ESERCIZI SETTIMANA 3

Determinare il dominio delle seguenti funzioni

$f(x) = \sqrt{\log(x-3)}$

$\begin{cases} x-3>0 \\ \log(x-3)\geq 0 \end{cases}$ $\begin{cases} x-3>0 \\ x-3\geq 1 \end{cases}$ $\begin{cases} x>3 \\ x\geq 4 \end{cases} \Rightarrow x\geq 4$

$D = [4, +\infty)$

$f(x) = \sqrt{\log(x^2-3)}$

$\begin{cases} x^2-3>0 \\ \log(x^2-3)\geq 0 \end{cases}$ $\begin{cases} x^2>3 \\ x^2-3\geq 1 \end{cases}$ $\begin{cases} x>\pm\sqrt{3} \\ x^2\geq 4 \end{cases}$ $\begin{cases} \sqrt{x^2}\geq \pm\sqrt{4} \end{cases}$ $\begin{cases} x\geq\sqrt{4} \end{cases}$

$+\ -2\ -\ +2\ +$ $\qquad$ $+\ -\sqrt{3}\ -\ +\sqrt{3}\ +$

$(-\infty, -2] \vee [2, +\infty)$ $\qquad$ $(-\infty, -\sqrt{3}) \vee (+\sqrt{3}, +\infty)$

se è $\geq$ bisogna mettere]

$-2 \quad -\sqrt{3} \quad \sqrt{3} \quad 2$

$(-\infty, -2] \vee [2, +\infty)$

6) $f(x) = \dfrac{\log\sqrt{x^2-3}}{|x^2-4|}$ $\qquad f(x) = \log\sqrt{x^2-3}$ $\qquad (x^2-4)\neq 0 \quad x^2\neq 4 \quad x\neq\pm\sqrt{4} \quad x\neq\pm 2$

$\sqrt{x^2-3}$

$\begin{cases} x^2-3>0 \\ \log(x^2-3)\geq 0 \end{cases}$ $\begin{cases} x^2>3 \\ x^2-3\geq 0 \end{cases}$ $\begin{cases} x>\pm\sqrt{3} \\ x^2\geq 4 \end{cases}$ $\begin{cases} x\geq\pm\sqrt{3} \\ x\geq\pm\sqrt{4} \end{cases}$

$+\ -2\ -\ +2\ +$ $\qquad$ $+\ -\sqrt{3}\ -\ \sqrt{3}\ +$

$(-\infty, -2] \vee [2, +\infty)$ $\qquad$ $(-\infty, -3) \vee (3, +\infty)$

$-2 \quad \sqrt{3} \quad \sqrt{3} \quad 2$

$(-\infty, -2] \vee [2, +\infty)$

6) $f(x) \quad \dfrac{\log\sqrt{x^2-3}}{|x^2-4|}$

se positiva $x^2 \to \cup$
se negativa $x^2 \to \cap$

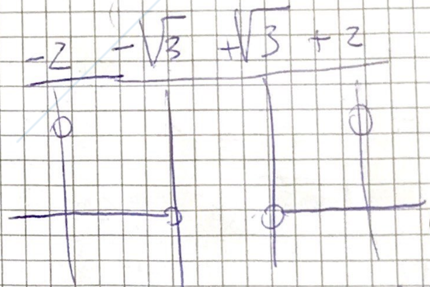
$|x^2-4| \neq 0 \qquad x^2 \neq 4 \qquad x \neq \pm\sqrt{4} = x \neq \pm 2$

