FORMULE DI TRIGONOMETRIA

Relazioni tra funzioni goniometriche

$$(\sin x)^2 + (\cos x)^2 = 1$$

$$(\sin x)^2 = \frac{(\tan x)^2}{1+(\tan x)^2} \qquad (\cos x)^2 = \frac{1}{1+(\tan x)^2}$$

$$\sin(\tfrac{\pi}{2} - x) = \sin(\tfrac{\pi}{2} + x) = \cos x \qquad \cos(\tfrac{\pi}{2} - x) = -\cos(\tfrac{\pi}{2} + x) = \sin x$$

$$\sin(\pi - x) = -\sin(\pi + x) = \sin x \qquad \cos(\pi - x) = \cos(\pi + x) = -\cos x$$

Formule di addizione e sottrazione

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \qquad \tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Formule di duplicazione e bisezione

$$\sin(2x) = 2\sin x\cos x \qquad \cos(2x) = (\cos x)^2 - (\sin x)^2$$

$$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-\cos x}{2}}, \quad x \in (0, 2\pi) \qquad \cos\left(\frac{x}{2}\right) = \sqrt{\frac{1+\cos x}{2}}, \quad x \in (-\pi, \pi)$$

$$\tan\left(\frac{x}{2}\right) = \sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}, \quad x \in (0, \pi)$$

Formule parametriche ($t = \tan(\frac{x}{2})$)

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad \tan x = \frac{2t}{1-t^2}$$

Formule di prostaferesi

$$\sin\alpha + \sin\beta = 2\sin(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2}) \qquad \sin\alpha - \sin\beta = 2\cos(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})$$

$$\cos\alpha + \cos\beta = 2\cos(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2}) \qquad \cos\alpha - \cos\beta = -2\sin(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})$$

$$\tan\alpha \pm \tan\beta = \frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}$$

LIMITI NOTEVOLI

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

$$\lim_{x\to 0}\frac{e^x-1}{x}=1 \qquad \lim_{x\to 0}\frac{\log(1+x)}{x}=1$$

$$\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x=e \qquad \lim_{x\to 0}\frac{\log_a(1+x)}{x}=\frac{1}{\log a}, \qquad a>0,\ a\neq 1$$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\log a, \qquad a>0 \qquad \lim_{x\to 0}\frac{(1+x)^\alpha-1}{x}=\alpha, \qquad \alpha\in\mathbb{R}.$$

DERIVATE

$$\frac{d}{dx}\sin x=\cos x \qquad \frac{d}{dx}\arcsin x=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos x=-\sin x \qquad \frac{d}{dx}\arccos x=-\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x=1+(\tan x)^2=\frac{1}{(\cos x)^2} \qquad \frac{d}{dx}\arctan x=\frac{1}{1+x^2}$$

$$\frac{d}{dx}x^\alpha=\alpha x^{\alpha-1} \qquad \frac{d}{dx}f(x)^{g(x)}=f(x)^{g(x)}\left[g'(x)\log f(x)+\frac{g(x)}{f(x)}f'(x)\right]$$

$$\frac{d}{dx}e^x=e^x \qquad \frac{d}{dx}a^x=a^x\log a$$

$$\frac{d}{dx}\log x=\frac{1}{x} \qquad \frac{d}{dx}\log_a x=\frac{1}{x\log a}$$

$$\frac{d}{dx}\sinh x=\cosh x \qquad \frac{d}{dx}\cosh x=\sinh x$$

SVILUPPI DI TAYLOR

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots + \frac{1}{n!}x^n + o\left(x^n\right)$$

$$\log\left(1+x\right) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots \pm \frac{1}{n}x^n + o\left(x^n\right)$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots \pm \frac{1}{(2n-1)!}x^{2n-1} + o\left(x^{2n}\right)$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots \pm \frac{1}{(2n)!}x^{2n} + o\left(x^{2n+1}\right)$$

$$\tan x = x + \frac{1}{3}x^3 + o\left(x^4\right)$$

$$\arctan x = x - \frac{1}{3}x^3 + o\left(x^4\right)$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + o\left(x^3\right)$$