3} = 2p \tag{15}$$

which leads to:

$$p = \frac{5}{6} \tag{16}$$

$$q_l = \frac{1}{6} \tag{17}$$

$$q_5 = \frac{7}{6} \tag{18}$$

$$\pi = \left[\frac{1}{3} \times \frac{1}{6} + \frac{2}{3} \times \frac{7}{6}\right] \frac{5}{6} = \frac{25}{36}$$

(b) With second degree price discrimination, the monopolist sets the tariff $T$ equal to the surplus of the low demand consumer. Therefore:

$$T = \frac{1}{2}(1 - p)^2 \tag{19}$$

The profit function is then:

$$\pi = \left[\frac{1}{3}(1 - p) + \frac{2}{3}(2 - p)\right] p + T = \frac{1}{3}\left(p - p^2\right) + \frac{2}{3}\left(2p - p^2\right) + \frac{1}{2}(1 - p)^2$$

The FOC of the monopolist are then:

$$\frac{2}{3} - p = 0 \tag{20}$$

which leads to:

$$p = \frac{2}{3} \tag{21}$$

$$q_l = \frac{1}{3} \tag{22}$$

$$q_h = \frac{4}{3} \tag{23}$$

$$\pi = \frac{1}{3} \times \left(\frac{1}{3} \times \frac{2}{3}\right) + \frac{2}{3} \times \left(\frac{4}{3} \times \frac{2}{3}\right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3}\right) = \frac{13}{18}$$

(c) Obviously, the profit of the monopolist increases with discrimination: $\frac{13}{18} > \frac{25}{36}$ As price decreases $\left(\frac{2}{3} < \frac{5}{6}\right)$ and weighted quantity increases $\left(1 > \frac{5}{6}\right)$, consumer surplus must increase. In fact, without discrimination it is equal to:

$$CS = \frac{1}{3} \times \left(\frac{1}{2} \times \frac{1}{6} \times \frac{1}{6}\right) + \frac{2}{3} \times \left(\frac{1}{2} \times \frac{7}{6} \times \frac{7}{6}\right) = \frac{11}{24}$$

wherease in case of discrimination it is equal to:

$$CS = \frac{1}{3} \times \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3}\right) - \frac{1}{18} + \frac{2}{3} \times \left(\frac{1}{2} \times \frac{4}{3} \times \frac{4}{3}\right) = \frac{5}{9}$$

**Exercise 4 (7.5 points)** Suppose a researcher wants—by using a cross-section of firm level data within a given industry—to assess the relative performance, in terms of productivity, of private and public (i.e. State) owned firms controlling for the localization (North, Center, South) of the firms. To this end, he estimates the following Cobb-Douglas production function augmented with a dummy variable—*State Owned*—for public ownership (i.e. the dummy *State Owned* equals 1 if the firm is under public ownership and 0 otherwise) and two dummies for firms' localization in the North (dummy *North* which equals 1 if the firm is localized in the North of the country and 0 otherwise) and in the Center (dummy *Center* which equals 1 if the firm is localized in the Center of the country and 0 otherwise):

$$log(Y)_i = \beta_0 + \beta_K log(K)_i + \beta_L log(L)_i + \beta_3 (State\ Owned)_i + \beta_4 (North)_i + \beta_5 (Center)_i + u_i \tag{24}$$

(1) Carefully interpret the coefficients;

(2) Suggest the likely sign of the coefficients, providing the economic explaination for your suggestions;

(3) Explain why the researcher did not include in the model the third dummy for firms' localization (dummy *South* which equals 1 if the firm is localized in the South of the country and 0 otherwise);

(4) How would you modify the model to allow the relative performance, in terms of productivity, of private and public firms to depend on firms' localization and how would you test that the relative performance, in terms of productivity, of private and public firms does not depend on firms' localization?

(5) How would you address the same question by using the cost function instead of the production function?



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**Solution**

(1) The coefficients for $log(K)$ and $log(L)$ have to be interpreted as elasticities of the dependent variable with respect to each of the input, Capital and Labour. So, a 1% increase in Labour (resp., Capital) will lead to a $\beta_L\%$ (resp., $\beta_K\%$) in output $Y$. The intercept, $\beta_0$ is the expected level of the dependent variable when the regressors are equal to 0, i.e. for a firm with $K = L = 1$ under private ownership (i.e. $State\ Owned = 0$) and located in the South. $exp\,(\beta_0)$ is therefore the average productivity for firms under private ownership and located in the South whereas $exp\,(\beta_0 + \beta_3)$ is the average productivity for those firms under public ownership nd located in the South. $exp\,(\beta_0 + \beta_4)$ is the average productivity of those firms under private ownership and located in the North whereas $exp\,(\beta_0 + \beta_5)$ is the average productivity of those firms under private ownership and located in the South.

(2) As for the likely sign of the coefficients, $\beta_L$ and $\beta_K$ will be positive, as the marginal productivity of factors is positive. For the coefficient $\beta_0$ we cannot predict the sign. The coefficient $\beta_3$ is likely to be negative as State Owned firms are usually (in developed countries) badly managed for political interference problems, so that they are less productive than privately owned firms. The sign of the coefficients $\beta_4$ and $\beta_5$ depend on the specific country under analysis. In the case of Italy, it might be argued they are both positive as firms in the South suffer from a less efficient buraucracy, less trasport infrastructure, less educated workforce and the like.

(3) The researcher does not include the dummy for South as there would be perfect collinearity among the three geografical dummies and the constant: $1 = North_i + Center_i + South_i$. In other terms, the information contained in the dummy *South* is already contained in the other two dummies and the constant.

(4) The model should be augmented with two interactions: $(State\ Owned \times North)$ and $(State\ Owned \times Center)$. To test that the relative performance, in terms of productivity, of private and public firms does not depend on firms' localization a joint test that the coefficients of these two interactions are equal to zero should be performed. This test uses the F statistics.

(5) Differential productivity analyses can be run also by estimating a cost function instead of the production function. In this case, the rationale is that a firm will be more (resp., less) productive than another firm if it has a lower (resp., higher) cost of production, holding the output and the price of factor fixed. So the corresponding equation would be:

$$log(C)_i = \gamma_0 + \gamma_K log(price_K)_i + \gamma_L log(price_L)_i + \gamma_Y log(Y)_i + \gamma_3 (State\ Owned)_i + \gamma_4 (North)_i + \gamma_5 (Center)_i + \varepsilon_i \qquad (25)$$

In this case, a positive value of the coefficient $\gamma_3$ has to be expected. In fact, a positive value of the coefficient entail higher costs and hence lower productivity. Likewise for $\gamma_4$ and $\gamma_5$ we expect a negative sign, opposite to the one of $\beta_4$ and $\beta_5$.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2017-18**
**Exam June, $26^{th}$ 2018**

**Instructions**: answer to all questions in 1 hour and 45 minutes

**Exercise 1 (8 points)**

Suppose a competitive market with 100 firms. The total (internal) cost function for each of these firms is $TC(q) = 15q + 25q^2$. Alongside the previous internal total cost, production also entails *external* costs represented by the following function $EC(q) = 15q + 25q^2$ where $q$ is the quantity produced by each firm (in units). Market demand is given by $Q = 30 - \frac{p}{2}$ where $Q$ is total (market) quantity.

Find:

(a) The competitive equilibrium (price and quantity);

(b) The efficient quantity (i.e. the quantity produced if producers take into account the externality);

(c) The welfare loss due to the externality;

(d) The optimal Pigouvian tax which corrects for the externality.

(e) Suppose now that the market is monopolized by a single firm whose cost function is $TC(Q) = 15Q + \frac{Q^2}{4}$. Compute the welfare loss due to the externality. Explain the result.

**NB**: the internal and external costs have been set equal only to facilitate computations.

(a) To find the competitive equilibrium (price and quantity) we have to find the intersection of the market demand and the market supply. The latter is the sum of the individual firm supply functions. The individual firm supply function is found by setting price equal marginal cost and solving for the quantity:

$$p = 15 + 50q \Rightarrow q = \frac{p}{50} - \frac{3}{10} \tag{1}$$

The market supply is the sum of the quantities given price:

$$Q = 100 \times \left(\frac{p}{50} - \frac{3}{10}\right) = 2p - 30 \tag{2}$$

By setting market supply equal to marked demand we find the equilibrium values for price and quantity: $p^* = 24\$$ and $Q^* = 18$.

(b) To find the efficient quantity (i.e. the quantity produced if producers take into account the externality) we have to replicate the previous steps but using the marginal social cost and not the marginal internal cost to derive firms' supply function. The marginal social cost is the sum of the marginal internal cost and of the marginal external cost.

$$p = (15 + 50q) + (15 + 50q) = 30 + 100q \Rightarrow q = \frac{p - 30}{100} \tag{3}$$

The market supply is the sum of the quantities given price:

$$Q = 100 \times \left(\frac{p - 30}{100}\right) = p - 30 \tag{4}$$

By setting market supply equal to marked demand we find the equilibrium values for price and quantity: $p^e = 40\$$ and $Q^e = 10$. Needless to say, the efficient quantity (resp. price) is lower (resp. higher) than the competitive one, the externality being *negative*.

(c) To find the welfare loss due to the externality, it is sufficient to notice it correspond to the area of a triangle with height the difference between the optimal and the competitive quantities and as base the marginal external cost evaluated at the competitive quantity produced by each firm.
Given that at the competitive quantity each firm is producing $\frac{18}{100} = 0.18$ units, the marginal external cost is equal to $MEC(0.18) = 15 + 50 \times 0.18 = 24$. The difference between competitive and efficient production is $18 - 10 = 6$ so that the loss is:

$$Loss = \frac{1}{2} \times 8 \times 24 = 96\$ \tag{5}$$

per year.

(d) To find the optimal Pigouvian tax which corrects for the externality it is sufficient to equate the market demand depending on price to the market supply depending to the price minus the tax:

$$Q^D = 30 - \frac{P}{2} \tag{6}$$

$$Q^S = 2(P - T) - 30 \tag{7}$$

from which

$$60 = \frac{5}{2}P - 2T \tag{8}$$

It is a single equation in two unknowns. It can be solved by setting the price equal to $p = 40$ (the efficient price found before) so that the quantity exchanged is the efficient one previously found, 10. The optimal tax amounts to 20$.

(e) In presence of negative externalities, the competitive output is larger than the efficient one. It can happen, then, that a non discriminating monopolist, by producing less than the competitive output, is more efficient than perfect competition in maximising social welfare.
This is indeed the case here. The monopolist produces where marginal cost is equal to marginal revenue, i.e.

$$15 + 0.5Q = 60 - 4Q$$

from which $Q = 10$ and $p = 40$, i.e. the efficient quantity and price we found before. Hence, the cost of externality is 0 as the two market failures exactly compensate each other.

**Exercise 2 (8 points)**

A market is characterised by an inverse demand curve $p = 4 - Q$ where $Q$ is total quantity. Two firms, $A$ and $B$, are competing *à* la Cournot.

(a) Suppose first that the total cost functions of the two firms are $C_A(q_A) = q_A$ and $C_B(q_B) = q_B$. Find the equilibrium price, quantities and profits.

(b) Suppose now that firm $B$ has a cost function $C_B(q_B) = q_B^2$. Find the equilibrium price, quantities and profits.

(c) Suppose now that firm $A$ owns a single plant, whose total cost function is $C_A(q_A) = q_A$ whereas firm $B$ has two plants, characterised by the total cost functions $C_{B_1}(q_{B_1}) = q_{B_1}$ and $C_{B_2}(q_{B_2}) = q_{B_2}^2$. Find the equilibrium quantities.

(d) Suppose now that the plant whose total cost function is $C_{B_2}(q_{B_2}) = q_{B_2}^2$ is owned by a third firm, independent of the other two. In other terms, there are three firms in the market: firm $A$ with a plant whose total cost function is $C_A(q_A) = q_A$, firm $B$ with the plant whose total cost functions is $C_B(q_B) = q_B$ and firm $C$ whose total cost function is $C_C(q_C) = q_C^2$. Find the equilibrium quantities and compare with case (c) above.

**Solution**

(a) This is a special case of a Cournot duopoly with symmetric and constant marginal costs (and no fixed costs). Marginal cost is equal to 1 for both firms, the slope and the intercept of the inverse demand function are 1 and 4.
Therefore, we know that:

$$q_1^* = q_2^* = \frac{a-c}{3 \times b} = \frac{4-1}{3 \times 1} = 1$$

Total quantity is

$$Q^* = q_1^* = q_2^* = 2$$

so that equilibrium price is

$$p^* = 4 - Q^* = 4 - 2 = 2$$

At the equilibrium, profits are of course equal for the two firms and they amount to $\pi_i^* = (p^* - 1) \times q_i^* = (2-1) \times 1 = 1$ for $i = A, B$.

(b) This is a case of Cournot duopoly with asymmetric costs (and no fixed costs) where one firm, $A$, has constant marginal costs whereas firm $B$ has variable marginal costs. To find the equilibrium quantity we have to find the intersection of the reaction functions.
The reaction function of firm $A$, the one with constant marginal cost, is the standard one:

$$q_A^*(q_B) = \frac{a-c}{2b} - \frac{q_B}{2} = \frac{3}{2} - \frac{q_B}{2} \tag{9}$$

The reaction function of firm $B$, the one with non constant marginal costs, must be found by deriving the profit function with respect to $q_B$ and setting it equal to 0:

$$\frac{\partial \pi_B}{q_B} = (4 - q_A - q_B) - q_B - 2q_B = 4 - q_A - 4q_B = 0$$

from which

$$q_B^*(q_A) = 1 - \frac{q_A}{4} \tag{10}$$

Solving the system of the two reaction functions we get:
$q_A^* = \frac{8}{7}$
$q_B^* = \frac{5}{7}$
$Q^* = q_1^* = q_2^* = \frac{13}{7}$
$p^* = 4 - Q^* = 4 - \frac{13}{7} = \frac{15}{7}$ $\pi_A^* = \frac{64}{49}$
$\pi_B^* = \frac{50}{49}$

(c) With firm $A$ having a single plant and firm $B$ having two plants, the profit of the two firms become:
$\pi_A = q_A \times (4 - q_A - q_{B_1} - q_{B_2}) - q_A$
$\pi_B = (q_{B_1} + q_{B_2}) \times (4 - q_A - q_{B_1} - q_{B_2}) - q_{B_1} - q_{B_2}^2$

The FOC are:

$$\frac{\partial \pi_A}{q_A} = 0 \Leftrightarrow (4 - q_A - q_{B_1} - q_{B_2}) - q_A - 1 = 0 \Leftrightarrow q_A = \frac{3 - q_{B_1} - q_{B_2}}{2}$$

$$\frac{\partial \pi_B}{q_{B_1}} = 0 \Leftrightarrow (4 - q_A - q_{B_1} - q_{B_2}) - q_{B_1} - q_{B_2} - 1 = 0 \Leftrightarrow q_{B_1} = \frac{3 - q_A - 2q_{B_2}}{2}$$

$$\frac{\partial \pi_B}{q_{B_2}} = 0 \Leftrightarrow (4 - q_A - q_{B_1} - q_{B_2}) - q_{B_1} - q_{B_2} - 2q_{B_2} = 0 \Leftrightarrow q_{B_2} = \frac{4 - q_A - 2q_{B_1}}{4}$$

Rearranging the FOC and solving the system we get: $q_A^* = 1$
$q_{B_1}^* = q_{B_2}^* = \frac{1}{2}$
$Q^* = q_1^* + q_{B_1}^* + q_{B_2}^* = 2$
$p^* = 4 - Q^* = 2$
$\pi_A^* = 1$
$\pi_B^* = 2 - \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{5}{4}$

(d) With there firms, the profit of the three firms become:
$\pi_A = q_A \times (4 - q_A - q_B - q_C) - q_A$
$\pi_B = q_B \times (4 - q_A - q_B - q_C) - q_B$
$\pi_C = q_C \times (4 - q_A - q_B - q_C) - q_C$

The FOC are:

$$\frac{\partial \pi_A}{q_A} = 0 \Leftrightarrow (4 - q_A - q_B - q_C) - q_A - 1 = 0 \Leftrightarrow q_A = \frac{3 - q_B - q_C}{2}$$

$$\frac{\partial \pi_B}{q_B} = 0 \Leftrightarrow (4 - q_A - q_B - q_C) - q_B - 1 = 0 \Leftrightarrow q_B = \frac{3 - q_A - q_C}{2}$$

$$\frac{\partial \pi_C}{q_C} = 0 \Leftrightarrow (4 - q_A - q_B - q_C) - q_C - 2q_C = 0 \Leftrightarrow q_C = \frac{4 - q_A - q_B}{4}$$

Rearranging the FOC and solving the system we get:
$q_A^* = q_B^* = \frac{4}{5}$
$q_C^* = \frac{3}{5}$
$Q^* = q_1^* + q_B^* + q_B^* = \frac{11}{5}$
$p^* = 4 - Q^* = \frac{9}{5}$
$\pi_A^* = \pi_B^* = \left(\frac{11}{5} - 1\right) \times \frac{4}{5} = \frac{24}{25} < 1$
$\pi_C^* = \frac{11}{5} \times \frac{3}{5} - \left(\frac{3}{5}\right)^2 = \frac{24}{25} < 1$
What is interesting in this equilibrium is that firm $C$ produces more than firm $B$ did with the plant whose total cost was increasing. This is because in case of two plants the firm can swith to the other one wherease the firm $C$ cannot.

**Exercise 3 (8 points)** A firm serves two kind of consumers, whose demands are $Q_1 = 60 - 0.25P_1$ and $Q_2 = 100 - 0.50P_2$ and $Q_1$ and $Q_2$ represent the number of units of the good sold per year (in thousands) and $P$ is price. The cost function is $C = 1,000 + 40Q$ where $Q = Q_1 + Q_2$.

(a) Suppose that the firm wants to price discriminate among the two groups and that the firm can distinguish the two groups of consumers. What are the profit-maximizing prices and quantities for the two markets?

(b) Suppose now that the firm does not want to price discriminate, i.e. it wants to set a single price for the two markets. What price should it charge, and what quantities will it sell in the two markets?

(c) In which of the above situations, (a) or (b), is the firm better off? In terms of consumer surplus, which situation do consumers in the two groups prefer? What about total surplus?

(d) Repeat step (a) considering the following cost function $C = 1,000 + 4Q^2$;

(e) Suppose that the firm, although it knows the two demand functions, cannot distinguish the two groups of consumer and decides to apply a unique (i.e. for both type of users) two part-tariff $T + pq$, where $p$ is the per-unit price of the good and $T$ the fixed fee. Do you expect the profit of the firm to be higher or lower with respect to the profit in cases (a) and (b) above (in doing so, consider the cost function $C = 1,000 + 40Q$)?

**Solution**

(a) The firm should pick quantities in each market so that the marginal revenues are equal to one another and equal to marginal cost. To determine marginal revenues in each market, first solve for price as a function of quantity:

$$P_1 = 240 - 4Q_1$$

and

$$P_2 = 200 - 2Q_2$$

Since marginal revenue curves have twice the slope of their demand curves, the marginal revenue curves for the respective markets are

$$MR_1 = 240 - 8Q_1$$

and

$$MR_2 = 200 - 4Q_2$$

Setting each marginal revenue equal to marginal cost, which is \$40, we can determine the profit-maximizing quantity in each submarket: $40 = 240 - 8Q_1$, or

$$Q_1 = 25 \text{ thousand}$$

and $40 = 200 - 4Q_2$, or

$$Q_2 = 40 \text{ thousand}$$

so that total quantity is $Q_T = 40 + 25 = 65$. To determine the price in each submarket, substitute the profit-maximizing quantity into the respective demand equation

$$P_1 = 240 - 4 \times 25 = \$140$$

and

$$P_2 = 200 - 2 \times 40 = \$120$$

(b) Firm's combined demand function is the horizontal summation of the two demand functions. Above a price of \$200 (the vertical intercept of the second demand function), the total demand is just the first demand function, whereas below a price of \$200, we add the two demands $Q_T = 60 - 0.25P + 100 - 0.50P$, or

$$Q_T = 160 - 0.75P$$

Solving for price gives the inverse demand function $P = 213.33 - 1.333Q$ so that

$$MR = 213.33 - 2.667Q$$

Setting marginal revenue equal to marginal cost $213.33 - 2.667Q = 40$, or

$$Q = 65 \text{ thousand}$$

Substitute $Q = 65$ into the inverse demand equation to determine price $P = 213.33 - 1.333 \times 65$, or

$$P = \$126.67$$

Although a price of \$126.67 is charged in both markets, different quantities are purchased in each market: $Q_1 = 60 - 0.25 \times 126.67 = 28.3$ thousand and $Q_2 = 100 - 0.50 \times 126.67 = 36.7$ thousand. Together, 65 thousand units (as before) are purchased at a price of \$126.67 each.

