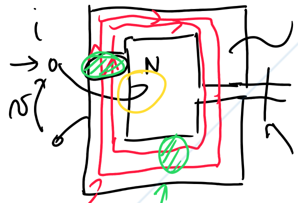
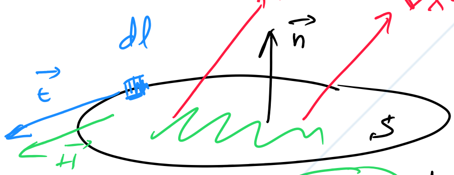
I CIRCUITI MAGNETICI

$$\nabla \times \vec{H} = \vec{J} + \cancel{\frac{\partial \vec{D}}{\partial t}} \qquad \nabla \cdot \vec{B} = 0$$



FERROMAGN. $\mu_{fe} \gg \mu_0$

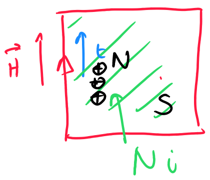
DIR ↑ → REGOLA CAVATAPPI

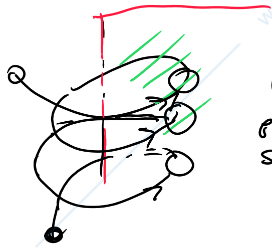


Scegliamo la linea di campo (media) come linea di int.

$$\int_S \underline{\nabla \times \vec{H} \cdot \vec{n}\, dS} = \int_S \vec{J}\, dS \overset{\text{teo rot}}{=} \oint_\ell \underline{\vec{H} \cdot \vec{t}\, d\ell}$$

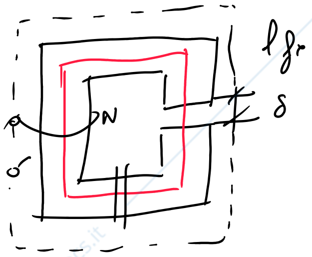
Ni $\qquad$ $H \cdot \ell$





⊕ entrante sup

⊙ uscente dalla sup.



$$\oint_\ell \vec{H} \cdot \vec{t}\, d\ell = Ni = \underbrace{H \ell_{fe}}_{U_{fe}} + \underbrace{H \cdot \delta}_{U_\delta}$$

M

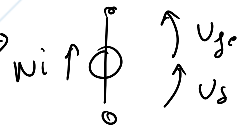
tensione magnetica

Forza magneto motrice $[A_{sp}]$ f.m.m.

cadute di tensione magnetica

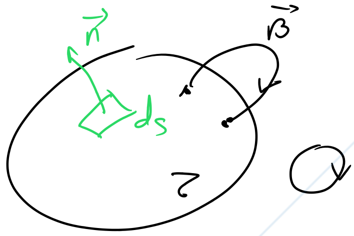
$H = [A/m]$

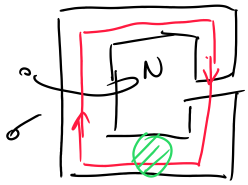
$U_{fe} + U_\delta = NI \Leftarrow$ LKT MAGN. $\Rightarrow$



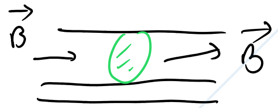
$\nabla\cdot\vec B = 0$

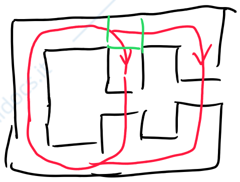
$$\int_\tau \nabla\cdot\vec B\, d\tau = 0$$



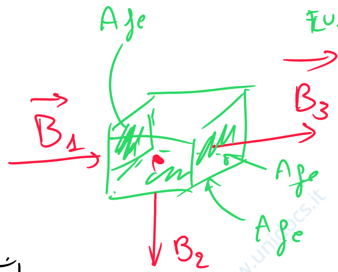


$$\int_\tau \nabla\cdot\vec B\, d\tau = \underbrace{\oint_S \vec B\cdot\vec n\, dS}_{\text{FLUSSO ATT. LA SUP. } S} = 0$$

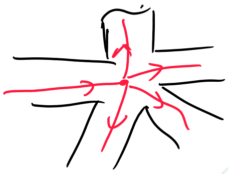




FLUSSO IND. MAGNETICA $= \varphi$

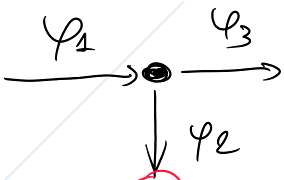


$$\int_{A_1} \vec B_1\cdot\vec n\, dS + \int_{A_{fe}} \vec B_2\cdot\vec n\, dS + \int_{A_{fe}} \vec B_3\cdot\vec n\, dS = 0$$



