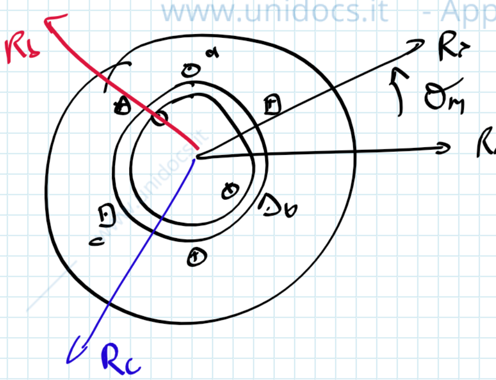


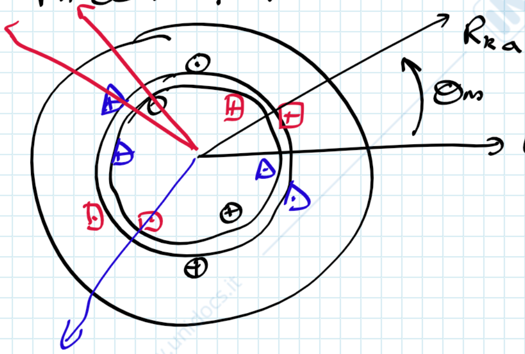
MACCHINA SINCRONA

$\omega_m = \omega / p$

$R_a \Rightarrow \left| \theta_m = \omega_m t \Rightarrow \omega_m = \omega \right|$

R_b MOTORE $\Rightarrow f$ variabile. (BRUSHLESS) M. PERMANENTI (ROT)

MACCHINA ASINCRONA. – MACCHINA AD INDUZIONE



$v_r = 0 \Leftarrow$ corto circuito le bobine di rotore

$R_{sa} \Rightarrow R_r$ il rotore è immerso in un campo magnetico $\Rightarrow$ legge di Faraday $\Rightarrow v$

P ∥

$$1 \quad v_{sa} = R_s i_{sa} + \frac{d\psi_{sa}}{dt}$$
$$+ \quad \alpha v_{sb} = R_s i_{sb} + \frac{d\psi_{sb}}{dt} \qquad \sqrt{\tfrac{2}{3}}$$
$$+ \quad \alpha^2 v_{sc} = R_s i_{sc} + \frac{d\psi_{sc}}{dt}$$

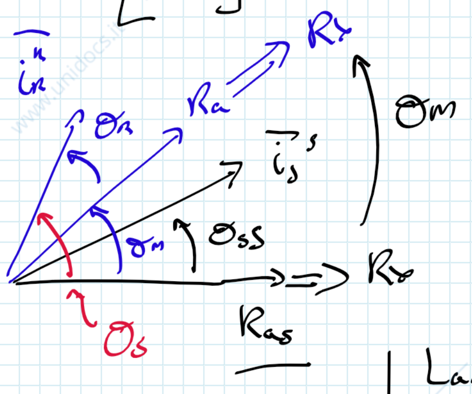
$$v_{ra} = R_r i_{ra} + \frac{d\psi_{ra}}{dt} \quad 1 \; +$$
$$v_{rb} = R_r i_{rb} + \frac{d\psi_{rb}}{dt} \quad \alpha \; + \quad \sqrt{\tfrac{2}{3}}$$
$$v_{rc} = R_r i_{rc} \quad \frac{d\psi_{rc}}{dt} \quad \alpha^2$$

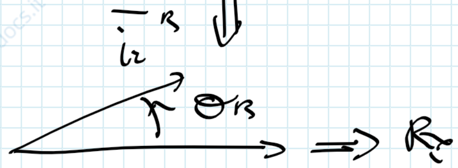
⇓

$$\sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix} = \bar{v}_s^s = R_s \bar{i}_s^s + \frac{d\bar{\psi}_s^s}{dt}$$

