

26 settembre

Standard formula → applied to every insurance company; even with great differences in risk profiles.

ORSA: own risk and solvency assessment. What is the meaning of this ORSA? Each company is required to apply their own analysis, quantitative and qualitative in order to see if the risk profile of each company is really taken in a good approximation by the Standard formula. If this is not true you have to see why and you have to quantify ORSA and you have to report to the supervisory authority.

And obviously it is better if you report to the supervisory authority before that supervisory authority discovers that. The SA has the power on the European legislation to require a capital add-on, very often used also in banking supervision (Basel 2 and Basel 3). The main point to analyse is that when there is not a good comprehension from the company of its own risk profile the SA can impose a capital add-on. This was typical of the bank sector, now starts to be done also in insurance sector.

We have to remember that SII entered in force in 2016 and so we have just 2 years' experience and so it is clear that there will emerge some drawback of the Standard formula. Also regarding the risk governance in the company the SA can apply capital add-on → not only on quantitative but also on qualitative basis.

Data quality → qualitative requirement regarding data. It's a requirement from the insurance legislation for SII and you have to show and prove to the SA that your quantitative analysis are based on data that respect that data quality and if you cannot prove it you can have some problems (capital add-on)

We can see that the capital add-on is not only for quantitative requirements but also qualitative.

We will speak about short-term analysis, touching in particular two points: estimation of the ruin probability of and the estimation of the Capital at Risk.

The concept of capital at risk is connected to the capital requirement for non-life company, especially for non-life underwriting risk.

Solvency II has many sources of risks: main of these are six: non-life underwriting risk, life underwriting risk, health underwriting risk and then there are the same present in banking sector (credit risk also known as counterpart default risk, operational risk and market risk).

Market risk is the main source of risk for an insurance company, especially if in life; for non-life depend in fact sometimes it's larger non-life underwriting risk sometimes the market risk (depend in particular on the LoB).

Inside the non-life underwriting risk there are three main sources of risk: premium risk, reserve risk and CAT risk.

Premium risk: claims and expenses are much more than written premium; we have to use probability to estimate this kind of risk.

We will see stochastic models and especially in life because in particular for mortality and longevity we will use them in order to compute the risk.

Why stochastic models are useful? We already have the Standard Formula, is necessary the study of these stochastic models? Yes, because there are important innovations in solvency II with respect to Solvency I; the insurance companies can use internal models and apply the results of your internal models, stochastic or not, instead of the Standard Formula. Obviously, the internal model must be approved by the SA

Big groups already use internal models for the full risk (or for the major part) → Allianz, Generali, AXA, Aviva.

If an internal model is used for the headquarter for a group and you want to apply it also all over the branches of the group, in other countries, you have to receive the approval also from the other SAs. This is the reason why we'll see in the documents issued by EIOPA that exists the "college of supervisors". Ex: if Generali wants

to apply the internal model not only for the headquarter but also for all the branches, the college is composed by German supervisor, French supervisor,...

EIOPA: European guardian of the insurance supervisors but it cannot make a sort of inspection on the local basis. It can try to harmonise all the legislations, all the guidelines for Solvency II.

We will see a long-term analysis for capital at risk. We are not only interested in 1 year term but also 3, 5 or more years term.

Other two important things are the risk measure (we have already analyse the VaR, most popular but exists a lot of risk measures, one of this is the TVaR) + measures of profitability (ex. RoE) because is clear that the matter is not only related to risk but also to profitability because exists a trade-off between the two because you cannot have very low risk with very high profitability. What is important for us is how to compute these risk adjustment measures.

Very important is also the impact that reinsurance has on risk and profitability; the reinsurers can apply a price in order to get a profit because he is risk-taker and so apply the price for the risk. Otherwise is useless an investment without a return on the investment of capital. The point is what is the right price? Depend on what is asking to you the supervisors (the SA is not really interested in profitability, is better for "him" that you have enough OF to cover the risks, if your net asset value is enough to cover the majority of your risks, what is Solvency II is called SCR, minimum level required by the SA). On the other side, we have the company and the shareholders because they are more interested in profitability; the point is that there is a trade-off. Obviously, the SA prefers to have much more OF, much more than the minimum required but, on the other side, is necessary have also enough profitability, more than the minimum. The company would like to have the maximum profitability but it's clearly impossible because it means having also the maximum risk. And are the OF enough to cover the maximum risk? Depend on how strong the company is and if the shareholders are strong to cover the risks.

- ➔ Trade-off between the SA and the top management.
- ➔ Which is the best strategy in order to respect both the parts? This is a problem of optimization with constraint. And in reality the optimal solution doesn't exist. Is possible to find the optimal solution under the constraint of the legislation, of internal organization,...