(c) Of course, the firm is better off in the price discrimination case. Under price discrimination, profit is equal to $\pi = P_1Q_1 + P_2Q_2 - [1000 + 40 \times (Q_1 + Q_2)]$, or

$$\pi = \$140 \times 25 + \$120 \times 40 - [1000 + 40 \times (25 + 40)] = \$4,700 \text{ thousand}$$

Under the market conditions in (b), profit is: $\pi = PQ_T - [1000 + 40Q_T]$, or

$$\pi = \$126.67 \times 65 - [1000 + 40 \times 65] = \$4,633.33 \text{ thousand}$$

Therefore, the firm is better off when the two markets are separated.
Under the market conditions in (a), the consumer surpluses in the two groups are $CS_1 = 0.5 \times 25 \times)(240 - 140) = \$1250$ thousand, and $CS_2 = 0.5 \times 40 \times (200 - 120) = \$1600$ thousand. Under the market conditions in b, the respective consumer surpluses are: $CS_1 = 0.5 \times 28.3 \times (240 - 126.67) = \$1603.67$ thousand, and $CS_2 = 0.5 \times 36.7 \times (200 - 126.67) = \$1345.67$ thousand. The first group of consumers (low elasticity of demand) prefer (b) because their price is \$126.67 instead of \$140, giving them a higher consumer surplus. The second group of consumers (low elasticity of demand) prefer (a) because their price is \$120 instead of \$126.67, and their consumer surplus is greater in (a).
In term of total welfare, the situation in (b) is better: $W_a = 7550$ thousand $< W_b = 7582.67$ thousand. This is hardly surprising as the total quantity produced has not increased.

(d) If $C = 1000 + 4Q^2$, $MC = 8Q$. To find the total quantity the firm has to optimally produce, set the usual $MR = MC$ cost condition, where $MR$ is the marginal revenue computed in point (b), i.e. using the aggregate demand function. Therefore: $MR = 213.33 - 2.667Q = 8Q = MC$. This leads to $Q = 20$ so that MC at the optimum is 160. By using this value of the marginal cost, we can find how the firm shares the total output for the two groups of consumers. In fact, $MR_1 = MC$ and $MR_2 = MC$. This leads to $MR_1 = 240 - 8Q_1 = 160$ and $MR_2 = 200 - 4Q_2 = 160$. Solving we get $Q_1^* = Q_2^* = 10$ and the two prices are $P_1 = 200$ and $P_2 = 180$.
Alternatively, one can set a system of first order conditions (either in prices or in quantities) which take into account the quadratic nature of the cost function and solve the system.

(e) If the firm operates a two-part tariff, its profits are of course higher than in case (b), i.e.

no price discrimination. As a general statement, there is no ranking in terms of monopolist's profits between two-part tariff and third-degree price discrimination. This depends on the exact demands of the two groups of consumers. Put differently, in the third degree price discrimination the firm has the advantage of distinguishing the two groups whereas in the two part tariff the tariff allows the monopolist to take the all the surplus of the low demand group of consumers.
In this case, noting that the low demand group of consumer is the first group, an optimal tariff would be $T = 3,612.5$ thousand with a price of 70. The profits would be 9,450 thousand, higher than in the third degree price discrimination case.

**Exercise 4 (6 points)** A researcher, by using a sample of 1,254 transactions of residential houses in the Torino area, wants to estimate how the price of the houses are affected by a set of regressors. To this end, he estimates several hedonic models. The first one regresses the price of the house in thousands euro per square meter, *price*, on the area of the house in square meters, *area*:

$$price_i = \alpha_0 + \alpha_1\ area_i + u_i \tag{11}$$

The second one uses as regressor the total number of rooms in the house, *rooms*:

$$price_i = \gamma_0 + \gamma_1\ rooms_i + u_i \tag{12}$$

The third one inserts both previous regressors:

$$price_i = \delta_0 + \delta_1\ area_i + \delta_2\ rooms_i + u_i \tag{13}$$

The last one also inserts the area of the house in square meters squared, $area^2$:

$$price_i = \beta_0 + \beta_1\ area_i + \beta_2\ area_i^2 + \beta_3\ rooms_i + u_i \tag{14}$$

The OLS estimates are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{price}_i = \underset{(0.052)}{1.107} + \underset{(0.0005)}{0.0078}\ area_i \qquad R^2 = 0.153 \tag{15}$$

$$\widehat{price}_i = \underset{(0.074)}{1.083} + \underset{(0.022)}{0.233}\ rooms_i \qquad R^2 = 0.080 \tag{16}$$

$$\widehat{price}_i = \underset{(0.072)}{1.190} + \underset{(0.0008)}{0.009}\ area_i - \underset{(0.035)}{0.0592}\ rooms_i \qquad R^2 = 0.155 \tag{17}$$

$$\widehat{price}_i = \underset{(0.096)}{1.084} + \underset{(0.003)}{0.013}\ area_i - \underset{(0.000006)}{0.00001}\ area_i^2 - \underset{(0.045)}{0.105}\ rooms_i \qquad R^2 = 0.157 \tag{18}$$

(1) Carefully interpret the coefficients in (11);

(2) Using the results in (15), test at the $\alpha = 5\%$ level that the coefficient of *area* is equal to 0.01 against a bilateral alternative, assuming the 4 OLS assumptions hold;

(3) Explain, both statistically and economically, the surprising (at first glance) result that the coefficient of *rooms* is positive in (16) and negative in (17);

(4) How would you test that the area of the house has no impact on price in model (14)?

(5) How would you extend model (14) in order to take into account that the effect of the area depends on the number of rooms? How would you then test that the number of rooms has no effect on price?

**Solution**

(1) The intercept $\alpha_0$ is the expected price when all regressors are equal to 0. As there is no flat with a 0 area, it has just a geometric, and not an economic, interpretation. $\alpha_1$ is the expected change in price when area increases of 1 unit.

(2) By performing an individual $t$ test of the hypothesis that the coefficient $\alpha$ is equal to 0.01 against a bilateral alternative, we get $t = \frac{0.0078-0.01}{0.0005} \approx -4.4$. As this value is outside the acceptance interval $(-1.96, +1.96)$ we reject the null hypothesis.

(3) The apparently surprising result can be explained, statistically, on the ground that in the regression without *area* an omitted variable bias is at work. Indeed, the area and the number of rooms in an apartment are positively correlated and in (17) the coefficient of *area* is positive. In turn, there is a positive bias in (16) which pushes up the estimated coefficient of *rooms*. As for the economic explanation, results in (17) tell us that holding the number of room fixed individuals prefer to have more area whereas holding the area fixed individuals prefer to have less, and hence larger, rooms.

(4) To test that the area of the house has no impact on price in model (14), a joint test using the F-statistics should be employed.

(5) To extend model (14) in order to take into account that the effect of the area depends on the number of rooms, the interactions $area \times rooms$ and $area^2 \times rooms$ should be included. This is because if there is a polynomial, the interaction must be done with all the terms of the polynomial (otherwise, you impose an arbitrary restriction in the parameters of your model). To test that the number of rooms has no effect on price, then a joint test testing that the coefficients of $rooms$, $area \times rooms$ and $area^2 \times rooms$ are equal to 0 should be employed.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2017-18**
**Exam July, $12^{th}$ 2018**

**Instructions**: answer to all questions in 1 hour and 45 minutes

**Exercise 1 (8 points)** Consider an industry where the technology is such that the total cost for producing the good is equal to $TC = cq^2 + A$, where $c$ and $A$ are positive constants and $q$ is the quantity produced by each firm in the market.

(a) Find the values of $q$ for which the market can be a natural monopoly.

(b) Provide an economic explanation why the size of the interval of $q$ for which we can have a natural monopoly depends negatively on the constant $c$ and positively on the constant $A$.

(c) Find the values of $q$ for which the market can be a natural duopoly.

(d) What is the optimal number of firms in the market for a value of $q = 7 \times \sqrt{\frac{A}{c}}$ ? Motivate your answer.

**Solution**

(1) To assess the range of $q$ such that the market is a natural monopoly we have to check the condition of cost sub-addititivy:

$$cq^2 + A < 2 \times \left[ c\left(\frac{q}{2}\right)^2 + A \right] \tag{1}$$

Solving this inequality we get:

$$q < \sqrt{\frac{2 \times A}{c}} \tag{2}$$

(2) The optimal number of firms in the market crucially depends on two opposing factors. One is the existence of fixed costs, which pushes towards a single firm being in the market in order to avoid duplication of fixed costs. The other is increasing marginal costs, which calls for the split of the production among the highest possible number of firms. The higher the parameters $A$ and $c$ the higher will be the relative importance of each component, so that the interval in which a natural monopoly exists depends positively on $A$ and negatively on $c$.

(3) By setting the following inequality:

$$2 \times \left[ c\left(\frac{q}{2}\right)^2 + A \right] < 3 \times \left[ c\left(\frac{q}{3}\right)^2 + A \right] \tag{3}$$

we get:

$$q < \sqrt{\frac{6 \times A}{c}} \tag{4}$$

so that the industry is a natural duopoly for the interval

$$\sqrt{\frac{2 \times A}{c}} < q < \sqrt{\frac{6 \times A}{c}} \tag{5}$$

(4) The total cost to produce the quantity $q$, as a function also of the number of firms in the market $n$, is equal to:

$$TC(n,q) = n \times \left[ c \left( \frac{q}{n} \right)^2 + A \right] \tag{6}$$

Minimizing this expression with respect to $n$ we find the number of firms which minimizes the cost of producing the quantity $q$, given $c$ and $A$. This solution is:

$$n = \sqrt{\frac{q^2 \times c}{A}} \tag{7}$$

Substituting the quantity $q = 7 \times \sqrt{\frac{A}{c}}$ in the previous expression we get $n = \sqrt{49} = 7$.

**Exercise 2 (8 points)**

A market is characterised by an inverse demand curve $p = 2 - Q$ where $Q$ is total quantity.

(a) Suppose first that two firms, $A$ and $B$, are competing *à* la Cournot and have a total cost function $C_i(q_i) = q_i \;\; i{=}A,B$. Find the Cournot equilibrium (price, quantities, profits), consumer surplus and compute the HHI and CR4 concentration indexes.

(b) Suppose now the firms in the market are three. The new firm, $C$, is identical to the previous three i.e. $C_i(q_i) = q_i \;\; i{=}A,B,C$. Find the Cournot equilibrium (price, quantities, profits), consumer surplus and compute the HHI and CR4 concentration indexes. Compare these values with those computed at the previous point (a).

(c) Suppose now that the three firms in the market, instead of competing *à* la Cournot, collude. Find the optimal collusive equilibrium (price, quantities, profits), consumer surplus and compute the HHI and CR4 concentration indexes. Compare these values with those computed at the previous point (b).

(d) Compute the discount factor that sustains the collusive equilibrium found in point (c) above as a SPNE solution to the supergame played an infinite number of times.

**Solution**

(a) This is a special case of a Cournot duopoly with symmetric and constant marginal costs (and no fixed costs). Marginal cost is equal to 1 for both firms, the slope and the intercept of the inverse demand function are 1 and 2.
Therefore, we know that:

$$q_1^* = q_2^* = \frac{a - c}{(N+1) \times b} = \frac{2-1}{3 \times 1} = \frac{1}{3}$$

Total quantity is

$$Q^* = q_1^* + q_2^* = \frac{2}{3}$$

so that equilibrium price is

$$p^* = 2 - Q^* = 2 - \frac{2}{3} = \frac{4}{3}$$

At the equilibrium, profits are of course equal for the two firms and they amount to $\pi_i^* = (p^* - 1) \times q_i^* = \left(\frac{4}{3} - 1\right) \times \frac{1}{3} = \frac{1}{9}$ for $i = 1, 2$.
Consumer's surplus is equal to:

$$CS = \frac{1}{2} \times (a - p^*)\, Q^* = \frac{1}{2} \times \left(2 - \frac{4}{3}\right) \frac{2}{3} = \frac{1}{2} \times \frac{2}{3} \times \frac{2}{3} = \frac{2}{9}$$

Market shares are $ms_i = \frac{q_i^*}{Q^*} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ for $i = 1, 2$
Therefore, concentration indexes are

$$HHI = \sum_1^2 (ms_i)^2 = 2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

and

$$CR_4 = \sum_1^2 (ms_i) = 2 \times \left(\frac{1}{2}\right) = 1$$

(b) This is a special case of a Cournot triopoly with symmetric and constant marginal costs (and no fixed costs). Marginal cost is equal to 1 for both firms, the slope and the intercept of the inverse demand function are 1 and 2.
Therefore, we know that:

$$q_1^* = q_2^* = q_3^* = \frac{a - c}{(N+1) \times b} = \frac{2-1}{4 \times 1} = \frac{1}{4}$$

Total quantity is

$$Q^* = q_1^* + q_2^* + q_3^* = \frac{3}{4}$$

so that equilibrium price is

$$p^* = 2 - Q^* = 2 - \frac{3}{4} = \frac{5}{4}$$

At the equilibrium, profits are of course equal for the two firms and they amount to $\pi_i^* = (p^* - 1) \times q_i^* = \left(\frac{5}{4} - 1\right) \times \frac{1}{4} = \frac{1}{16}$ for $i = 1, 2, 3$.
Consumer's surplus is equal to:

$$CS = \frac{1}{2} \times (a - p^*)\, Q^* = \frac{1}{2} \times \left(2 - \frac{5}{4}\right) \frac{3}{4} = \frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{32}$$

Market shares are $ms_i = \frac{q_i^*}{Q^*} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$ for $i = 1, 2, 3$.
Therefore, concentration indexes are

$$HHI = \sum_1^3 (ms_i)^2 = 3 \times \left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

and

$$CR_4 = \sum_{1}^{3} (ms_i) = 3 \times \left(\frac{1}{3}\right) = 1$$

As we could expect, increasing the number of firms leads to a decrease in individual quantities, an increase in total quantity and hence a decrease in price, a decrease in individual profits and an increase in total consumer surplus. The market is clearly less concentrated but only the $HHI$ decreases whereas $CR_4$ does not. This highlights how $CR_4$ is a worse index than $HHI$.

(c) If the three firms collude, they will operate as a single monopolist. Given that marginal cost is constant and equal for the three firms, we can find the monopolitic quantity and then split evenly the total quantity among the three firms.
By setting the usual condition marginal cost equal to marginal revenue we get: $MR = MC \Leftrightarrow 2 - 2Q = 1$ so that $Q^* = \frac{1}{2}$ and $p^* = 2 - Q^* = 2 - \frac{2}{2} = \frac{3}{2}$. Individual quantities are $q_1^* = q_2^* = q_3^* = \frac{\frac{1}{2}}{3} = \frac{1}{6}$ for $i = 1, 2, 3$ and profits are $\pi_i^* = (p^* - 1) \times q_i^* = \left(\frac{3}{2} - 1\right) \times \frac{1}{6} = \frac{1}{12}$ for $i = 1, 2, 3$.
Consumer's surplus is equal to:

$$CS = \frac{1}{2} \times (a - p^*)\, Q^* = \frac{1}{2} \times \left(2 - \frac{3}{2}\right) \frac{1}{2} = \frac{1}{2} \times \frac{1}{1} \times \frac{1}{2} = \frac{1}{8}$$

Market shares are $ms_i = \frac{q_i^*}{Q^*} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$ for $i = 1, 2, 3$.
Therefore, concentration indexes are

$$HHI = \sum_{1}^{3} (ms_i)^2 = 3 \times \left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

and

$$CR_4 = \sum_{1}^{3} (ms_i) = 3 \times \left(\frac{1}{3}\right) = 1$$

As we could expect, when firm collude instead of competing *à* la Cournot, they decrease the individual quantities and the total quantity produced, thereby increasing price and individual profits. Total consumer's surplus decreases. The market is equally concentrated so that $HHI$ and $CR_4$ do not change. This highlights that $HHI$ is able to be a measure of profitability in the industry only in the Cournot setting but it is not able to capture the kind of competition in the industry.

(b) If only one firm deviates from the collusion (say, firm 1) whereas the other two do not, firm 1 will produce the quantity according to its reaction function. Therefore:

$$q_1^D = \frac{1 - q_2 - q_3}{2} = \frac{1 - \frac{1}{6} - \frac{1}{6}}{2} = \frac{1}{3}$$

Total quantity will be $Q^* = q_1^* + q_2^* + q_3^* = \frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{2}{3}$ so that equilibrium price is

$$p^* = 2 - Q^* = 2 - \frac{2}{3} = \frac{4}{3}$$

and the profit for firm 1 (the one which deviates) is $\pi_1^* = (p^* - 1) \times q_1^* = \left(\frac{4}{3} - 1\right) \times \frac{1}{3} = \frac{1}{9}$. We can now compute the lowest discount factor which sustains the collusive equilibrium:

$$\overline{\delta} = \frac{\pi^D - \pi^{collusion}}{\pi^D - \pi^C} = \frac{\frac{1}{9} - \frac{1}{12}}{\frac{1}{9} - \frac{1}{16}} = \frac{\frac{12-9}{108}}{\frac{16-9}{144}} = \frac{4}{7} = 0.571 \quad (8)$$

**Exercise 3 (8 points)** Consider a linear city whose lenght is 1 with two firms, $A$ and $B$. Consumers are uniformly distributed along the interval $x \in [0,1]$. Firm $A$ is located in point $x_A = \frac{1}{4}$ and firm $B$ in $x_B = \frac{3}{4}$. Every consumer buys one unit of the good. Marginal costs are constant and equal to $c$ for both firms: $TC_A = cq_A$ and $TC_B = cq_B$. Consumption of the good gives to consumers a utility equal to 1.5 for both goods.

(1) Suppose first that transportation costs are equal to the distances between consumers and firms, i.e. $\tau = 1$. Find firms' equilibrium prices and profits.

(2) Suppose now that transportation costs are equal to square of the distances between consumers and firms, i.e. transportation costs are $d^2$ where $d$ is the distance between consumers and each of the two firms. Find firms' equilibrium prices and profits and compare with the previous results.

(3) Modify point (2) supposing that firm $B$ is located in $x_B = \frac{1}{2}$. Find firms' equilibrium prices and profits and compare with the results obtained in point (2).

**Solution**

(1) This is a standard Hotelling price game with linear transportation costs and locations $x_A = \frac{1}{4}$ and $x_B = \frac{3}{4}$, $\tau = 1$ and constant and equal marginal cost of production for the two firms, $c$. The two goods provide to the consumers the same groos utility, 1.5.
The indifferent consumer, located in $x$, will be indifferent between purchasing the good from firm $A$ or purchasing the good of firm $B$. This leads to the following equality:

$$1.5 - p_A - 1\left(x - \frac{1}{4}\right) = 1.5 - p_B - 1\left(\frac{3}{4} - x\right)$$

The indifferent consumer will then be located in:

$$x = \frac{1 + p_B - p_A}{2}$$

so that the demands for firms $A$ and $B$ will be, respectively, $\frac{1+p_B-p_A}{2}$ and $1 - \frac{1+p_B-p_A}{2}$. This leads to the following profit functions:

$$\pi_A = (p_A - c)\left(\frac{1 + p_B - p_A}{2}\right)$$
$$\pi_B = (p_B - c)\left(1 - \frac{1 + p_B - p_A}{2}\right)$$

The FOCs are:

$$\frac{\partial \pi_A}{\partial p_A} = \frac{1}{2} + \frac{p_B - p_A}{2} - \frac{p_A - c}{\frac{1}{2}} = 0 \Rightarrow p_A = \frac{1 + p_B + c}{2}$$
$$\frac{\partial \pi_B}{\partial p_B} = \frac{1}{2} + \frac{p_A - p_B}{2} - \frac{p_B - c}{\frac{1}{2}} = 0 \Rightarrow p_B = \frac{1 + p_A + c}{2}$$

Solving the system of two FOCs we obtain:

$$p_A = p_B = 1 + c > c$$
$$x = \frac{1}{2}$$
$$\pi_A = \frac{1}{2} > 0$$
$$\pi_B = \frac{1}{2} > 0$$

(2) The only difference now is that transportation costs are quadratic, i.e. transportation costs are a quadratic function of the distance between consumers and firms. The indifferent consumer, located in $x$, will be indifferent between purchasing the good from firm $A$ or purchasing the good from firm $B$. This leads now to the following equality:

$$1.5 - p_A - 1\left(x - \frac{1}{4}\right)^2 = 1.5 - p_B - 1\left(\frac{3}{4} - x\right)^2$$

The indifferent consumer will then be located in:

$$x = \frac{1}{2} + p_B - p_A$$

so that the demands for firms $A$ and $B$ will be, respectively, $\frac{1}{2} + p_B - p_A$ and $1 - \left(\frac{1}{2} + p_B - p_A\right)$.
This leads to the following profit functions:

$$\pi_A = (p_A - c)\left(\frac{1}{2} + p_B - p_A\right)$$
$$\pi_B = (p_B - c)\left(1 - \left(\frac{1}{2} + p_B - p_A\right)\right)$$

The FOCs are:

$$\frac{\partial \pi_A}{\partial p_A} = \left(\frac{1}{2} + p_B - p_A\right) - (p_A - c) = 0 \Rightarrow p_A = \frac{1}{4} + \frac{p_B + c}{2}$$
$$\frac{\partial \pi_B}{\partial p_B} = \left(\frac{1}{2} + p_A - p_B\right) - (p_B - c) = 0 \Rightarrow p_B = \frac{1}{4} + \frac{p_A + c}{2}$$

Solving the system of two FOCs we obtain:

$$p_A = p_B = \frac{1}{2} + c > c$$
$$x = \frac{1}{2}$$
$$\pi_A = \frac{1}{4} > 0$$
$$\pi_B = \frac{1}{4} > 0$$

The two firms have clearly the same price and profit (the model is symmetric) but now they have lower price and profit than before. This is because quadratic transportation costs are lower than linear ones (the distance is less than 1) so that product differentiation is lower (transportation costs operate as product differentiation between the two firms) and product differetiation is directly proportional to profits.

