Limiti notevoli

Limiti di funzioni

Forma standard	**Forma con gli "o" piccolo**
$\lim_{x\to 0} \frac{\sin x}{x} = 1$	$\sin x = x + o(x)\,, \quad x \to 0$
$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$	$\cos x = 1 - \frac{1}{2}x^2 + o\left(x^2\right), \quad x \to 0$
$\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log a}$	$\log_a(1+x) = \frac{1}{\log a}x + o(x), \quad x \to 0,\ \forall a > 0,\ a \neq 1$
$\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$	$\log(1+x) = x + o(x), \quad x \to 0$
$\lim_{x\to 0} \frac{a^x - 1}{x} = \log a$	$a^x = 1 + x\log a + o(x), \quad x \to 0,\ \forall a > 0$
$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$	$e^x = 1 + x + o(x), \quad x \to 0$
$\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha$	$(1+x)^\alpha = 1 + \alpha x + o(x), \quad x \to 0,\ \forall \alpha \in \mathbb{R}$
$\lim_{x\to +\infty} \frac{x^k}{a^x} = 0$	$x^k = o\left(a^x\right), \quad x \to +\infty,\ \forall a > 1,\ \forall k \in \mathbb{R}$
$\lim_{x\to -\infty} \lvert x\rvert^k a^x = \lim_{x\to -\infty} \frac{a^x}{\frac{1}{\lvert x\rvert^k}} = 0$	$a^x = o\left(\frac{1}{\lvert x\rvert^k}\right), \quad x \to -\infty,\ \forall a > 1,\ \forall k > 0$
$\lim_{x\to -\infty} \frac{\lvert x\rvert^k}{a^x} = 0$	$\lvert x\rvert^k = o\left(a^x\right), \quad x \to -\infty,\ \forall\, 0 < a < 1,\ \forall k \in \mathbb{R}$
$\lim_{x\to +\infty} x^k a^x = \lim_{x\to +\infty} \frac{a^x}{\frac{1}{x^k}} = 0$	$a^x = o\left(\frac{1}{x^k}\right), \quad x \to +\infty,\ \forall\, 0 < a < 1,\ \forall k > 0$
$\lim_{x\to +\infty} \frac{\log_a x}{x^k} = 0$	$\log_a x = o\left(x^k\right), \quad x \to +\infty,\ \forall a, k > 0,\ a \neq 1$
$\lim_{x\to 0^+} x^k \log_a x = \lim_{x\to 0^+} \frac{\log_a x}{\frac{1}{x^k}} = 0$	$\log_a x = o\left(\frac{1}{x^k}\right), \quad x \to 0^+,\ \forall a, k > 0,\ a \neq 1$
$\lim_{x\to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$	
$\lim_{x\to \pm\infty} \left(1 + \frac{a}{x}\right)^x = e^a, \qquad \forall a \in \mathbb{R}$	
$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$	
$\lim_{x\to 0} (1+ax)^{\frac{1}{x}} = e^a, \qquad \forall a \in \mathbb{R}$	

Limiti di successioni

Forma standard	**Forma con gli "o" piccolo**
$\lim_{n\to+\infty} \frac{n^k}{a^n} = 0$	$n^k = o\left(a^n\right), \quad n \to +\infty, \forall a > 1, \forall k \in \mathbb{R}$
$\lim_{n\to+\infty} n^k\, a^n = \lim_{n\to+\infty} \frac{a^n}{\frac{1}{n^k}} = 0$	$a^n = o\left(\frac{1}{n^k}\right), \quad n \to +\infty, \forall\, 0 < a < 1, \forall k > 0$
$\lim_{n\to+\infty} \frac{\log_a n}{n^k} = 0$	$\log_a n = o\left(n^k\right), \quad n \to +\infty, \forall a, k > 0,\ a \neq 1$
$\lim_{n\to+\infty} \left(1 + \frac{1}{n}\right)^n = e$	
$\lim_{n\to+\infty} \left(1 + \frac{a}{n}\right)^n = e^a, \qquad \forall a \in \mathbb{R}$	
$\lim_{n\to+\infty} \sqrt[n]{n^k} = 1, \qquad \forall k \in \mathbb{R}$	
$\lim_{n\to+\infty} \sqrt[n]{a^k} = 1, \qquad \forall a > 0, \forall k \in \mathbb{R}$	
$\lim_{n\to+\infty} \sqrt[n]{n!} = +\infty$	