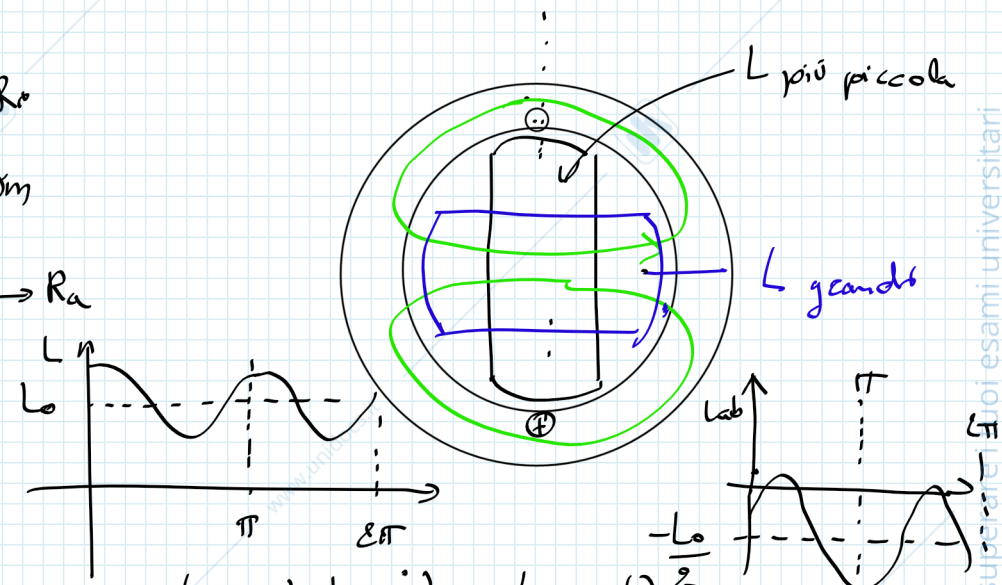
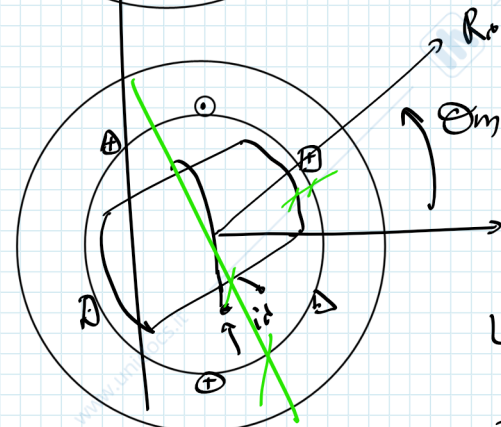


$$\vec{v}_s = R_s \vec{i}_s + j X_s \vec{i}_s + \vec{e}$$

$$\vec{v}_s = \sqrt{\frac{3}{2}} I_m \cos \omega t$$

$$\vec{i}_s = \sqrt{\frac{3}{2}} I_m \cos \omega t$$

$\theta_m = \omega t + \theta$



$$L_{aa} = L_0 + L_m \cos(2\theta_m)$$

$$L_{bb} = L_0 + L_m \cos 2(\theta_m - \frac{2}{3}\pi)$$

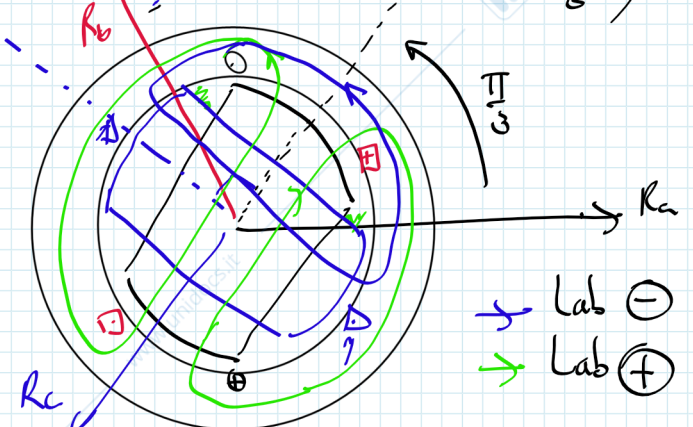
$$L_{cc} = L_0 + L_m \cos 2(\theta_m + \frac{2}{3}\pi)$$