We have said that we will see the risk theory applied to life insurance. In risk theory 1, we have seen the collective approaches and the individual approaches. Collective approaches, under specific assumptions, are used in non-life while is better not use them for life insurance. For life is better the application of individual approaches.

Ex: In the portfolio of a life company, we have term insurance, I have short capital and in case of death the company has to pay. ➔ the company is exposed to mortality risk; but each of us has a different capital issue according to the perception of the risk, if I insure high capital I have higher premium.

Omans Formula ➔ provides us the technical results of a life insurance company. It provides the demographical result and investment result; then the profit (or losses) for lapses and for expenses loadings. Medium-long term insurance business (different from the MTPL, generally of 1 year) where generally the coverages are of 10, 20 years and if are present also some annuities we have to consider a duration of 40, 50 years maybe.

Sum at risk: important concept in life insurance. It's the capital insured less the mathematical reserve. It's easy as a concept but it changes according to the duration, if you use it at the beginning or at the end of the contract. For term insurance is easy; for pure endowment (if I survive at the end of the contract I get the capital insured)

Pure endowment + term insurance = endowment ➔ the Sum at risk is always the capital insured less the mathematical reserve

The issue is understand how the mathematical reserve moves. If I have very small sum at risk means that I have very small demographic risk and vice versa.

For pure endowment and annuities is not so easy the sum at risk, also because the contracts have a complete different structures.

Then we will see the International Actuarial Association: association of all the national actuarial associations. In Italy we have the Istituto Italiano degli Attuari (IIA) and the National Order of Actuaries. In UK for example there is the Institute of Actuaries (for England) and the Faculty of Actuary (for Scotland).

International Actuarial Association is very important and very active in particular on the insurance regulation and on the requirements.

We will see also Solvency II. It's based on three points:

- Directive of 2009
- Delegated act of 2014 (in force starting from 2015)
- Quantitative impact studies (QIS): studies made by EIOPA (before 2011, it was named CEIOPS, Committee of European Insurance and Occupational Pensions Supervisors) in order to analyse the structure Standard Formula and to see which was the result for the market. In practise, in order to make a sort of exercise with some insurance companies in order to see is which way the formula was working in reality, to see if they were overestimating or underestimating some risks. The number 1 so not so important, while the number 2 (for premium and reserve risk) was very well structured.

We will mainly focus on Standard Formula in order to compute the MCR but much more the SCR (in particular because when we speak about solvency ratio we make reference on the ratio between OF and SCR).

We will see also some internal models that use stochastic models to estimate the risks. We have also something in the middle between Standard Formula and internal model that are the USP, undertaking specific parameters. If the insurance company doesn't want to apply an internal model because takes a lot of time and money and it's not easy to have all the structures for applying an internal model, but the company wants something in the middle because the Standard Formula is not applying very well to your own business, it can use the USP (that are allowed only for premium and reserve risk).

If the company is allowed to use USP instead of Standard Formula and in the calculation for premium and reserve risk for specific LoB for which the company has required the authorization, it can use its own data in order to see the results about the volatility. It's not a partial internal model, the underlying model is still the regulation. The rules are inside the delegated act, the company is not free to use whatever it likes. The company cannot choose its own model but can use the own data inside the model that is fixed.

In particular there is just one model for premium risk and two models for reserve risk; one of these two for reserve risk is very relevant in actuarial technique because is the ?????????? formula, included inside the Delegated Act. This formula is based on the use of all the data in the run-off triangle, under the Chain-Ladder framework.

We can start with a review.

On the long-term period, we have studied in TDR1 the expectation of capital ratio.

$$\text{Capital ratio} = \frac{U}{B}$$

Where U risk reserve, B amount of written premium.

Risk reserve (amount, called OF in Solvency II, is the available capital to face the risk) \neq Reserve risk (risk).

The expectation of the capital risk and now we'll analyse the volatility, the standard deviation and the skewness.

In the upper figure, we have 5 years, medium-long term analysis and in generally we don't take more than 5 years because the assumptions that we make are strong and the probability that the process will change after 5 years is very high.

FIGURE 2a: results of 500 simulations of the ratio $u=U/B$ for the Small Insurer ($u_0=25\%$, $n_0=1.000$ and $c_z=10$) – Results gross of reinsurance.