(3) Now transportation costs are quadratic but firm $B$ is better located, i.e. it occupies the position $x_B = \frac{1}{2}$. The indifferent consumer, located in $x$, will be indifferent between purchasing the good from firm $A$ or purchasing the good from firm $B$. This leads now to the following equality:

$$1.5 - p_A - 1\left(x - \frac{1}{4}\right)^2 = 1.5 - p_B - 1\left(\frac{1}{2} - x\right)^2$$

The indifferent consumer will then be located in:

$$x = \frac{3}{8} + 2p_B - 2p_A$$

so that the demands for firms $A$ and $B$ will be, respectively, $\frac{3}{8} + 2p_B - 2p_A$ and $1 - \left(\frac{3}{8} + 2p_B - 2p_A\right)$.
This leads to the following profit functions:

$$\pi_A = (p_A - c)\left(\frac{3}{8} + 2p_B - 2p_A\right)$$
$$\pi_B = (p_B - c)\left(1 - \left(\frac{3}{8} + 2p_B - 2p_A\right)\right)$$

The FOCs are:

$$\frac{\partial \pi_A}{\partial p_A} = \frac{3}{8} + 2p_B - 4p_A + 2c = 0 \Rightarrow p_A = \frac{2p_B + 2c + \frac{3}{8}}{4}$$
$$\frac{\partial \pi_B}{\partial p_B} = \frac{5}{8} + 2p_A - 4p_B + 2c = 0 \Rightarrow p_B = \frac{2p_A + 2c + \frac{5}{8}}{4}$$

Solving the system of two FOCs we obtain:

$$p_A = \frac{11}{48} + c > c \tag{9}$$
$$p_B = \frac{13}{48} + c > c$$
$$x = \frac{11}{24}$$
$$\pi_A = 0.105 < \frac{1}{4}$$
$$\pi_B = 0.15 < \frac{1}{4}$$

Firm B now enjoys, as expected, higher profits than firm A. This is because it is better located being a sort of "monopolist" in the $\left[\frac{1}{2}, 1\right]$ interval. However, profits are lower than before (even for firm B) as with quadratic transportation cost the *competition effect* dominates the *market size effect* so that it is better to be located the further, and not the closer, with respect to the competitor.

**Exercise 4 (6 points)** A researcher, by using a sample of 3,294 USA individuals observed in 1987 wants to estimate the gender wage gap between males and females. To this end, he estimates several models. The first one regresses the log of hourly wage (in 1980 US dollars), *logw*, on a gender dummy, *male*, which takes value of 1 if the worker is male and 0 if female:

$$logw_i = \beta_0 + \beta_1\, male_i + u_i \tag{10}$$

and the OLS estimates are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{logw}_i = \underset{(0.015)}{1.475} + \underset{(0.021)}{0.215}\, male_i \qquad R^2 = 0.030 \tag{11}$$

In the second model, the researcher includes an additional regressor, the working experience (in years) *exper*. The OLS estimates are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{logw}_i = \underset{(0.039)}{1.326} + \underset{(0.022)}{0.203}\, male_i + \underset{(0.005)}{0.019}\, exper_i \qquad R^2 = 0.035 \tag{12}$$

In a third model, the researcher includes a quadratic term for experience, *exper*2. The OLS estimates are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{logw}_i = \underset{(0.087)}{0.767} + \underset{(0.021)}{0.209}\, male_i + \underset{(0.021)}{0.166}\, exper_i - \underset{(0.001)}{0.009}\, exper2_i \qquad R^2 = 0.05 \tag{13}$$

Finally, the researcher includes the interactions between the gender dummy and the linear and quandratic terms for experience, *exper_male* and *exper2_male*. The OLS estimates are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{logw}_i = \underset{(0.131)}{0.548} + \underset{(0.179)}{0.633}\, male_i + \underset{(0.034)}{0.220}\, exper_i - \underset{(0.002)}{0.012} exper2$$
$$- \underset{(0.044)}{0.099}\, exper\_male_i + \underset{(0.003)}{0.005}\, exper2\_male_i \qquad R^2 = 0.051 \tag{14}$$

(1) Carefully interpret the coefficients in (10);

(2) Using the results in (11), test at the $\alpha = 5\%$ level that the coefficient of *male* is equal to 0.1 first against a bilateral alternative and then against the unilateral alternative $\beta_1 > 0.1$, assuming the 4 OLS assumptions hold;

(3) Comparing the results in (11) and (12), female have more or less years of experience of males? Explain.

(4) How would you test in (14) that the effect of experience is the same for males and females? And that there is no gender wage gap?

(5) Suppose a female and a male worker with 15 years of experience. Using the results in (14), which one has the highest incentive, in terms of additional salary, to remain at work an additional year?

**Solution**

(1) The intercept $\beta_0$ is the expected log wage when all regressors are equal to 0, i.e. for a female worker (when the dummy male is equal to 0). Therefore, it has not only a geometric but also an economic interpretation. $\beta_0 + \beta_1$ is the expected log wage when the dummy male takes a value of 1, i.e. for a male worker. Therefore, $\beta_1$ is the expected difference in log wage between a male and a female worker. As in any log-lin models, $\beta_1 \times 100$ is also the expected percentage difference in wage when the dummy increases of 1 unit, i.e. between a male and a female worker. Given the dummy nature of the regressor, a better approximation for the expected percentage difference in wage between a male and a female worker is $(exp(\beta_1) - 1) \times 100$.

(2) By performing an individual $t$ test of the null hypothesis $\beta_1 = 0.1$ against the bilateral alternative $\beta_1 \neq 0.1$, we get $t = \frac{0.215-0.1}{0.021} \approx 5.5$. As this value is outside the acceptance interval $(-1.96, +1.96)$ we reject the null hypothesis. As for the test of the hypothesis $\beta_1 = 0.1$ against the unilateral alternative $\beta_1 > 0.1$, the value of the statistics is as before 5.5 but the rejection region contains only the value of the statistics that lend support to the alternative. So the rejection region is $[1.64; +\infty]$. As the value is inside the rejection region, we reject the null hypothesis also versus the unilateral alternative.

(3) Comparing the results in (11) and (12), we see that in the first one there is an omitted varible bias: $a$) inclusion of experience in (12) decreases the coefficient of males and $b$) experience exerts a positive effect on log wage. Therefore, there is a positive bias, which suggests a positive correlation between male and experience. This means that, on average, when the variable male is high (i.e. equal to 1, a male worker) also experience is high and when the variable male is low (i.e. equal to 0, a female worker) also experience is low. In other terms, we expect males to have more experience than females. This is also what we expect as female workers do stop working for temporary periods due to family reasons. As a matter of fact, the sample average of experience is 7.73 years for female and 8.33 years for male workers.

(4) To test in (14) that the effect of experience is the same for males and females, we have to jointly test that all the coefficients of the interactions between male and experience (representing the differential effect of experience for males with respect to females) are equal to zero. The null hypothesis is then $\beta_4 = \beta_5 = 0$ vs a bilateral alternative. The

F-statistics should be employed to perform the test.
To test for that there is no gender wage gap, a test of the null hypothesis $\beta_1 = 0$ vs a bilateral alternative should be performed. In fact, gender wage gap is the difference in wage between males and females taking into account observable characteristics (in this case, experience). A null hypothesis like $\beta_1 = \beta_4 = \beta_5 = 0$ is more general and represents a test that gender has no effect whatsoever on salary, neither directly (the effect of the dummy *male*) nor indirectly (the different coefficients of experience for males with respect to females).

(5) To evaluate, by using the results in (14), whether a female worker with 15 years of experience has the highest incentive, in terms of additional salary, to remain at work an additional year, we have to compute the estimated log wage for both categories of workers at two levels of experience, 15 and 16 years.
The incentive of a male worker is:

$$\begin{aligned}&\left(\widehat{logw_i}|male, exper = 16\right) - \left(\widehat{logw_i}|male, exper = 15\right) = \\ &\left(0.548 + 0.633 \times 1 + (0.220 - 0.099) \times 16 - (0.012 - 0.005) \times 16^2\right) - \\ &\left(0.548 + 0.633 \times 1 + (0.220 - 0.099) \times 15 - (0.012 - 0.005) \times 15^2\right) = 1.865 - 1.95\end{aligned} \tag{15}$$

whereas for a female worker it is:

$$\begin{aligned}&\left(\widehat{logw_i}|female, exper = 16\right) - \left(\widehat{logw_i}|female, exper = 15\right) = \\ &\left(0.548 + 0.220 \times 16 - 0.012 \times 16^2\right) - \\ &\left(0.548 + 0.220 \times 15 - 0.012 \times 15^2\right) = 1.45 - 1.60\end{aligned} \tag{16}$$

As we can see, both changes in log wages are negative (in terms of wages, the difference is $-0.57\$$ for males and $-0.70\$$ for females) due to the quadratic relationship between experience and log wage (and hence on wage) with a negative coefficient for $exper^2$ (this means not only that the effect of experience on log wage (and hence on wage) is decreasing but that over time it becomes negative). Given that the absolute value of the decrease in log wage (and hence on wage) is lower for male workers, this category is the most incentivated one to remain at work.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2017-18**
**Exam September, $17^{th}$ 2018**

**Instructions**: answer to all questions in 1 hour and 45 minutes

**Exercise 1 (8 points)** Consider a duopolistic market in which two firms (A and B) compete in quantity. The inverse demand curve is $p = \alpha - Q$, with $Q = q_A + q_B$. The cost for both firms are linear in quantities: total cost functions are $TC_i(q_i) = c_i q_i$.

(a) Assume first $\alpha > c_B = c_A$. Find the equilibrium quantities, price and profits;
(b) Assume now $\alpha > c_B > c_A$. Find the equilibrium quantities, price and profits;
(c) Assuming the cost structure in point (b), find the condition on marginal costs for which the least efficient firm (firm $B$) is not obliged to exit the market and provide the economic intuition;
(d) Provide the economic intuition about the role played by $\alpha$ in the survival of the least efficient firm (firm $B$).

**Solution**

(1) This is a standard Cournot model with linear and symmetric cost functions. The optimal quantities are in this case $q_i^* = \frac{\alpha - c}{3}$ so that $p^* = \frac{\alpha + 2c}{3} > c$ and $\pi_i^* = \left(\frac{\alpha - c}{3}\right)^2$, $i = A, B$.
(2) This is a standard Cournot model with linear and asymmetric cost functions. The optimal quantities are in this case $q_i^* = \frac{\alpha - 2c_i + c_j}{3}$ so that $p^* = \frac{\alpha + c_i + c_j}{3} > c$ and $\pi_i^* = \left(\frac{\alpha - 2c_i + c_j}{3}\right)^2$, $i = A, B$.
(3) The least efficient firm, firm $B$ will not exit the market insofar the optimal quantity is strictly positive, i.e. $q_B = \frac{\alpha - 2c_B + c_A}{3} > 0 \Leftrightarrow c_B < \frac{\alpha + c_A}{2}$. The intuition is that the least efficient firm must have, to remain in the market, a cost disadvantage which is not too large with respect to the other firm. Needless to say, the same condition hold if we want to look at profit and not at quantity.
(4) In this kind of models, $\alpha$ is the size of the market (or the highest reservation wage for the good). The higher $\alpha$, the larger the market and the room for the least efficient firm. So an increase in $\alpha$ leads to an increase in the maximum cost disadvantage which still leads firm $B$ to remain in the market.

**Exercise 2 (8 points)** In a market two firms compete in prices. The demand curve for each firm are the following:

$$q_1 = 28 - p_1 + p_2$$

and

$$q_2 = 28 + p_1 - 2p_2$$

The two firms have the same total cost function $TC_i = 10$, $i = 1, 2$.

(1) Which kind of differentiation between the two product exists?

(2) Find the equilibrium prices, quantities and profits.

(3) Suppose the two firms merge, i.e. they become a single firm. Find the new equilibrium prices and compare with the previous ones;

(4) What would happen if firm 2 has a first mover advantage and hence chooses the price before the other? Comment the results and compare with those obtained in point (2)

**Solution**

(1) This is a non address model of horizontal product differentiation (representative consumer approach): consumers buy more than one units of the goods. Goods are substitutes: firm 1 has a demand where the own price effect (i.e. the coefficient of $p_1$) is equal than the cross price effect (i.e. the coefficient of $p_2$). Firm 2 has a demand where the own price effect (i.e. the coefficient of $p_2$) is larger than the cross price effect (i.e. the coefficient of $p_1$).

(2) We have:
$\pi_1 = p_1(28 - p_1 + p_2) - 10$ and $\pi_2 = p_2(28 + p_1 - 2p_2) - 10$.
Deriving with respect to $p_1$ and $p_2$, respectively, and setting equal to 0 the first derivatives we get $p_1 = 20$, $p_2 = 12$ so that $\pi_1 = 390$ and $\pi_2 = 278$. Total profits are then $\Pi = 668$.

(3) In case the two firms merge, the new firm behaves as a monopolist in the two markets where the two products are imperfectly substitutes. Therefore, the firm will maximise the following profit function:
$\Pi = \pi_1 + \pi_2 = p_1(28 - p_1 + p_2) + p_2(28 + p_1 - 2p_2) - 20$
Taking the first derivatives, setting them equal to 0 and solving the system we get $p_1 = 42$, $p_2 = 28$. Notice how prices increase in both markets leading to a higher profit equal to $980 > 668$.

(4) If firm 2 has a first mover advantage, it will use the reaction function of the other firm $p_1^* = \frac{28+p_2}{2}$ and insert it in its profit function, which becomes:

$$\pi_2 = p_2(28 - 2p_2 + (28 + p_2)/2) - 10 = p_2((84 - 3p_2)/2) - 10.$$

So that $p_2 = 14 > 12$ and firm 2 has a higher profit than in case (2): $284 > 278$. Notice that also firm 1 has higher price and hence profits than before: $p_1 = 21 > 20$ and $\pi_1 = 431 > 390$). The reason here is that reaction functions slope upward so that higher prices from firm 2 leads to higher prices also for the other firm. In so doing, they approach the monopolistic prices and hence they both enjoy higher profits.

**Exercise 3 (8 points)** Consider a firm with monopoly power that faces the *inverse* demand curve $p = 40 - 3Q + 2A^{1/2}$ and has the total cost function $TC = 2Q^2 + 8Q + A$ where $A$ is the level of advertising expenditures, and $p$ and $Q$ are price (in US dollars, \$) and output.

(1) Suppose at first that the monopolist decides not to advertise and uses only price (or quantity) as strategic variable. Find the equilibrium value of quantity, price and profit;

(2) Suppose now that the monopolist wants to maximise profits using adverstising alongside price (or quantity) as strategic variables. Find the values of $A$, $Q$, and $p$ that maximize the firm's profit;

(3) Verify the Dorfman-Steiner condition and provide its economic interpretation;

(4) How would you expect the optimal quantity, price and advertising found in point (2) above change if the demand curve becomes $p = 40 - 3Q + \alpha A^{1/2}$ with $\alpha > 2$? What about the Advertising/Sales ratio found in point (3)?

**Solution**

(1) In case the monopolist does not consider advertising, so he sets $A = 0$, then the optimization problem is the standard one of a monopolist which maximises profit only with respect to quantity, i.e. $MC = MR$. Therefore

$$4Q + 8 = 40 - 6Q$$

which leads to $Q^* = 3.2$. In turn, price is $p^* = 40 - 3Q^* + 2A^{1/2} = 40 - 3 \times 3.2 + 0 = 30.4$. In turn, profits are

$$30.4 \times 3.2 - 2 \times 3.2^2 - 8 \times 3.2 + 0 = 97.3 - 20.5 - 25.6 = 51.2\$$$

(2) Total Revenue (TR) is given by $p \times Q$, i.e.

$$TR(Q, A) = \left(40 - 3Q + 2A^{\frac{1}{2}}\right) \times Q = 40Q - 3Q^2 + 2A^{\frac{1}{2}}Q \tag{1}$$

Total cost function is $TC(Q, A) = 2Q^2 + 8Q + A$ so that the profit function is:

$$\pi(Q, A) = 40Q - 3Q^2 + 2A^{\frac{1}{2}}Q - 2Q^2 - 8Q - A = 32Q - 5Q^2 + 2A^{\frac{1}{2}}Q - A \tag{2}$$

The first order conditions, with respect to both $Q$ and $A$, are:

$$\begin{cases} \frac{\partial \pi}{\partial Q} = 32 - 10Q + 2A^{\frac{1}{2}} = 0 \\ \frac{\partial \pi}{\partial A} = QA^{-\frac{1}{2}} - 1 = 0 \end{cases}$$

From the second FOC we get $A^{\frac{1}{2}} = Q$. Inserting this expression in the first FOC we get

$$32 - 10 \times Q + 2 \times Q = 0 \Leftrightarrow 32 = 8Q$$

so that $Q^* = 4$ and, in turn, $A^* = 16$ and $p^*(Q^*, A^*) = 40 - 3 \times 4 + 2 \times 16^{1/2} = 40 - 12 + 2 \times 4 = 36\$$. Firm's profit are equal to

$$\pi(Q^*, A^*) = 32 \times 4 - 5 \times 4^2 + 2 \times 16^{\frac{1}{2}} \times 4 - 16 = 128 - 80 + 32 - 16 = 64\$$$

As $51.2 < 64$, it is verified that profits are higher if the firm takes into account the role of advertising.

(3) The Dorfman-Steiner condition states that, at the optimal choice of $Q$ and $A$

$$\frac{A}{pQ} = \frac{\varepsilon_{QA}}{|\varepsilon_{Qp}|}$$

where $\varepsilon_{QA}$ is the advertising elasticity of demand and $\varepsilon_{Qp}$ is the price elasticity of demand, inserted in the equation in absolute value in order to have a non-negative value.
The advertising-to-sales ratio $\frac{A}{pQ}$ is equal to:

$$\frac{A}{pQ} = \frac{16}{36 \times 4} = \frac{16}{144} = \frac{1}{9} = 0.111$$

Inverting the *inverse* demand, we get the *direct* demand:

$$Q = \frac{40}{3} - \frac{p}{3} + \frac{2}{3}A^{1/2}$$

from which

$$\varepsilon_{QA} = \frac{\partial Q}{\partial A} \times \frac{A}{Q} = \left[\frac{2}{3} \times \frac{1}{2}A^{-1/2}\right] \times \frac{A}{Q} = \frac{1}{3} \times \frac{A^{1/2}}{Q} = \frac{1}{3} \times \frac{4}{4} = \frac{1}{3}$$

and

$$\varepsilon_{Qp} = \frac{\partial Q}{\partial p} \times \frac{p}{Q} = -\frac{1}{3} \times \frac{36}{4} = -\frac{36}{12} = -3$$

Therefore, we have

$$\frac{A}{pQ} = \frac{1}{9} = \frac{\frac{1}{3}}{3} = \frac{1}{9} = \frac{\varepsilon_{QA}}{|\varepsilon_{Qp}|}$$

so the Dorfmain-Steiner condition is verified.

(4) When $\alpha > 2$, quantity, advertising, price, and Advertising/Sales ratio found in points (2) and (3) will increase. The rationale is that the parameter $\alpha$ represents the sensitivity of consumers to adevertising. When this sensitivity increases, it is of interest for the firm to increase the advertising expenditures. Prices will also increase but also quantity sold will increase (despite the higher price).
To verify this, you can repeat the steps above considering the additional parameter $\alpha$ and find the derivatives of equilibrium quantity, advertising, price, and Advertising/Sales ratio with respect to $\alpha$. Alternatively, you can notice that what you found in (1) were the optimal quantity and price when $\alpha = 0$ (advertising and Advertising/Sales ratio are optimally equal to 0 in this case). When $\alpha = 2$, $p^* = 36 > 30.4$, i.e. the price found in point (1) and $Q^* = 4 > 3.2$, i.e. the quantity found in point (1). So we have an (informal) proof that increasing $\alpha$ those optimal values will increase as well.