$$L_{ba} = L_{ab} = -\frac{L_0}{2} + L_m \cos(\dots)$$

$$L_{ab} = -\frac{L_0}{2} + L_m \cos 2(\theta_m - \frac{\pi}{3})$$

$$L_{bc} = -\frac{L_0}{2} + L_m \cos 2(\theta_m - \frac{\pi}{3} - \frac{2\pi}{3})$$

$$L_{ca} = -\frac{L_0}{2} + L_m \cos 2(\theta_m - \frac{\pi}{3} + \frac{2\pi}{3})$$



$$L_{ab} = -\frac{L_0}{2} + L_m \cos 2(\theta_m - \frac{\pi}{3})$$

$$L_{bc} = -\frac{L_0}{2} + L_m \cos 2(\theta_m - \pi)$$

$$L_{ca} = -\frac{L_0}{2} + L_m \cos 2(\theta_m + \frac{\pi}{3})$$

$$v_a = R_s i_a + \frac{d\psi_a}{dt} \quad v_b = \frac{d\psi_b}{dt} \quad v_c = \frac{d\psi_c}{dt}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{d}{dt} \begin{pmatrix} \begin{bmatrix} L_{aa} & L_{ab} & L_{ca} \\ L_{ab} & L_{bb} & L_{bc} \\ L_{ca} & L_{bc} & L_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_f \cos \theta_m \\ L_f \cos(\theta_m - \frac{2\pi}{3}) \\ L_f \cos(\theta_m + \frac{2\pi}{3}) \end{bmatrix} I_0 \end{pmatrix} \begin{pmatrix} 1 \\ +\alpha \\ +\alpha^2 \end{pmatrix} \sqrt{\frac{2}{3}}$$

$$\sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} = \frac{d}{dt} \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} L_f \cos \theta_m \\ L_f \cos(\theta_m - \frac{2\pi}{3}) \\ L_f \cos(\theta_m + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} I_p$$

$L_{aa} + L_{bb} + L_{cc} = 0$

$$\psi_a = \left(L_0 + L_m \cos 2\theta_m \right) i_a + \left(-\frac{L_0}{2} + L_m \cos 2\left(\theta_m - \frac{\pi}{3}\right) \right) i_b + \left(-\frac{L_0}{2} + L_m \cos 2\left(\theta_m + \frac{\pi}{3}\right) \right) i_c$$

$$= \underbrace{L_0 i_a}_{\frac{3}{2} L_0 i_a} - \frac{L_0}{2} i_b - \frac{L_0}{2} i_c + L_m i_a \cos 2\theta_m + L_m \cos 2\left(\theta_m - \frac{\pi}{3}\right) i_b + L_m \cos 2\left(\theta_m + \frac{\pi}{3}\right) i_c$$

$$\psi_b = \left(-\frac{L_0}{2} + L_m \cos 2\left(\theta_m - \frac{\pi}{3}\right) \right) i_a + \left(L_0 + L_m \cos 2\left(\theta_m - \frac{2\pi}{3}\right) \right) i_b + \left(-\frac{L_0}{2} + L_m \cos 2\left(\theta_m + \frac{\pi}{3}\right) \right) i_c$$

$$= \frac{3}{2} L_0 i_b + L_m \cos\left(2\theta_m - \frac{2\pi}{3}\right) i_a + L_m \cos\left(2\theta_m + \frac{2\pi}{3}\right) i_b + L_m \cos 2\theta_m i_c$$

$$\psi_c = \frac{3}{2} L_0 i_c + L_m \cos\left(2\theta_m + \frac{2\pi}{3}\right) i_a + L_m \cos 2\theta_m i_b + L_m \cos\left(2\theta_m - \frac{2\pi}{3}\right) i_c$$

$$\frac{1}{\sqrt{3}} \psi_a = \frac{3}{2} L_0 i_a + L_m \cos 2\theta_m i_a + L_m \cos\left(2\theta_m - \frac{2\pi}{3}\right) i_b + L_m \cos\left(2\theta_m + \frac{2\pi}{3}\right) i_c$$