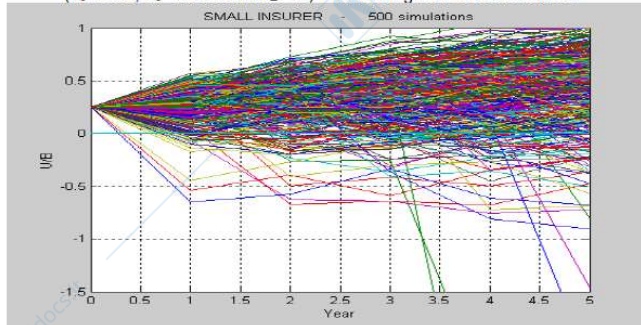
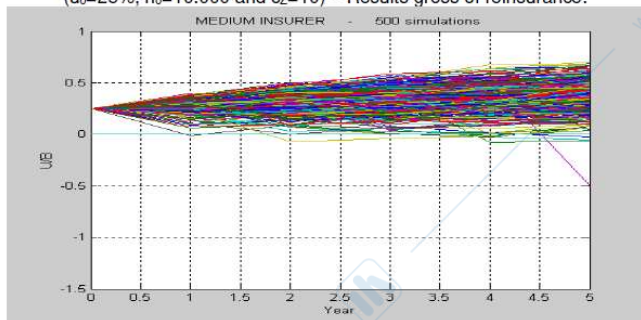


FIGURE 2b: results of 500 simulations of the ratio $u=U/B$ for the Medium Insurer ($u_0=25\%$, $n_0=10.000$ and $c_z=10$) – Results gross of reinsurance.



For listed company is required 3 years analysis.

Here we have 5 years simulation. How is moving the risk reserve divided by the amount of premium. The value starts from more or less 25%. We have 200 simulations. Not enough, a company cannot base the capital requirement analysis on 200 simulations.

X: amount of claims

When we speak about risk we make reference on the downside risk, the risk of unfavourable results and so you ruin. But, in general, is possible to have both an upper side of the risk. And if you are a shareholder you are also interested in volatility of the upper side.

We can see than some years, you are very lucky and there is an increase in the capital

ratio; you can also be very unlucky and be on the downside.

The capital at risk measure is just measuring the downside risk, is measuring, if you have very unfavourable event, how much capital you must have to get available initially to cover this unfavourable event.

n: expected number of claims in the portfolio.

Here we have 1000 (very small).

We can see the difference from the lower figure, where we have more or less the same parameters but with a higher $n=10.000$. We have lower volatility, and this is clear from the fact that the very low number of paths go under the zero.

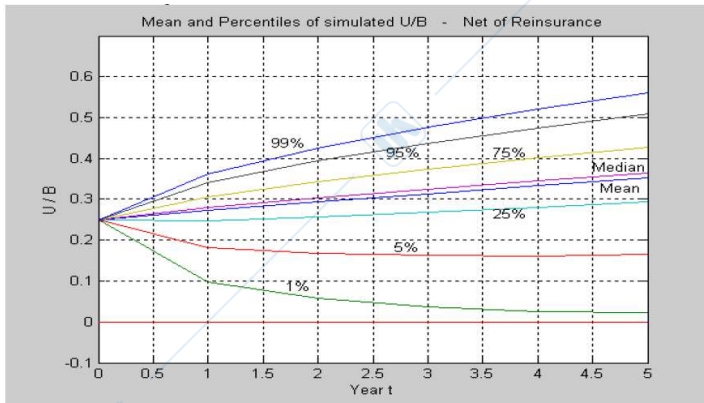
Why we have such a difference? We have the same parameters except for the number of the expected claims.

The initial value of the capital ratio is the same, more or less 25%. This because is proved that the ratio doesn't depend on the dimension, but on the parameters, lambda for example, the safety loading,...

Obviously if the two company start from the same level of capital (U, not the ratio U/B), the largest company is riskier because for example 10.000 million are a lot for the small company but not for the medium company. This because the two companies have the same amount to cover different risks, the greatest company has larger risk because it has $n=10.000$.

When we have the capital ratio we have the situation of the figure because there is diversification, it's working on the relative index. → you are not starting with the same capital but with the same ratio (25% of the volume of gross premium). So, thanks to diversification, we have smaller volatility.

We will also analyse the confidence region of the capital ratio in order to understand the percentiles of the distribution. I will have 5 probability distributions of my capital ratio. I can construct simulated distribution for time 1 based on 200 simulations; than the same for the time 2, 3 and so on.



You can also construct the confidence region using the percentile of the 5 distributions, according to your level of confidence (1%, 5%, 95%, 99%,...). If you take the percentile 1% and 99% I know that 1% of the simulations are under that value, 1% upper the 99% percentile and so 98% inside the region.

Obviously, we can see that the mean is in the middle between the 25° and 75° percentile.