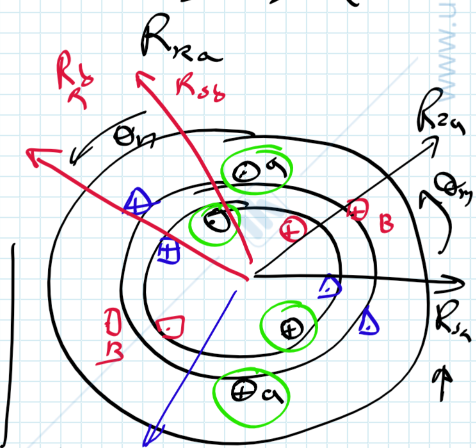
$$\bar{v}_R^R = R_R \bar{i}_r^R + \frac{d\bar{\psi}_r^R}{dt}$$

$\bar{i}_r^R$ ⇓





$$\boxed{\begin{aligned} \theta_r &= \theta_s - \theta_m \\ \omega_r t &= \omega_s t - \omega_m t \end{aligned}}$$



$$\left| \begin{aligned} & L_{aa} = L_{bb} = L_{cc} = L_s \\ & L_{ab} = L_s \cos\left(\tfrac{2}{3}\pi\right) = -L_s/2 \\ & L_{aa_R} = L_m \cos(\theta_m) \\ & L_{ab_R} = L_m \cos\left(\theta_m + \tfrac{2}{3}\pi\right) \end{aligned} \right|$$

L_m quando $a_s \equiv a_R$

$$\bar{v}_s^s = R_s \bar{i}_s^s + \frac{d\bar{\psi}_s^s}{dt}$$

$$\psi_{sa} = L_{aa} i_{sa} + L_{ab} i_{sb} + L_{ac} i_{sc} + L_{aa_R} i_{ra} + L_{ab_R} i_{rb} + L_{ac_R} i_{cR}$$

$$\psi_{sb} = \ldots \quad L_{bb} i_{sb} \quad + L_{ba_R} i_{ra} + \ldots -$$

$$\psi_{sc} = \ldots\ldots \quad L_{cc} i_{sc} +$$

$$-i_{sb} - i_{sc} = i_{sa}$$

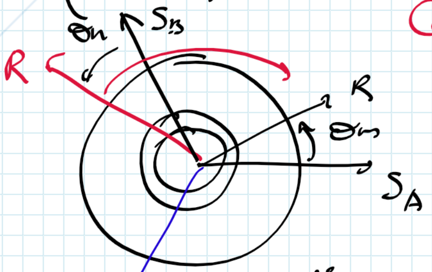
$$\begin{cases} \psi_{sa} = L_s i_{sa} - \frac{L_s}{2} i_{sb} - \frac{L_s}{2} i_{sc} + L_m \cos(\vartheta_m) i_{ra} + L_m \cos\left(\vartheta_m + \frac{2}{3}\pi\right) i_{rb} + L_m \cos\left(\vartheta_m - \frac{2}{3}\pi\right) i_{rc} \\ \psi_{sb} = -\frac{L_s}{2} i_{sa} + L_s i_{sb} - \frac{L_s}{2} i_{sc} + L_m \cos\left(\vartheta_m - \frac{2}{3}\pi\right) i_{ra} + L_m \cos(\vartheta_m) i_{rb} + L_m \left(\vartheta_m + \frac{2}{3}\pi\right) i_{rc} \\ \psi_{sc} = -\frac{L_s}{2} i_{sa} - \frac{L_s}{2} i_{sb} + L_s i_{sc} + L_m \cos\left(\vartheta_m + \frac{2}{3}\pi\right) i_{ra} + L_m \cos\left(\vartheta_m - \frac{2}{3}\pi\right) i_{rb} + L_m \cos\vartheta_m i_{rc} \end{cases}$$

1, α, α^2

① ② ③

$$\bar{\psi}_s^s = \underbrace{\frac{3}{2} L_s}_{\text{SINCRONA}} \vec{i}_s^{\,s} + \dots$$

STATORE



$$i_{sa} + i_{sb} + i_{sc} = 0$$

$$i_{ra} + i_{rb} + i_{rc} = 0$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$e^{-j\frac{2}{3}\pi} \qquad e^{j\frac{2}{3}\pi}$$

