# Inference

21 June 2021

## Exercise 1

Let $\mathbf{x}^* = (3, 4, 2, 7, 4, 5, 8, 1, 0, 0)$ a realization of i.i.d random variables from a population with pmf

$$f(x, \theta) = \exp(-\theta)\frac{\theta^x}{x!} \quad x = 0, 1, 2, \ldots, \quad \theta > 0$$

1. Compute the maximum likelihood estimator of the parameter $\theta$;
2. Compute the probability of the event $x_1^* = 1$ and, observing that it is a function of the unknown parameter $\theta$, compute the maximum likelihood estimate of this quantity.

## Exercise 2

Let $X_1, \ldots, X_n$ be a random sample of size $n = 3$ from a Poisson population of parameter $\theta$. We want to test

$$H_0 : \theta = \theta_0 = 2 \quad \text{against} \quad H_1 : \theta = \theta_1 = 1$$

1. Find a best rejection region of size $\alpha$ for testing the above system of hypothesis
2. Compute $\alpha$ and the power of the test assuming a threshold $c_\alpha$ equal to 1.
3. Given the value of $\alpha$ computed in the previous point, build a GLRT test

$$H_0 : \theta = \theta_0 = 2 \quad \text{against} \quad H_1 : \theta \neq \theta_0$$

4. apply the test to the observed sample $\mathbf{x}^* = (10, 5, 6)$ and report your conclusions.

## Exercise 3

Define the Cramér-Rao Inequality and the Expected Fisher Information, specifying their role in the statistical inference.