Politecnico di Milano – School of Industrial and Information Engineering

**Logistics Management**

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**COLLECTION OF FURTHER EXERCISES – 1st PART (WAREHOUSE DESIGN)**

## TRADITIONAL WAREHOUSES - SOLUTIONS

### Exercise 1

1. Required storage capacity of the system

   The required storage capacity can be calculated from Table 1. Let *j* denotes the generic month and *i* the generic product family:

$$SC_{req} = \max_j \sum_{i=F1}^{F12} SC_{ij} = 8{,}050\ pallet\ loads$$

2. Storage system configuration

$$H\ (bay\ height) = H_{PL} + beam\ height + c = 1.4 + 0.1 + 0.15 = 1.65\ m$$

$$L(bay\ width) = NPB \cdot L_{PL} + (NPB + 1) \cdot d + upright\ width = 3 \cdot 0.8 + 4 \cdot 0.08 + 0.1 = 2.82\ m$$

$$D = D_{PL} + \frac{e}{2} = 1.2 + \frac{0.16}{2} = 1.28\ m$$

$$Module\ area = Module\ width \cdot L = (2 \cdot D + AW) \cdot L = (2 \cdot 1.28 + 2.5) \cdot 2.82 = 14.27\ m^2$$

$$NL = \left\lfloor \frac{H\ max_{fork}}{H} \right\rfloor = \left\lfloor \frac{8}{1.65} \right\rfloor = 5\ levels$$

$$AUR = \frac{PL/module}{Module\ area} = \frac{2 \cdot NPB \cdot NL}{(2 \cdot D + AW) \cdot L} = \frac{30}{14.27} = 2.10\ \ PL/m^2$$

$$Area = \frac{SC_{req}}{AUR} = \frac{8{,}050}{2.10} = 3{,}833\ m^2$$

   As the I/O is located at the corner of the warehouse, the optimal shape is a square:

$$U = V = \sqrt{Area} = 61.88\ m$$

   The number of aisles can be calculated as:

$$NA = \left\lceil \frac{U}{Module\ width} \right\rceil = 13\ aisles$$

   The number of bay columns NC:

$$NC = \left\lceil \frac{SC}{2 \cdot\ NPB \cdot NA \cdot NL} \right\rceil = 21\ columns$$

   The real dimensions of the storage area are:

$$U_{real} = NA\ \cdot Module\ width =\ 13 \cdot\ (2 \cdot 1.28 + 2.5) = 65.8\ m$$

$$V_{real} = NC \cdot\ L = 21 \cdot 2.82 = 59.2\ m$$

   The real storage capacity is:

$$SC_{real} = 2 \cdot NPB \cdot NL \cdot NC \cdot\ NA = 2 \cdot 3 \cdot 5 \cdot 21 \cdot 13 =\ 8{,}190\ pallet\ loads$$

3. Number of straddle reach trucks

   To assess the number of required trucks we need to assess the average time required to complete a single command cycle, which consists of two components: fixed and variable time. The fixed time is given (i.e. 55 s/cycle). As regards the variable time:

$$VT_{SC} = \frac{P}{S_H} + \frac{S}{S_V} = \frac{2 \cdot (\frac{U_{real}}{2} + \frac{V_{real}}{2} + cross\ aisle\ width)}{S_H} + \frac{H \cdot (NL - 1)}{S_V} = \frac{131}{2.2} + \frac{6.6}{0.3} = 81.5\ s$$

The total time is:

$$T_{SC} = VT_{SC} + FT_{SC} = 81.5 + 55 = 136.5\ s$$

Based on the total time, the average throughput capacity of a single truck is:

$$TC_{truck} = UF \cdot \frac{3{,}600}{T_{SC}} = 1 \cdot \frac{3{,}600}{136.5} = 26.36\ single\ command\ cycles/h$$

And the overall number of trucks required:

$$N_{trucks} = \left\lceil \frac{TC_{req}}{TC_{truck}} \right\rceil = \left\lceil \frac{110}{26.36} \right\rceil = 5\ trucks$$

$$TC_{real} = TC_{truck} \cdot N_{trucks} = 26.36 \cdot 5 = 132\ single\ command\ cycles/h$$