$$\boxed{\varphi_1 + \varphi_2 + \varphi_3 = 0}$$ LKCM

$$\varphi_1 = \int_{A_{fe}} \vec B_1\cdot n\, dS = B_1 A_{fe}$$



[Wb] weber $\quad B = \mu_{fe} H$

$$\int_{l_{fe}} \vec H\cdot\vec t\, dl = U_{fe} = \int_{l_{fe}} H\, dl = \int_{l_{fe}} \frac{B}{\mu_{fe}}\, dl = \int_{l_{fe}} \frac{\overbrace{B A_{fe}}^{\varphi}}{\mu_{fe} A_{fe}}\, dl = \frac{1}{\mu_{fe}}\frac{l_{fe}}{A_{fe}}\varphi_{fe}$$

$$\boxed{U_{fe} = \theta_{fe}\cdot\varphi_{fe}}$$ LΩM

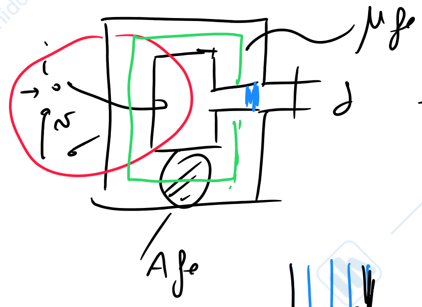
RILUTTANZA θ

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \vec{i}\left(\frac{\partial F_z}{\partial y} + \frac{\partial F_y}{\partial z}\right) +$$

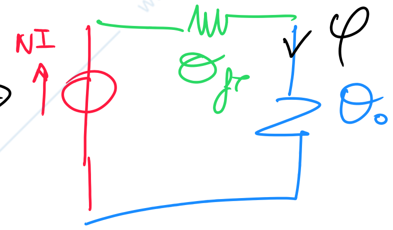
$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

$$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

DANIEL FLEISH → GUIDA ALLE EQ. DI MAXWELL

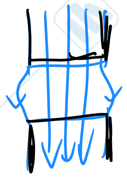


$\Rightarrow$

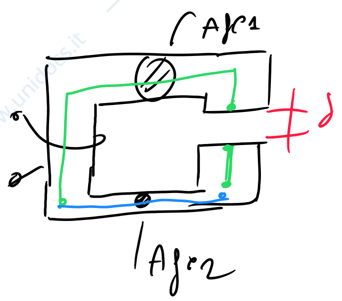


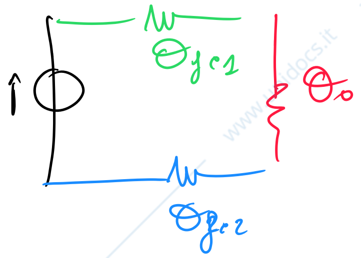
$$\Theta_{fe} = \frac{1}{\mu_{fe}} \cdot \frac{l_{fe}}{A_{fe}}$$

$$\Theta_0 = \frac{1}{\mu_0} \cdot \frac{\delta}{A_{fe}}$$



$$\varphi = B A_{fe}$$

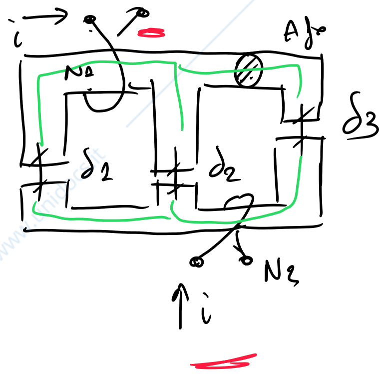




$$\Theta_{fe1} = \frac{1}{\mu_{fe}} \cdot \frac{l_{fe1}}{A_{fe1}}$$

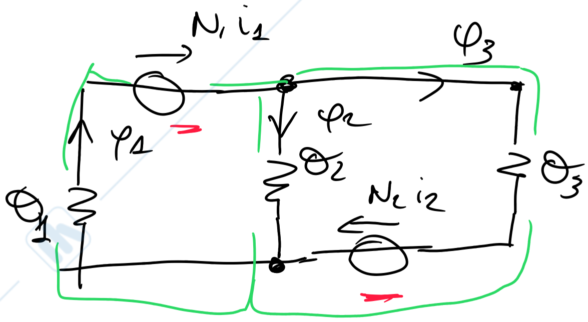
$$\Theta_{fe2} = \frac{1}{\mu_{fe}} \cdot \frac{l_{fe2}}{A_{fe2}}$$

$$\Theta_0 = \frac{1}{\mu_0} \cdot \frac{\delta}{A_{fe}}$$



$\mu_{fe} >> \mu_0 \rightarrow \mu_{fe} = \infty$

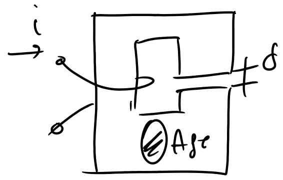
$$\Theta = \frac{1}{\mu} \cdot \frac{l}{A} = 0 \qquad \Theta_1 = \frac{1}{\mu_0} \cdot \frac{\delta_1}{A_{fe}} \; ----$$