**Exercise 4 (6 points)** Suppose a researcher wants—by using cross-sectional data on 252 industries—to model Price Cost Margin ($PCM$, measured as Sales minus Total Variable Costs over Sales) as a function of: concentration in the market (measured by a concentration index, $HHI$, ranging from 0 to $10,000$) and the extent of entry barriers due to product differentiation, measured by *R&D Expenditures over Sales* ($R\&D$), and to the absolute cost advantage,

measured by the *Absolute Capital Requirement* ($ACR$) (the amount of money necessary to build an optimally sized plant, in million US \$). $ACR$ and $R\&D$ enter the model non-linearly as $ACR$ enters in log and interaction between $R\&D$ and $log(ACR)$ is included.
The model is therefore:

$$PCM_i = \beta_0 + \beta_1 R\&D_i + \beta_2 log(ACR)_i + \beta_3 HHI_i + \beta_4 \left(R\&D \times log(ACR)\right)_i + u_i \qquad (3)$$

and the OLS estimate are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{PCM}_i = \underset{(0.05)}{0.03} + \underset{(0.08)}{0.20} R\&D_i + \underset{(0.02)}{0.04} log(ACR)_i + \underset{(0.02)}{0.01} HHI_i$$
$$+ \underset{(0.001)}{0.0018} \left(R\&D \times log(ACR)\right)_i \quad R^2 = 0.47$$

(1) Carefully interpret the coefficients;

(2) Provide an economic explanation for the sign of the estimated coefficients;

(3) Test at the $\alpha = 5\%$ level, that the coefficients of $HHI$ is equal to $-0.1$ against a bilateral alternative, assuming the 4 OLS assumptions hold;

(4) Knowing that $\hat{\rho}_{t_1,t_2} = 0.83$, test at the $\alpha = 5\%$ level that the coefficients of $log(ACR)$ and $R\&D \times log(ACR)$ are jointly equal to 0 against a bilateral alternative, assuming the 4 OLS assumptions hold. (Hint: the critical value at the 5% level is 3.0);

(5) How would you expect the $R^2$ to change if we estimate model (1) by dropping the $R\&D \times log(ACR)$ variable? Provide the sign and an approximate magnitude of the change.

**Solution**

(1) The intercept ($\beta_0$) is the expected level of the dependent variable when the regressors are equal to 0, i.e. for an industry with $R\&D = 0$, $log(ACR) = 1$ and $HHI = 0$ (notice that when $R\&D = 0$ or $log(ACR) = 1$ also the interaction $R\&D \times log(ACR) = 0$). The intercept ($\beta_0$) has, in this case, only a geometrical interpretation. The coefficient $\beta_1$ has to be interpreted as the expected change in the dependent variable for a unit increase in $R\&D$ holding the other regressors constant and when $log(ACR) = 1$. The coefficient $\frac{\beta_2}{100}$ is the expected change in the dependent variable for a 1% increase in $ACR$ holding the other regressors constant and when $R\&D = 0$. The coefficients $\beta_3$ is the expected change in the dependent variable for a unit increase in $HHI$ holding the other regressors constant. The coefficient $\beta_4$ is the increment of the effect of $R\&D$ (resp. $log(ACR)$) when $log(ACR)$ (resp. $R\&D$) increases of one unit. In fact,

$$\frac{\partial PCM}{\partial R\&D} = \beta_1 + \beta_4 \times log(ACR) \qquad (4)$$

and

$$\frac{\partial PCM}{\partial log(ACR)} = \beta_2 + \beta_4 \times R\&D \qquad (5)$$

(2) Barriers to entry lead to higher market power, so the sign of both $\beta_1$ and $\beta_2$ is expected to be positive. Furthermore, we expect the effect of an increase in market concentration to

be positive i.e. $\beta_3$ to be positive. As for $\beta_4$, it is reasonable to expect that the two forms of entry barriers reinforce each other, so we expect that $\beta_4$ is positive as well.

(3) By performing individual $t$ tests, we get $t_1 = \frac{0.01-(-0.1)}{0.02} = 5.5$. As this value is larger than the critical value 1.96 so we reject the null hypothesis.

(4) To permorm a joint test, we have to use the $F$ statistics whose formula is:

$$F = \frac{1}{2}\frac{t_1^2 + t_2^2 - 2 \times \hat{\rho}_{t_1,t_2} t_1 \times t_2}{1 - \hat{\rho}_{t_1,t_2}^2} \tag{6}$$

By inserting the value of $\hat{\rho}_{t_1,t_2} = 0.83$ and computing $t_1 = \frac{0.04}{0.02} = 2$ and $t_2 = \frac{0.0018}{0.001} = 1.8$, we get a value of the statistics 2.03 which does not exceed the critical value of 3.0. So we do not reject the null hypothesis of equality to zero of both coefficients. In economic terms, $ACR$ appears not to affect significantly $PCM$.

(5) By dropping a regressor, the $R^2$ cannot increase. Given we suppose to drop the interaction between $R\&D = 0$ and $log(ACR)$, the information contained in this variable is to a great extent also present in the individual variables. So by dropping $R\&D \times log(ACR)$ from the regression, we expect the $R^2$ to decrease of a small amount, say to the value of 0.43.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2017-18**
**Exam January, $24^{th}$ 2019**

**Instructions**: answer to all questions in 1 hour and 45 minutes

**Exercise 1 (8 points)** In a market with homogeneous products and Cournot competition two types of firms compete: 1 E-type firm and $M$ I-type firms. The aggregated market demand is $p = \alpha - \beta Q$, where $Q$ is the aggregated production of the $M+1$ firms. The cost function of the firms is linear: firm E(fficient) has a marginal cost $c^E$ and firms I(nefficient) have a marginal cost $c^I$, where $c^I > c^E$.

(a) Find equilibrium quantities;

(b) Discuss the level of the production of $E$ with respect to the production of $I$;

(c) How the entry of a new inefficient firm will affect the quantities produced at equilibrium by the two kinds of firms?

(d) Compute the $CR4$ concentration index for the case with 1 efficient firm and 4 inefficient ones.

**Solution**

(a) The profit functions of E-type and I-type firms are:

$$\Pi^E = q^E\left(\alpha - \beta\left[q^E + \sum_{m=1}^{M} q_m^I\right]\right) - c^E q^E \tag{1}$$

$$\Pi_i^I = q_i^I\left(\alpha - \beta\left[q^E + \sum_{m=1}^{M} q_m^I\right]\right) - c^I q_i^I \quad i = 1,\ldots,M \tag{2}$$

The FOCs are:

$$\frac{\partial \Pi^E}{\partial q^E} = \left(\alpha - \beta\left[q^E + \sum_{m=1}^{M} q_m^I\right]\right) - \beta q^E - c^E = 0 \tag{3}$$

$$\frac{\partial \Pi_i^I}{\partial q_i^I} = \left(\alpha - \beta\left[q^E + \sum_{m=1}^{M} q_m^I\right]\right) - \beta q_i^I - c^I = 0 \quad i = 1,\ldots,M \tag{4}$$

Invoking symmetry among inefficient firms $\left(q_1^I = q_2^I = \ldots = q_M^I = q^I\right)$:

$$\begin{cases} \alpha - 2\beta q^E - \beta M q^I - c^E = 0 \\ \alpha - \beta q^E - \beta\left(M+1\right) q^I - c^I = 0 \end{cases}$$

Solving for $q^E$ and $q^I$, we get:

$$q^E = \frac{\alpha + Mc^I - (M+1)\,c^E}{\beta\,(2+M)} \tag{5}$$

$$q^I = \frac{\alpha + c^E - 2c^I}{\beta\,(2+M)} \tag{6}$$

(b) Obviously, the quantities produced by the two kinds of firms are equal only if $c^E = c^I$. In this case the usual formula for a Cournot-Nash Equilibrium apply. If $c^I > c^E$, then the E-type firm will produce more than I-type firms, as $(M+2)(c^I - c^E) > 0$. This is in line with the general result that in a Cournot model the firm with the lowest marginal cost will produce more.

(c) Like in any Cournot game, equilibrium quantities for both kind of firms are negatively affected by the increase in $M$, the number of firms in the market:

$$\frac{\partial q^I}{\partial M} = \frac{\alpha + c^E - 2c^I}{\beta}\frac{-1}{(2+M)^2} < 0 \tag{7}$$

$$\frac{\partial q^E}{\partial M} = \frac{(c^I - c^E)\beta(2+M) - (\alpha + Mc^I - (M+1)c^E)\beta}{\beta^2(2+M)^2} < 0 \tag{8}$$

(d) In the case with 1 efficient firm and 4 inefficient ones (i.e. $M = 4$), the quantity produced by each kind of firms will be:

$$q^E = \frac{\alpha + 4c^I - (4+1)\,c^E}{\beta\,(2+4)} = \frac{\alpha + 4c^I - 5c^E}{6\beta} \tag{9}$$

$$q^I = \frac{\alpha + c^E - 2c^I}{\beta\,(2+4)} = \frac{\alpha + c^E - 2c^I}{6\beta} \tag{10}$$

In turn, the total quantity produced will be:

$$Q = \frac{\left(\alpha + 4c^I - 5c^E\right) + 4\left(\alpha + c^E - 2c^I\right)}{6\beta} = \frac{5\alpha - 4c^I - c^E}{6\beta} \tag{11}$$

The $CR4$ concentration index will be the total quantity minus the quantity produced by one of the inefficient firm divided by total quantity:

$$CR4 = 1 - \frac{\frac{\alpha + c^E - 2c^I}{6\beta}}{\frac{5\alpha - 4c^I - c^E}{6\beta}} = 1 - \frac{\alpha + c^E - 2c^I}{5\alpha - 4c^I - c^E} \tag{12}$$

**Exercise 2 (8 points)** Consider a standard linear city *à la Hotelling* (i.e unitary length with uniform distribution of the consumers) with linear transportation costs $\theta$ and three firms $(a,b,c)$ localized in $x_a = 0$, $x_c = 1$, and $x_b = x$, with $0 < x < 1$. The three firms compete in prices and their production costs are null.

(a) Verify that at equilibrium the profit of firm $b$ does not depend on its position $x$. Provide the economic rationale of this result.

(b) How the equilibrium found in point (a) would change if firm $b$ has a constant production cost $c > 0$ whereas firms $a$ and $c$ still have null production costs?

(c) How would your previous results modify if firm $a$ is located $x_a = \frac{1}{4}$?

Assume an equilibrium exists.

**Solution**

(a) If an equilibrium $[p_a^*, p_b^*, p_c^*]$ exists, then two indifferent consumers are localized in $x_{ab}$, between $a$ and $b$, and in $x_{bc}$, between $b$ and $c$.

Consider the consumer who is indifferent between firm $a$ and firm $b$. For this consumer it must hold the following equality:

$$\theta x_{ab} + p_a = \theta \left(x - x_{ab}\right) + p_b \tag{13}$$

whereas for the consumer who is indifferent between firm $b$ and firm $c$ the following equality must hold:

$$\theta \left(x_{bc} - x\right) + p_b = \theta \left(1 - x_{bc}\right) + p_c \tag{14}$$

From (13):

$$2\theta x_{ab} = \theta x + p_b - p_a \tag{15}$$

and from (14):

$$2\theta x_{bc} = \theta \left(1 + x\right) + \left(p_c - p_b\right) \tag{16}$$

The coordinates of the position of the indifferent consumers are therefore:

$$x_{ab} = \frac{x}{2} + \frac{p_b - p_a}{2\theta} \tag{17}$$

$$x_{bc} = \frac{1+x}{2} + \frac{p_c - p_b}{2\theta} \tag{18}$$

The payoffs of the firms are:

$$\pi_a = p_a x_{ab} = p_a \times \left(\frac{x}{2} + \frac{p_b - p_a}{2\theta}\right) \tag{19}$$

$$\pi_b = p_b \left(x_{bc} - x_{ab}\right) = p_b \left[\left(\frac{1+x}{2} + \frac{p_c - p_b}{2\theta}\right) - \left(\frac{x}{2} + \frac{p_b - p_a}{2\theta}\right)\right]$$

$$= p_b \times \left(\frac{1}{2} + \frac{p_a + p_c}{2\theta} - \frac{p_b}{\theta}\right) \tag{20}$$

$$\pi_c = p_c(1 - x_{bc}) = p_c \times \left[1 - \left(\frac{1+x}{2} + \frac{p_c - p_b}{2\theta}\right)\right] = p_c \times \left(\frac{1}{2} - \frac{x}{2} - \frac{p_c - p_b}{2\theta}\right) \tag{21}$$

To obtain the Nash equilibrium in prices we have to derive the profit functions, set equal to 0 and solve the system of the reaction functions.

$$\frac{\partial \pi_a}{p_a} = \frac{x}{2} + \frac{p_b}{2\theta} - \frac{p_a}{\theta} = 0 \tag{22}$$

$$\frac{\partial \pi_b}{p_b} = \left(\frac{1}{2} + \frac{p_a + p_c}{2\theta} - \frac{p_b}{\theta}\right) - \frac{p_b}{\theta} = 0 \tag{23}$$

$$\frac{\partial \pi_c}{p_c} = \left(\frac{1}{2} - \frac{x}{2} - \frac{p_c - p_b}{2\theta}\right) - \frac{p_c}{2\theta} = \frac{1}{2} - \frac{x}{2} - \frac{p_c}{\theta} + \frac{p_b}{2\theta} = 0 \tag{24}$$

From (22) and (24):

$$p_a = \frac{\theta x}{2} + \frac{p_b}{2} \tag{25}$$

$$p_c = \theta\left(\frac{1-x}{2}\right) + \frac{p_b}{2} \tag{26}$$

If we substitute in (23) and then back in (22) and (24) we get:

$$p_a^* = \theta\left(\frac{1+2x}{4}\right) \tag{27}$$

$$p_b^* = \frac{\theta}{2} \tag{28}$$

$$p_c^* = \theta\left(\frac{3-2x}{4}\right) \tag{29}$$

As it can be seen, at the equilibrium both $(x_{bc} - x_{ab})$ and $p_b$ do not depend on $x$. In particular, $(x_{bc} - x_{ab}) = \frac{1}{2}$, and then $\pi_b = \frac{\theta}{4}$. This result can be explained as—at the equilibrium—a variation of $x$ determines a decrease of the demand in the direction of the variation, and, at the same time, an *equal* increase in the opposite direction. This explains why prices and profits of $b$ are independent on $x$.

(b) If firm $b$ incurs in a positive marginal cost $c$, its payoff (eq. (20)) is modified into

$$(p_b - c) \times \left(\frac{1}{2} + \frac{p_a + p_c}{2\theta} - \frac{p_b}{\theta}\right) \tag{30}$$

In turn, the derivative of payoff (eq. (23)) becomes

$$\frac{\partial \pi_b}{p_b} = \left(\frac{1}{2} + \frac{p_a + p_c}{2\theta} - \frac{p_b}{\theta}\right) - \frac{p_b - c}{\theta} = \frac{1}{2} + \frac{p_a + p_c}{2\theta} - \frac{2p_b}{\theta} + \frac{c}{\theta} = 0 \tag{31}$$

Formulas (27) and (28) are unchanged. By inserting them into equation (31) we get:

$$p_a^* = \theta\left(\frac{1+2x}{4}\right) + \frac{c}{3} \tag{32}$$

$$p_b^* = \frac{\theta}{2} + \frac{2c}{3} \tag{33}$$

$$p_c^* = \theta\left(\frac{3-2x}{4}\right) + \frac{c}{3} \tag{34}$$

As we can see, equilibrium prices are higher than before for all firms. Profits are higher now for firms $a$ and $c$ but lower for firm $b$. Indifferent consumer $x_{ab}$ will move to the right and indifferent consumer $x_{bc}$ will move to the left.

(c) If firm $a$ is located $x_a = \frac{1}{4}$, previous results are affected. In particular, firm $a$ will enjoy a sort of monopoly power over the $[0; \frac{1}{4}]$ interval leading to an increase in their profits. As for firms $b$ and $c$, they will have lower profits than before.

**Exercise 3 (8 points)** A firm produces three products, the demands for which are independent. Consumers (or group of consumers) have the following reservation prices:

| *Consumer* | *Good* 1 | *Good* 2 | *Good* 3 |
|---|---|---|---|
| *A* | 25 | 95 | 60 |
| *B* | 40 | 80 | 60 |
| *C* | 80 | 40 | 60 |
| *D* | 95 | 25 | 60 |

(a) Suppose first that all three products are produced at 0 marginal cost. Consider three alternative pricing strategies:

- selling the three goods separately;
- pure bundling (i.e. selling only the three goods together);
- mixed bundling (i.e. selling the three goods together but also the goods separately or in bundle of two goods).

For each strategy, determine the optimal prices to be charged and the resulting profits. Which strategy would be best?

(b) Now suppose that the production of each good entails a marginal cost of 35$. How this information changes your answer to point (a)? Carefully explain why the optimal strategy is now different.

(c) Suppose now that the production of each good entails a marginal cost of 20$. How this information changes your answer to point (b)? Carefully explain.

## Solution

(a) Notice first that the sum of the three reservation prices, for all four consumers, is constant (and equal to 180). This means that the three reservation prices lie on a plane. This is equivalent to the two good case when the reservation prices lie on a straight line. Given the assumption that all three products are produced at 0 marginal cost, we have the two conditions which make pure bundling the preferred choice.

To verify this, consider first the strategy of selling the three goods separately. The optimal price for each of the good must be set according to the standard rule $MC = MR$. This leads to the optimal prices $p_1 = 80$, $p_2 = 80$, and $p_3 = 60$. This leads to a revenue of 560 (equal to the profit, as marginal costs are 0). The pure bundling strategy has as optimal price for each bundle $p_B = 180$, the sum of the reservation prices of the goods for each of the four consumers. This leads to a revenue (profit) of 720. As for mixed bundling, no matter which good is bundled and which is unbundled, the profit will be less than the one of pure bundling. Just as an example, consider the following strategy (which will prove to be the optimal one under point (b) below): to sell the bundle of 3 goods at $p_B = 180$ and to sell the bundle of goods 1 and 3 at $p_{13} = 155 - \varepsilon$ and the bundle of goods 2 and 3 at $p_{23} = 155 - \varepsilon$. Consumers $B$ and $C$ will buy the bundle of three goods, consumer $A$ will buy the bundle composed of goods 2 and 3 whereas consumer $D$ will buy the bundle composed of goods 1 and 3. The profit (revenue) of this strategy is $670 - 2\varepsilon$, which is less than the profit of pure bundling.

(b) If the production of each good entails a marginal cost of 35\$, there is room—at least in principle—for mixed bundling to be the best strategy.

To see whether this is the case, consider first the strategy of selling the three goods separately. The optimal price for each of the good are equal than before: $p_1 = 80$, $p_2 = 80$, and $p_3 = 60$. This leads to a revenue of 560. Now the cost is equal to $35 \times 8 = 280$, so the profit is 280. The pure bundling strategy—as before—has as optimal price for each bundle $p_B = 180$ and a revenue of 720. Now the cost is equal to $35 \times 12 = 420$, so the profit is 300. As for mixed bundling, the optimal strategy entails to induce consumer $A$ not to buy good 1 and consumer $D$ not to buy good 2, as the marginal willingness to pay the two consumers have for those goods is less than the marginal cost of production. Therefore, the optimal strategy is to sell the bundle of 3 goods at $p_B = 180$ and to sell the bundle of goods 1 and 3 at $p_{13} = 155 - \varepsilon$ and the bundle of goods 2 and 3 at $p_{23} = 155 - \varepsilon$. Consumers $B$ and $C$ will buy the bundle of three goods, consumer $A$ will buy the bundle composed of goods 2 and 3 whereas consumer $D$ will buy the bundle composed of goods 1 and 3. The profit of this strategy is $670 - 2\varepsilon - 10 \times 35 = 320 - 2\varepsilon$, which is now more than the profit of pure bundling. The mixed bundling is now the best strategy as it allows the firm not to produce some units of the goods which are evaluated by some consumers less than the production cost. In fact, the difference in profit of the two alternative (pure and mixed bundling) is equal (not considering the $2\varepsilon$) to the loss pure bundle entails by selling products 1 and 2 to consumers $A$ and $D$: $2 \times (35 - 25) = 20$.