① $$\frac{L_m}{2} i_{ra} \left(e^{j\vartheta_m} + \cancel{e^{-j\vartheta_m}} + e^{j\vartheta_m} \cancel{e^{-j\frac{2}{3}\pi}} \cancel{e^{j\frac{2}{3}\pi}} + \cancel{e^{-j\vartheta_m} e^{j\frac{2}{3}\pi} e^{j\frac{2}{3}\pi}} + e^{j\vartheta_m} \cancel{e^{j\frac{2}{3}\pi}} \cancel{e^{-j\frac{2}{3}\pi}} + \cancel{e^{-j\vartheta_m} e^{-j\frac{2}{3}\pi} e^{-j\frac{2}{3}\pi}} \right)$$

$$\boxed{\frac{L_m}{2} i_{ra}\, 3 e^{j\vartheta_m}} = \boxed{\frac{3}{2} L_m e^{j\vartheta_m} i_{ra}}$$

$$e^{j\frac{2}{3}\pi} = \alpha$$

② $$\frac{L_m}{2} i_{rb} \left(e^{j\vartheta_m} e^{j\frac{2}{3}\pi} + \cancel{e^{-j\vartheta_m} e^{-j\frac{2}{3}\pi}} + e^{j\vartheta_m} e^{j\frac{2}{3}\pi} + \cancel{e^{-j\vartheta_m} e^{j\frac{2}{3}\pi}} + e^{j\vartheta_m} e^{-j\frac{2}{3}\pi} e^{j\frac{2}{3}\pi} + \cancel{e^{-j\vartheta_m} e^{j\frac{2}{3}\pi} e^{-j\frac{2}{3}\pi}} \right)$$

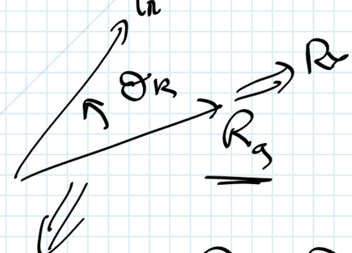
$$= \frac{L_m}{2} i_{rb}\, 3 e^{j\vartheta_m} \alpha = \boxed{\frac{3}{2} L_m e^{j\vartheta_m} i_{rb} \cdot \alpha}$$

$$\alpha(1 + \alpha + \alpha^2) = 0$$

③ $$\boxed{= \frac{3}{2} L_m e^{j\vartheta_m} i_{rc} \alpha^2}$$

$$\sqrt{\frac{2}{3}}(i_{sa} + \alpha i_{sb} + \alpha^2 i_{sc})$$

$$\bar{i}_R^R$$



$$\bar{\psi}_s^s = \frac{3}{2} L_s \bar{i}_s^s + \sqrt{\frac{2}{3}} \left(\frac{3}{2} L_m e^{j\vartheta_m} (i_{ra} + \alpha i_{rb} + \alpha^2 i_{rc}) \right)$$

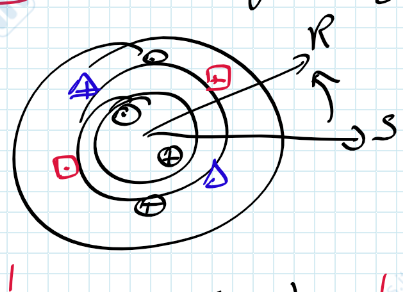
$$\vartheta_R = \vartheta_s - \vartheta_m$$

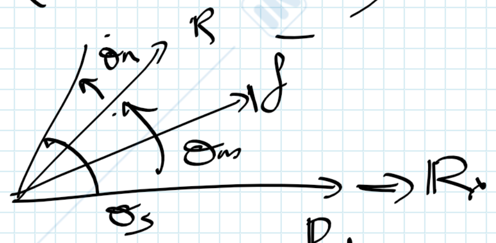
$$\boxed{\bar{\psi}_s^s = \frac{3}{2} L_s \bar{i}_s^s + \frac{3}{2} L_m e^{j\vartheta_m} \bar{i}_R^R}$$

$$\bar{f} = \sqrt{\frac{2}{3}} (f_a + f_b \alpha + f_c \alpha^2)$$

$$v_{Ra} = R_r i_r + \frac{d\psi_{ra}}{dt}$$

...