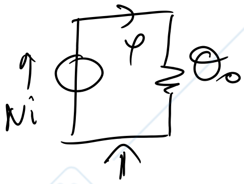


INDUTTANZA

$\Theta_0 = \frac{1}{\mu_0} \cdot \frac{\delta}{A_{fe}}$

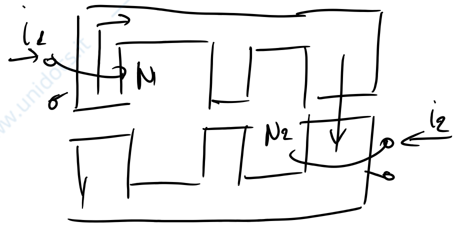
$L = \frac{\Psi}{i}$ ← FLUSSO CONC. [H]





$\Psi = N\varphi$ [Wb]

$L = \frac{N\varphi}{i}$ ← $\varphi = \frac{Ni}{\Theta_0}$ $\Rightarrow$ $\boxed{L = \frac{\frac{N^2 \cancel{i}}{\Theta_0}}{\cancel{i}} = \frac{N^2}{\Theta_0}}$ ← Dip solo dai par. geom., sist.

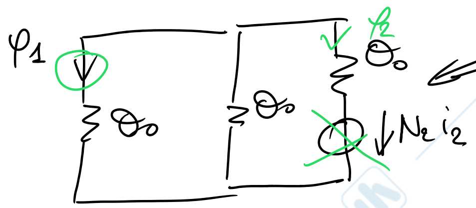


$L = \frac{\Psi}{i}$ AUTOIND. $L_{11} = \frac{\Psi_1}{i_1}\Big|_{i_2=0}$

$L_{22} = \frac{\Psi_2}{i_2}\Big|_{i_1=0}$

$\Phi = ?$

MUTUE IND.



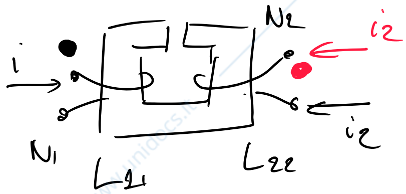
$L_{12} = \frac{\Psi_1}{i_2}\Big|_{i_1=0}$ ($N\varphi_1$)

$\parallel$

$L_{21} = \frac{\Psi_2}{i_1}\Big|_{i_2=0}$ ($N\varphi_2$)

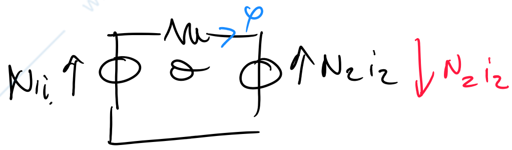
DIPENDONO SOLO DALLA GEOM.

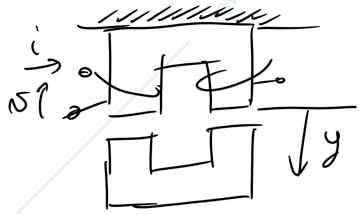
$$\begin{cases} \Psi_1 = L_{11} \cdot i_1 + L_{12}\, i_2 \\ \Psi_2 = L_{21}\, i_1 + L_{22}\, i_2 \end{cases}$$



$\varphi = \frac{N_1 i_1 - N_2 i_2}{\Theta}$

$\varphi = \frac{N_1 i_1 + N_2 i_2}{\Theta}$





$$[i] = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \end{bmatrix} \qquad [v] = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \end{bmatrix}$$

$$dW = \delta L_e - \delta L_m = \underbrace{i \cdot v \cdot dt}_{\delta L_e} - \underbrace{F \cdot dy}_{\delta L_m} = [i]^t [v]\, dt - F\, dy$$

$$W = f(\text{stato}) = W([\psi], y)$$

i_L

$\varphi = L i$

$\psi = N \varphi$

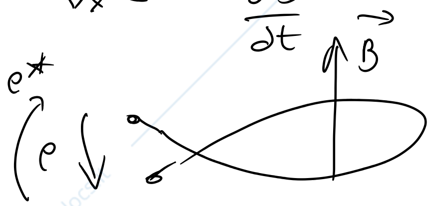
$$dW = \frac{\partial W}{\partial [\psi]} d[\psi] + \frac{\partial W}{\partial y} dy = [i]^t \overbrace{[v]\, dt}^{d[\psi]} - F\, dy$$