4. Labour cost for a single command cycle

The labour cost for a single command cycle is:

$$Labour\ cost = \frac{136.5\ [s/cycle]}{3{,}600\ [s/h]} \cdot 12\ [€/h] = 0.46\ €/single\ command\ cycle$$

**Exercise 2**

In this case, $U$ is given ($U = aisle\ lenght \cdot 2 + central\ aisle\ width = 24 \cdot 2 + 6 = 54\ m$), while $V$ has to be calculated as follows:

$$\frac{\#\ Pallet\ Loads}{aisle} = 2 \cdot NPB \cdot NC \cdot NL = 2 \cdot 4 \cdot \frac{24}{4} \cdot 5 = 240\ pallet\ loads/aisle$$

$$NA = \frac{SC_{req}}{\frac{\#PLs}{aisle}} = \frac{12{,}000}{240} = 50\ aisles\ (25\ aisles\ per\ each\ side)$$

$$SC_{real} = 2 \cdot NPB \cdot NL \cdot NC \cdot NA = 2 \cdot 4 \cdot 5 \cdot 6 \cdot 50 = 12{,}000\ pallet\ loads$$

$$Module\ width = 2 \cdot D + AW = 2 \cdot 1.3 + 2.5 = 5.1\ m$$

$$V = \frac{NA}{2} \cdot Module\ width = 25 \cdot (2 \cdot 1.3 + 2.5) = 127.5\ m$$

To assess the required number of trucks we need to assess the average time required to complete a single command cycle, which consists of two components: fixed and variable time. The fixed time is given (i.e. 40 s/cycle). As regards the variable time, the formula to get $P$ has to be adjusted according to the layout features:

$$P = 2 \cdot (\frac{central\ cross\ aisle\ width}{2} + \frac{aisle\ lenght}{2} + \frac{V}{2})$$

$$VT_{SC} = \frac{P}{S_H} + \frac{S}{S_V} = \frac{2 \cdot (\frac{6}{2} + \frac{24}{2} + \frac{127.5}{2})}{2.2} + \frac{2 \cdot (5 - 1)}{0.3} = \frac{157.5}{2.2} + \frac{8}{0.3} = 98.26\ s$$

The total time is:

$$T_{SC} = VT_{SC} + FT_{SC} = 98.26 + 40 = 138.26\ s$$

Based on the total time, the average throughput capacity of a single truck is:

$$TC_{truck} = UF \cdot \frac{3{,}600}{T_{SC}} = 0.97 \cdot \frac{3{,}600}{138.26} = 25.3\ single\ command\ cycles/h$$

And the overall number of trucks required:

$$N_{trucks} = \left\lceil \frac{TC_{req}}{TC_{truck}} \right\rceil = \left\lceil \frac{100}{25.3} \right\rceil = 4\ trucks$$

$$TC_{real} = TC_{truck} \cdot N_{trucks} = 25.3 \cdot 4 = 101.2\ single\ command\ cycles/h$$

## Exercise 3

1. Storage system configuration

The system depth (*V*) is given (i.e. 30 m). As regards the system width:

**CASE a) *Straddle reach trucks***

$$NL = \left\lfloor \frac{H\ max_{fork}}{H} \right\rfloor = \left\lfloor \frac{9}{1.6} \right\rfloor = 6\ levels$$

$$NC = \left\lfloor \frac{Rack\ length}{L} \right\rfloor = \left\lfloor \frac{30}{2} \right\rfloor = 15\ columns$$

$$NA = \left\lceil \frac{SC_{req}}{2 \cdot NPB \cdot NL \cdot NC} \right\rceil = \left\lceil \frac{3{,}600}{2 \cdot 2 \cdot 6 \cdot 15} \right\rceil = 10\ aisles$$

$$U = NA \cdot Module\ width = 10 \cdot (2 \cdot 1.3 + 2.5) = 51\ m$$

**CASE b) *Counterbalance forklift trucks***

$$NL = \left\lfloor \frac{H\ max_{fork}}{H} \right\rfloor = \left\lfloor \frac{6}{1.6} \right\rfloor = 4\ levels$$