(c) If the production of each good entails a marginal cost of 20\$, instead of 35\$, the reservation price of each good for each consumer is higher than the marginal cost of production. In turn, pure bundling is again the best strategy. Put differently, when the reservation prices are perfectly correlated, mixed bundling emerges as the best strategy if the marginal cost are not only strictly positive but also higher than the willingness to pay (reservation price) of some consumers.

**Exercise 4 (6 points)** Suppose a researcher wants—by using a cross-section of firm level data—to assess the relative performance, in terms of productivity, of domestic owned and foreign owned firms. To this end, he estimates the following Cobb-Douglas production function augmented with a dummy variable—*Foreign Owned*—for foreign ownership (i.e. the dummy *Foreign Owned* equals 1 if the firm is under foreing ownership and 0 otherwise):

$$log(Y)_i = \beta_0 + \beta_K log(K)_i + \beta_L log(L)_i + \beta_3 (Foreign\ Owned)_i + u_i \tag{35}$$

(1) Carefully interpret the coefficients;

(2) Suggest the sign of the coefficients $\beta_K$ and $\beta_L$, providing the economic explaination for your suggestions;

(3) Suggest the sign of the coefficient $\beta_3$ if you believe that foreign-owned firms are more productive than domestic owned ones;

(4) Suppose now that the researcher wants to estimate a separate productivity differential for firms under US and under foreign ownership different from US. Write the corresponding model and suggest the sign of the coefficients if firms with US ownership are more productive than other foreign owned firms.

**Solution**

(1) The coefficients for $log(K)$ and $log(L)$ have to be interpreted as elasticities of the dependent variable with respect to each of the input, Capital and Labour. So, a 1% increase in Labour (resp., Capital) will lead to a $\beta_L\%$ (resp., $\beta_K\%$) in output $Y$. The intercept, $\beta_0$ is the expected level of the dependent variable when the regressors are equal to 0, i.e. for a firm with $K = L = 1$ under domestic ownership (i.e. $Foreign\ Owned = 0$). $exp\,(\beta_0)$ is therefore the average productivity for firms under domestic ownership whereas $exp\,(\beta_0 + \beta_3)$ is the average productivity for those firms under foreign ownership.

(2) As for the likely sign of the coefficients, $\beta_L$ and $\beta_K$ will be positive, as the marginal productivity of factors is positive.

(3) If we believe that foreign-owned firms are more productive than domestic owned ones, the coefficient of $\beta_3$ will be positive

(4) If the researcher wants to estimate a separate productivity differential for firms under US and under foreign ownership different from US, the corresponding model would be:

$$log(Y)_i = \beta_0 + \beta_K log(K)_i + \beta_L log(L)_i + \beta_3 (Foreign\ Owned_{USA})_i + \beta_4 (Foreign\ Owned_{OTHER})_i + u_i \quad (36)$$

The signs of $\beta_0$, $\beta_K$ and $\beta_L$ are unaffected. The sign of $\beta_3$ will be positive if we believe that firms under US ownership are more productive than other foreign owned firms whereas the sign of $\beta_4$ will depend on whether we believe that other foreign firms are more productive than domestic owned firms (positive sign) or we believe that other foreign firms are less productive than domestic owned firms (negative sign).

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2018-19**
**Exam June, $28^{th}$ 2019**

## EXAM RULES

You have already been given the two points bonus right after your inscription. If you submit your copy, it will be corrected, the mark cannot be refused but you can keep it on hold ("freeze") and try a second time. If you decide for this second test to be corrected, then the old mark is eliminated, and the final mark is the one of the second test. If you froze the previous mark and then you fail the second test, then you can try again only next academic year.
After September 2019, all frozen marks are cancelled.

**Instructions**: answer to all questions in 1 hour and 50 minutes

**Exercise 1 (8.5 points)** Consider a market with homogeneous products with inverse demand $p = 1-Q$ with $Q = \sum_i^n q_i$. Firms compete à la Cournot.

(a) Suppose first two firms (1 and 2) with constant marginal cost $c_1 = c_2 = \frac{1}{2}$ and no fixed costs. Find the market equilibrium (quantity, price, profits);

(b) Suppose now that the two firms still have no fixed costs but they have different marginal cost: $c_1 = \frac{1}{2}$ and $c_2 = \frac{1}{3}$. Find the market equilibrium (quantity, price, profits);

(c) Suppose now that there are three firms in the market. Alongside the previous two firms, there is now a the third firm identical to firm 2 of the previous point, i.e. firm 3 has a marginal cost $c_3 = \frac{1}{3}$. Supposing no fixed costs for all firms, find the market equilibrium (quantity, price, profits);

(d) Suppose now that firms 2 and 3 merge of the previous point merge, i.e. they become a single entity. Suppose also that the merger induces cost reductions so that the marginal cost of the new entity is $c_{2,3} = \frac{1}{8}$. Supposing no fixed costs for all firms, find the market equilibrium (quantity, price, profits). Comment whether the merger is detrimental for consumer welfare explaining your result;

(e) Compute for all the previously found equilibria the Herfindahl-Hirschman and the CR4 concentration indexes. Comment about the results you obtain.

### Solution

(a) This is a standard case of Cournot duopoly with symmetric and constant marginal costs ($c_1 = c_2 = \frac{1}{2}$). The equilibrium can be found by solving the system of reaction functions, which are obtained by finding the profit maximization quantities for each firms. The reaction function for the first firm is:

$$q_1(q_2) = \frac{1}{4} - \frac{q_2}{2} \tag{1}$$

and symmetrically for firm 2:

$$q_2(q_1) = \frac{1}{4} - \frac{q_1}{2} \tag{2}$$

Invoking symmetry, we get $q_1^* = q_2^* = \frac{1}{6}$. Quantities could have been obtained by the formula $q_i = \frac{a-c}{(N+1)b}$ setting $a = 1$, $b = 1$ and $c = \frac{1}{2}$. $Q^* = \frac{1}{3}$ and $p^* = \frac{2}{3}$. In turn, $\pi_1^* = \pi_2^* = \left(\frac{2}{3} - \frac{1}{2}\right) \times \frac{1}{6} = \frac{1}{36}$.

(b) If the duopoly becomes asymmetric, with $c_2 = \frac{1}{3}$, we cannot invoke simmetry anymore. The reaction function of the first firm remains the same as before (equation (1)) whereas the one of the second firm becomes:

$$q_2(q_1) = \frac{1}{3} - \frac{q_1}{2} \tag{3}$$

Solving the system, we get $q_1^* = \frac{1}{9} < \frac{1}{6}$ and $q_2^* = \frac{5}{18} > \frac{1}{6}$. $Q^* = \frac{7}{18} > \frac{1}{3}$ and $p^* = \frac{11}{18} < \frac{2}{3}$. $\pi_1^* = \left(\frac{11}{18} - \frac{1}{2}\right) \times \frac{1}{9} = \frac{1}{81} < \frac{1}{36}$ and $\pi_2^* = \left(\frac{11}{18} - \frac{1}{3}\right) \times \frac{5}{18} = \frac{25}{324} > \frac{1}{36}$. It is confirmed that if one firms becomes more efficient (i.e. there is a cost reduction) that firm produces more and has higher profit than the other firm. Overall quantity increases so that price decreases. Consumers benefit from part of the cost reduction as price has decreased.

(c) If the previous duopoly becomes a triopoly, with $c_1 = \frac{1}{2}$ and $c_2 = c_3 = \frac{1}{3}$, we have to solve the system composed by the three reaction functions:

$$q_1(q_2, q_3) = \frac{1}{4} - \frac{q_2 + q_3}{2} \tag{4}$$

$$q_2(q_1, q_3) = \frac{1}{3} - \frac{q_1 + q_3}{2} \tag{5}$$

$$q_3(q_1, q_2) = \frac{1}{3} - \frac{q_1 + q_2}{2} \tag{6}$$

Invoking symmetry between $q_2^*$ and $q_3^*$, we get: $q_1^* = \frac{1}{24} < \frac{1}{9}$ and $q_2^* = q_3^* = \frac{5}{24} < \frac{5}{18}$. $Q^* = \frac{11}{24} > \frac{7}{18}$ and $p^* = \frac{13}{24} < \frac{11}{18}$. $\pi_1^* = \left(\frac{13}{24} - \frac{1}{2}\right) \times \frac{1}{24} = \frac{1}{576} < \frac{1}{81}$ and $\pi_2^* = \pi_3^* = \left(\frac{13}{24} - \frac{1}{3}\right) \times \frac{5}{24} = \frac{25}{576} < \frac{25}{324}$. It is confirmed that if one additional firms enters the market, all firms produce less and hence have lower profits but overall quantity increases so that price decreases.

(d) If firms 2 and 3 merge, so that they become a single entity enjoying cost reductions ($c_{2,3} = \frac{1}{8} < \frac{1}{3}$, we are back to an asymmetric duopoly. The system of reactions functions to solve is:

$$q_1(q_{2,3}) = \frac{1}{4} - \frac{q_{2,3}}{2} \tag{7}$$

$$q_{2,3}(q_1) = \frac{7}{16} - \frac{q_1}{2} \tag{8}$$

Solving the system, we get $q_1^* = \frac{1}{24}$ and $q_{2,3}^* = \frac{5}{12}$. $Q^* = \frac{11}{24}$ and $p^* = \frac{13}{24}$. $\pi_1^* = \left(\frac{13}{24} - \frac{1}{2}\right) \times \frac{1}{24} = \frac{1}{576}$ and $\pi_{2,3} = \left(\frac{13}{24} - \frac{1}{8}\right) \times \frac{5}{12} = \frac{25}{144} > \frac{25}{576} \times 2 = \frac{25}{288}$. Merger is profitable for the two firms as the profit for the merged firm is higher than the sum of the profit for the separate firms. This is not always the case in a Cournot setting but it happens here due to cost reductions. Furthermore, overall quantity and price are unaffected by the merger. This is surprising as in a Cournot setting the merger, by reducing the number of firms in the market, should reduce total quantity and hence increase the price. Here this does not happen because the merger has two effects which perfectly compensate each other: the first one is the reduction in the number of firms (which increases the price) and the second is cost reductions, which lead (like in previous point (b)) to a decrease in price. Given that price is unchanged with respect to the previous point, consumers are not negatively affected by the merger as it is the case in mergers without cost reductions.

(e) To compute for all the previously found equilibria the Herfindahl-Hirschman and the CR4 concentration indexes, we have first to compute market shares.

In point (a), it is a symmetric duopoly so that $s_1 = s_2 = \frac{1}{2}$. In turn, $HHI = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$ and $CR4 = \frac{1}{2} + \frac{1}{2} = 1$

In point (b), $s_1 = q_1^*/Q^* = \frac{\frac{1}{9}}{\frac{7}{18}} = \frac{2}{7} < \frac{1}{2}$ and $s_2 = q_2^*/Q^* = \frac{\frac{5}{18}}{\frac{7}{18}} = \frac{5}{7} = 1 - \frac{2}{7} > \frac{1}{2}$. Firm 2 has now a larger market share than firm 1. $HHI = \left(\frac{2}{7}\right)^2 + \left(\frac{5}{7}\right)^2 = \frac{29}{49} > \frac{1}{2}$ and $CR4 = \frac{2}{7} + \frac{5}{7} = 1$. The Herfindahl-Hirschman index is able to catch the increased concentration in the market whereas the CR4 index is not.

In point (c), $s_1 = q_1^*/Q^* = \frac{\frac{1}{24}}{\frac{11}{24}} = \frac{1}{11} < \frac{2}{7}$ and $s_2 = s_3 = q_2^*/Q^* = \frac{\frac{5}{24}}{\frac{11}{24}} = \frac{5}{11} > \frac{5}{7}$. The entry of the third firm decreases the market share of the existing firms. $HHI = \left(\frac{5}{11}\right)^2 + \left(\frac{5}{11}\right)^2 + \left(\frac{1}{11}\right)^2 = \frac{51}{111} < \frac{29}{49}$ and $CR4 = \frac{5}{11} + \frac{5}{11} + \frac{1}{11} = 1$. As before, the Herfindahl-Hirschman index is able to catch the decreased concentration in the market whereas the CR4 index is not.

In point (e), $s_1 = q_1^*/Q^* = \frac{\frac{1}{24}}{\frac{11}{24}} = \frac{1}{11}$ and $s_{2,3} = q_{2,3}^*/Q^* = \frac{\frac{10}{24}}{\frac{11}{24}} = \frac{10}{11}$. In turn, $HHI = \left(\frac{1}{11}\right)^2 + \left(\frac{10}{11}\right)^2 = \frac{101}{111} > \frac{29}{49}$ and $CR4 = \frac{1}{11} + \frac{10}{11} = 1$. The merger, needless to say, increases the market concentration and, as before, the Herfindahl-Hirschman index is able to catch the increased concentration in the market whereas the CR4 index is not.

**Exercise 2 (8.5 points)** Suppose a duopoly where the inverse demand is given by

$$p = 1 - Q \tag{9}$$

with $Q = q_1 + q_2$

The cost function is

$$TC(q_i) = \frac{1}{64} \tag{10}$$

i.e. there are null marginal costs and a fixed cost equal to $\frac{1}{64}$.

a) Find the Stackelberg equilibrium (quantities, price, profits) supposing firm 1 is the leader;
b) Find the limit output;
c) What is the SPNE for the entry game with the following timing: in the first-stage firm 1 can commit to its output (either the Stackelberg or the limit output); in the second stage firm 2 can enter and choose its output;
d) What is the SPNE for the entry game with the following timing: in the first-stage firm 2 can commit to enter and, in case of entry, chooses its output; in the second stage firm 1 decides how much to produce.

**Solution**

(a) To find the Stackelberg equilibrium, we have to find the reaction function of firm 2 (the follower). Starting from the profit function of the follower

$$\pi_2(q_1, q_2) = [(1 - (q_1 + q_2)] \times q_2 - \frac{1}{64} \tag{11}$$

we get a standard reaction function of the form $q_2^*(q_1) = \frac{a-bq_1-c}{2b}$ from which $q_2^*(q_1) = \frac{1}{2} - \frac{q_1}{2}$. Inserting the reaction function of the follower in the profit function of the leader:

$$\pi_1(q_1) = \left(1 - q_1 - \frac{1}{2} + \frac{q_1}{2}\right) \times q_1 - \frac{1}{64} \tag{12}$$

and finding the first order condition:

$$\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = 0 \Leftrightarrow \frac{1}{2} - q_1 = 0 \tag{13}$$

so that $q_1^* = \frac{1}{2}$ and by inserting this value in the reaction function above we get:

$$q_2^*(q_1^*) = \frac{1}{2} - \frac{q_1}{2} = \frac{1}{4} < \frac{1}{2} \tag{14}$$

In turn:

$$Q^* = q_1^* + q_2^* = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \tag{15}$$

$$p^* = 1 - Q^* = 1 - \frac{3}{4} = \frac{1}{4} \tag{16}$$

$$\pi_1^* = \frac{1}{2} \times \frac{1}{4} - \frac{1}{64} = \frac{1}{8} - \frac{1}{64} = \frac{7}{64} \tag{17}$$

$$\pi_2^* = \frac{1}{4} \times \frac{1}{4} - \frac{1}{64} = \frac{1}{16} - \frac{1}{64} = \frac{3}{64} < \frac{7}{64} \tag{18}$$

b) To find the limit output, i.e. the minimum quantity of the leader that leads the follower to produce 0 output, we have to solve the following equation:

$$\pi_2\left(R_2\left(q_1^L\right), q_1^L\right) = 0 \tag{19}$$

where $q_1^L$ is the limit output of the leader.

The profit function of the follower is:

$$\pi_2(q_1, q_2) = (1 - q_1 - q_2) \times q_2 - \frac{1}{64} \tag{20}$$

so that:

$$\pi_2(R_2(q_1), q_1) = \left[1 - q_1 - \left(\frac{1}{2} - \frac{q_1}{2}\right)\right] \times \left(\frac{1}{2} - \frac{q_1}{2}\right) - \frac{1}{64} = \left(\frac{1}{2} - \frac{q_1}{2}\right)^2 - \frac{1}{64} \tag{21}$$

Solving this equation we get:

$$q_1^L = \frac{3}{4} \tag{22}$$

which is a special case of the general solution

$$q_1^L = \frac{a - c - \sqrt{4bf}}{b} = \frac{1 - 0 - \sqrt{4 \times 1 \times \frac{1}{64}}}{1} = 1 - \sqrt{\frac{1}{16}} = 1 - \frac{1}{4} = \frac{3}{4} \tag{23}$$

If the leader plays this quantity, $p^L = \frac{1}{4}$ and $\pi_1^L = \left(1 - \frac{3}{4}\right) \times \frac{3}{4} - \frac{1}{64} = \frac{3}{16} - \frac{1}{64} = \frac{12-1}{64} = \frac{11}{64} > \frac{7}{64}$. For the follower, $\pi_2^L = 0$.

c) To find the SPNE for the entry game, we have to solve the game by backward induction. Firm 1 has two strategies: to play the Stackelberg quantity or to play the limit output whereas firm 2 has four strategies: (Enter, Enter), (Enter, Not Enter), (Not Enter, Enter), (Not Enter, Not Enter) where the first action refers to the node after the first firm has played the Stackelberg quantity and the second action to the node after the first firm has played the limit output.
As firm 1 knows that firm 2 will play the reaction function, firm 1 has to choose, in the first stage, whether to play the limit output or the Stackelberg output depending on the relative profit of the two outputs.

The profit for firm 1 if it produces the Stackelberg quantity is $\frac{7}{64}$ whereas if it produces the limit quantity is $\frac{11}{64}$. Therefore it is better to play in the first stage the limit output. So the SPNE is for firm 1 to play the limit output and for firm 2 (Enter, Not Enter) and the path entailed by the SPNE is: firm 1 produces $q_1^L$ in the first stage, firm 2 does not enter the market.

d) To find the SPNE for the entry game where in the first-stage firm 2 can commit to enter and, in case of entry, chooses its output and in the second stage firm 1 decides how much to produce, we have – as before – to use backward induction. Now the role of the two firms are reversed. In the first stage firm 2 takes into account what firm 1 is going to do in stage 2.
We know already what will be the SPNE of the game: firm 2 will enter, committing to produce the limit output as it delivers higher profits than producing the Stackelberg quantity. In the second stage, firm 1 will produce a null amount of output. The bottom line is that to be incumbent or entrant is not important. What is important is the ability to committ to a given output so that the other firm has to behave accordingly.

**Exercise 3 (8.5 points)** A monopolistic firm faces the following inverse demand function for its products: $p = 70 - 0.5Q$. Suppose no fixed costs and that constant marginal costs is equal to 10.

(a) Suppose first that the monopolist cannot (or does not want) to price discriminate. Compute the monopolistic equilibrium (quantity, price, profit);

(b) Suppose now that the firm can implement a perfect first degree price discrimination. Compute the monopolistic equilibrium (quantity, price, profit);

(c) Suppose now that the firm wants to implement a second degree price discrimination. In particular, the firms wants to introduce the "block pricing". The first block is the one found in previous point (a). Find the optimal second block given the first block;

(d) Provide an economic explanation why parking in Caselle airport adopt a second degree price discrimination by decreasing prices the longer you park (25 euros the first day, 14 euros the second day, 13 euros the third day, 5 euros the third day,... *Multipiano Parking, 1,800 lots*) whereas the parking in Piazza San Carlo does not (2 euros per hour from 9:00am to 8:00pm).

**Solution**

(a) If the firm does not price discriminate, it will choose the standard monopolistic solution: $MR = MC$. $MR = 70 - Q = 10$ so that $Q^* = 60$, $p^* = 40$ and $\Pi^* = 60 \times (40 - 10) = 1,800$.

(b) If the firm perfectly first degree price discriminate, it will set the quantity to produce according to $p = 70 - 0.5Q = MC$. $70 - 0.5Q = 10$ so that $Q^* = 120$. There is no unique price as the firm charges a different price for each consumer. The firm will capture all the consumer surplus so that now $\Pi^* = \frac{1}{2} \times (70 - 10) \times 120 = 3,600$. Unsurprisingly, the firm now enjoys a higher profit.

(c) If the firm wants to implement a second degree price discrimination, notably a "block pricing" with as first block the quantity found in previous point (a) (i.e. $Q^* = 60$ and $p^* = 40$), the optimal second block will be given by the monopolistic quantity on the residual demand. Put differently, after selling the first block the firm is left with a residual demand on which it is monopolist. The residual demand will be given by a demand which has the same slope as the original one but with an intercept equal to the price of the first block: $p = 40 - 0.5Q$. In turn, the second block will be sold according to $MR = MC$ where $MR$ is computed from the residual demand, i.e. $40 - Q = 10$ so that $Q^{**} = 30$ and $p^{**} = 25 > MC = 10$. Unsurprisingly, this leads to an additional profit equal to $(p^{**} - MC) \times Q^{**} = (25 - 10) \times 30 = 450$.