$v_1 i_1 + v_2 i_2 + \ldots$

$$[i]^t = \frac{\partial W}{\partial [\psi]}$$

$$F = -\frac{\partial W([\psi], y)}{\partial y}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \int_S \nabla \times \vec{E} \cdot \vec{n}\, dS = \oint_l \vec{E} \cdot \vec{t}\, dl = -\int_S \frac{\partial}{\partial t} \vec{B} \cdot \vec{n}\, dS$$



f.e.m = e = lavoro campo elettrico

$$\boxed{e = -\frac{d}{dt}\varphi}$$ FARADAY

$$e = -N\frac{d\varphi}{dt} = -\frac{d\psi}{dt}$$

$e^* = -e$

$$\boxed{e^* = \frac{d\psi}{dt}} \qquad \boxed{e\, dt = d\psi}$$

$$dW = \delta L_e - \delta L_m = [i]^t \overbrace{[v]dt}^{d[\psi]} - F\,dy \leftarrow \text{legge cons.}$$

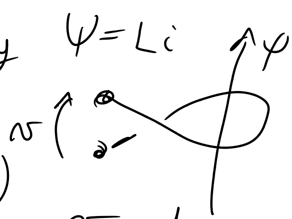
× Faraday

$$= \frac{\partial W}{\partial[\psi]} d[\psi] + \frac{\partial W}{\partial y} dy \leftarrow \text{def mat.}$$

$$F = -\frac{\partial W([\psi], y)}{\partial y} \qquad [i]^t = \frac{\partial W}{\partial [\psi]} \qquad [v] = \frac{d[\psi]}{dt}$$

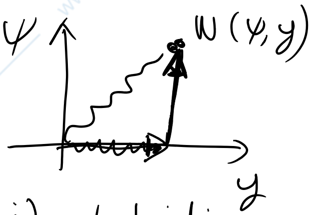
$$[\psi] = [L][i]$$

$$\int dW(\psi, y) = \int \left(i \underbrace{v\,dt}_{d\psi} - F\,dy \right) = \frac{\partial W}{\partial \psi} d\psi + \frac{\partial W}{\partial y} dy \qquad \psi = Li$$

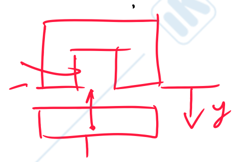


$$v = \frac{d\psi}{dt}$$

$$i = \frac{\partial W}{\partial \psi} \qquad F = -\frac{\partial W}{\partial y}$$



$$\int dW = W = \int i\,d\psi = \int i\,d(Li) = L \int i\,di = \frac{1}{2} L i^2$$

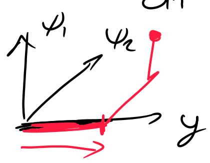


$$\int dW([\psi], y) = \int [i]^t [v] dt - F\,d[y] =$$

$$= [i]^t d[\psi] - F\,d[y]$$

$$[\psi] = [L][i] \qquad \frac{d[\psi]}{dt} = [v]$$

$$W = \int_{[\psi]} [i]^t d[\psi] = \int [i]^t [L] d[i]$$



$$= \frac{1}{2} [i]^t [L][i] = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + \frac{1}{2} L_{12} i_1 i_2 + \frac{1}{2} L_{21} i_1 i_2$$

$$L_{12} = L_{21} = L_m$$

$$\boxed{= \frac{1}{2} \underbrace{L_{11} i_1^2}_{>0} + \frac{1}{2} \underbrace{L_{22} i_2^2}_{>0} + \underbrace{L_m i_1 i_2}_{\gtrless 0}}$$

$$\int dW(\psi_1, \psi_2, \cancel{x}) = \int \frac{\partial W}{\partial \psi_1} d\psi_1 + \frac{\partial W}{\partial \psi_2} d\psi_2 \qquad \frac{\partial W}{\partial [\psi]} = [i]^t$$

$$\frac{\partial W}{\partial \psi_1} = i_1 \qquad \frac{\partial W}{\partial \psi_2} = i_2$$

$$= i_1 \, d\psi_1 + i_2 \, d\psi_2$$

$$\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\frac{\partial i_1}{\partial \psi_2}$$

$$\frac{\partial^2 W}{\partial \psi_1 \partial \psi_2} = \frac{\partial^2 W}{\partial \psi_2 \partial \psi_1} \Longrightarrow \frac{\partial i_1}{\partial \psi_2} = \frac{\partial i_2}{\partial \psi_1}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} L_{22} & -L_{12} \\ -L_{21} & L_{11} \end{bmatrix} [\psi]$$

$$i_1 = \frac{1}{D} \left(L_{22} \psi_1 - L_{12} \psi_2 \right)$$

$$i_2 = \frac{1}{D} \left(-L_{21} \psi_1 + L_{11} \psi_2 \right)$$

DETERMINANTE

RECIPROCITÀ MUT. IND.

$$\frac{\partial i_1}{\partial \psi_2} = -\frac{L_{12}}{\cancel{D}} = \frac{\partial i_2}{\partial \psi_1} \quad -\frac{L_{21}}{\cancel{D}}$$