

FORMULARIO di TRIGONOMETRIA

$$\sin^2 x + \cos^2 x = 1 \quad -1 \leq \sin x \leq 1 \quad -1 \leq \cos x \leq 1 \quad \forall x \in \mathbf{R};$$

$$\begin{array}{ccccccccc} \sin 0 = 0 & \sin \frac{\pi}{2} = 1 & \sin \pi = 0 & \sin \frac{3}{2}\pi = -1 & \sin \frac{\pi}{6} = \frac{1}{2} & \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} & \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \\ \cos 0 = 1 & \cos \frac{\pi}{2} = 0 & \cos \pi = -1 & \cos \frac{3}{2}\pi = 0 & \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} & \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} & \cos \frac{\pi}{3} = \frac{1}{2} \end{array}$$

seno è funzione **dispari**: $\sin(-x) = -\sin x \quad \forall x \in \mathbf{R};$

coseno è funzione **pari**: $\cos(-x) = \cos x \quad \forall x \in \mathbf{R}.$

$$\begin{array}{cccc} \sin\left(\frac{\pi}{2} - x\right) = \cos x & \sin\left(\frac{\pi}{2} + x\right) = \cos x & \sin(\pi - x) = \sin x & \sin(\pi + x) = -\sin x \\ \cos\left(\frac{\pi}{2} - x\right) = \sin x & \cos\left(\frac{\pi}{2} + x\right) = -\sin x & \cos(\pi - x) = -\cos x & \cos(\pi + x) = -\cos x \end{array}$$

Formule di addizione e sottrazione

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Formule di prostaferesi

$$\sin \alpha \pm \sin \beta = 2 \sin \left(\frac{\alpha \pm \beta}{2} \right) \cos \left(\frac{\alpha \mp \beta}{2} \right)$$

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$$

Formule parametriche

Posto $t = \tan \frac{x}{2}$ si ha:

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\tan x = \frac{2t}{1-t^2}$$

Formule di duplicazione

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$$

dalla precedente si ottiene:

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

Formule di Werner

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

Formule di bisezione

$$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}, \quad x \in (0, 2\pi)$$

$$\cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}}, \quad x \in (-\pi, \pi)$$

$$\tan \frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}, \quad x \in (0, \pi)$$