$$\frac{1}{\sqrt{3}} \psi_b = \frac{3}{2} L_0 i_b + L_m \cos\left(2\theta_m - \frac{2\pi}{3}\right) i_a + L_m \cos\left(2\theta_m + \frac{2\pi}{3}\right) i_b + L_m \cos 2\theta_m i_c$$

$$\frac{1}{\sqrt{3}} \psi_c = \frac{3}{2} L_0 i_c + L_m \cos\left(2\theta_m + \frac{2\pi}{3}\right) i_a + L_m \cos 2\theta_m i_b + L_m \cos\left(2\theta_m - \frac{2\pi}{3}\right) i_c$$

$$\psi_s = L_s i_s \quad \uparrow \quad b(1 + \alpha + \alpha^2) \quad \uparrow \quad \left[e^{-j2\theta_m} + e^{-j2\theta_m} e^{-j\frac{2\pi}{3}} + e^{-j2\theta_m} e^{-j\frac{4\pi}{3}} \right]$$

$$\frac{\sqrt{2}}{\sqrt{3}} \frac{L_m}{2} \left(e^{j2\theta_m} + e^{-j2\theta_m} + e^{j2\theta_m} e^{-j\frac{2\pi}{3}} + e^{-j2\theta_m} e^{j\frac{2\pi}{3}} + e^{j2\theta_m} e^{-j\frac{4\pi}{3}} + e^{-j2\theta_m} e^{j\frac{4\pi}{3}} \right) i_a$$

$$\frac{\sqrt{2}}{\sqrt{3}} \frac{L_m}{2} \left(3 e^{j2\theta_m} \right) i_a \quad \psi_s = 1^a \text{ colonna} = \frac{\sqrt{2}}{\sqrt{3}} \frac{3}{2} L_m e^{j2\theta_m} i_a$$

$$\frac{\sqrt{2}}{\sqrt{3}} \frac{L_m}{2} \left(e^{j2\theta_m} e^{-j\frac{2\pi}{3}} + e^{-j2\theta_m} e^{j\frac{2\pi}{3}} + e^{j2\theta_m} e^{j\frac{2\pi}{3}} + e^{-j2\theta_m} e^{-j\frac{2\pi}{3}} + e^{j2\theta_m} e^{-j\frac{4\pi}{3}} + e^{-j2\theta_m} e^{j\frac{4\pi}{3}} + e^{j2\theta_m} e^{j\frac{4\pi}{3}} + e^{-j2\theta_m} e^{-j\frac{4\pi}{3}} \right)$$

$$\frac{\sqrt{2}}{\sqrt{3}} \frac{L_m}{2} \left(e^{j2\theta_m} e^{-j\frac{2\pi}{3}} + 0 \right)$$

$$\sqrt{\frac{2}{3}} \frac{3}{2} L_m e^{j2\theta_m} i_a$$

$$\sqrt{\frac{2}{3}} \frac{3}{2} L_m e^{j2\theta_m} i_b \alpha^2$$

$$\sqrt{\frac{2}{3}} \frac{3}{2} L_m e^{j2\theta_m} i_c \alpha$$

$$1 + \alpha + \alpha^2 = 0$$

$$\bar{i}_s = (i_a + \alpha i_b + \alpha^2 i_c) = (i_a + e^{j\frac{2\pi}{3}} i_b + e^{-j\frac{2\pi}{3}} i_c)$$

$$\bar{\psi}_s = \frac{3}{2} L_o \bar{i}_s + \frac{3}{2} L_m e^{j2\theta_m} \sqrt{\frac{2}{3}} (i_a + i_b \alpha^2 + i_c \alpha) + L_f \bar{i}_s$$

$$\bar{i}_s = (i_a + i_b e^{-j\frac{2\pi}{3}} + i_c e^{j\frac{2\pi}{3}})$$

$$\bar{\psi}_s = \frac{3}{2} L_o \bar{i}_s + \frac{3}{2} L_m e^{j2\theta_m} \bar{i}_s + L_f \bar{i}_s$$

$I_s = |\bar{i}_s|$
 $\sqrt{\frac{3}{2}} I_m e^{j\theta_m}$

$$\bar{v}_s = R_s \bar{i}_s + \frac{d}{dt} \bar{\psi}_s = R_s \bar{i}_s + \frac{d}{dt} \left(\frac{3}{2} L_o \bar{i}_s + \frac{3}{2} L_m e^{j2\theta_m} \bar{i}_s + L_f \bar{i}_s \right)$$