$$NC = \left\lfloor \frac{Rack\ length}{L} \right\rfloor = \left\lfloor \frac{30}{2} \right\rfloor = 15\ columns$$

$$NA = \left\lceil \frac{SC_{req}}{2 \cdot NPB \cdot NL \cdot NC} \right\rceil = \left\lceil \frac{3{,}600}{2 \cdot 2 \cdot 4 \cdot 15} \right\rceil = 15\ aisles$$

$$U = NA \cdot Module\ width = 15 \cdot (2 \cdot 1.3 + 3) = 84\ m$$

2. Estimate the investment costs (i.e. truck, land and building costs)

**CASE a) *Straddle reach trucks***

In order to assess the truck costs we need to calculate the number of trucks:

$$VT_{SC} = \frac{P}{S_H} + \frac{S}{S_V} = \frac{2 \cdot (\frac{U}{3} + \frac{V}{2} + cross\ aisle\ width)}{S_H} + \frac{H \cdot (NL-1)}{S_V} = \frac{74}{2} + \frac{8}{0.5} = 53\ s$$

$$T_{SC} = VT_{SC} + FT_{SC} = 53 + 40 = 93\ s$$

$$TC_{truck} = \frac{3{,}600}{T_{SC}} = \frac{3{,}600}{93} = 38.7\ single\ command\ cycles/h$$

$$N_{trucks} = \left\lceil \frac{TC_{req}}{TC_{truck}} \right\rceil = \left\lceil \frac{150}{38.7} \right\rceil = 4\ trucks$$

$I_0 = I_{Land} + I_{Building} + I_{Trucks} = 51 \cdot (30 + 5 + 5)\ [m^2] \cdot (500 + 370)\ [€ / m^2] + 4\ [trucks] \cdot 30{,}000\ [€ / truck] = 1{,}894{,}800\ €$

**CASE b) *Counterbalance forklift trucks***

$$VT_{SC} = \frac{P}{S_H} + \frac{S}{S_V} = \frac{2 \cdot (\frac{U}{3} + \frac{V}{2} + cross\ aisle\ width)}{S_H} + \frac{H \cdot (NL-1)}{S_V} = \frac{96}{2} + \frac{4.8}{0.4} = 60\ s$$

$$T_{SC} = VT_{SC} + FT_{SC} = 60 + 40 = 100\ s$$

$$TC_{truck} = \frac{3{,}600}{T_{SC}} = \frac{3{,}600}{100} = 36\ single\ command\ cycles/h$$

$$N_{trucks} = \left\lceil \frac{TC_{req}}{TC_{truck}} \right\rceil = \left\lceil \frac{150}{36} \right\rceil = 5\ trucks$$

$I_0 = I_{Land} + I_{Building} + I_{Trucks} = 84 \cdot (30 + 5 + 5)\ [m^2] \cdot (500 + 340)\ [€ / m^2] + 5\ [trucks] \cdot 22{,}000\ [€ / truck] = 2{,}932{,}400\ €$

# AUTOMATED WAREHOUSES - SOLUTIONS

## Exercise 1.

1. Storage system configuration

We can start from sizing the generic bay:

$$H\ (bay\ height) = H_{PL} + beam\ height + c = 1.5 + 0.1 + 0.15 = 1.75\ m$$

$$L\ (bay\ width) = NPB \cdot L_{PL} + (NPB + 1) \cdot d + upright\ width = 3 \cdot 0.8 + 4 \cdot 0.08 + 0.09 = 2.81\ m$$

$$D = D_{PL} + \frac{e}{2} = 1.2 + \frac{0.2}{2} = 1.3\ m$$

Warehouse dimensions:

$$NL_{max} = NL = \left\lfloor \frac{Rack\ height}{H} \right\rfloor = \left\lfloor \frac{22}{1.75} \right\rfloor = 12\ levels$$

$$NC_{max} = \left\lfloor \frac{Rack\ lenght}{L} \right\rfloor = \left\lfloor \frac{90}{2.81} \right\rfloor = 32\ columns$$

