Tavole e Formulari

Formule notevoli

$\cos^2 x + \sin^2 x = 1, \quad \forall x \in \mathbb{R}$

$\sin x = 0 \quad \text{se } x = k\pi\,, \quad \forall k \in \mathbb{Z}\,,$ $\qquad \cos x = 0 \quad \text{se} \quad x = \frac{\pi}{2} + k\pi$

$\sin x = 1 \quad \text{se} \quad x = \frac{\pi}{2} + 2k\pi\,,$ $\qquad \cos x = 1 \quad \text{se} \quad x = 2k\pi$

$\sin x = -1 \quad \text{se} \quad x = -\frac{\pi}{2} + 2k\pi\,,$ $\qquad \cos x = -1 \quad \text{se} \quad x = \pi + 2k\pi$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\sin 2x = 2\sin x\cos x\,,$ $\qquad \cos 2x = 2\cos^2 x - 1$

$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$

$\cos x - \cos y = -2\sin\frac{x-y}{2}\sin\frac{x+y}{2}$

$\sin(x+\pi) = -\sin x\,,$ $\qquad \cos(x+\pi) = -\cos x$

$\sin(x+\frac{\pi}{2}) = \cos x\,,$ $\qquad \cos(x+\frac{\pi}{2}) = -\sin x$

$a^{x+y} = a^x a^y\,,$ $\qquad a^{x-y} = \frac{a^x}{a^y}\,,$ $\qquad (a^x)^y = a^{xy}$

$\log_a(xy) = \log_a x + \log_a y\,, \quad \forall x, y > 0$

$\log_a \frac{x}{y} = \log_a x - \log_a y, \quad \forall x, y > 0$

$\log_a(x^y) = y\log_a x, \quad \forall x > 0\,, \ \forall y \in \mathbb{R}$

Limiti notevoli

$$\lim_{x\to+\infty} x^\alpha = +\infty\,, \qquad \lim_{x\to 0^+} x^\alpha = 0\,, \qquad \alpha > 0$$

$$\lim_{x\to+\infty} x^\alpha = 0\,, \qquad \lim_{x\to 0^+} x^\alpha = +\infty\,, \qquad \alpha < 0$$

$$\lim_{x\to\pm\infty} \frac{a_n x^n + \ldots + a_1 x + a_0}{b_m x^m + \ldots + b_1 x + b_0} = \frac{a_n}{b_m} \lim_{x\to\pm\infty} x^{n-m}$$

$$\lim_{x\to+\infty} a^x = +\infty\,, \qquad \lim_{x\to-\infty} a^x = 0\,, \qquad a > 1$$

$$\lim_{x\to+\infty} a^x = 0\,, \qquad \lim_{x\to-\infty} a^x = +\infty\,, \qquad a < 1$$

$$\lim_{x\to+\infty} \log_a x = +\infty\,, \qquad \lim_{x\to 0^+} \log_a x = -\infty\,, \qquad a > 1$$

$$\lim_{x\to+\infty} \log_a x = -\infty\,, \qquad \lim_{x\to 0^+} \log_a x = +\infty\,, \qquad a < 1$$

$$\lim_{x\to\pm\infty} \sin x\,, \qquad \lim_{x\to\pm\infty} \cos x\,, \qquad \lim_{x\to\pm\infty} \tan x \qquad \text{non esistono}$$

$$\lim_{x\to\left(\frac{\pi}{2}+k\pi\right)^\pm} \tan x = \mp\infty\,,\ \forall k\in\mathbb{Z}\,, \quad \lim_{x\to\pm\infty} \arctan x = \pm\frac{\pi}{2}$$

$$\lim_{x\to\pm 1} \arcsin x = \pm\frac{\pi}{2} = \arcsin(\pm 1)$$

$$\lim_{x\to+1} \arccos x = 0 = \arccos 1\,, \qquad \lim_{x\to-1} \arccos x = \pi = \arccos(-1)$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1\,, \qquad \lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x\to\pm\infty} \left(1+\frac{a}{x}\right)^x = \mathrm{e}^a\,, \quad a\in\mathbb{R}\,, \qquad \lim_{x\to 0} (1+x)^{\frac{1}{x}} = \mathrm{e}$$

$$\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log a}\,,\ a>0; \quad \text{in particolare,} \quad \lim_{x\to 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{x\to 0} \frac{a^x-1}{x} = \log a\,, \quad a>0; \qquad \text{in particolare,} \quad \lim_{x\to 0} \frac{\mathrm{e}^x-1}{x} = 1$$

$$\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha\,, \quad \alpha\in\mathbb{R}$$