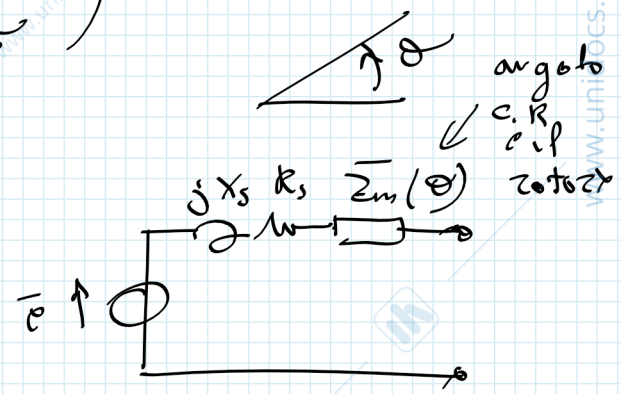
$$\bar{v}_s = R_s \bar{i}_s + j\omega \frac{3}{2} L_o \bar{i}_s + j\omega L_f \bar{i}_s + \frac{d}{dt} \left(\frac{3}{2} L_m e^{j2\theta_m} \sqrt{\frac{3}{2}} I_m e^{-j\omega t} \right) \theta_m = \omega t + \theta$$

$$= R_s \bar{i}_s + jX_s \bar{i}_s + \bar{e} + \frac{d}{dt} \left(\frac{3}{2} L_m e^{j2\omega t + 2\theta} I_s e^{-j\omega t} \right) =$$

$$= R_s \bar{i}_s + jX_s \bar{i}_s + \bar{e} + \frac{d}{dt} \left(\frac{3}{2} L_m e^{j2\theta} \underbrace{I_s}_{I_s} e^{j\omega t} \right) =$$

$$\bar{v}_s = R_s \bar{i}_s + jX_s \bar{i}_s + \bar{e} + j\omega \frac{3}{2} L_m e^{j2\theta} \bar{i}_s$$

$\underbrace{\hspace{10em}}_{Z_m}$



$$T = p = R_o \left(\frac{\bar{v}_s \bar{i}_s}{\omega} \right)$$

$R_s i_s^2 = p \text{ joule}$
 $jX_s i_s^2 = \text{energ. acc.}$

$$p = \text{Re} \left(R_s \bar{i}_s \bar{i}_s + jX_s \bar{i}_s \bar{i}_s + j\omega L_f i_b \bar{i}_s + j\omega \frac{3}{2} L_m e^{j2\theta} \bar{i}_s \bar{i}_s \right) =$$

$\underbrace{\hspace{10em}}_{\text{coppia isotropo}}$

$$P = \text{Re} \left(\underbrace{R_s \bar{i}_s i_s}_{\substack{\text{perditi} \\ \downarrow \text{calore}}} + \underbrace{j X_s \bar{i}_s i_s}_{\substack{\uparrow \\ P=0}} + \underbrace{j \omega L_f \bar{i}_b i_s}_{\text{coppia isotropa}} + \underbrace{j \omega \frac{3}{2} L_m e^{j2\theta} \bar{i}_s i_s}_{\text{coppia anisotropa}} \right) =$$

$$P = \text{Re} \left(j \omega L_f \bar{i}_b i_s + j \omega \frac{3}{2} L_m e^{j2\theta} I_s^2 \right) = - \text{Im} \left(\dots \right)$$

$$= - \text{Im} \left(\omega L_f I_s^* I_s e^{j(\omega t + \theta)} \cdot I_s e^{-j\omega t} + \omega \frac{3}{2} L_m e^{j2\theta} I_s^2 \right) = \text{Re}(jx) = - \text{Im}(x)$$

$$P = \underbrace{- \omega L_f I_s^* I_s \sin \theta}_{P = \text{coppia isotropa}} - \underbrace{\omega \frac{3}{2} L_m I_s^2 \sin 2\theta}_{P/\omega = \text{coppia di anisotropia}}$$

$$\rightarrow \frac{P}{\omega} = \boxed{\text{SINCRONI A MP}} \rightarrow \boxed{\text{BRUSHLESS}} \quad \boxed{L_f I_s^* = \psi_{MP}}$$