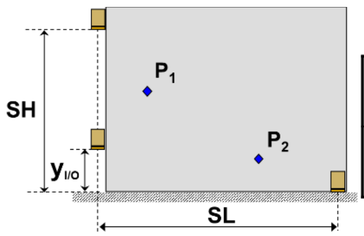
$$NA_{min} = \left\lceil \frac{SC_{req}}{2 \cdot NPB \cdot NL \cdot NC} \right\rceil = \left\lceil \frac{16{,}000}{2 \cdot 3 \cdot 12 \cdot 32} \right\rceil = 7\ aisles$$

In order to minimize space and S/R machine travel distance, we can verify whether the minimum number of columns such that the required SC is satisfied is lower than 32:

$$NC = \left\lceil \frac{SC_{req}}{2 \cdot NPB \cdot NL \cdot NA_{min}} \right\rceil = \left\lceil \frac{16{,}000}{2 \cdot 3 \cdot 12 \cdot 7} \right\rceil = 32\ columns$$

$$SC_{real} = 2 \cdot NPB \cdot NL \cdot NC \cdot NA = 2 \cdot 3 \cdot 12 \cdot 32 \cdot 7 = 16{,}128\ pallet\ loads$$

Knowing the main parameters related to the warehouse design, we can proceed to the cycle time assessment (using the FEM 9851, as reported in the figure and table below), to make sure that the identified system configuration respects also the required Throughput Capacity:




| | X | Y |
|---|---|---|
| $P_1$ | 1/5SL | 2/3SH + 1/3$y_{I/O}$ |
| $P_2$ | 2/3SL | 1/5SH+ 1/3$y_{I/O}$ |

| Coordinates [m] | X | Y |
|---|---|---|
| y(I/O) | 0 | 3.50 |
| P1 | 17.98 | 14.00 |
| P2 | 59.95 | 5.02 |

Average time to perform a single command cycle:

$$T(I \rightarrow P1) = T(P1 \rightarrow O) = max\left(\left|\frac{17.98}{2}\right|; \left|\frac{14 - 3.5}{0.4}\right|\right) = 26.25\ s$$

$$T(I \rightarrow P2) = T(P2 \rightarrow O) = max\left(\left|\frac{59.95}{2}\right|; \left|\frac{5.02 - 3.5}{0.4}\right|\right) = 29.98\ s$$

$$AVTSC = [T(I \rightarrow P1) + T(I \rightarrow P2)] = 56.23\ s$$

$$FTSC = 2 \cdot (5 + 8 + 10) = 46\ s$$

$$ATSC = 102.23\ s/SC$$

Average time to perform a dual command cycle:

$$T(P1 \rightarrow P2) = max\left(\left|\frac{59.95 - 17.98}{2}\right|; \left|\frac{5.02 - 14.00}{0.4}\right|\right) = 22.45\ s$$

$$AVTDC = [T(I \rightarrow P1) + T(P1 \rightarrow P2) + T(P2 \rightarrow O)] = 78.68\ s$$

$$FTDC = 3 \cdot (5 + 8) + 4 \cdot 10 = 79\ s$$

$$ATDC = 157.68\ s/DC$$

Therefore, the Throughput Capacity of a single S/R machine is:

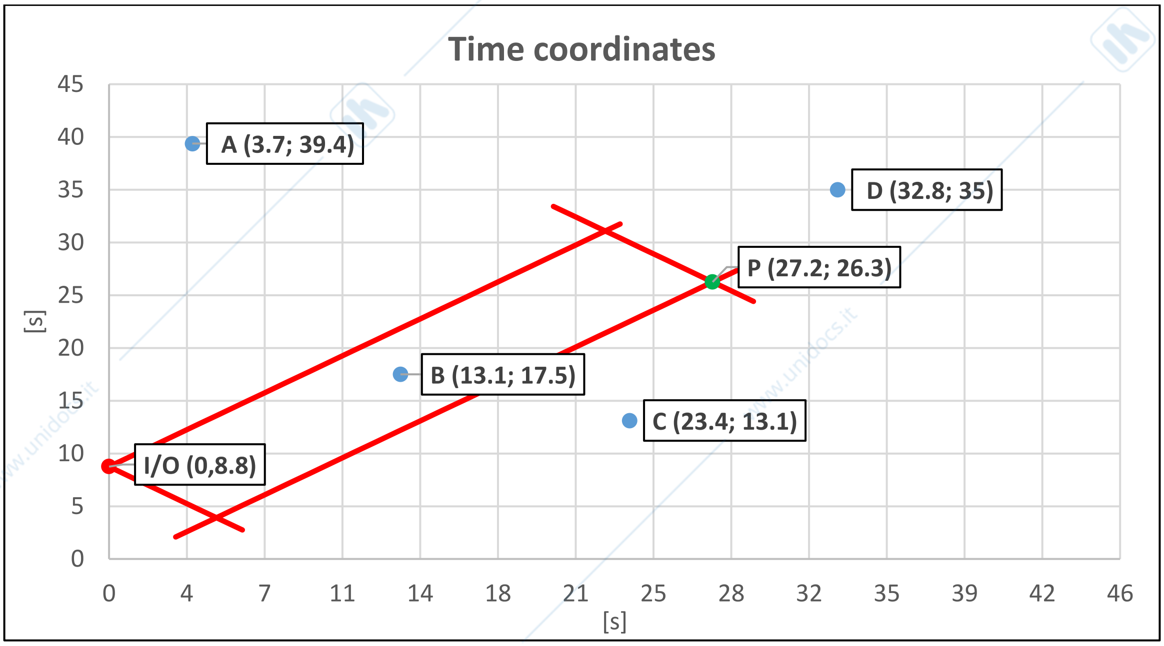
$$TC_{S/R\ machine} = \frac{3600 \cdot \left(\frac{\%SC}{2} + \%DC\right)}{\%SC \cdot ATSC + \%DC \cdot ATDC} = 3600 \cdot \frac{\frac{0.6}{2} + 0.4}{0.6 \cdot 102.23 + 0.4 \cdot 157.68} = 20.26\ PL/h$$

And the system TC is:

$TC = NA \cdot TC_{S/R\ machine} = 7 \cdot 20.26 = 141.8\ PLs/h$ (higher than the target value = 130 PL/h)

2. Selection of the best storage location among those proposed for performing the dual command cycle

To identify the best pallet location we apply the "No-cost zone" criteria. Apply the methodology consists in checking which (if any) of the four points A, B, C and D falls into the NCZ.
Using the graphical method (i.e. drawing the NCZ made of 45° degree lines), results show that B is the only point falling into the NCZ, and therefore it is the most suitable candidate for optimizing the dual command cycle.




The same conclusion can be reached also analytically, by verifying the equation reported below for each of the four points (denoted as $Q$ in the formula):

$$VT(I \rightarrow P) = VT(I \rightarrow Q) + VT(Q \rightarrow P)$$

$$VT(I \rightarrow P) = max(|0 - 27.2|; |8.8 - 26.3|) = 27.2\ s$$

$$VT(I \rightarrow A) + VT(A \rightarrow P) = max(|0 - 3.7|; |8.8 - 39.4|) + max(|3.7 - 27.2|; |39.4 - 26.3|) = 54.1\ s$$

$$VT(I \rightarrow B) + VT(B \rightarrow P) = 27.2\ s$$

$$VT(I \rightarrow C) + VT(C \rightarrow P) = 36.6\ s$$

$$VT(I \rightarrow D) + VT(D \rightarrow P) = 41.5\ s$$

**Exercise 2.**

1. Dimension of the single bay

$$H\ (bay\ height) = H_{PL} + beam\ height + c = 1.4 + 0.1 + 0.15 = 1.65\ m$$

$$L\ (bay\ width) = NPB \cdot L_{PL} + (NPB + 1) \cdot d + upright\ width = 2 \cdot 0.8 + 3 \cdot 0.1 + 0.1 = 2\ m$$