(d) Parking in Caselle airport has an incentive to adopt a second degree price discrimination as it has almost always *unused capacity*, i.e. there are available parking lots (in fact, *Multipiano Parking* is very large, 1,800 lots). By decreasing prices the longer you park, the Caselle parking induces additional demand which, due to almost null marginal cost, increases profits. Put differently: suppose you are the owner of the parking in Caselle. Would you keep the price high knowing that, at that price, you will have some lots unused or you will decrease the price (needless to say to a level higher than marginal cost) in order to fill those places?
The parking in Piazza San Carlo has no incentive to do so. It is almost always full, i.e. it is working at *full capacity*. Full capacity means vertical marginal costs so that decreasing prices will simply lead to lower profits. Put differently: suppose you are the owner of the parking in Piazza San Carlo. Would you decrease the price of the parking for the second hour from 2 euros to, say, 1 euro, thereby inducing the consumer to park for the second hour at a lower price, if you were sure that another consumer will immediately occupy that place paying 2 euros?

**Exercise 4 (4.5 points)** A researcher, by using a sample of 1,254 transactions of residential houses in the Torino area, wants to estimate how the price of the houses are affected by a set of regressors. To this end, he estimates two hedonic models.
The first one regresses the price of the house in thousands euro, *price*, on the dummy variable "zone" indicating if the house is in a high-value zone of the town (e.g. city center or Crocetta):

$$price_i = \alpha_0 + \alpha_1 \; zone_i + u_i \tag{24}$$

The second one adds as additional regressor the area of the house in square meters, *area*:

$$price_i = \delta_0 + \delta_1 zone_i + \delta_2 \; area_i + u_i \tag{25}$$

The OLS estimates of (24) are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{price}_i = \underset{(4.3)}{152.7} + \underset{(14.2)}{306.8} \; zone_i \qquad R^2 = 0.271 \qquad SER = 145.03 \tag{26}$$

(1) Carefully interpret the coefficients in equations (24) and (25);

(2) Comment the results shown in equation (26);

(3) Using the results in equation (26), test at the $\alpha = 5\%$ level that the coefficient of *zone* is equal to 0 against a bilateral alternative, assuming the 3 OLS assumptions hold;

(4) How would you expect your results in equation (26) will change if you measure the price of the house in euros instead of thousands euros?

(5) Starting from results in equation (26), suggest the expected results if you estimate equation (25).

**Solution**

(1) The intercept $\alpha_0$ is the expected price when the regressor is equal to 0, i.e. when the house is located in a zone different from a high-value zone. $\alpha_1$ is the expected change in price for houses located in high-value zones with respect to non high-value zones.
The intercept $\delta_0$ is the expected price when the regressors are equal to 0, i.e. when the house is located in a zone different from a high-value zone and the area of the house is 0. Needless to say, it only has a geometrical interpretation. $\delta_1$ is the expected change in price for houses located in high-value zones with respect to non high-value zones, holding fixed the area of the house. $\delta_2$ is the expected change in the price of the house when area increases of 1 unit (one square meter) holding fixed the zone of the house.

(2) Results show that the average price of a house located in a non high-value zone is around $150,000$ euros whereas the average price of a house located in a high-value zone is around $460,000 = 152,000 + 307,000$ euros. The fit of the model ($R^2 = 0.271$) is not bad considering that the dummy is the only regressor but of course there are many other variables which explain the price of the houses different from the zone (first of all, the area of the house) contained in $u$.

(3) By performing a $t$ test of the hypothesis that the coefficient $\alpha_1$ is equal to 0 against a bilateral alternative, we get $t = \frac{306.8-0}{14.2} \approx 22$. As this value is outside the acceptance interval $(-1.96, +1.96)$ for $\alpha = 5\%$ we reject the null hypothesis.

(4) If you measure the price of the house in euros instead of thousands euros, both coefficients will be multiplied by $1,000$. This is clearly seen by considering that the expected price in euros is just $1,000$ times the price in thousands euros, so both coefficients must be multiplied by the same amount. Mathematically, $\hat{\alpha_1} = \frac{cov(y,x}{var(x)}$ so that if multiply $y$ by $1,000$ and we estimate again by OLS we get $\hat{\alpha_1^*} = \frac{cov(y^*,x}{var(x)} = \frac{cov(1{,}000y,x}{var(x)} = 1,000\frac{cov(y,x}{var(x)} = 1,000\hat{\alpha_1}$ where $y^*$ is price in euros and $\hat{\alpha_1^*}$ the OLS coefficient estimated using $y^*$ as dependent variable. Likewise $\hat{\alpha_0} = \bar{y} - \hat{\alpha_1}\bar{x}$. If we estimate using $y^*$ we get $\hat{\alpha_0^*} = \bar{y^*} - \hat{\alpha_1^*}\bar{x} = 1,000\bar{y} - 1,000\hat{\alpha_1}\bar{x} = 1,000\hat{\alpha_0}$. Notice that also standard errors will be multiplied by $1,000$. The $R^2$ will be unaffected (it is unitless) whereas the $SER$, which has the same unit of measurement of the dependent variable, will be multiplied by $1,000$ as well.

(5) If we estimate model (25) we expect: 1) to have a positive estimate for for $\delta_2$ as increasing the area of the house will lead to a higher price; 2) to have a lower estimate for $\delta_1$ with respect to $\alpha_1$ as houses in high valued zone are larger than those in other zones (at least in Torino) so that the difference in price captured by $\hat{\alpha_1}$ also contains the effect of larger houses; 3) to have a lower estimate for $\delta_0$ with respect to $\alpha_0$ as the price of a house in a low-value zone with 0 area will be lower than the price of a house in a low-value zone with an average area; 4) to have a significant increase in the fit of the model (i.e. an large increase in $R^2$ and a large decrease in $SER$) as the area of the house is a crucial determinant of the price. As a matter of fact, estimation of model (25) leads to the following results:

$$\widehat{price}_i = \underset{(5.24)}{-98.3} + \underset{(8.3)}{157.8}\ zone + \underset{(0.5)}{2.9}\ area_i \qquad R^2 = 0.779 \qquad SER = 79.91 \tag{27}$$

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2018-19**
**Exam July, $12^{th}$ 2019**

## EXAM RULES

You have already been given the two points bonus right after your inscription. If you submit your copy, it will be corrected, the mark cannot be refused but you can keep it on hold ("freeze") and try a second time. If you decide for this second test to be corrected, then the old mark is eliminated, and the final mark is the one of the second test. If you froze the previous mark and then you fail the second test, then you can try again only next academic year.

After February 2020, all frozen marks are cancelled.

**Instructions**: answer to all questions in 1 hour and 45 minutes

**Exercise 1 (8.5 points)** A competitive refining industry produces one unit of waste for each unit of refined product. The industry disposes of the waste by releasing it into the atmosphere. The inverse demand curve for the refined product (which is also the marginal benefit curve) is $P^d = 24 - Q$, where $Q$ is the quantity consumed when the price consumers pay is $P^d$. The inverse supply curve (also the marginal private cost curve) for refining is $MPC = 2Q$, where MPC is the marginal private cost when the industry produces $Q$ units. The marginal external cost curve is $MEC = 0.5Q$, where MEC is the marginal external cost when the industry releases $Q$ units of waste.

(a) What are the equilibrium price and quantity for the refined product when there is no correction for the externality?

(b) How much of the chemical should the market supply at the social optimum?

(c) How large is the deadweight loss from the externality?

(d) Suppose the government imposes an emissions fee of T$ per unit of emissions (Pigouvian tax). How large should the emissions fee be if the market is to produce the economically efficient amount of the refined product?

(e) Suppose you are the economic advisor of the government. Would you suggest the Pigouvian tax if the refining industry was not competitive but monopolistic? Motivate your answer.

### Solution

(a) To find the equilibrium price and quantity for the refined product when there is no correction for the externality, it is sufficient to equate supply and demand. Inverse demand is given by $P^d = 24 - Q$ and direct demand is $Q^d = 24 - P$. The inverse supply curve, equal to the marginal private cost curve, is $P = MPC = 2Q$ so that the direct supply curve is $Q^s = \frac{P}{2}$. Solving the equality $24 - P = \frac{P}{2}$ we find $P^* = 16$ so that $Q^* = 24 - 16 = 8$.

(b) To find the optimal social quantity we have to consider the external costs. The marginal external cost curve is $MEC = 0.5Q$. It has to be summed to the marginal private cost curve to form the marginal social cost curve: $MSC = 2Q + 0.5Q = 2.5Q$. Inverting this curve we have the direct supply curve when we consider the externality: $Q^s = \frac{P}{2.5}$. Solving the equality $24 - P = \frac{P}{2.5}$ we get $p^* = \frac{120}{7} \approx 17.14$ so that $Q^* = 24 - \frac{120}{7} = \frac{48}{7} \approx 6.86 < 8$. As expected, in the presence of a negative externality the market equilibrium quantity is too large.

(c) To compute the deadweight loss from the externality we have to compute the area of the triangle which has the difference between the private and the social quantities as height and as base either the difference of the supply curves with and without considering the externality evaluated at the private optimum or the marginal external cost evaluated at the competitive quantity. By using the first option:

$$DWL = \frac{1}{2}\left(8 - \frac{48}{7}\right)(20 - 16) = \frac{16}{7} \approx 2.28$$

(d) If the government wants to impose a Pigouvian tax $T$ in order to completely remove the DWL due to the externality (i.e. let the market to produce the economically efficient amount of the refined product), we have to solve for $T$ the following equality: $Q^d(P) = 24 - P = \frac{P-T}{2} = Q^s(P - T)$ setting $P = \frac{120}{7}$. This leads to $24 - \frac{120}{7} = \frac{120}{14} + \frac{T}{2} \Leftrightarrow T = 2 \times \frac{12}{7} = \frac{24}{7} \approx 3.42$.

(e) The presence of two market failures (externality and market power) can compensate each other. In particular, the presence of a negative externality (so that the production is higher than the social optimal) can be compensated by the presence of market power which instead leads to produce a lower than optimal quantity. Therefore, if you were the economic advisor of the government you should not necessarily suggest the Pigouvian tax if the refining industry was not competitive but monopolistic.

**Exercise 2 (8.5 points)** In a market with homogeneous products and Cournot competition two types of firms compete: $J$ E-type firm and $M$ I-type firms. The aggregated inverse market demand is $p = 2 - Q$, where $Q$ is the aggregated production of the $J + M$ firms. The cost function of the firms is linear: firms E(fficient) have marginal cost $c^E$ whereas firms I(nefficient) have a marginal cost $c^I > c^E$.

(a) Suppose $J = M = 1$. Find equilibrium quantities;

(b) Discuss the level of the production of $E$ with respect to the production of $I$;

(c) Suppose $J = M = 2$. Find equilibrium quantities;

(d) Starting from the quantities found in point (c), find the cost difference between the two kinds of firms that still allows inefficient firms to produce a strictly positive amount. Comment your result;

(e) Starting from the value found in point (d), how the increase of $J$ from 2 to 3 (holding $M = 2$) changes this value? And how the increase of $M$ from 2 to 3 (holding $J = 2$) changes this value? Comment your result;

(f) How would you expect the value found in (d) to depend on the intercept and the slope of the inverse demand function? Explain.

**Solution**

(a) This is a standard problem of asymmetric Cournot duopoly with linear demand. The solution quantities are found by solving the system composed of the two reaction functions. The reaction function for the efficient firm (i.e. the one with marginal cost $c^E$) is:

$$q^E(q^I) = \frac{2 - c^E}{2} - \frac{q^I}{2} \tag{1}$$

whereas the reaction function for the inefficient firm (i.e. the one with marginal cost $c^I$) is:

$$q^I(q^E) = \frac{2 - c^I}{2} - \frac{q^E}{2} \tag{2}$$

Solving the system, we get $q^{E*} = \frac{2}{3} - \frac{2}{3}c^E + \frac{1}{3}c^I$ and $q^{I*} = \frac{2}{3} - \frac{2}{3}c^I + \frac{1}{3}c^E$.

Alternatively, we could have solved the general system formed considering the general formula with $J$ and $M$ firms with asymmetric costs and generic demand parameters $\alpha$ and $\beta$. The profit functions of E-type and I-type firms are:

$$\Pi_j^E = q_j^E \left( \alpha - \beta \left[ \sum_{j=1}^{J} q_j^E + \sum_{m=1}^{M} q_m^I \right] \right) - c^E q_j^E \quad j = 1, \ldots, J \tag{3}$$

$$\Pi_i^I = q_i^I \left( \alpha - \beta \left[ \sum_{j=1}^{J} q_j^E + \sum_{m=1}^{M} q_m^I \right] \right) - c^I q_i^I \quad i = 1, \ldots, M \tag{4}$$

The FOCs are:

$$\frac{\partial \Pi_j^E}{\partial q_j^E} = \left( \alpha - \beta \left[ \sum_{j=1}^{J} q_j^E + \sum_{m=1}^{M} q_m^I \right] \right) - \beta q_j^E - c^E = 0 \quad j = 1, \ldots, J \tag{5}$$

$$\frac{\partial \Pi_i^I}{\partial q_i^I} = \left( \alpha - \beta \left[ \sum_{j=1}^{J} q_j^E + \sum_{m=1}^{M} q_m^I \right] \right) - \beta q_i^I - c^I = 0 \quad i = 1, \ldots, M \tag{6}$$

Invoking symmetry among the group of efficient firms $\left(q_1^E = q_2^E = \ldots = q_J^E = q^E\right)$ and the one of inefficient firms $\left(q_1^I = q_2^I = \ldots = q_M^I = q^I\right)$:

$$\begin{cases} \alpha - \beta(J+1)q^E - \beta M q^I - c^E = 0 \\ \alpha - \beta J q^E - \beta\,(M+1)\,q^I - c^I = 0 \end{cases}$$

Solving for $q^E$ and $q^I$, we get:

$$q^E = \frac{\alpha + Mc^I - (M+1)\,c^E}{\beta\,(J+M+1)} \tag{7}$$

$$q^I = \frac{\alpha + Jc^E - (J+1)c^I}{\beta\,(J+M+1)} \tag{8}$$

Setting $J = M = 1$, $\alpha = 2$ and $\beta = 1$ we go back to previous formula.

(b) As for the level of the production of $E$ with respect to the production of $I$, the quantities produced by the two kinds of firms are equal only if $c^E = c^I$. If $c^I > c^E$, then the E-type firm will produce more than I-type firms, as $(M + J + 1)(c^I - c^E) > 0$. This is in line with the general result that in a Cournot model the firm with the lowest marginal cost will produce more.

(c) If $J = M = 2$, we have an oligopoly composed of 4 firms, 2 of which efficient and 2 inefficient. To find equilibrium quantities, it is convenient to impose symmetry between the two efficient firms and between the two inefficient ones. Notice that symmetry must be imposed in the first order conditions and not in the profit function. After imposing symmetry, the reaction function for the efficient firm (i.e. the one with marginal cost $c^E$) is:

$$q^E = \frac{2 - c^E}{2} - \frac{q^E + 2q^I}{2} \tag{9}$$

whereas the reaction function for the inefficient firm (i.e. the one with marginal cost $c^I$) is:

$$q^I = \frac{2 - c^I}{2} - \frac{2q^E + q^I}{2} \tag{10}$$

Solving the system composed of (9) and (10), we get

$$q^{E*} = \frac{2}{5} - \frac{3}{5}c^E + \frac{2}{5}c^I$$

and

$$q^{I*} = \frac{2}{5} - \frac{3}{5}c^I + \frac{2}{5}c^E$$

Of course, we could have derived the same result from the general formulas (7) and (8) found in point (a) above.

(d) Starting from the quantities found in previous point, we have to find the cost difference between the two kinds of firms that still allows inefficient firms to produce a strictly positive amount, i.e. $q^{I*} > 0$. This "survival condition" is equivalent to

$$2 - 3c^I + 2c^E > 0 \Leftrightarrow c^I < \frac{2}{3}(1 + c^E)$$

Interpretation is that inefficient firms can survive in a market where also efficient ones exist only if they are not "too inefficient", i.e. if the level of their cost is not too high with respect to the efficient ones.
Of course, we could have used the general formula to find that $q^{I*} > 0 \Leftrightarrow \alpha + Jc^E - (J+1)c^I \Leftrightarrow c^I < \frac{\alpha}{J+1} + \frac{J}{J+1}c^E$. Inserting the values $\alpha = 2$ and $J = 2$ we are back to the previous result.

(e) Starting from the value found in point (d), to find how the increase of $J$ from 2 to 3 (holding $M = 2$) changes the survival condition, we can recalculate this value for $J = 3$. Doing so, we find $q^{I*} > 0 \Leftrightarrow 2 - 4c^I + 3c^E > 0 \Leftrightarrow c^I < \frac{1}{2} + \frac{3}{4}c^E$. As $\frac{1}{2} + \frac{3}{4}c^E < \frac{2}{3}(1 + c^E)$ for $c^E < 2$ (the "survival condition" for efficient firms), we find that the increase of the number of efficient firms from 2 to 3 makes the life of the inefficient ones more difficult. More generally, we could have used the general formula $c^I < \frac{\alpha + Jc^E}{J+1}$. As $\frac{\partial \frac{\alpha + Jc^E}{J+1}}{\partial J} > 0$ for $c^E < \alpha$, we can conclude that the increase in the number of efficient firms, no matter their initial number, makes the survival condition of inefficient ones more binding.

Differently from the case of an increase in $J$, an increase in $M$ from 2 to 3 (holding $J = 2$) does not affect the survival condition of inefficient firms. More generally, the survival condition of inefficient firms does not depend on the number of inefficient firms. The intuition is that the entry of an additional inefficient firm also steals market share to the group of existing *efficient* firms leaving unaffected the survival condition of inefficient ones.

(f) Intuitively, the value found in (d) depends on the intercept of the inverse demand function as this value represents the size of the market. Larger this size, everything else equal, larger the room for inefficient firms. In turn, we expect that the surviving condition for inefficient firms, found in

point (d), becomes less binding for higher values of $\alpha$. This is indeed the case if we consider the general formula.
Instead, there is no reason to think that $\beta$ affects this condition. This can be verified by looking at the general formula $\frac{\alpha}{J+1} + \frac{J}{J+1}c^E$.

**Exercise 3 (8.5 points)** A monopolistic company faces two kinds of consumers, characterised by low and high demand. Low demand is $Q_L^d = 800 - 100P$ whereas high demand is $Q_H^d = 1600 - 100P$. Marginal cost is 2 euros for each unit and fixed costs are equal to 100 euros.

(a) Find prices, quantities, profit, consumers' and total welfare if the company can discriminate the two types of consumers (third degree price discrimination);

(b) Suppose now that the company cannot discriminate the two kind of consumers and has to set a unique price. Find price, quantities, profit, consumers' and total welfare and compare with previous results;

(c) How your previous results would change if low demand was $Q_L^d = 400 - 100P$? Explain your results.

**Solution**

(a) If the company can discriminate, it will charge $p^H = 9$ and $p^L = 5$. The quantities are $q^H = 700$ and $q^L = 300$. The profit is $\pi = 700 \times 9 + 300 \times 5 - 1,000 \times 2 - 100 = 5,700$ (notice that fixed costs have to be considered only once and not twice, once for each demand group). Consumer surpluses are $CS^H = \frac{(16-9)700}{2} = 2,450$ and $CS^L = \frac{(8-5)300}{2} = 450$, so that total consumers' surplus is $2,900$. Total welfare is $5,700 + 2,900 = 8,600$.

(b) Without price discrimination, the company faces an aggregate demand, given by the sum of the two demands. This is $Q = 2,400 - 200p$. Inverting the demand we find $p = 12 - 0.005Q$. Marginal revunue is $MR = 12 - 0.01Q$ and setting equal to the marginal cost leads to: $Q^* = 1,000$ and $p^* = 7$. The profit is $\pi = 7,000 - 2,000 - 100 = 4,900$. Inverse demands for each category are: $p^H = 16 - 0.01q^A$ and $p^L = 8 - 0.01q^S$. Consumers' surpluses are therefore: $CS^H = \frac{(16-7)900}{2} = 4,050$ and $CS^L = \frac{(8-7)100}{2} = 50$, so that total consumers' surplus is $4,100$ (notice that consumers surpluses have to be computed by using individual and not aggregate demand). Total welfare is $4,900 + 4,100 = 9,000$. As we can see, the low demand consumer are better off whereas the high demand consumer is worse off. This is unsurprising as the consumer who is penalised by third degree discrimination is the one with more rigid demand. In fact, the elasticities of the two demands are $-100 \times \frac{7}{100} = -7$ for low demand and $-100 \times \frac{7}{900} = -0.78$ for high demand. Without price discrimination profits have decreased and total welfare increased (as quantity is remained the same but it has been redistributed from high to low demand consumers).