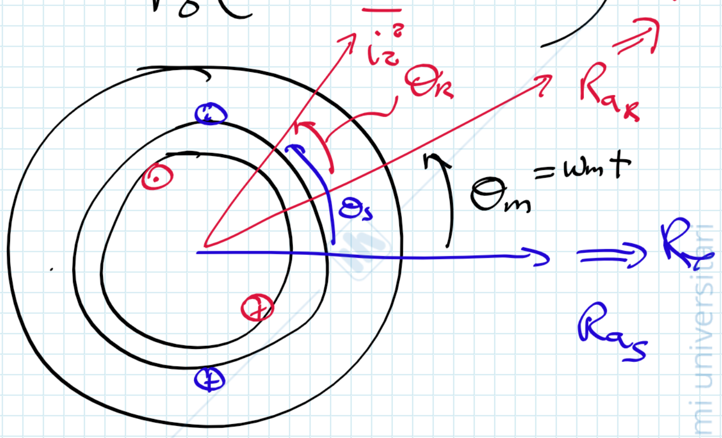
$$\begin{cases} \psi_{Ra} = L_r i_{ra} - \frac{L_r}{2} i_{rb} - \frac{L_r}{2} i_{rc} + L_m \cos(-\vartheta_m) i_{sa} + L_m \cos\left(-\vartheta_m + \frac{2}{3}\pi\right) i_{sb} + L_m\left(-\vartheta_m - \frac{2}{3}\pi\right) i_{sc} \\ \psi_{Rb} = -\frac{L_r}{2} i_{ra} + L_r i_{rb} - \frac{L_r}{2} i_{rc} + L_m \cos\left(-\vartheta_m - \frac{2}{3}\pi\right) i_{sa} + L_m \cos(-\vartheta_m) i_{sb} + L_m\left(-\vartheta_m + \frac{2}{3}\pi\right) i_{sc} \\ \psi_{Rc} = -\frac{L_r}{2} i_{ra} - \frac{L_r}{2} i_{rb} + L_r i_{rc} + L_m \cos\left(-\vartheta_m + \frac{2}{3}\pi\right) i_{sa} + L_m \cos\left(-\vartheta_m - \frac{2}{3}\pi\right) i_{sb} + L_m \cos(-\vartheta_m) i_{sc} \end{cases}$$

$$\begin{cases} \bar{\psi}_s^s = \frac{3}{2} L_s \bar{i}_s^s + \frac{3}{2} L_m e^{j\theta_m} \bar{i}_r^s \\ \bar{\psi}_R^R = \frac{3}{2} L_r \bar{i}_R^R + \frac{3}{2} L_m e^{-j\theta_m} \bar{i}_s^s \end{cases}$$

$$I_R e^{j\theta_R} = \bar{i}_R^R = \sqrt{\frac{2}{3}}\left(i_{ra} + \alpha i_{rb} + \alpha^2 i_{rc}\right)$$

$$\bar{i}_s^s = \sqrt{\frac{2}{3}}\left(i_{sa} + \alpha i_{sb} + \alpha^2 i_{sc}\right)$$

$$\underbrace{\omega_s t}_{\theta_s} = \underbrace{\omega_m t}_{\theta_m} + \omega_R t$$