$$D = D_{PL} + \frac{e}{2} = 1.2 + \frac{0.2}{2} = 1.3\ m$$

2. System configuration (i.e. number of the storage levels, columns of bays, and aisles)

$$NL_{max} = NL = \left\lfloor \frac{Rack\ height}{H} \right\rfloor = \left\lfloor \frac{30}{1.65} \right\rfloor = 18\ levels$$

$$NC_{max} = \left\lfloor \frac{Rack\ lenght}{W} \right\rfloor = \left\lfloor \frac{100}{2} \right\rfloor = 50\ columns$$

$$NA_{min} = \left\lceil \frac{SC_{req}}{2 \cdot NPB \cdot NL \cdot NC} \right\rceil = \left\lceil \frac{16{,}000}{2 \cdot 2 \cdot 18 \cdot 50} \right\rceil = 5\ aisles$$

In order to minimize space and S/R machine travel distance, we can verify whether the minimum number of columns such that the required SC is satisfied is lower than 50:

$$NC = \left\lceil \frac{SC_{req}}{2 \cdot NPB \cdot NL \cdot NA_{min}} \right\rceil = \left\lceil \frac{16{,}000}{2 \cdot 2 \cdot 18 \cdot 5} \right\rceil = 45\ columns$$

$$SC_{real} = 2 \cdot NPB \cdot NL \cdot NC \cdot NA = 2 \cdot 2 \cdot 18 \cdot 45 \cdot 5 = 16{,}200\ pallet\ loads$$

3. Warehouse dimensions

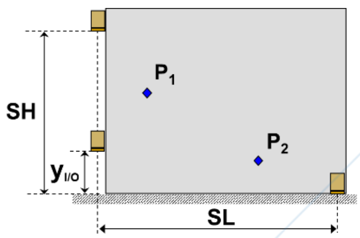
$$RL = NC \cdot L = 45 \cdot 2 = 90\ m$$

$$RH = NL \cdot H = 18 \cdot 1.65 = 29.7\ m$$

$$RFW = NA \cdot Module_{width} = 5 \cdot (2 \cdot 1.3 + 1.4) = 20\ m$$

4. Average single and dual command cycle time

Knowing the main parameters related to the warehouse design, we can proceed to the cycle time assessment (using the FEM 9851, as reported in the figure and table below), to make sure that the identified system configuration respects also the required Throughput Capacity:




| | X | Y |
|---|---|---|
| $P_1$ | 1/5SL | 2/3SH + 1/3$y_{I/O}$ |
| $P_2$ | 2/3SL | 1/5SH+ 1/3$y_{I/O}$ |

| Coordinates [m] | X | Y |
|---|---|---|
| y(I/O) | 0 | 6.6 |
| P1 | 18 | 20.9 |
| P2 | 60 | 7.81 |

Average time to perform a single command cycle:

$$T(I \to P1) = T(P1 \to O) = max\left(\left|\frac{18}{2}\right|; \left|\frac{20.9 - 6.6}{0.6}\right|\right) = 23.83\ s$$

$$T(I \to P2) = T(P2 \to O) = max\left(\left|\frac{60}{2}\right|; \left|\frac{7.81 - 6.6}{0.6}\right|\right) = 30\ s$$

$$AVTSC = [T(I \to P1) + T(I \to P2)] = 53.83\ s$$

$$FTSC = 2 \cdot (4 + 10 + 6) = 40\ s$$

$$ATSC = 93.83\ s/SC$$

Average time to perform a dual command cycle:

$$T(P1 \rightarrow P2) = max\left(\left|\frac{60 - 18}{2}\right|; \left|\frac{20.9 - 7.81}{0.6}\right|\right) = 21.82\ s$$

$$AVTDC = [T(I \rightarrow P1) + T(P1 \rightarrow P2) + T(P2 \rightarrow O)] = 75.65\ s$$