(c) If the low demand was $Q_L^d = 400 - 100P$, things would change dramatically. Intuitively, when the low demand decreases in size price discrimination will lead to lower quantity for this group and lower profits for the firm with respect to the case (a) above. However, when low demand is *too low* with respect to the high demand one, as it is the case here, the company has no interest in considering an aggregate demand composed as the sum of the two. Put differently, if the firm cannot price differentiate it will only consider the high demand, setting $p^H = 9$ and selling $q^H = 700$ only. As an example, consider a theater which faces a group of very high demand consumers and a group of very low demand consumers. Clearly, if the theater cannot price discriminate it has no interest in considering an "average" consumer and will target the very high demand consumers only. Formally, this can be verified noticing that there is a discontinuity in the demand and hence

in the marginal revenue function. This discontinuity is not relevant if the two demands are similar but it is if they are different.
If the company can price differentiate it can charge a different, much lower, price for the low demand and increase its profits. In fact, if the firm can price differentiate it will set $p^L = 3$ and selling $q^L = 100$. By doing so, the company will increase its profits by $(3-2) \times 100 = 100$ and the low demand consumers are better off: $CS^L = \frac{1}{2}(4-3) \times 100 = 50$. Notice that in this case price discrimination increases total welfare as it increases the quantity sold.

**Exercise 4 (4.5 points)** A researcher, by using a sample of 5,128 full time workers in Italy observed in 2005, wants to estimate how the wage of the worker is affected by the working experience, i.e. the number of years the worker has already been working, and nationality, i.e. if the worker is Italian or foreigner. To this end, he estimates two models.
The first one regresses the yearly wage in euro, *wage*, on the variable *lab_exper* indicating the number of years the worker has already been working:

$$wage_i = \alpha_0 + \alpha_1\ lab\_exper_i + u_i \tag{11}$$

The second one adds as additional regressor the origin of the worker, $foreign$, which takes value 0 if the worker is native Italian and 1 if foreign born:

$$wage_i = \delta_0 + \delta_1\ lab\_exper_i + \delta_2\ foreign_i + u_i \tag{12}$$

The OLS estimates of (11) are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{wage}_i = \underset{(272.3)}{11,190.8} + \underset{(21.3)}{355.7}\ lab\_exper_i \qquad R^2 = 0.067 \qquad SER = 13,909 \tag{13}$$

(a) Carefully interpret the coefficients in equations (11) and (12);

(b) Comment the results shown in equation (13);

(c) Using the results in equation (13), test at the $\alpha = 5\%$ level that the coefficient of *lab_exper* is equal to 200 against a bilateral alternative, assuming the 3 OLS assumptions hold;

(d) Starting from results in equation (13), suggest the expected results if you estimate equation (12);

(e) Suppose now that for a computer mistake the original value of the variable *lab_exper* is transformed into $lab\_exper + 2$. How would you expect results in equation (13) will change if the transformed variable is used instead of the original one? Motivate your answer.

**Solution**

(1) The intercept $\alpha_0$ is the expected wage when the regressor is equal to 0, i.e. for a worker with no previous experience. It has both an economic and a geometrical interpretation. $\alpha_1$ is the expected change in wage for an additional year of previous working experience.
The intercept $\delta_0$ is the expected wage when the regressors are equal to 0, i.e. when the worker has no previous experience and he is Italian. As before, it has both an economic and a geometrical interpretation. $\delta_1$ is the expected change in wage for any additional year of previous working experience, holding fixed the nationality of the worker. $\delta_2$ is the expected change in wage for a foreign born worker with respect to an Italian one, holding fixed the number of years of previous working experience.

(2) Results show that the predicted yearly wage for a worker with no previous working experience is 11,190.8 euros and that an additional year of previous experience increases predicted wage of approximately 350 euros per year. The fit of the model ($R^2 = 0.067$) is low, thereby showing that many other variables which explain wage different from the previous working experience (education, tenure, gender, type of task performed, industry where the firm operates, and the like...) are contained in $u$.

(3) By performing a $t$ test of the hypothesis that the coefficient $\alpha_1$ is equal to 200 against a bilateral alternative, we get $t = \frac{355.7-200}{21.3} \approx 7.3$. As this value is outside the acceptance interval $(-1.96, +1.96)$ for $\alpha = 5\%$ we reject the null hypothesis.

(4) If we estimate model (12) we expect: 1) to have a negative estimate for $\delta_2$ as in Italy foreign workers are usually paid less than Italian ones (notice that in other countries, expecially developing countries, foreign workers could be paid more than domestic ones); 2) to have a higher estimate for $\delta_0$ with respect to $\alpha_0$ as the average wage for an Italian worker with no experience is higher than the average wage for a foreign worker with no experience; 3) to have a lower estimate for $\delta_1$ with respect to $\alpha_1$ as foreign workers usually have less experience than Italian ones being usually younger; 4) to have a small increase in the fit of the model (i.e. a small increase in $R^2$ and a small decrease in $SER$) as—among all the factors affecting wages different from experience—nationality of the worker does not play a very important role. As a matter of fact, estimation of model (12) leads to the following results:

$$\widehat{wage}_i = \underset{(520.2)}{11,540} + \underset{(26.2)}{345.9}\ lab\_exper - \underset{(137.0)}{345.5}\ area_i \qquad R^2 = 0.068 \qquad SER = 13,908 \tag{14}$$

(Notice that the value of $\widehat{\delta_1}$ and $\widehat{\delta_2}$ are very similar in absolute values only by hazard. With other datasets the values could be very different.)

(5) If we use the wrong measure of labour experience, the only thing that changes is the estimated intercept. You can verify by considering that $\widehat{\alpha_1^*} = \frac{cov(y,x^*)}{var(x^*)} = \frac{cov(y,x+2)}{var(x+2)} = \frac{cov(y,x)}{var(x)} = \widehat{\alpha_1}$ as adding a constant to a variable does not change neither the variance nor the covariance. Furthermore, $\widehat{\alpha_0} = \bar{y} - \widehat{\alpha_1}\bar{x}$. If we estimate using $x^*$ we get $\widehat{\alpha_0^*} = \bar{y} - \widehat{\alpha_1}\bar{x^*} = \bar{y} - \widehat{\alpha_1}(\bar{x}+2) = \widehat{\alpha_0} - 2 \times \widehat{\alpha_1}$. Estimation of the model using the wrong regressor leads indeed to $\widehat{\alpha_0^*} = 10,479.4 = 11,190,8 - 2 \times 355.7$.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2018-19**
**Exam September, $10^{th}$ 2019**

**EXAM RULES**

You have already been given the two points bonus right after your inscription. If you submit your copy, it will be corrected, the mark cannot be refused but you can keep it on hold ("freeze") and try a second time. If you decide for this second test to be corrected, then the old mark is eliminated, and the final mark is the one of the second test. If you froze the previous mark and then you fail the second test, then you can try again only next academic year.

After February 2020, all frozen marks are cancelled.

**Instructions**: answer to all questions in 1 hour and 45 minutes

**Exercise 1 (8.5 points)** Consider an industry where the technology is such that the total cost for producing the good is equal to $TC = cq^2 + A$, where $c$ and $A$ are positive constants and $q$ is the quantity produced by each firm in the market.

(a) Find the values of $q$ for which the market can be a natural monopoly.

(b) Provide an economic explanation why the size of the interval of $q$ for which we can have a natural monopoly depends negatively on the constant $c$ and positively on the constant $A$.

(c) Find the values of $q$ for which the market can be a natural duopoly.

(d) How would you expect the values found in point (c) be affected if the total cost function becomes $TC = cq^\alpha + A$, with $\alpha > 2$? Motivate your answer.

(e) How the interval previously found can be affected by technological progress?

(f) Explain why it is important for a government to know if an industry is a natural monopoly or not.

**Solution**

(a) By setting the following inequality:

$$cq^2 + A < 2 \times \left[c\left(\frac{q}{2}\right)^2 + A\right] \tag{1}$$

we get:

$$q < \sqrt{\frac{2 \times A}{c}} \tag{2}$$

(b) The optimal number of firms in the market crucially depends on two opposing factors. One is the existence of fixed costs, which pushes towards a single firm being in the market. The other is increasing marginal costs, which calls for a plethora of firms. The higher the parameter $c$ the higher will be the relative importance of this second component, so that the interval in which a natural monopoly exists shrinks.

(c) By setting the following inequality:

$$2 \times \left[c\left(\frac{q}{2}\right)^2 + A\right] < 3 \times \left[c\left(\frac{q}{3}\right)^2 + A\right] \quad (3)$$

we get:

$$q < \sqrt{\frac{6 \times A}{c}} \quad (4)$$

so that the industry is a natural duopoly for

$$\sqrt{\frac{2 \times A}{c}} < q < \sqrt{\frac{6 \times A}{c}} \quad (5)$$

(d) The values found in point (c) (but also those in point (a)) will be affected if the total cost function becomes $TC = cq^\alpha + A$, with $\alpha > 2$. In particular, the interval will move to the left. The reason is that the marginal cost component will become more important and – as stated in point (b) – this leads towards a higher number of firms in the industry for a given quantity $q$.

(e) Technological progress affects the previously found results as it changes the total cost function, in particular by decreasing the total cost of producing a given amount of output. The decrease can concern both fixed cost A and marginal costs. In some cases, the effect on A will prevail whereas in other the effect on marginal costs will be the most important. This must be considered on a case by case basis. For instance, in the telecommunication industry, the decrease in fixed costs has trasformed the natural monopoly of the short distance calls into an industry where several players could coexist.

(f) For a government it is crucial to know if an industry is a natural monopoly or not as if it is, then it necessary to regulate that market, for instance through price regulation to limit the use of market power. If the industry is not a natural monopoly, the government can leave competition between firms to operate without intervention.

**Exercise 2 (8.5 points)** Suppose a market in which a firm is monopolist (incumbent) and another firm is considering whether to enter or not in the market (potential entrant). Suppose that the incumbent—in case the potential entrant enters the market—can either trigger a price war or it can accommodate entry. The payoff for the firms are: $\Pi^m = 7$ (monopoly profit for the incumbent if the potential entrant does not enter); $\Pi^d = 2$ (duopoly profit for both firms if the potential entrant enters and the incumbent accommodates entry); $\Pi^w = -2$ (price war profit for both firms if the potential entrant enters and the incumbent triggers the price war); 0 (profit of the potential entrant if it does not enter the market).

(a) Model the game in normal form and find the Nash Equilibria;

(b) Model the game in extensive form and find the Subgame Perfect Nash Equilibria. Clearly explain why one of the Nash Equilibria found in point (a) is not a SPNE;

(c) Suppose the incumbent can—before the potential entrant decides whether to enter or not—make an irreversible investment $c$ (such as capacity or advertising) which prepares the firm in case of price war. Include this additional stage in the game and find the values of $c$ that can lead the potential entrant not to enter the market. Discuss the role of $\Pi^m$, $\Pi^d$ and $\Pi^w$ for determining these values.

**Solution**

(a) The normal form can be derived by lectures notes. There are two Nash Equilibria: (Accommodate, Enter) and (price war, Not Enter).

(b) The extensive form can be derived by lectures notes. By using backward induction, the only Subgame Perfect Nash Equilibria is (Accommodate, Enter). The other Nash Equilibria (price war, Not Enter) is not SPNE as it is a Nash Equilibrium in the normal form due to the non credible threat that the incumbent will trigger a price war.

(c) The structure of the game if there is a pre entry additional stage in which the incumbent can make an irreversible investment preparing for the price war can be derived from the lectures notes. The value of $c$ which can make the incumbent strategy credible is $c \in (4, 5)$. The investment must be high enough to change the incentive of the incumbent not to trigger the price war ($\Pi^w > \Pi^d - c$) but not too high to alter the will of the incumbent to remain monopolist ($\Pi^m - c > \Pi^d$). The investment must be irreversible as it has to alter payoffs of the incumbent. In other terms, the payoff of the incumbent if it makes the investment, the potential entrant enters and the incumbent accommodates will be $\pi^d - c$ if the investment is irreversible but $\pi^d$ if it is not irreversible as the same investment can be used for other purposes.

**Exercise 3 (8.5 points)** Consider a duopolistic market in which two firms, $A$ and $B$, compete in prices. Demand curves for each firms are:

$$q_A = \frac{p_B}{(p_A)^3}$$

and

$$q_B = \frac{p_A}{(p_B)^3}$$

(a) Find the equilibrium prices, quantities and profits, considering that marginal cost of production are constant, symmetric and equal to $c$;

(b) Discuss qualitatively how your previous answer would change if the marginal costs of production of the two firms are still constant but unequal, with $c_A < c_B$;

(c) Discuss why the product sold in this market has to be considered as non homogenous;

(d) Is this model an address or non-address model of product differentiation?

(e) Compute the cross price elasticities for each product and discuss your result.

**Solution**

(a) For both producers, the profit function is

$$\pi_i = (p_i - c)p_j/p_i^3$$

In turn, the first order condition is

$$\frac{\partial \pi_i}{\partial p_i} = p_j/p_i^3 - 3(p_i - c)p_j/p_i^4 = (p_j/p_i^4)[3c - 2p_i] = 0$$

Imposing symmetry, for both firms the equilibrium price is $\frac{3}{2}c > c$. In turn, the equilibrium quantity is $\frac{4}{9c^2}$ and the profit at equilibrium is $\frac{2}{9c}$.

(b) Intuitively, if $c_A < c_B$, firm $A$ will enjoy of some form of cost advantage which will lead, in equilibrium, to higher profits. However, differently from a situation of product homogeneity, the cost advantage will not translate into a monopolistic situation as the two firms will coexist (unless the cost advantage is very large).

(c) The product is differentiated as even in the case the two firms will set a different price ($p_A \neq p_B$) the demand for both will be strictly positive, a situation impossible in the case of price competition with product homogeneity.

(d) This model belongs to the non address category: it uses a representative consumer which buys a bundle of goods which contains both goods A and B

(e) Cross price elasticity is, for both firms, given by:

$$\varepsilon_{q_i p_j} = \frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i} = +1$$

Unsurprisingly, the value is positive showing that the two goods are substitutes and not complements.

**Exercise 4 (4.5 points)** A researcher, by using a sample of 638 managers, wants to assess whether managers in multinational firms (i.e. firms operating in more than one country) earn more than managers in domestic only firms. To this end, he collects data on managers' yearly wage (*wage*, in thousands US \$), whether the firms is multinational or not (*multinational*, equal to 1 if the firm is multinational and 0 otherwise) and on firm profitability (*ROE*, in percentage terms). The first models regresses the yearly wage on the variable *multinational*:

$$wage_i = \alpha_0 + \alpha_1 \; multinational_i + u_i \tag{6}$$

The second one adds as additional regressor firms' profitability:

$$wage_i = \delta_0 + \delta_1 \; multinational_i + \delta_2 \; ROE_i + u_i \tag{7}$$

The OLS estimates of (6) are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{wage}_i = \underset{(35.8)}{275.3} + \underset{(24.1)}{78.8} \; multinational_i \qquad R^2 = 0.11 \tag{8}$$

(a) Carefully interpret the coefficients in equations (6) and (7);

(b) Comment the results shown in equation (8);

(c) Using the results in equation (8), test at the $\alpha = 5\%$ level that the difference between managers' wage in multinational and domestic firms is equal to 100 against a bilateral alternative, assuming the 3 OLS assumptions hold;

(d) Starting from results in equation (8), suggest the expected results if you estimate equation (7). Motivate your answer;

(e) Suppose now that, for an unvoluntary informatic mistake, the researcher uses only a randomly selected portion of the available sample, namely one half. How would you expect the result of equation (8) to be affected?

(f) Suggest a reasonable value for the *Standard Error of the Regression* (*SER*) in equation (8) knowing that the standard deviation of *wage* is 431?

**Solution**

(a) The intercept $\alpha_0$ is the expected wage when the regressor is equal to 0, i.e. for a manager working in a firm operating only in the domestic market. It has both an economic and a geometrical interpretation. $\alpha_1$ is the expected change in wage for managers working in multinational firms w.r.t. managers working in domestic firms.
The intercept $\delta_0$ is the expected wage when the regressors are equal to 0, i.e. for a manager working in a firm operating only in the domestic market and with no profitability, i.e. a ROE of 0%. As before, it has both an economic and a geometrical interpretation as firms can have null or even negative ROE. $\delta_1$ is the expected change in wage for managers working in multinational firms w.r.t. managers working in domestic firms, holding fixed the profitability of the firm. $\delta_2$ is the expected change in wage for every percentage increase in ROE, holding fixed the nationality of the firm (domestic or multinational).

(b) Results show that the predicted yearly wage for a manager working in a firm operating only in the domestic market is approximately 275 thousands dollars wheareas the predicted yearly wage for a manager working in a multinational firm is approximately $275 + 79 = 354$ thousands dollars. The fit of the model ($R^2 = 0.11$) is low, thereby showing that many other variables which explain wage different from the multinationality of the firm (profitability, ability of the manager, and the like) are contained in $u$.

(c) By performing a $t$ test of the hypothesis that the difference between managers' wage in multinational and domestic firms, i.e. the coefficient $\alpha_1$, is equal to 100 against a bilateral alternative we get $t = \frac{78.8-100}{24.1} \approx -0.8$. As this value is inside the acceptance interval $(-1.96, +1.96)$ for $\alpha = 5\%$ we do not reject the null hypothesis.

(d) If we estimate model (7) we expect: 1) to have a positive estimate for $\delta_2$ as managers' wage is usually linked with the performance of the firms (for instance, through bonuses); 2) to have a lower estimate for $\delta_0$ with respect to $\alpha_0$ as the average wage for a manager when the firm has 0 profitability is less than the average wage when the firms has an average profitability; 3) to have a lower estimate for $\delta_1$ with respect to $\alpha_1$ as multinational firms have usually a higher profitability than domestic firms; 4) to have an increase in the fit of the model (i.e. an increase in $R^2$ and a decrease in $SER$) as—among all the factors affecting managers' wages different from multinationlity of the firm—bonus linked to firm performance can constitute a significant component of managers' wage.

(e) If the sample size is randomly reduced by one half, the only sistematically affected result will be the standard errors of the estimated coefficients. As we have less information in the reduced sample, the parameters will be estimated less precisely. There is no other change which can be predicted a priori, i.e. all the other results will be only be affected randomly and they will be "similar" to the previous ones.

(f) The poor fit of the model ($R^2 = 0.11$) will lead the $SER$ to be closer to its upper bound, the standard deviation of the dependent variable. In turn, as the standard deviation of *wage* is 431, we expect the $SER$ to be around 415.

**Industrial Economics (02OJAPH)**
**Prof. Luigi Benfratello**
**Academic year 2018-19**
**Exam February, $21^{st}$ 2020**

## EXAM RULES

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After this attempt, all frozen marks will be cancelled.

**Instructions**: answer to all questions in 1 hour and 45 minutes

**Exercise 1 (8.5 points)** Consider an industry where the technology is such that the total cost for producing the good is equal to $TC = cq^2 + A$, where $c$ and $A$ are positive constants and $q$ is the quantity produced by each firm in the market.

(a) Find the values of $q$ for which the market can be a natural monopoly.

(b) Provide an economic explanation why the size of the interval of $q$ for which we can have a natural monopoly depends negatively on the constant $c$ and positively on the constant $A$.

(c) Find the values of $q$ for which the market can be a natural duopoly.

(d) Find the values of $q$ for which the market can be a natural triopoly.

(e) What is the optimal number of firms in the market for a value of $q = 20$, $c = 3$ and $A = 48$? Motivate your answer.

**Solution**

(a) By setting the following inequality:

$$cq^2 + A < 2 \times \left[c\left(\frac{q}{2}\right)^2 + A\right] \tag{1}$$

we get:

$$q < \sqrt{\frac{2 \times A}{c}} \tag{2}$$

(b) The optimal number of firms in the market crucially depends on two opposing factors. One is the existence of fixed costs, which pushes towards a single firm being in the market. The other is increasing marginal costs, which calls for a plethora of firms. The higher the parameter $c$ the higher will be the relative importance of this second component, so that the interval in which a natural monopoly exists shrinks.