$\theta_s = \theta_m + \theta_R$

$$\omega_s = \omega_m + \omega_R$$

$$\bar{v}_R^R = R_r \bar{i}_r^R + \frac{d\bar{\psi}_R^R}{dt}$$

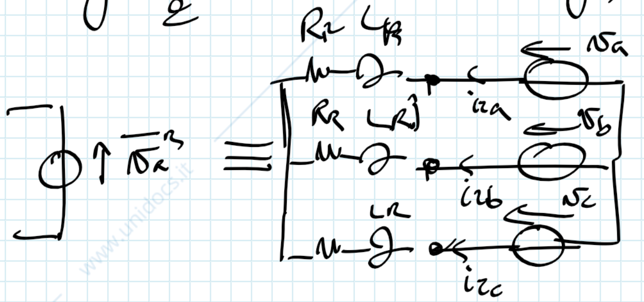
$\bar{i}_R^R = 0 \implies$ teniamo gli avv. di rotore aperti - alimento con tre corr. eq. a ω lo statore

$$\bar{i}_s^s = \sqrt{\frac{3}{2}} I_m e^{j\omega t} = I_s e^{j\omega t}$$

$$\theta_m = 0$$

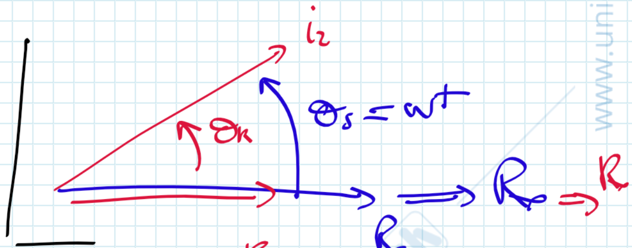
$$\bar{v}_R^R = \frac{d\bar{\psi}_R^R}{dt} = \frac{d}{dt}\left(\frac{3}{2} L_m e^{-j\theta_m} \bar{i}_s^s\right)$$

$$\bar{v}_R^R = j\omega \frac{3}{2} L_m e^{-j\theta_m} \bar{i}_s^s \Rightarrow j\omega \frac{3}{2} L_m \bar{i}_s^s$$



tre tens. simm. a puls. ω

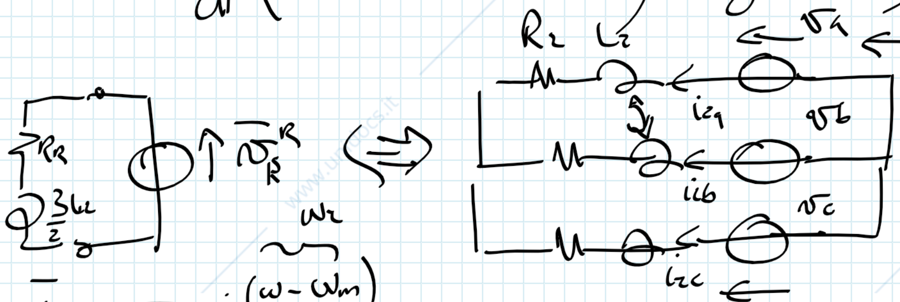
$$i_R^R = I_R e^{j\omega t}$$

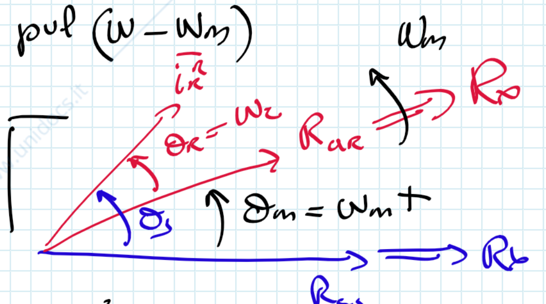


$\theta_m \neq 0 = \omega_m t \quad i_r = 0$ rotore aperto

$$\bar{v}_R^R = \frac{d}{dt}\left(\frac{3}{2} L_m e^{-j\omega_m t} I_s e^{j\omega t}\right) = j(\omega - \omega_m)\frac{3}{2} L_m I_s e^{j(\omega-\omega_m)t}$$

$\leftarrow$ a pul $(\omega - \omega_m)$





$$\bar{i}_R^R = I_R e^{j\overbrace{(\omega-\omega_m)}^{\omega_r}}$$

$$\omega_s = \omega_m + \omega_r = \cancel{\omega_m} + \omega - \cancel{\omega_m} = \omega$$