$x^2-3 > 0 \qquad x^2 > 3 \qquad x > \pm\sqrt{3}$

$+ \; -\sqrt{3} \; - \; +\sqrt{3} \; +$

$(-\infty, -\sqrt{3}) \vee (\sqrt{3}, +\infty)$

prendiamo dove passa senza contare -2 e +2



$D = (-\infty, -2) \vee (-2, -\sqrt{3}) \cup (\sqrt{3}, +2)(+2 +\infty)$

8) $f(x = \left(\frac{1}{2}x\right)^{\log(-x)}$

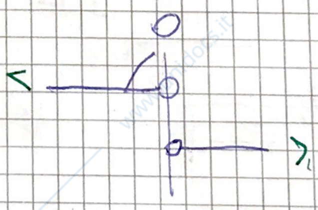
esponenziale

$-x > 0 \qquad x < 0$

$\frac{1}{2}x > 0 \qquad$ ~~x > 2 > 0~~ ~~x > 2~~

$\frac{1}{2} : \frac{1}{2} x > 0 : \frac{1}{2} \qquad x > 0$

da sto lato perché minore $\to$ <



lato destra perché maggiore

$\emptyset \to$ insieme nullo

12) $f(x) = \frac{|x|}{x+|x|}$

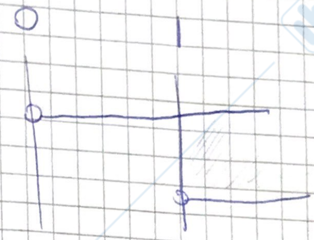
D: $x > 0$

$(0, +\infty)$

18) $f(x) = (\log x)^x$

$x > 0$

$\log x > 0 \quad x > 1$



$(1, +\infty)$

20) $f(x) = \frac{3\log x^2}{\sqrt{x^2}}$

$\sqrt{x^2} \neq 0 \quad x \neq 0$

~~$x>0$~~ ~~$x>0$~~

D: $x \neq 0$

~~$\log x$~~ ~~$3\log x^2$~~

22) $f(x) = e^{\frac{2-x}{1-x}}$

numero di nepero

$y = e^{\frac{2-x}{1-x}}$

$\begin{cases} x = 0 \\ y = e^{\frac{2}{1}} = e^2 \end{cases}$

int. asse y

$\begin{cases} y = 0 \\ e^{\frac{2-x}{1-x}} = 0 \end{cases}$

" " asse x

$\emptyset$ ~~$2-x \neq 1$~~ ~~$1-x \neq 1$~~

$a^b = 0$

$b = 1$

24) $f(x) = \dfrac{x^2+2x+5}{x+2}$ $\quad y = \dfrac{x^2+2x+5}{x+2}$

$x+2 \neq 0 \quad x \neq -2$

$\begin{cases} x=0 \\ y = \frac{5}{2} \end{cases}$ asse delle y

$\begin{cases} y=0 \\ \frac{x^2+2x+5}{x+2} = 0 \end{cases}$ asse x

$x^2+2x+5=0$

x^2+2x+5

$\begin{cases} y=0 \\ \frac{x^2+2x+5}{x+2} = 0 \end{cases}$

$x+2=0 \quad x=-2$

a 1

b 2

c 5

$\Delta = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad \dfrac{-2 \pm \sqrt{4-4(5)}}{2} = \sqrt{-16}$

Delta negativo $\nexists$

essendo indifinito al numeratore non ci sono intersezioni

26) $f(x) = \dfrac{\log x}{x^2}$ $\quad y = \dfrac{\log x}{x^2}$

$\begin{cases} x=0 \\ y = \frac{\log x}{x^2} \end{cases}$ $\quad \begin{cases} x=0 \\ y = \frac{\log 0}{0} \end{cases}$

0 → non ho intersezioni con l'asse delle y perché denominatore non può essere 0

$\begin{cases} y=0 \\ \frac{\log x}{x^2} = 0 \end{cases}$ $\quad \begin{cases} y=0 \\ \log x = 0 \cdot x^2 \end{cases}$ $\quad \begin{cases} y=0 \\ \log x = 0 \end{cases}$ $\quad \begin{cases} y=0 \\ x=1 \end{cases}$

N.B → $\log x = 0$ → $x=1$

$P(1,0)$

Segno

N $\log(x) \geq 0 \quad x \geq 1$

D $x^2 > 0$

→ sempre positiva quindi una linea continua

$f(x) > 0$ per $x \geq 1$

$f(x) < 0$ per < 1