(c) By setting the following inequality:

$$2 \times \left[ c \left( \frac{q}{2} \right)^2 + A \right] < 3 \times \left[ c \left( \frac{q}{3} \right)^2 + A \right] \tag{3}$$

we get:

$$q < \sqrt{\frac{6 \times A}{c}} \tag{4}$$

so that the industry is a natural duopoly for

$$\sqrt{\frac{2 \times A}{c}} < q < \sqrt{\frac{6 \times A}{c}} \tag{5}$$

(d) By setting the following inequality:

$$3 \times \left[ c \left( \frac{q}{3} \right)^2 + A \right] < 4 \times \left[ c \left( \frac{q}{4} \right)^2 + A \right] \tag{6}$$

we get:

$$q < \sqrt{\frac{12 \times A}{c}} \tag{7}$$

so that the industry is a natural triopoly for

$$\sqrt{\frac{6 \times A}{c}} < q < \sqrt{\frac{12 \times A}{c}} \tag{8}$$

(e) The total cost to produce the quantity $q$, as a function also of the number of firms in the market $n$, is equal to:

$$TC(n, q) = n \times \left[ c \left( \frac{q}{n} \right)^2 + A \right] \tag{9}$$

Miniming this expression with respect to $n$ we find the number of firms which minimizes the cost of producing the quantity $q$, given $c$ and $A$. This solution is:

$$n = \sqrt{\frac{q^2 \times c}{A}} \tag{10}$$

Plugging in the previous equation the values $q = 20$, $c = 3$ and $A = 48$ we get $n = 5$.

**Exercise 2 (8.5 points)**

A market is characterised by an inverse demand curve $p = 4 - Q$ where $Q$ is total quantity. Two firms, $A$ and $B$, are competing *à* la Cournot.

(a) Suppose first that the total cost functions of the two firms are $TC_A(q_A) = q_A$ and $TC_B(q_B) = q_B$. Find the equilibrium price, quantities and profits.

(b) Suppose now that firm $B$ has a total cost function $TC_B(q_B) = C_B + q_B^2$, where $C_B$ is a fixed cost. Find the equilibrium price, quantities and profits.

(c) Suppose now that firm $B$ has a total cost function $TC_B(q_B) = d \times q_B^2$, where $d$ is a positive constant. Find the value of $d$ which makes the quantity produced by firm B the same as the quantity produced it produces in point (b) above. Find the value of $d$ which makes the profits of firm B the same as the profits it has in point (b) above.

(d) Suppose now that a third firm enters the market with a total cost function $TC_C(q_C) = C_C + q_C$, where $C_C$ is a positive constant. Find the equilibrium price, quantities and profits.

## Solution

(a) This is a special case of a Cournot duopoly with symmetric and constant marginal costs (and no fixed costs). Marginal cost is equal to 1 for both firms, the slope and the intercept of the inverse demand function are 1 and 4.
Therefore, we know that:

$$q_1^* = q_2^* = \frac{a-c}{3 \times b} = \frac{4-1}{3 \times 1} = 1$$

Total quantity is

$$Q^* = q_1^* = q_2^* = 2$$

so that equilibrium price is

$$p^* = 4 - Q^* = 4 - 2 = 2$$

At the equilibrium, profits are of course equal for the two firms and they amount to $\pi_i^* = (p^* - 1) \times q_i^* = (2-1) \times 1 = 1$ for $i = A, B$.

(b) This is a case of Cournot duopoly with asymmetric costs where one firm, $A$, has constant marginal costs and no fixed costs whereas firm $B$ has variable marginal costs and fixed costs. To find the equilibrium quantity we have to find the intersection of the reaction functions.
The reaction function of firm $A$, the one with constant marginal cost, is the standard one:

$$q_A^* (q_B) = \frac{a-c}{2b} - \frac{q_B}{2} = \frac{3}{2} - \frac{q_B}{2} \quad (11)$$

The reaction function of firm $B$, the one with non constant marginal costs, must be found by deriving the profit function with respect to $q_B$ and setting it equal to 0 (notice that the reaction function does not depend on the fixed costs):

$$\frac{\partial \pi_B}{q_B} = (4 - q_A - q_B) - q_B - 2q_B = 4 - q_A - 4q_B = 0$$

from which

$$q_B^* (q_A) = 1 - \frac{q_A}{4} \quad (12)$$

Solving the system of the two reaction functions we get:
$q_A^* = \frac{8}{7}$
$q_B^* = \frac{5}{7}$
$Q^* = q_1^* = q_2^* = \frac{13}{7}$
$p^* = 4 - Q^* = 4 - \frac{13}{7} = \frac{15}{7}$
$\pi_A^* = \frac{64}{49}$
$\pi_B^* = \frac{50}{49} - C_B$

(c) The optimal quantity only depends on marginal costs. In turn, it is obvious that the answer to the first question is $d = 1$. This can be proved by solving the model finding a reaction function of firm $B$ which depends on $d$.
A bit more complex is the answer to the second point. To find the value of $d$ such as the profits are the same as in the previous case we have to find the profit function of firm $B$ as a function of

$d$ and equates it to the value $\frac{50}{49} - C_B$.
Analytically:

$$q_A(q_B) = \frac{3}{2} - \frac{q_B}{2}$$
$$\pi_B = (4 - q_a - q_B)q_B - dq_B^2$$
$$\frac{\delta\pi_B}{\delta q_B} = 0 \implies q_B(q_A) = \frac{4 - q_A}{2(1+d)}$$

By substituting $q_A$ inside the reaction function of firm B, we find:

$$q_B = \frac{5}{2}\left(\frac{1}{\frac{3}{2} + 2d}\right); q_A = \frac{3}{2} + \frac{5}{4}\left(\frac{1}{\frac{3}{2} + 2d}\right)$$

Now, we can equate firm B's profit with the one obtained in the previous point:

$$\left[\frac{5}{2} - \frac{5}{4}\left(\frac{1}{\frac{3}{2} + 2d}\right)\right]\left[\frac{5}{2}\left(\frac{1}{\frac{3}{2} + 2d}\right)\right] - d\left[\frac{5}{2}\left(\frac{1}{\frac{3}{2} + 2d}\right)\right]^2 = \frac{50}{49} - C_B$$

The previous expression should be solved wuth respect to $d$.

(d) With there firms, the profit of the three firms become:
$\pi_A = q_A \times (4 - q_A - q_B - q_C) - q_A$
$\pi_B = q_B \times (4 - q_A - q_B - q_C) - q_B^2 - C_B$
$\pi_C = q_C \times (4 - q_A - q_B - q_C) - q_C - C_C$

The FOC are:

$$\frac{\partial \pi_A}{q_A} = 0 \Leftrightarrow (4 - q_A - q_B - q_C) - q_A - 1 = 0 \Leftrightarrow q_A = \frac{3 - q_B - q_C}{2}$$
$$\frac{\partial \pi_B}{q_B} = 0 \Leftrightarrow (4 - q_A - q_B - q_C) - q_B - 2q_B = 0 \Leftrightarrow q_B = \frac{4 - q_A - q_C}{4}$$
$$\frac{\partial \pi_C}{q_C} = 0 \Leftrightarrow (4 - q_A - q_B - q_C) - q_C - 1 = 0 \Leftrightarrow q_C = \frac{3 - q_A - q_B}{2}$$

By using the symmetry between firms $A$ and $C$ we get:
$q_A^* = q_C^* = \frac{4}{5}$
$q_B^* = \frac{3}{5}$
$Q^* = q_1^* + q_B^* + q_B^* = \frac{11}{5}$
$p^* = 4 - Q^* = \frac{9}{5}$
$\pi_A^* = \left(\frac{11}{5} - 1\right) \times \frac{4}{5} = \frac{24}{25} < \frac{64}{49}$
$\pi_C^* = \left(\frac{11}{5} - 1\right) \times \frac{4}{5} - C_C = \frac{24}{25} - C_C < \frac{24}{25}$
$\pi_B^* = \frac{9}{5} \times \frac{3}{5} - \left(\frac{3}{5}\right)^2 - C_B = \frac{18}{25} - C_B < \frac{50}{49} - C_B$

**Exercise 3 (8.5 points)** Consider a standard linear city *à la Hotelling* (i.e unitary length with uniform distribution of the consumers) with linear transportation costs $\theta$. Suppose at first two firms ($a$ and $b$), localized in $x_a = \frac{1}{4}$ and $x_b = \frac{3}{4}$, competing in prices and with null production costs.

(a) Find firms' equilibrium prices and profits;

(b) How would your previous results modify if both firms have a strictly positive marginal cost $c$?

(c) Suppose now that a third firm enters the market. Where this third firm, call it firm $c$, will find it convenient to locate? Motivate your answer.

(d) How would your previous results modify if firm $a$ is located $x_a = \frac{1}{2}$?

**Solution**

(a) This is a standard Hotelling price game with linear transportation costs and locations $x_A = \frac{1}{4}$ and $x_B = \frac{3}{4}$ and null marginal cost of production for the two firms.
The indifferent consumer, located in $x$, will be indifferent between purchasing the good from firm $A$ or purchasing the good of firm $B$. This leads to the following equality:

$$p_A + \theta\left(x - \frac{1}{4}\right) = p_B + \theta\left(\frac{3}{4} - x\right) \tag{13}$$

The indifferent consumer will then be located in:

$$x = \frac{\theta + p_B - p_A}{2\theta} \tag{14}$$

so that the demands for firms $A$ and $B$ will be, respectively, $\frac{\theta+p_B-p_A}{2\theta}$ and $1 - \frac{\theta+p_B-p_A}{2\theta}$.
This leads to the following profit functions:

$$\pi_A = (p_A)\left(\frac{\theta + p_B - p_A}{2\theta}\right) \tag{15}$$

$$\pi_B = (p_B)\left(1 - \frac{\theta + p_B - p_A}{2\theta}\right) \tag{16}$$

The FOCs are:

$$\frac{\partial \pi_A}{\partial p_A} = \frac{1}{2} + \frac{p_B - p_A}{2\theta} - \frac{p_A}{2\theta} = 0 \tag{17}$$

$$\frac{\partial \pi_B}{\partial p_B} = \frac{1}{2} + \frac{p_A - p_B}{2\theta} - \frac{p_B}{2\theta} = 0 \tag{18}$$

Solving the system of two FOCs we obtain:

$$p_A = p_B = \theta \tag{19}$$

$$\pi_A = \pi_B = \frac{\theta}{2} > 0 \tag{20}$$

$$\tag{21}$$

(b) If firms incur in a positive marginal cost $c$, their payoff is modified into

$$\pi_A = (p_A - c) \times \left(\frac{1}{2} + \frac{p_B - p_A}{2\theta}\right)$$

$$\pi_B = (p_B - c) \times \left(\frac{1}{2} - \frac{p_A - p_B}{2\theta}\right)$$

In turn, by solving the system of the FOCs, we obtain that:

$$p_A = p_B = \theta + c$$

Since marginal costs are charged directly on the price, profits will be the same.

(c) Now a third firm $C$ is entering the market. In order to find its most convenient location, we should analyse some cases.
First of all, let us dwell on the the case according to which $C$ locates between $A$ and $B$. As the Hotelling model with three firms implies, the market is now characterized by two indifferent consumers, $x_1$, namely the one between $a$ and $c$, and $x_2$, the one between $c$ and $b$.
Let's find the expression of those indifferent consumers:

$$P_A + \theta\left(x_1 - \frac{1}{4}\right) = P_C + \theta(x_C - x_1)$$

Both firm A and firm B cannot modify their prices, thus $P_A = P_B = \theta$

$$\theta + \theta\left(x_1 - \frac{1}{4}\right) = P_C + \theta(x_C - x_1)$$
$$x_1 = \frac{4P_C + 4\theta x_C - 3\theta}{8\theta}$$

For what concerns $x_2$:

$$P_C + \theta(x_2 - x_C) = \theta + \theta\left(\frac{3}{4} - x_2\right)$$
$$x_2 = \frac{-4P_C + 4\theta x_C + 7\theta}{8\theta}$$

The overall demand faced by $C$ is given by $x_2 - x_1$:

$$D_C = x_2 - x_1 = \frac{10\theta - 8P_C}{8\theta}$$

Notice that $x_C$ does not affect the expression of the demand; practically it means that $C$ is indifferent among all the possible locations in the interval $\left(\frac{1}{4}; \frac{3}{4}\right)$. By inserting the expression of the demand in $c$'s profit, we can maximize it with respect to $P_C$:

$$\pi_C = P_C\left(\frac{10\theta - 8P_C}{8\theta}\right)$$
$$\frac{\delta\pi_C}{\delta p_C} = \frac{10\theta - 8P_C}{8\theta} - \frac{P_C}{\theta} = 0$$
$$\downarrow$$
$$P_C = \frac{5\theta}{8}; \pi_C = \frac{25\theta}{64}$$

The alternative available to $c$ is to locate exactly either on $1/4$ or $3/4$, by replacing one of the two incumbents. The new entrant can deploy this strategy if and only if it is willing to reduce the price below $\theta$, so to undercut its rival. Let's suppose, arbitrarily, that $C$ decides to replace firm $A$.

$$P_C + \theta\left(x - \frac{1}{4}\right) = P_B + \theta\left(\frac{3}{4} - x\right) \implies$$
$$x = 1 - \frac{P_C}{2\theta}$$

$x$ is exactly the demand faced by $c$. Then we can write its profit:

$$\pi_C = P_C\left(1 - \frac{P_C}{2\theta}\right)$$

The optimum price is $P_C = \theta$, but this price is unfeasible since it would rule out any undercut strategy. $P_C$ is then $\theta - \epsilon$ with $\epsilon$ suitably small.
We should now compare the two profits:

$$(\theta - \epsilon)\left(1 - \frac{(\theta - \epsilon)}{2\theta}\right) \geq \frac{25}{64}\theta$$

This inequality holds if:

$$\epsilon \leq \sqrt{\frac{7}{32}}\theta$$
$$\sqrt{\frac{7}{32}} < \frac{3}{5}$$

where 3/5 is the price reduction of firm $c$, if it locates between the incumbents. Thus is always convenient to replace one of the two firms.

(d) Point (a) modifies as follows:

$$P_A + \theta\left(x - \frac{1}{2}\right) = P_B + \theta\left(\frac{3}{4} - x\right)$$
$$x = \frac{4P_B - 4P_A + 5\theta}{8\theta}$$
$$D_A = x; D_B = 1 - x$$

As usual we plug the expressions of demands inside firms' profit and we maximize with respect to $P$

$$\pi_A = P_A\left(\frac{4P_B - 4P_A + 5\theta}{8\theta}\right)$$
$$\pi_B = P_B\left(\frac{4P_A - 4P_B + 3\theta}{8\theta}\right)$$

FOCs lead to:

$$P_A = \frac{13}{12}\theta$$
$$P_B = \frac{11}{12}\theta$$
$$D_A = \frac{13}{24}$$
$$D_B = \frac{11}{24}$$
$$\pi_A = \frac{169}{288}\theta$$
$$\pi_B = \frac{121}{288}\theta$$

Firm's $A$ profit is bigger than the one found at point (a), while the opposite occurs for firm $B$.

For what concerns point (b), marginal costs will be charged on the price:

$$P_A = \frac{13}{12}\theta + c$$

$$P_B = \frac{11}{12}\theta + c$$

Without diving in computations, point (c) can be solved intuitively. From new entrant's perspective, it is convenient to replace the firm in 1/2, and undercut.

**Exercise 4 (4.5 points)** A researcher, by using a sample of 982 firms operating in the chemical and textile industries, wants to assess how *R&D intensity* of the firm, i.e. the ratio expenditures in Research and Development *R&D* over sales (measured by the variable *R&D_int*, in percentage) is affected by firm size (variable *size*, i.e. sales measured in billions euros) and by the specific industry (i.e. chemical or textile) the firms is operating in (dummy variable *chemical*, equal to 1 if the firm operates in the chemical industry and 0 otherwise). The first model he estimates regresses the *R&D* intensity on firm size:

$$R\&D\_int_i = \alpha_0 + \alpha_1\ size_i + u_i \tag{22}$$

In the second one the researcher adds as additional regressor the chemical industry dummy:

$$R\&D\_int_i = \delta_0 + \delta_1\ size_i + \delta_2\ chemical_i + u_i \tag{23}$$

The OLS estimates of (22) are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{R\&D\_int}_i = \underset{(0.135)}{0.722} + \underset{(2.140)}{5.503}\ size_i \qquad R^2 = 0.01 \tag{24}$$

The OLS estimates of (23) are (standard errors in round brackets below the corresponding coefficient):

$$\widehat{R\&D\_int}_i = \underset{(0.156)}{0.344} + \underset{(2.173)}{3.207}\ size_i + \underset{(0.259)}{1.214}\ chemical_i \qquad R^2 = 0.03 \tag{25}$$

Sample means for the variables *size* e *R&D_int*, for the whole sample and for each industry are:

| | *size* | *R&D_int* |
|---|---|---|
| whole sample | – | 0.866 |
| chemical | – | 1.697 |
| textile | 0.016 | 0.397 |

(a) Carefully interpret the coefficients in equations (22) and (23);

(b) Comment the results shown in equations (24) and (25);

(c) Using the results in equation (25), test at the $\alpha = 5\%$ level that the difference in *R&D* intensity between chemical and textile is equal to 2% against a bilateral alternative, assuming the 3 OLS assumptions hold;

(d) Starting from results in equations (24) and (25), do you think the average size of the firms is larger or smaller in the chemical or in the textile industry? Motivate your answer on statistical grounds;

(e) Suppose now that, instead of estimating equation (24) with the full sample of the data you are estimating equation (24) with chemical firms only. Provide reasonable values for your results.

**Solution**

(a) The intercept $\alpha_0$ is the expected *R&D* intensity when the regressor is equal to 0, i.e. for a firm of with size equal to 0, i.e. without any sales. It only has a geometrical interpretation as economically cannot be a firm without sales. $\alpha_1$ is the expected change in *R&D* intensity when size increases of one unit, i.e. when it increases of one billion euros. All results must be interpreted in percentage as the dependent variable is in percentage terms.
The intercept $\delta_0$ is the expected wage when the regressors are equal to 0, i.e. for a firm with size equal to 0, i.e. without any sales, and operating in the textile industry. As before it only has a geometrical interpretation. $\delta_1$ is the expected change in *R&D* intensity when size increases of one unit, i.e. when it increases of one billion euros, holding the chemical dummy fixed, i.e. for a firm either in the textile or in the chemical industry. All results must be interpreted in percentage as the dependent variable is in percentage terms.

(b) Results in eq (24) show that for every increase in one billion euros in size, the predicted *R&D* intensity increases of 5.5%. The value of the estimated intercept (0.722) is not important economically. The fit of the model is quite poor, highlighting the presence of many other factors affecting the decision to perform *R&D* activities in $u$: the specific industry (textile or chemical), the age of the firm, the high/low quality of the product, the specific subindustries firms are operating in (e.g. pharmaceutical or base chemical).
Results in eq (25) show that for every increase in one billion euros in size, holding the industry constant, the predicted *R&D* intensity increases of 3.2%. Firms operating in the chemical industry have, holding size constant, a higher level of *R&D* intensity (1.2%) than firms in the textile industry. This is reasonable on economic grounds, as the textile industry relies less on *R&D* than chemical one (consider that pharmaceutical is a branch of the chemical industry). The fit of the model obviously does not decrease but it increases only of a moderate amount showing that the specific industry is only one of many other factors affecting *R&D* omitted in equation (24).

(c) By performing a $t$ test of the hypothesis that the difference in *R&D* intensity between chemical and textile is equal to 2% against a bilateral alternative we get a value of the t-statistics equal to $t = \frac{1.214-2}{0.259} \approx -3$. As this value is outside the acceptance interval $(-1.96, +1.96)$ for $\alpha = 5\%$ we do reject the null hypothesis.

(d) The average size of firms in the chemical industry is larger than the one in the textile industry. In fact, in equation (24) the omission of the chemical dummy leads to an upward bias of the coefficient of size (5.503 vs 3.207). As the coefficient of the dummy chemical in equation (25) is positive, there must be a positive correlation between the dummy chemical and size, i.e. firms in the chemical industry are on average larger than those in the textile industry. As a matter of fact, the average size of firms in the chemical industry is 0.043 which is larger than the average size of firms in the textile industry, 0.016.

(e) The starting point is the intercept estimated using only the chemical sample, the predicted value of the dependent variable for firms in the chemical industry when size is equal to 0. It will be approximately equal to the value $0.344 + 1.22 = 1.56$. We also know that the regression must pass through the center of the data and that the average size is larger for chemical firms than textile ones (as shown in the previous point). The slope will then be $\frac{R\&\bar{D}_i nt - \hat{\alpha}_0}{\bar{size}}$. Then it must then be $\frac{1.697-1.56}{\bar{size}} = \frac{0.137}{\bar{size}}$. As the size in chemical industry is larger than in textile industry, plugging at the denominator the value 0.016 leads to an upper bound of approximately 8, i.e. the slope must be less than 8. As a matter of fact, the estimated equation using only the chemical sample leads to the following results:

$$\widehat{R\&D\_int}_i = \underset{(0.369)}{1.521} + \underset{(3.939)}{4.087}\, size_i \qquad R^2 = 0.01 \tag{26}$$