Tavola delle derivate di funzioni elementari

$f(x)$	$f'(x)$
x^α	$\alpha x^{\alpha-1}, \qquad \forall \alpha \in \mathbb{R}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$1+\tan^2 x = \dfrac{1}{\cos^2 x}$
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\dfrac{1}{1+x^2}$
a^x	$(\log a)\, a^x$
$\log_a \lvert x\rvert$	$\dfrac{1}{(\log a)\, x}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$

Regole di derivazione

$$\big(\alpha f(x) + \beta g(x)\big)' = \alpha f'(x) + \beta g'(x)$$

$$\big(f(x)g(x)\big)' = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{\big(g(x)\big)^2}$$

$$\big(g(f(x))\big)' = g'(f(x))f'(x)$$

Sviluppi di Maclaurin notevoli

$$e^x = 1 + x + \frac{x^2}{2} + \ldots + \frac{x^k}{k!} + \ldots + \frac{x^n}{n!} + o(x^n)$$

$$\log(1+x) = x - \frac{x^2}{2} + \ldots + (-1)^{n-1}\frac{x^n}{n} + o(x^n)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^m \frac{x^{2m+1}}{(2m+1)!} + o(x^{2m+2})$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \ldots + (-1)^m \frac{x^{2m}}{(2m)!} + o(x^{2m+1})$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2m+1}}{(2m+1)!} + o(x^{2m+2})$$

$$\cosh x = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \ldots + \frac{x^{2m}}{(2m)!} + o(x^{2m+1})$$

$$\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \ldots + \left|\binom{-\frac{1}{2}}{m}\right| \frac{x^{2m+1}}{2m+1} + o(x^{2m+2})$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^m \frac{x^{2m+1}}{2m+1} + o(x^{2m+2})$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \ldots + \binom{\alpha}{n}x^n + o(x^n)$$

$$\frac{1}{1+x} = 1 - x + x^2 - \ldots + (-1)^n x^n + o(x^n)$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + o(x^3)$$

Tavola degli integrali di funzioni elementari

$f(x)$	$\int f(x)\,\mathrm{d}x$
x^α	$\dfrac{x^{\alpha+1}}{\alpha+1}+c\,,\qquad \alpha\neq -1$
$\dfrac{1}{x}$	$\log\lvert x\rvert + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$
e^x	$\mathrm{e}^x + c$
$\sinh x$	$\cosh x + c$
$\cosh x$	$\sinh x + c$
$\dfrac{1}{1+x^2}$	$\arctan x + c$
$\dfrac{1}{\sqrt{1-x^2}}$	$\arcsin x + c$
$\dfrac{1}{\sqrt{1+x^2}}$	$\log(x+\sqrt{x^2+1})+c = \operatorname{sett}\sinh x + c$
$\dfrac{1}{\sqrt{x^2-1}}$	$\log(x+\sqrt{x^2-1})+c = \operatorname{sett}\cosh x + c$

Regole di integrazione

$$\int \Big(\alpha f(x)+\beta g(x)\Big)\,\mathrm{d}x = \alpha\int f(x)\,\mathrm{d}x + \beta\int g(x)\,\mathrm{d}x$$

$$\int f(x)g'(x)\,\mathrm{d}x = f(x)g(x) - \int f'(x)g(x)\,\mathrm{d}x$$

$$\int \frac{\varphi'(x)}{\varphi(x)}\,\mathrm{d}x = \log\lvert\varphi(x)\rvert + c$$

$$\int f(\varphi(x))\varphi'(x)\,\mathrm{d}x = \int f(y)\,\mathrm{d}y \quad \text{con} \quad y=\varphi(x)$$