$$FTDC = 3 \cdot (4 + 6) + 4 \cdot 10 = 70\ s$$

$$ATDC = 145.65\ s/DC$$

5. Throughput capacity of the overall system

$$TC_{S/R\ machine} = \frac{3600 \cdot \left(\frac{\%SC}{2} + \%DC\right)}{\%SC \cdot ATSC + \%DC \cdot ATDC} = 3600 \cdot \frac{\frac{0.4}{2} + 0.6}{0.4 \cdot 93.83 + 0.6 \cdot 145.65} = 23.05\ PL/h$$

$$TC = NA \cdot TC_{S/R\ machine} = 5 \cdot 23.05 = 115.3\ PL/h$$

**Exercise 3.**

We can start from assessing the warehouse dimensions (the bay gross dimensions are given)

$$NL_{max} = NL = \left\lfloor\frac{Rack\ height}{H}\right\rfloor = \left\lfloor\frac{8}{0.55}\right\rfloor = 14\ levels$$

$$NC_{max} = \left\lfloor\frac{Rack\ lenght}{W}\right\rfloor = \left\lfloor\frac{50}{2.3}\right\rfloor = 21\ columns$$

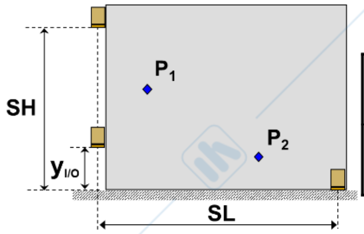
$$NA_{min} = \left\lceil\frac{SC_{req}}{2 \cdot NPB \cdot NL \cdot NC}\right\rceil = \left\lceil\frac{8{,}500}{2 \cdot 4 \cdot 14 \cdot 21}\right\rceil = 4\ aisles$$

In order to minimize space and S/R machine travel distance, we can verify whether the minimum number of columns such that the required SC is satisfied is lower than 21:

$$NC = \left\lceil\frac{SC_{req}}{2 \cdot NPB \cdot NL \cdot NA_{min}}\right\rceil = \left\lceil\frac{8{,}500}{2 \cdot 4 \cdot 14 \cdot 4}\right\rceil = 19\ columns$$

$$SC_{real} = 2 \cdot NPB \cdot NL \cdot NC \cdot NA = 2 \cdot 4 \cdot 14 \cdot 19 \cdot 4 = 8{,}512\ totes$$

Knowing the main parameters related to the warehouse design, we can proceed to the cycle time assessment (using the FEM 9851, as reported in the figure and table below), to make sure that the identified system configuration respects also the required Throughput Capacity:




| | X | Y |
|---|---|---|
| $P_1$ | 1/5SL | 2/3SH + 1/3$y_{I/O}$ |
| $P_2$ | 2/3SL | 1/5SH+ 1/3$y_{I/O}$ |

| Coordinates [m] | X | Y |
|---|---|---|
| y(I/O) | 0 | 1.10 |
| P1 | 8.74 | 5.13 |
| P2 | 29.13 | 1.8 |

Average time to perform a dual command cycle:

$$T(I \rightarrow P1) = T(P1 \rightarrow O) = max\left(\left|\frac{8.74}{4}\right|; \left|\frac{5.13 - 1.1}{1.5}\right|\right) = 2.69\ s$$

$$T(I \rightarrow P2) = T(P2 \rightarrow O) = max\left(\left|\frac{29.13}{4}\right|; \left|\frac{1.8 - 1.1}{1.5}\right|\right) = 7.28\ s$$

$$T(P1 \rightarrow P2) = max\left(\left|\frac{29.13 - 8.74}{4}\right|; \left|\frac{5.13 - 1.8}{1.5}\right|\right) = 5.1\ s$$

$$AVTDC = [T(I \rightarrow P1) + T(P1 \rightarrow P2) + T(P2 \rightarrow O)] = 15.07\ s$$

$$FTDC = 12\ s$$

$$ATDC = 27.07\ s/DC$$

Therefore, the Throughput Capacity of a single S/R machine is:

$$TC_{S/_R\,machine} = \frac{3{,}600}{ATDC} = \frac{3{,}600}{27.07} = 133\ totes/h$$

And the system TC is:

$TC = NA \cdot TC_{S/_R\,machine} = 4 \cdot 133 = 532\ totes/h$ (higher than the target value = 450 totes/h)